BY **SERENA ALEXANDER**

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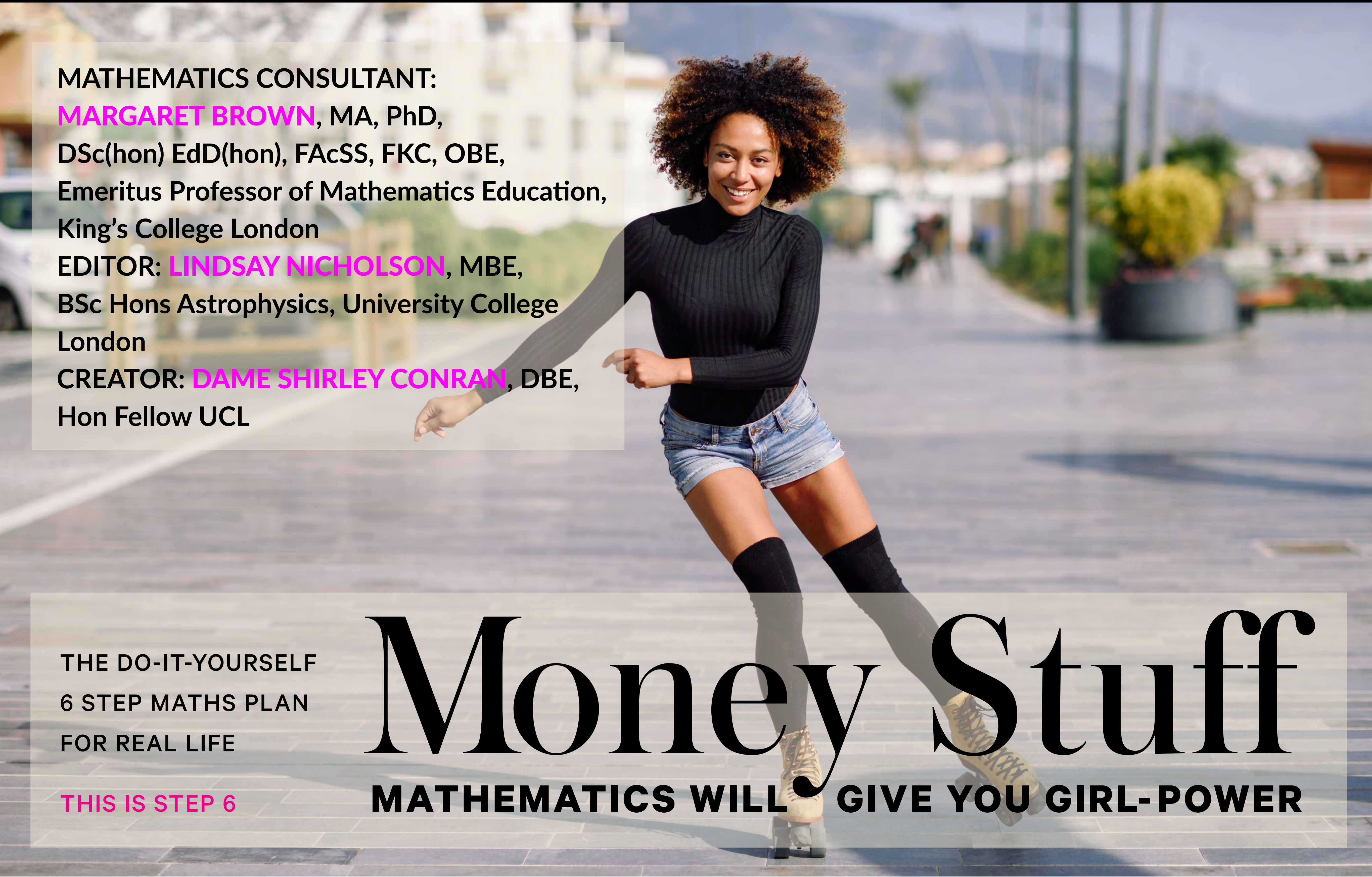


MATHEMATICS CONSULTANT: MARGARET BROWN, MA, PhD, DSc(hon) EdD(hon), FAcSS, FKC, OBE, Emeritus Professor of Mathematics Education, King's College London

EDITOR: LINDSAY NICHOLSON, MBE, BSc Hons Astrophysics, University College London

CREATOR: DAME SHIRLEY CONRAN, DBE, Hon Fellow UCL

THE DO-IT-YOURSELF
6 STEP MATHS PLAN
FOR REAL LIFE

THIS IS STEP 6

Money Stuff

MATHEMATICS WILL GIVE YOU GIRL-POWER

A BRIEF DESCRIPTION OF MONEY STUFF ONLINE MATHS COURSE

Maths is increasingly important in the modern world.

The **MONEY STUFF** Teach-Yourself Maths Course is online, **FREE**, and doesn't need a teacher.

This tested and proven Maths Course links maths to money to help people in real life. It was designed for girls who don't like maths, but actually it is suitable for anyone of any age, especially anyone who lacks confidence and is anxious about maths. It can be used on any tablet or computer and can also be individually printed.

Up to 15% of people in the UK are dyslexic. **MONEY STUFF has been specifically designed so that dyslexics can read it easily.** The entire computer production team was dyslexic.

!!! Watch out for prices !!!

(Another warning)

The cost of living has been zig-zagging upwards for hundreds of years. In the sixteenth century, Queen Elizabeth I worried about the increasing costs of feeding and equipping her army and navy. Today, you can still expect prices to rise unsteadily in the unforeseeable future.

What causes prices to rise? Many reasons, including bad weather, which increases farmers' food prices. So workers need higher wages, which means that the cost of the goods they make will increase. If the prices of bricks, cement and steel increase then so will the cost of housing and rents.

Sometimes the price rises are so small you don't notice them – but you will certainly notice if your home energy bill shoots up in a few months and mum starts switching off the lights and heating.

These rising prices are called **inflation**.

When I started to write this maths course, the prices I used in the exercises were the same as the prices in the shops – but by the time I had finished **Step 1**, the shop prices had risen - so the exercise prices were out-of-date. That is why the prices in **MONEY STUFF** are not current prices; they are historically correct prices, paid by your grandmother and mother in the early **21st** century.

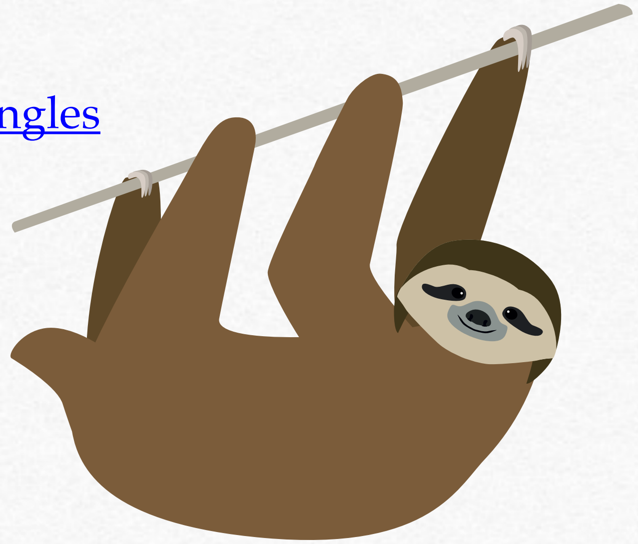
In maths, as in life, people have different ways to writing numbers. For example, you can write a fraction as either ½ with a diagonal line, as we do, or as $\frac{1}{2}$ which you may also see. Whichever you use, the meaning is the same. Likewise, some people write **1,000** or **1,000,000** as we do, with commas to break up the digits, others prefer just to leave a space, like this **1 000** or **1 000 000**. The choice is yours – that's the joy of maths!

Shop prices will alter throughout your life.

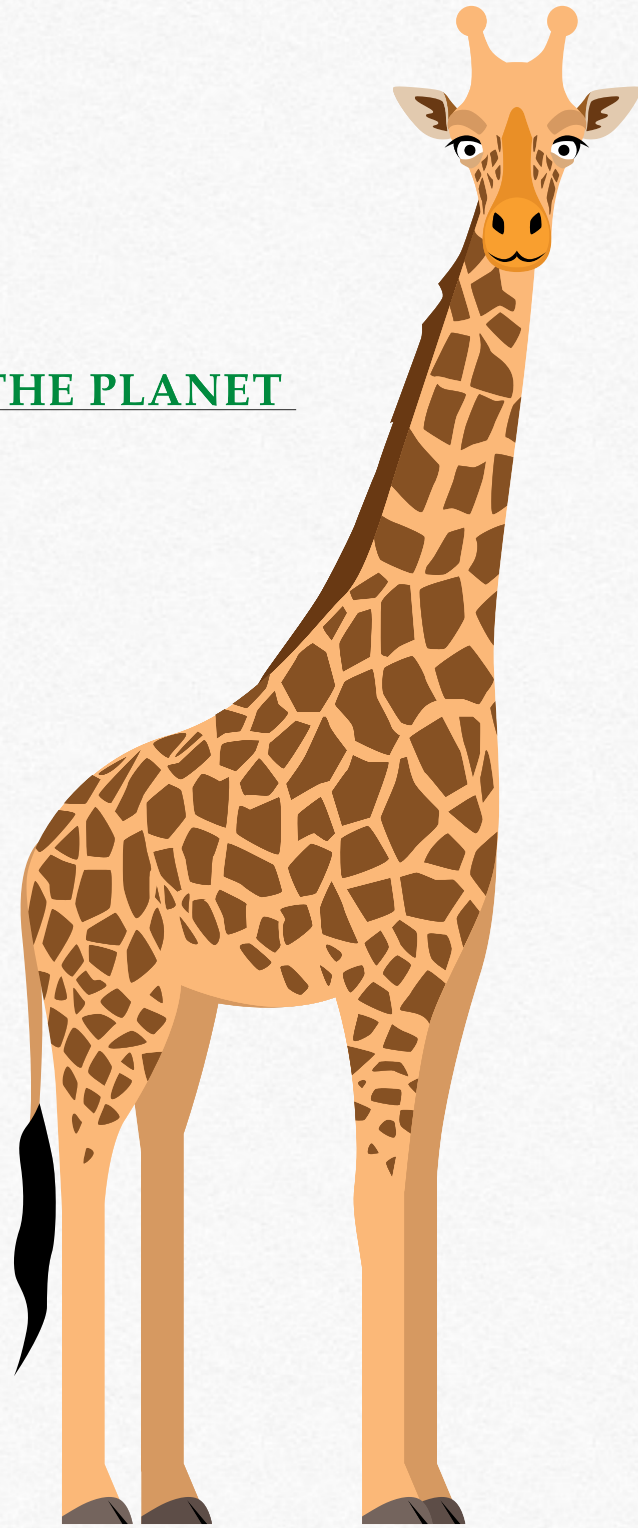
But the maths you need to shop will never alter.

Dame Shirley Conran

Contents



To make calculations – especially when hunting – animals have an innate calculation system.



Introduction and Thanks

When I was asked by a despairing mother to find a good maths textbook for her mutinous 14-year-old daughter, I went to my local Waterstones and discovered in one afternoon in 2004 how inefficient the UK maths learning system is: I was astonished by the badness of the maths textbooks - with one exception.

In my opinion far the best textbook writer – the only one – was maths school teacher and qualified architect **Serena Alexander** BA (Hons) Architecture, Dept of Engineering, University of Bristol, PGCE (Secondary Mathematics) University of Southampton. In her acknowledgements, Serena thanked all her students who had tested her work BEFORE it was printed.

I am an instructions writer – so I know that testing is exasperating, time consuming and very expensive; endless patience, tenacity and money is needed.

So I started a voluntary group, Maths Action, to improve maths learning in the UK and then I started the Maths Anxiety Trust, which produced the online, FREE, teach-yourself MONEY STUFF Maths Course with 4 Steps, all carefully tested.

Astonishingly, two 17th Century dishonest beliefs still persist in Britain today: that only boys are born with maths ability and that girls don't need maths in their adult lives. Believe me, many Year 9 girls still believe that rubbish.

So we decided to produce two further Steps for the MONEY STUFF COURSE. This was to teach girls – future women – the importance and the excitement of further mathematics, if they are to get the same chances as men. In 2022, the bosses of the FTSE – the hundred biggest businesses in Britain – numbered 7 women bosses and 93 men bosses. So which group has the most power?

The NHS then discovered that I had a tumour the size of a tennis ball in my brain. I was unable to write these important Steps and without them this series would not be a complete course at international level.

So Serena Alexander was asked to write **MONEY STUFF STEP 5** and **STEP 6**.

We all held our breath.

Serena said yes.

Nervously, I asked Serena what her fee would be.

Serena said, “Nothing. It needs doing.”

Members of the Maths Anxiety Trust know how long textbooks take to write and test, so we are very grateful to Serena.

Also working for no pay on **MONEY STUFF** were two distinguished women, who want girls to use maths to get a better life. The mathematics consultant is Margaret Brown and the editor-in-chief is Lindsay Nicholson.

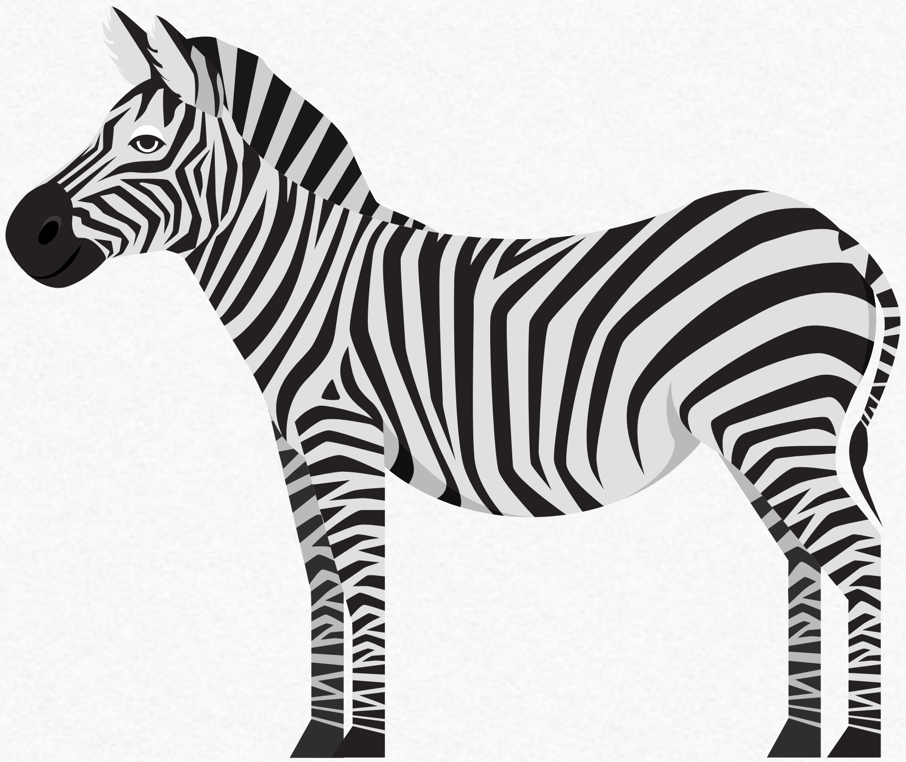
The Emeritus Professor of Mathematics at Kings College, London, **Margaret Brown** MA, PhD, DSc(hon) EdD(hon), FAcSS, FKC, OBE, has a mathematical background in teaching and writing good maths books. The modest and discreet Margaret seems to have her finger in every important maths pie in Britain, but she never talks about those pies. When recently I asked Margaret to come with me to a small business lunch with the Royal Society to discuss their important, Government-funded project to improve maths ability in Britain, I only discovered over the risotto that The Royal Society had already consulted Margaret.

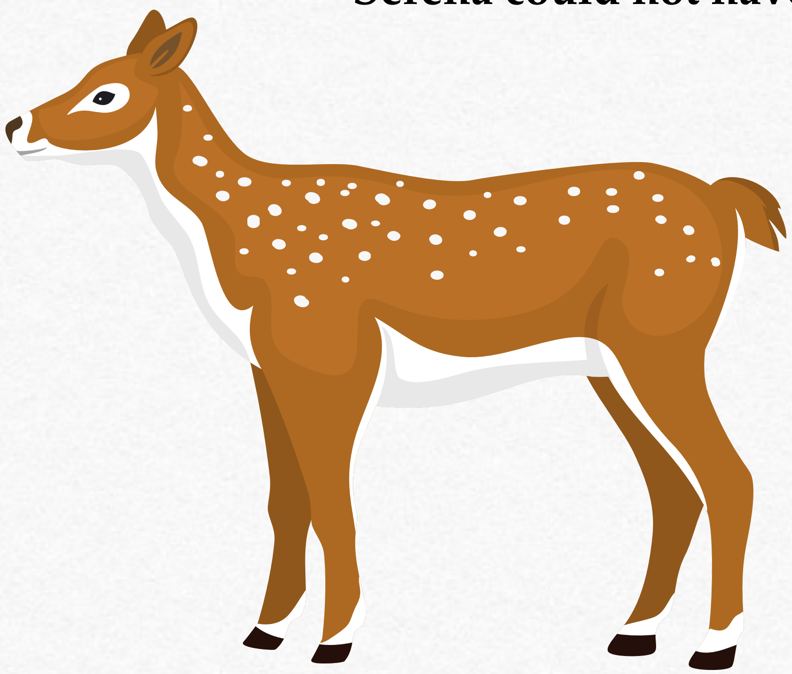
Lindsay Nicholson MBE, BSc Hons Astrophysics, University College London is a qualified astrophysicist and still works closely with her famous college. As well as being a top journalist, top editor and writer of a bestselling autobiography, Lindsay was editor-in-chief for the Hearst Publishing Empire in Europe and so is familiar with international finance at Board level.

Serena could not have had better champions at her side.

Serena is a maths textbook STAR and she is **OUR** star and we hope she realises how much we appreciate her abilities and her generosity. We hope that will soon be appreciated by many, many people who Serena will never meet – whose lives will be improved by the gifted and generous SERENA ALEXANDER.

Dame Shirley Conran





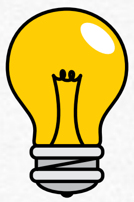
Stepping into Step 6

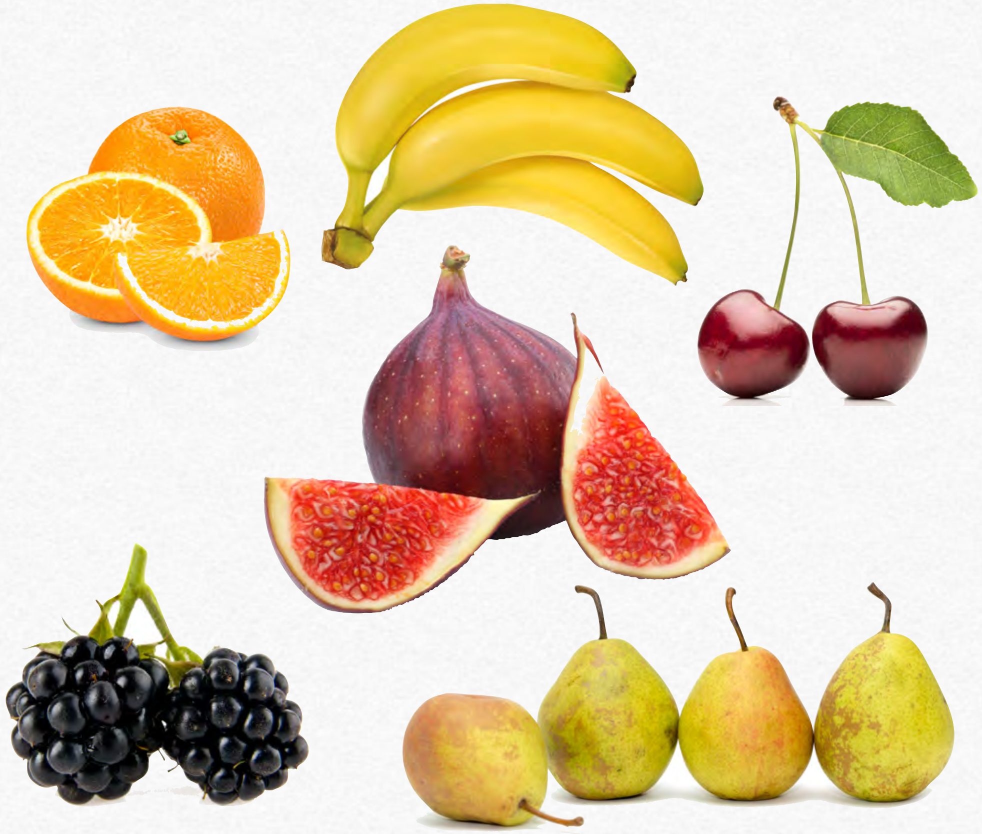
As you embark on STEP 6 you should be aware of the importance of numbers, mathematics and money.

Just as you have learnt some general knowledge on topics such as literature, geography, famous people, history, etc., you need to have some general knowledge on mathematics.

To help you with this you will see a new feature in addition to Notes and Quick Tips, called Food for Thought:

Note

and Tips:  *Quick tip*



Food for Thought

Food for Thought boxes give you additional general knowledge such as information about the history of mathematics, how mathematics and money may be used in different cultures and civilisations, information about facts and symbols that may be new to you and sometimes just a moment to reflect on the beauty of mathematics.

PART 1

MATHS FOR DESIGN

The Design Professional

There are many professions that involve design. Fashion design may be one that comes to mind first but consider these other professions:

- Graphic design
- Textile design
- Theatre, lighting and sound design
- Cinematography
- Furniture and toy design
- Food design
- Architecture
- Engineering
- Animation
- Photography
- Illustrator

And even teaching, a teacher has to ensure that every visual image that they put in front of you is well thought out and clear, getting the correct message across.

When you think about it, every profession needs to communicate visually and so all should have some awareness of design.

In this part you are going to explore some of the mathematical concepts that help designers with their work.

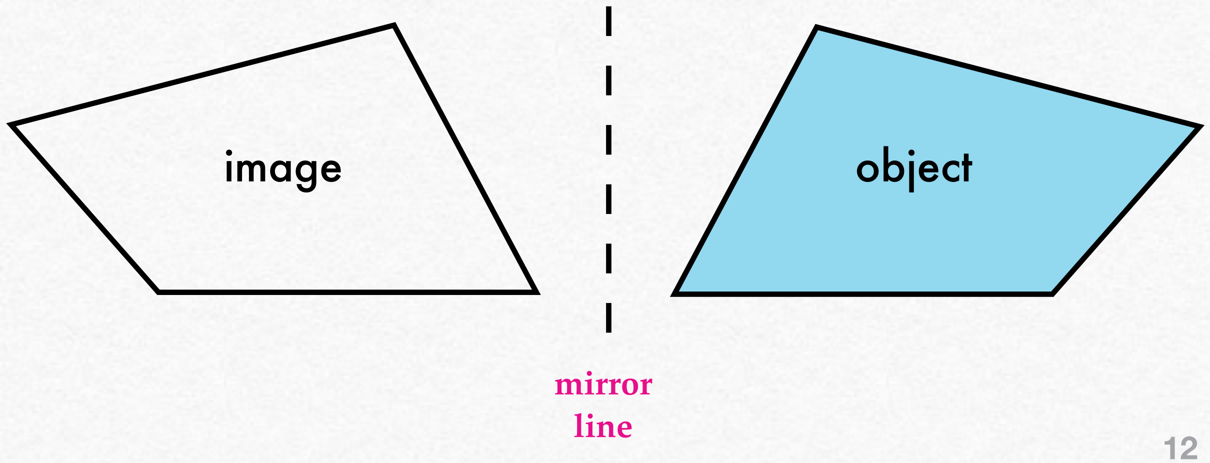
Reflection

Some images make you wonder at their beauty, but this may be to do with some different properties. Look at this photograph of Mont Blanc reflected in Lac Blanc:

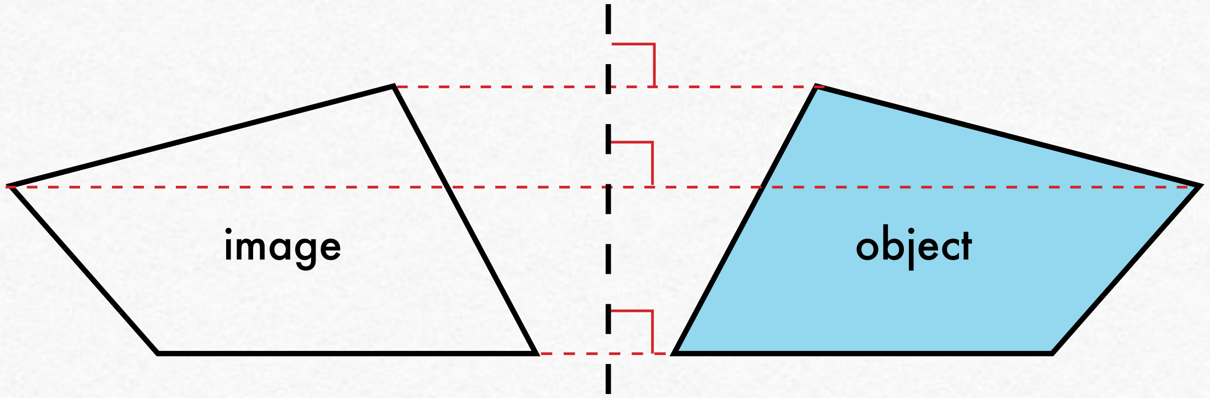
If you look around you at various designs, which may be on packaging, on fabrics or in pictures, you may see many examples of reflection.

A reflection is a copy of an image in a mirror line. It is so called because if you hold up a mirror to a shape you see its refection.

Here is a **2** dimensional shape and its reflection:

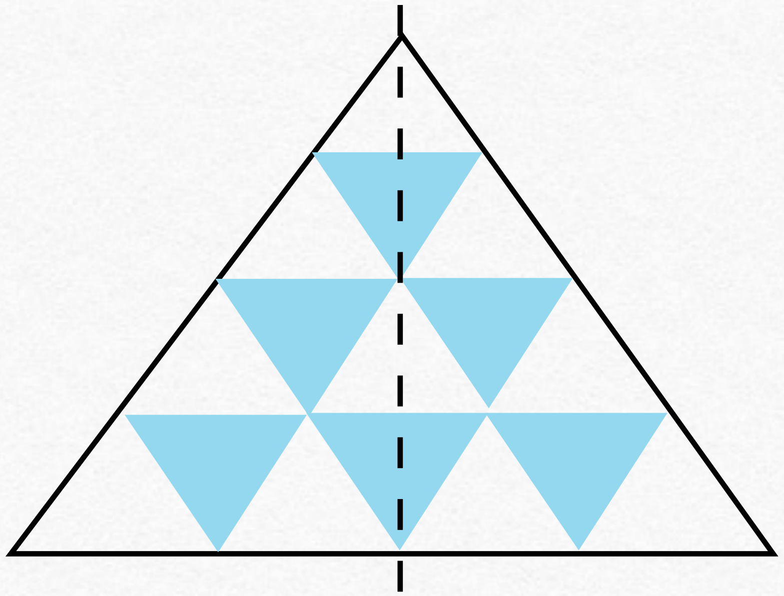


If you join the corresponding points of the object and its reflection or image you will see that then are all parallel and perpendicular to the mirror line:

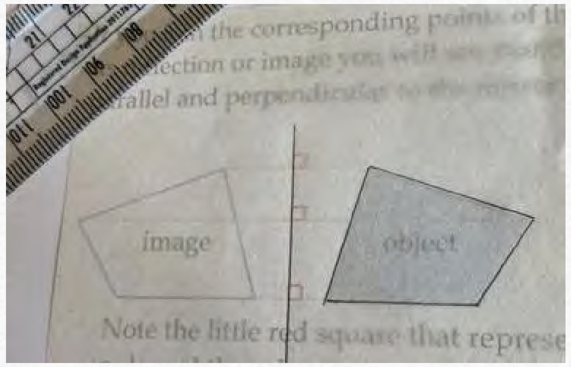


Note the little red square that represents a right angle and thus shows the lines are perpendicular.

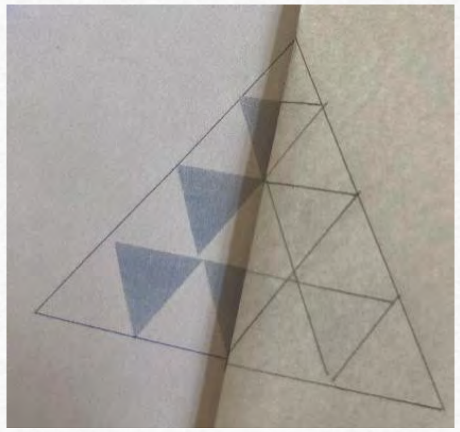
A shape or design can sometimes be divided into two equal parts, where one part is a reflection of the other, like this:



Tracing paper can be useful when working out reflections. You can draw reflections by tracing over the object and the mirror line, then flipping the paper and draw over the original image to create the reflection.



You could also trace over a shape and then fold the tracing paper to find the line of reflection.



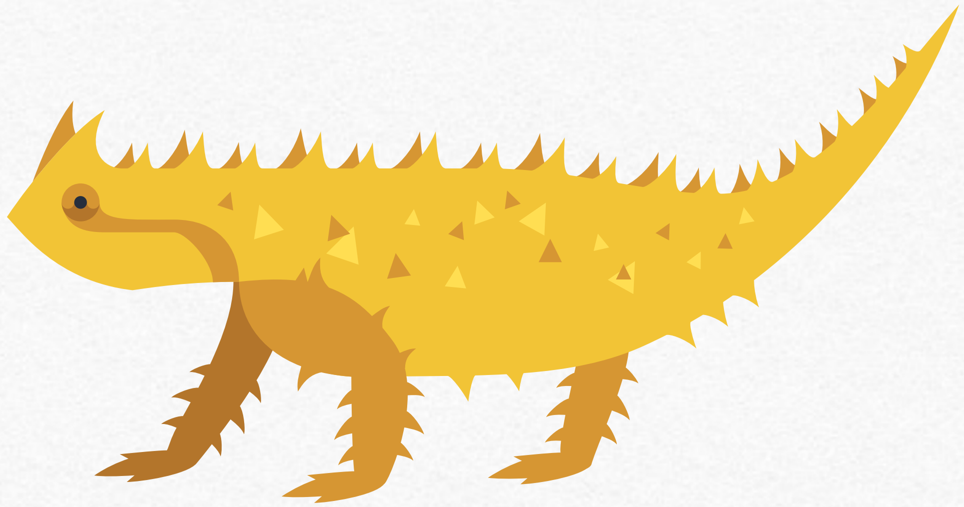
If you have no tracing paper, try using greaseproof paper in the kitchen which is just as good.

When working on reflections it can be useful to use a coordinate grid so that you can explain the position of the shapes and the line.

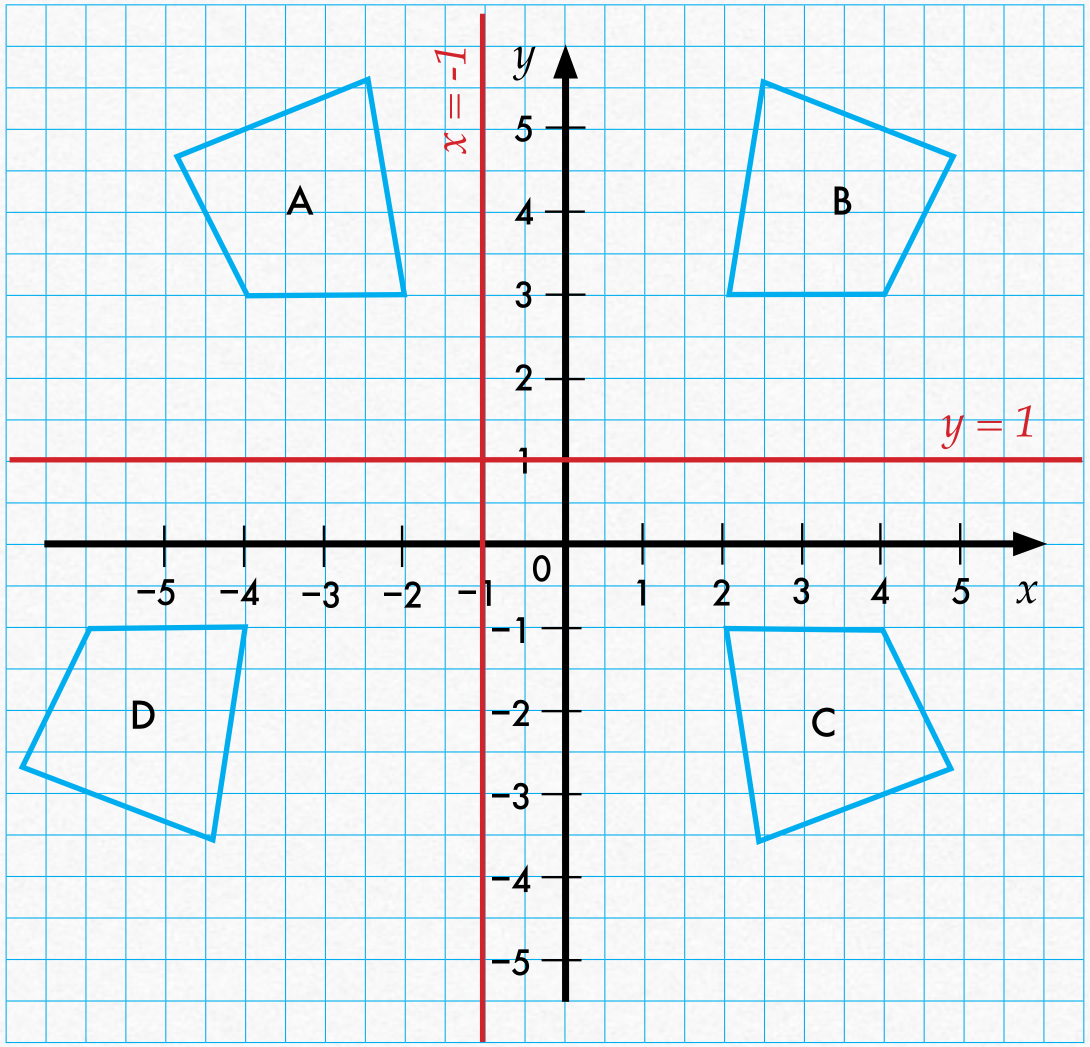
You may have studied this in school or college. It is called 'transformational geometry'.

In this section you will revise some of what you may already know – or may have forgotten. Then you will put it all together and see how you can use transformations to generate new designs – that are nothing to do with x and y coordinates.

However, the coordinates can be useful if you ever have to use a graphic design package to reproduce your design using a computer.



Look at shapes A, B, C and D on this grid:



You will also see some straight lines: the x-axis and the y-axis, as well as the lines $x = -1$ and $y = 1$

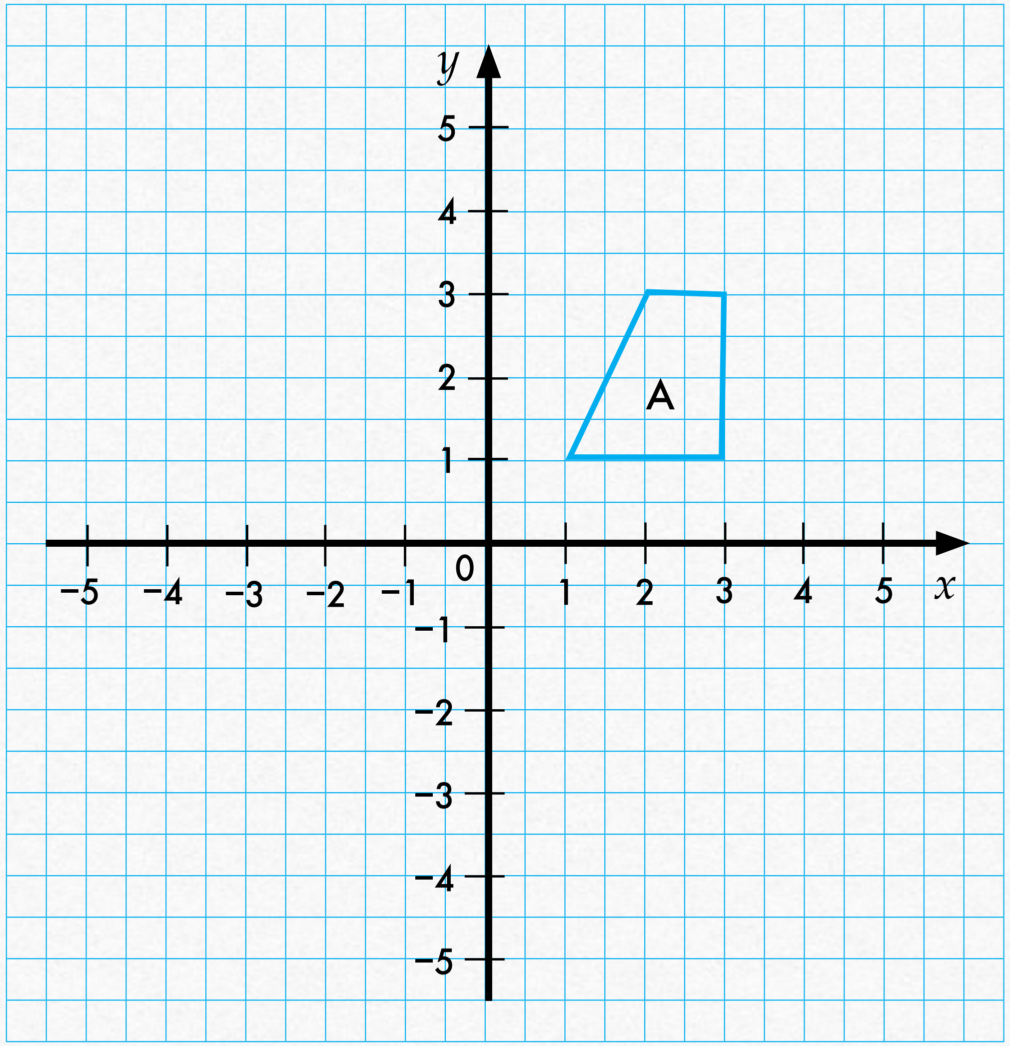
B is the image of A after a reflection in the y-axis.

C is the image of B after a reflection in the line $y = 1$.

D is the image of C after a reflection in the line $x = -1$.

Exercises

Copy the grid below and then draw the shapes described:



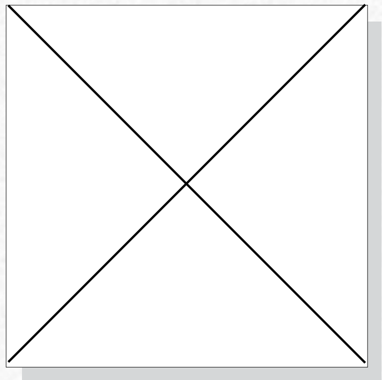
1. Draw B, the image of A after a reflection in the x-axis
2. Draw C, the image of A after a reflection in the line $x = -1$
3. Draw D, the image of A after a reflection in the line $y = 1$

Rotation

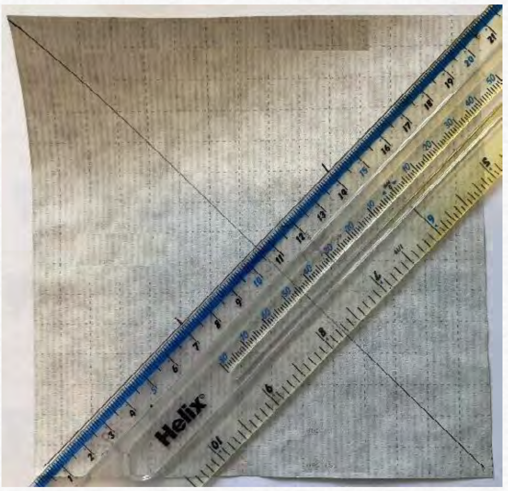
Now would be a good time to make a windmill.

All you need is a square of paper (15 cm by 15 cm is a good size), scissors and a pin.

On the square draw the diagonals:

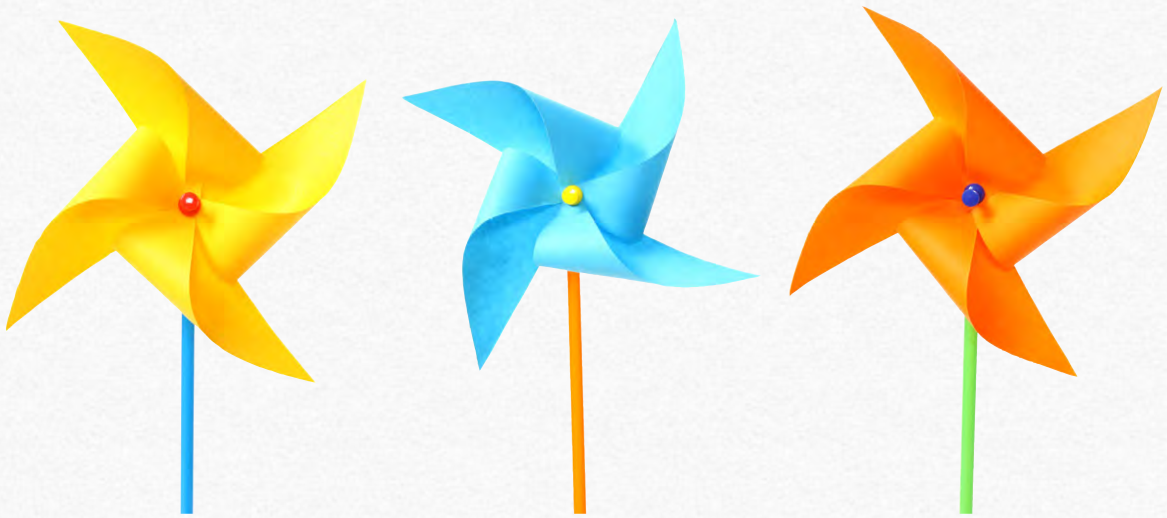


Measure the diagonals, divide the length by three and mark off the thirds.



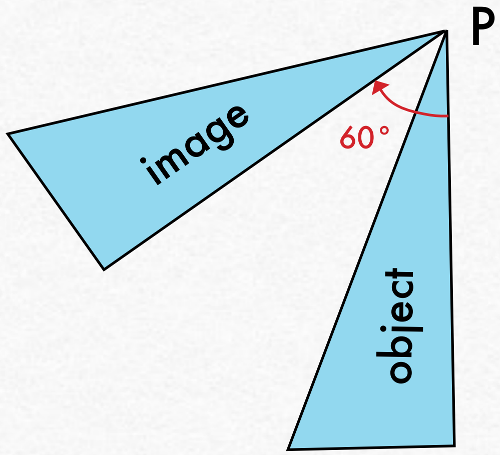
Cut from each corner, along the line to the third mark you just made.

Mark the left corners with an x, and then loosely fold the corners marked X into the centre. When you have done all 4, fasten them with a pin and pin the windmill to a stick (I used a pencil!)



Your windmill should rotate when you blow on it.

Many pleasing images are constructed by using rotation.



To draw or describe a rotation you need **3** pieces of information:

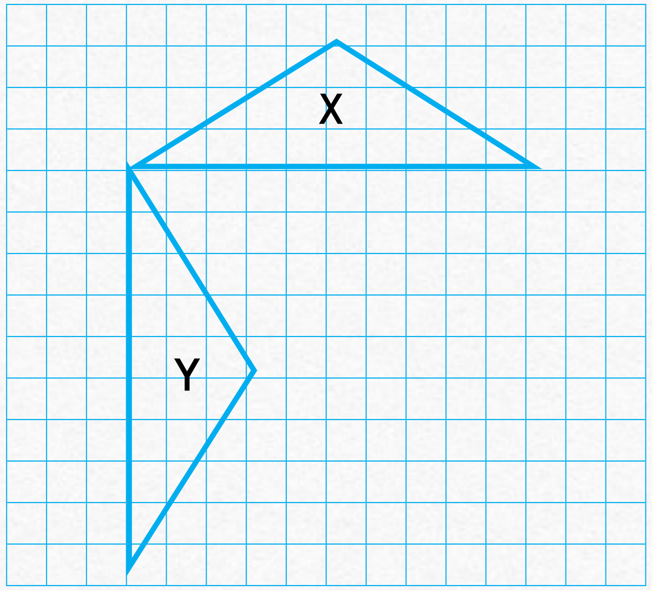
- The centre of rotation
- The angle of rotation
- The direction of rotation, either clockwise or anticlockwise (unless the angle is $180°$)

The rotation above can be described as:

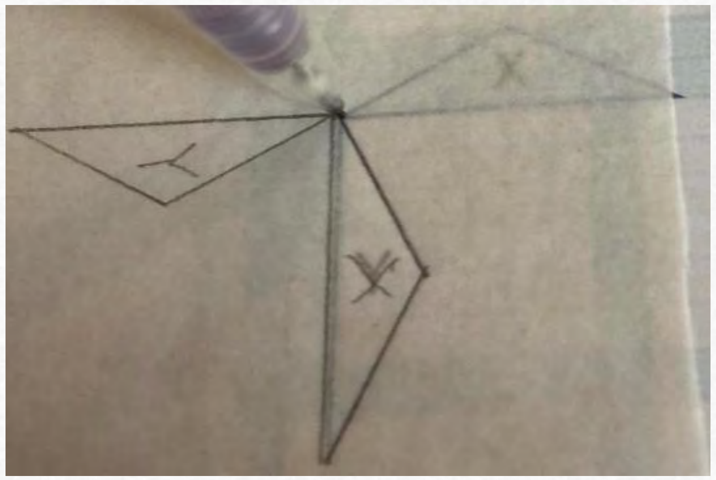
A rotation of $60°$ clockwise about the point P

Again, tracing paper can be helpful in determining the centre and angle of rotation.

Consider these triangles:



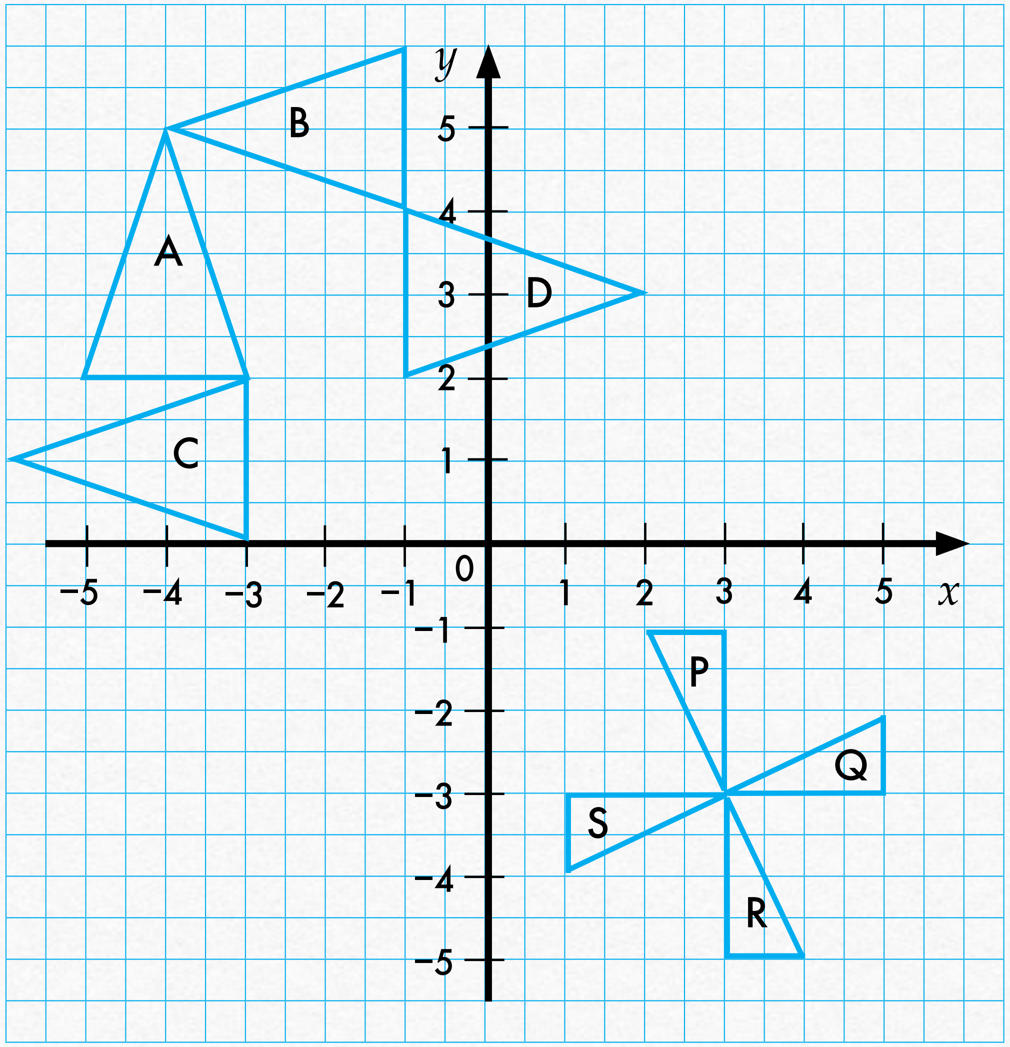
By tracing and rotating:



You can see that Y is the image of X after a rotation of $90°$ clockwise about the point of the pencil.

Just as with reflection, it can be helpful to draw rotations on a coordinate grid so that you can describe the rotation precisely:

Exercises



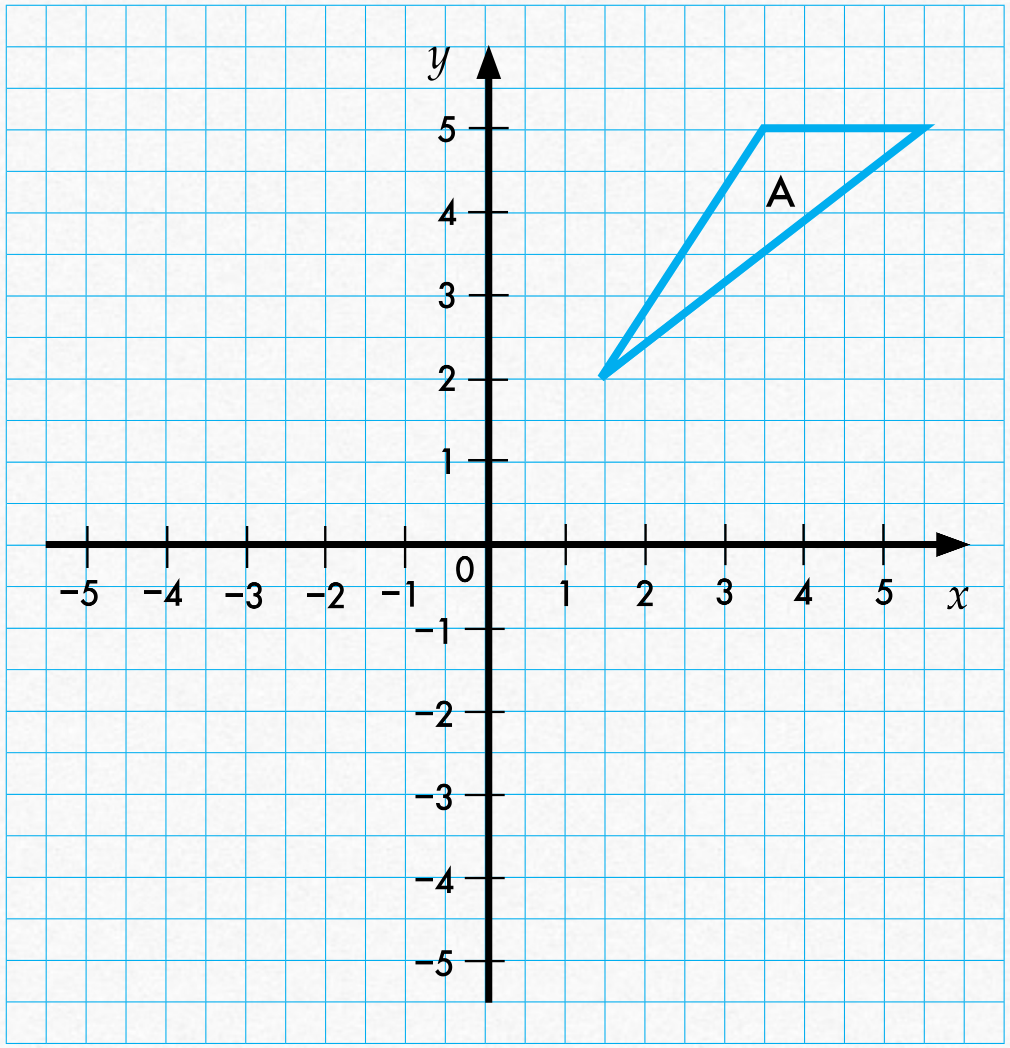
4 If B is the image of A after a rotation of 90° anticlockwise about the point (−4, 5) describe the image (i) C and (ii) D

5 If Q is the image of P after a rotation of 90° clockwise about the point (3, −3) describe the images (i) R and (ii) S

Rotation is an important skill for an ice skater.

You may have noted that you could have more than one answer to Q5, as a rotation of $90°$ clockwise is the same as a rotation of $270°$ anticlockwise. You also might note that a rotation of $180°$ does not need a direction, rotating either clockwise or anticlockwise will arrive in the same place.

Now try designing some rotations of your own.



Exercises

6 Copy the coordinate grid opposite and triangle A. Now design a pattern using various rotations of triangle A. (You could use another shape of your own choice if you wish)

When you are pleased with your final pattern, write down the instructions. Pass your instructions to a friend and ask them to follow them.

Has your friend drawn the same pattern that you designed?

"Animals seem to have their own mental maths ability".

“Learn where you feel like it.”

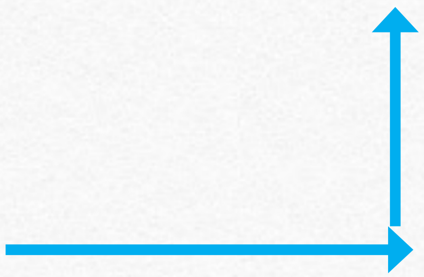
Translation

You might think of translation being when you work out the English meaning to a phrase or sentence written in another language:

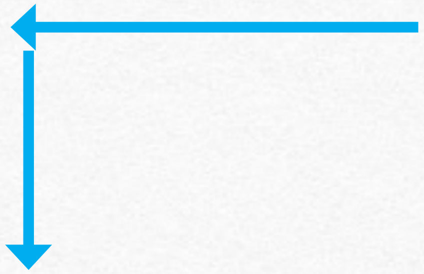
'La maison de mon ami' in French and *'hús vina minna'* in Icelandic both translate into 'my friend's house' in English.

In mathematics, a translation is a sliding movement.
It is described by the distance moved horizontally followed by the distance moved vertically.

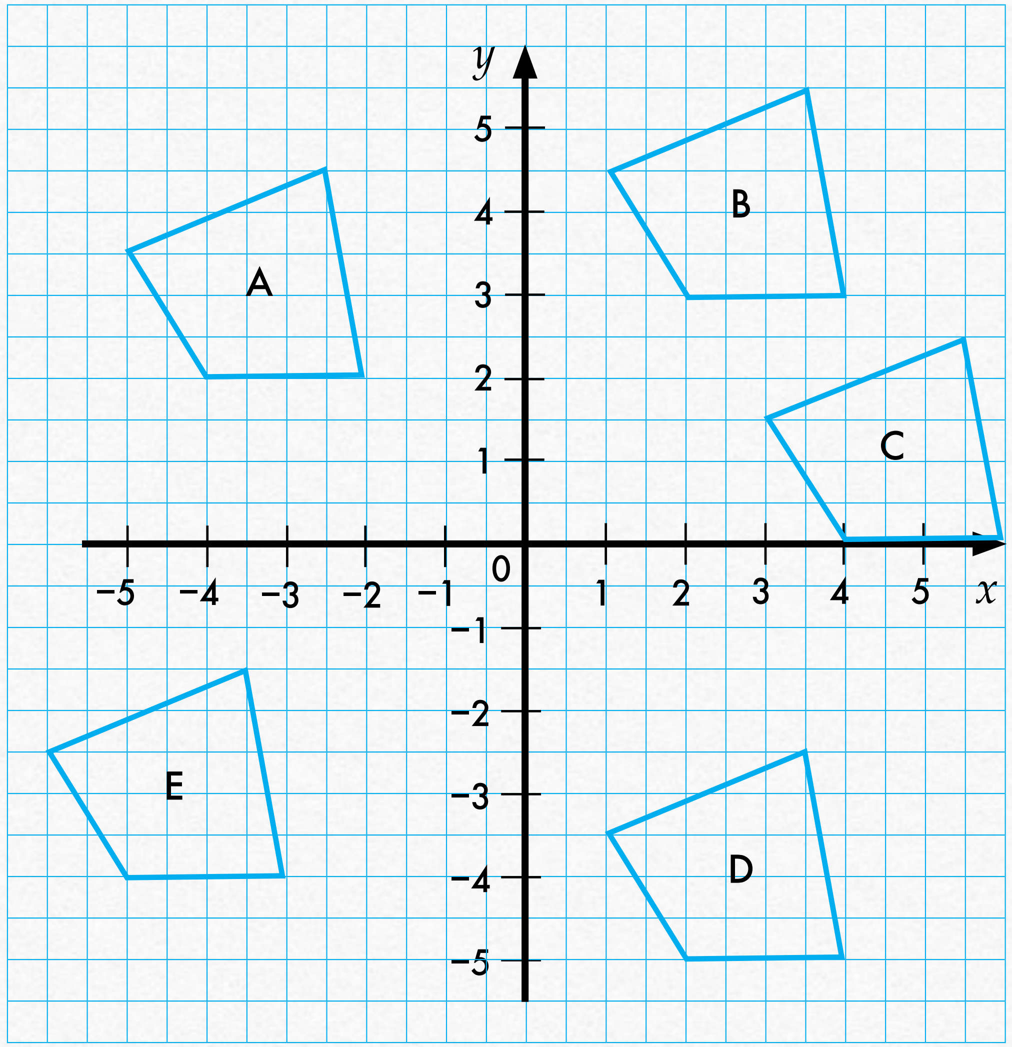
Maybe, a translation right and then up:



Or a translation left and then down:



Consider the shapes in this grid:



All the images, A, B, C, D and E are identical, they have not been flipped or rotated, they have simply been moved or translated.

We say: B is the image of A after a translation of 6 units to the right and 1 unit up. Note that a unit is defined by the numbers on the axes, in this example each unit is two squares.

Exercises

Describe the translations of these shapes on the grid on the previous page:

7 From A to C
8 From A to E
9 From C to B
10 From B to D

Reflections, rotations and translations are collectively known in mathematics as transformations. There is a fourth transformation called an enlargement, but we are not going to look at that here.

If you have to work with maps and scale drawings then these are like an enlargement except they have got smaller rather than larger, we will look at maps later in Maths for Explorers.

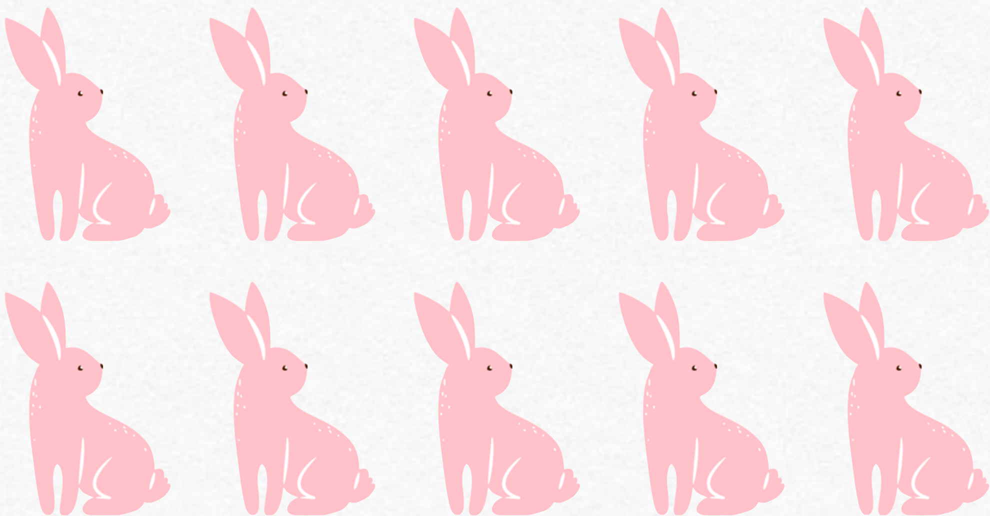
Transformations are useful in design because you can put them together to make patterns more interesting.

Try cutting a strip of paper, pleating it and then cutting out a shape:

Now you have a row of little men that are both reflections and translations:

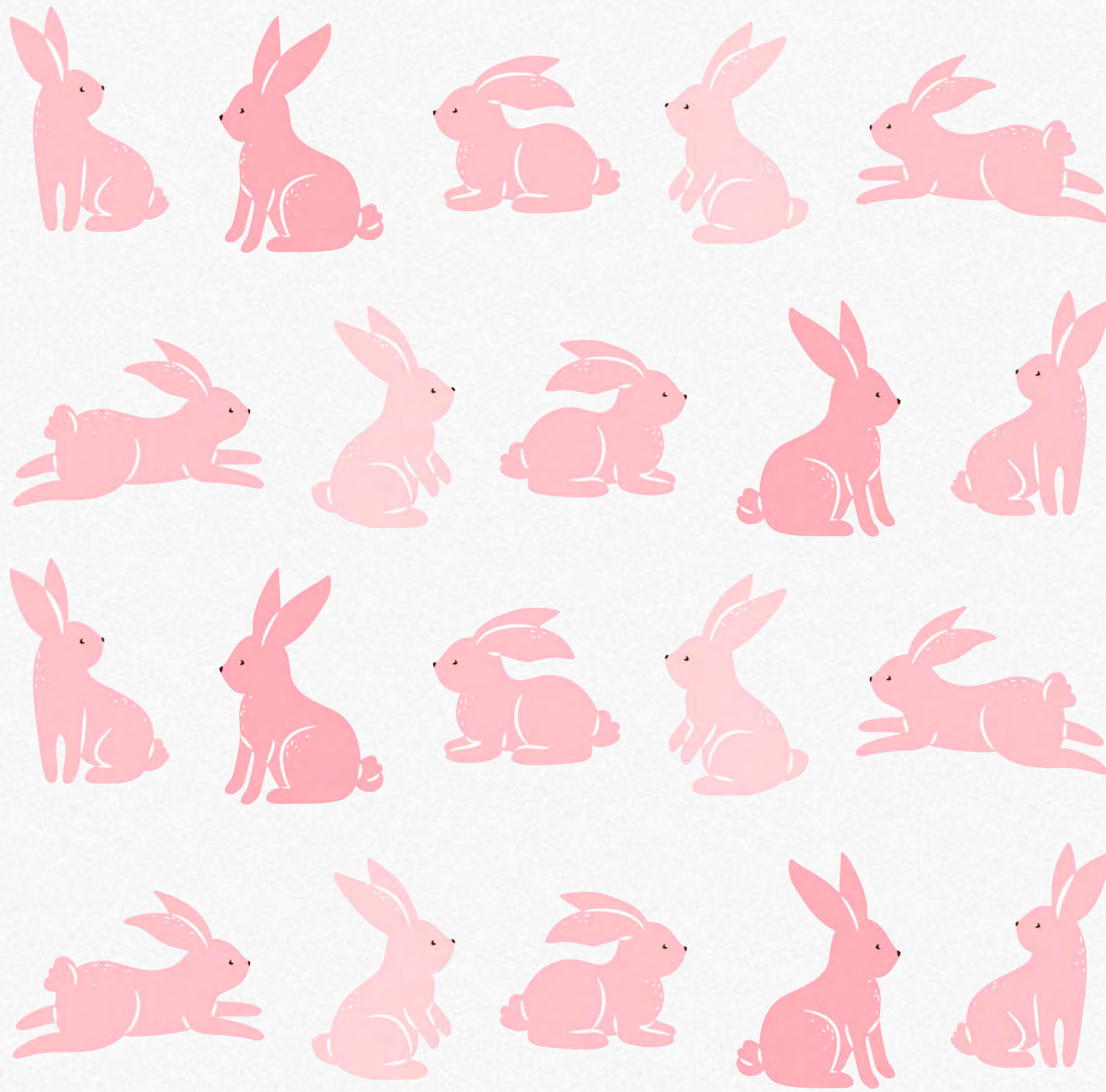
Consider this hare:

He is rather a good-looking hare and you could make an interesting textile for a child's bedroom by combining lots of hares in a pattern:



But you could also consider a pattern that includes a reflection of the hare:

You could also add some other hares and then put together a pattern of the hares with reflections and translations:



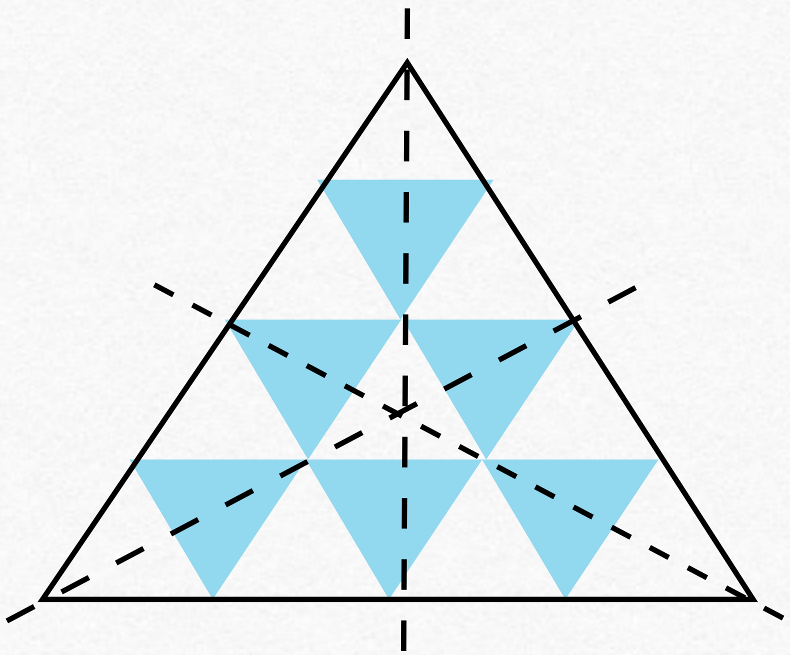
Which results in a much more interesting pattern. An interesting pattern is one that is more likely to sell and make you lots of money!

Symmetry – Line Symmetry

When you looked at reflections you described a mirror line. The mirror line can also be called a line of symmetry.

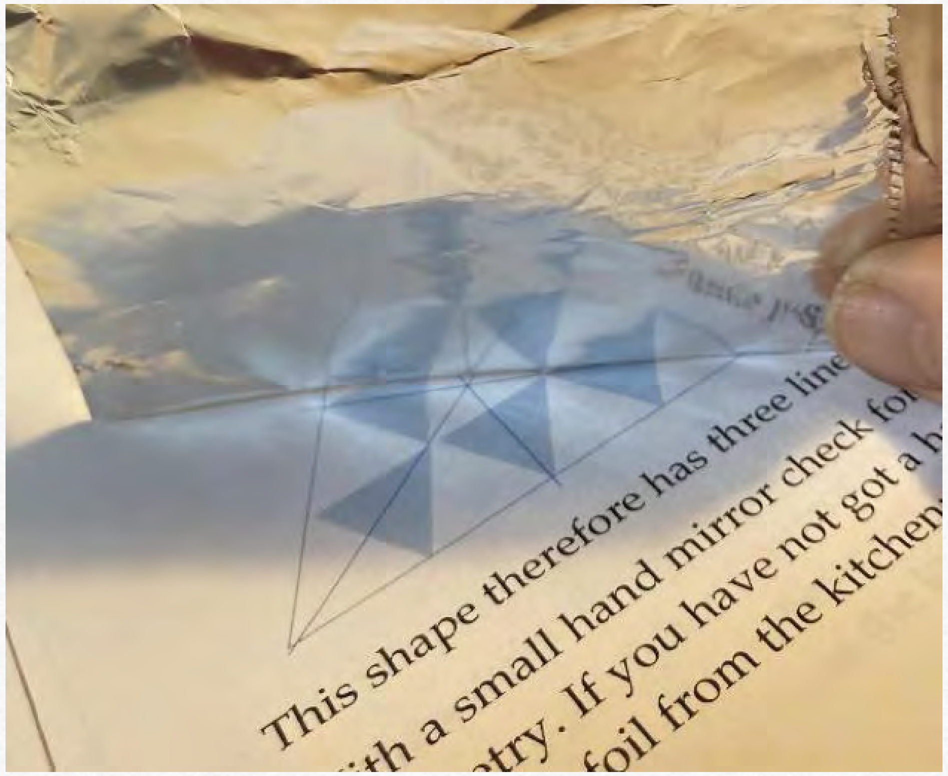
We say that a shape has line symmetry when you could draw a line going through it which divides it into two pieces that are mirror images of each other.

In the shape below, the mirror line can also be drawn in two other directions:



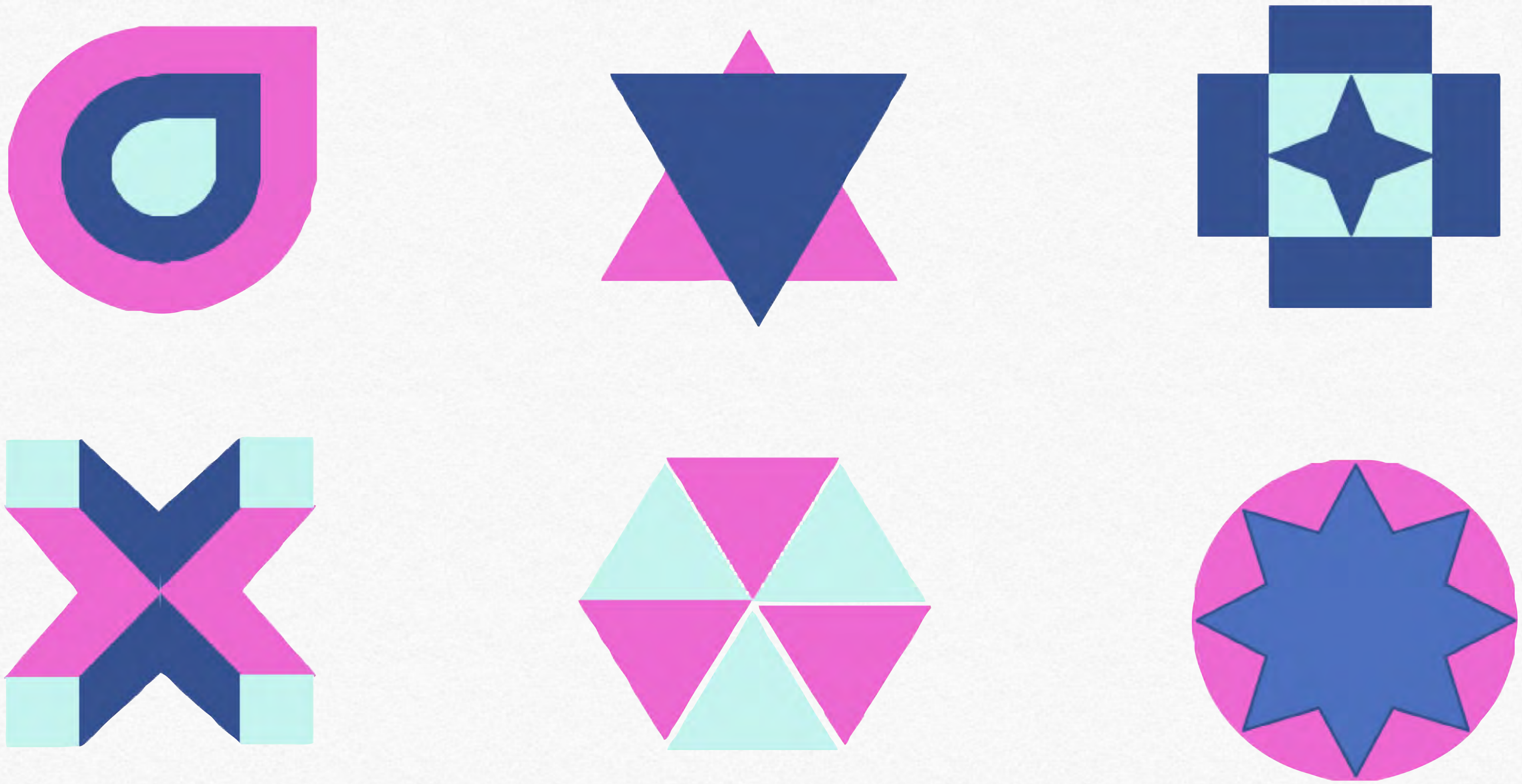
This shape therefore has three lines of symmetry.

With a small hand mirror check these lines of symmetry. If you do not have a hand mirror around then you can use silver foil from the kitchen:



Exercises

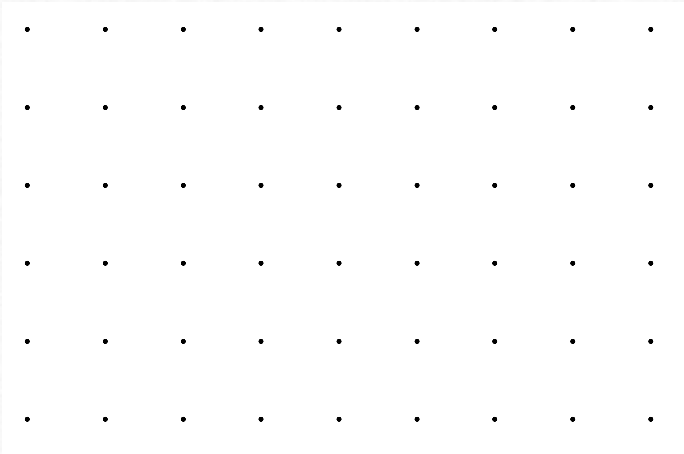
11 On a printout or on the screen, use a mirror or tracing paper to find all the lines of symmetry in these shapes:



Check your answers are correct before you continue. It is not always easy to spot all the lines of symmetry.

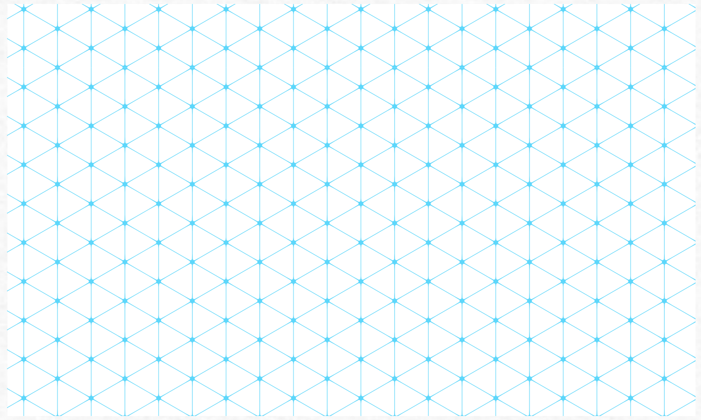
When constructing a design, either by hand or with a computer, designers often start with a grid. Here as some examples, you will find larger page sized versions of these at the end if this Part.

Square spotted

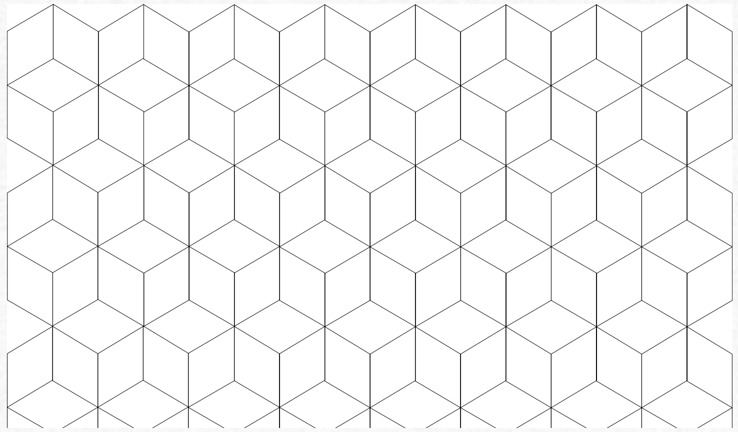


Triangular spotted

Isometric grid



Rhomboid grid



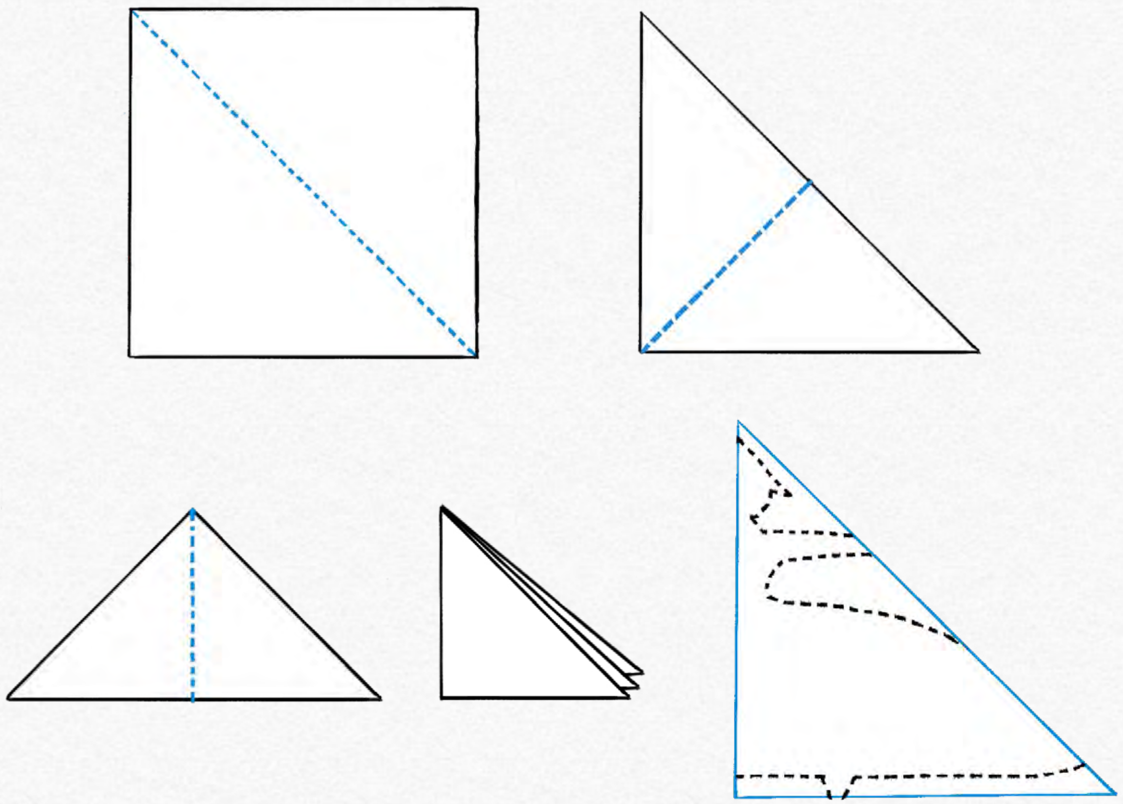
Food for Thought

Symmetry has been important since ancient times as it contributes to beautiful designs. This is a Celtic pendant made of silver with amber stones; you can see that it has both line symmetry and rotational symmetry

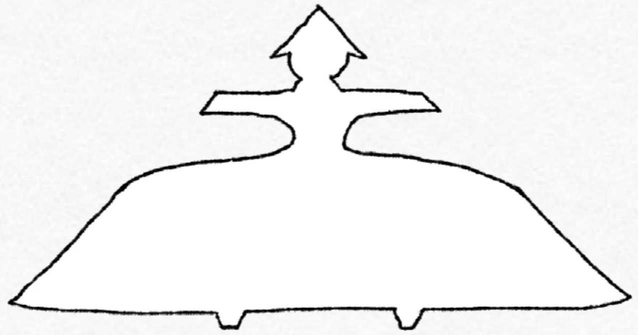
You can see that if you are designing a symmetrical pattern based on a square, then you would us the square spotted grid but if you wanted a pattern based on a triangle then you would use either the triangular spotted or the isometric grid. There are other grids you can find if you wanted something a bit different.

Try this before you start the exercises.

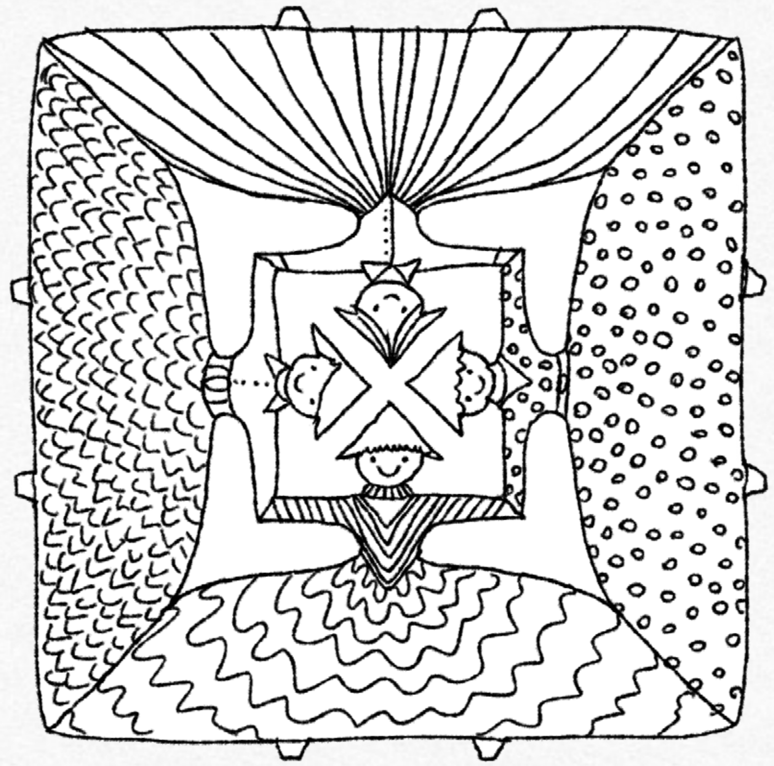
Fold a square four times until you have a triangle and then cut out the design shown:



Unfold your paper once; you have a lady with a wide skirt.



Keep unfolding and you have a pattern of ladies with wide skirts with four lines of symmetry.



Check the lines of symmetry with a mirror or by folding

Exercises

Using whichever grid you like, design a pattern to each of these specifications:

12 A square pattern with two lines of symmetry

13 A triangle pattern with one line of symmetry

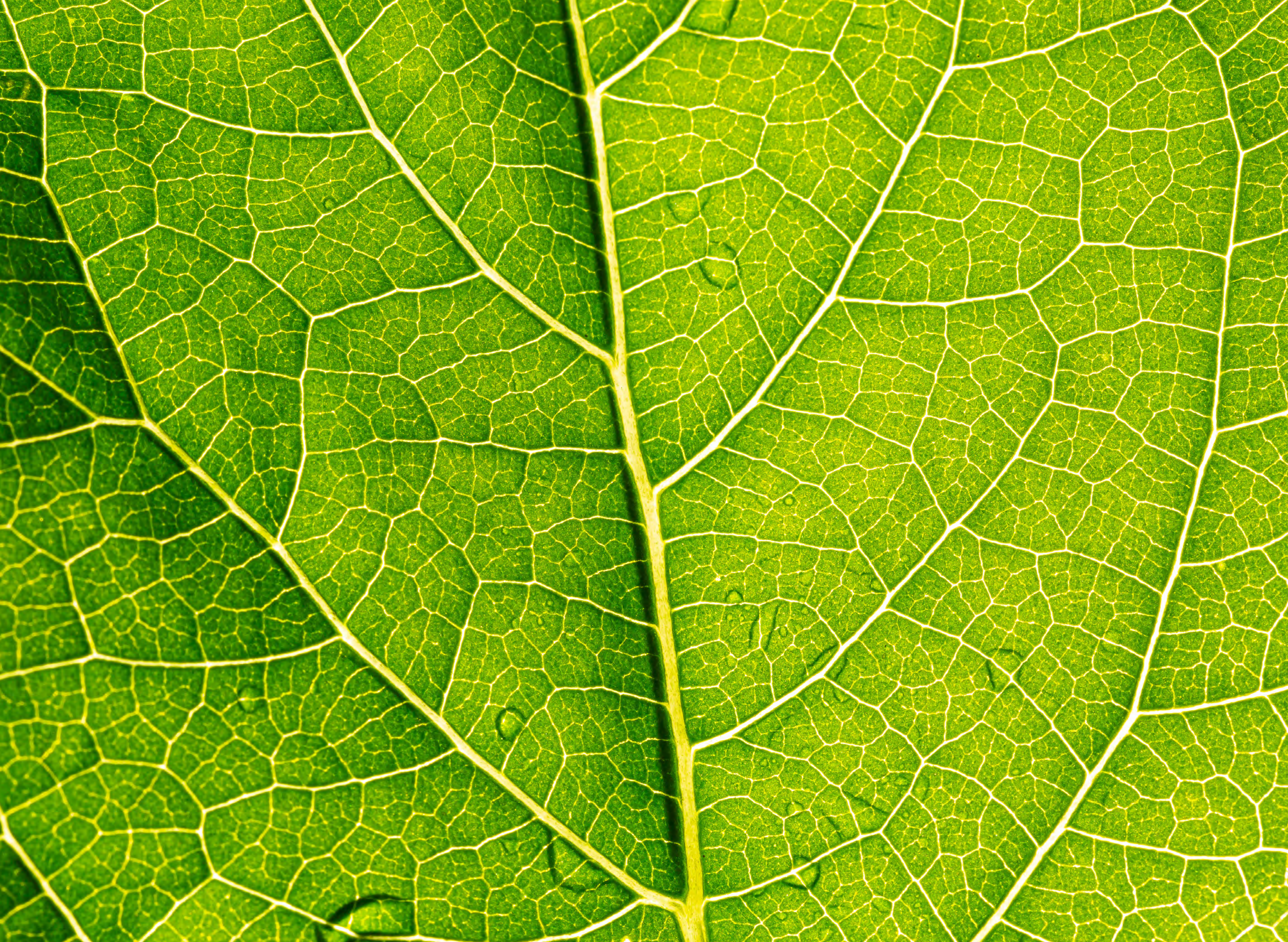
14 A square pattern with four lines of symmetry

15 A hexagonal pattern with six lines of symmetry

Food for Thought

You can find hexagonal patterns in nature such as the honeycomb found in a bees' nest.

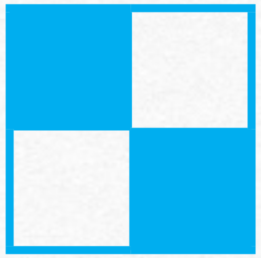
If you look in your wardrobe you will probably find several patterns that are based on squares.



Rotational Symmetry

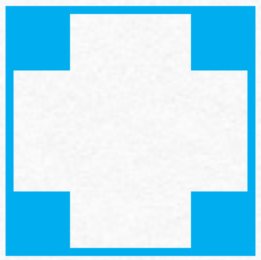
Just as a shape can have line symmetry, it may also have rotational symmetry.

A shape has rotational symmetry if it still looks the same after a part turn, that is not a full turn.



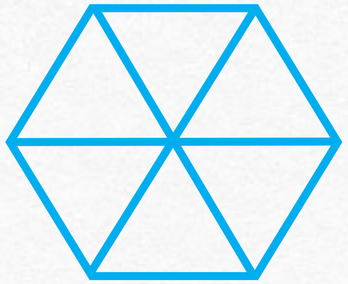
This is the same after ½ turn – Order **2**

This is the same after 1/3 turn – Order **3**

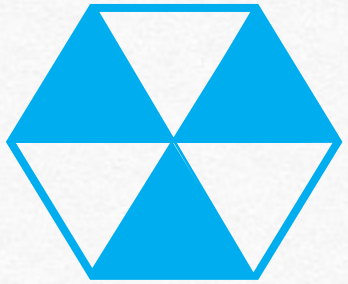


This is the same after ¼ turn – Order **4**

This is the same after 1/5 turn – Order **4**



This is the same after 1/6 turn – Order **6**



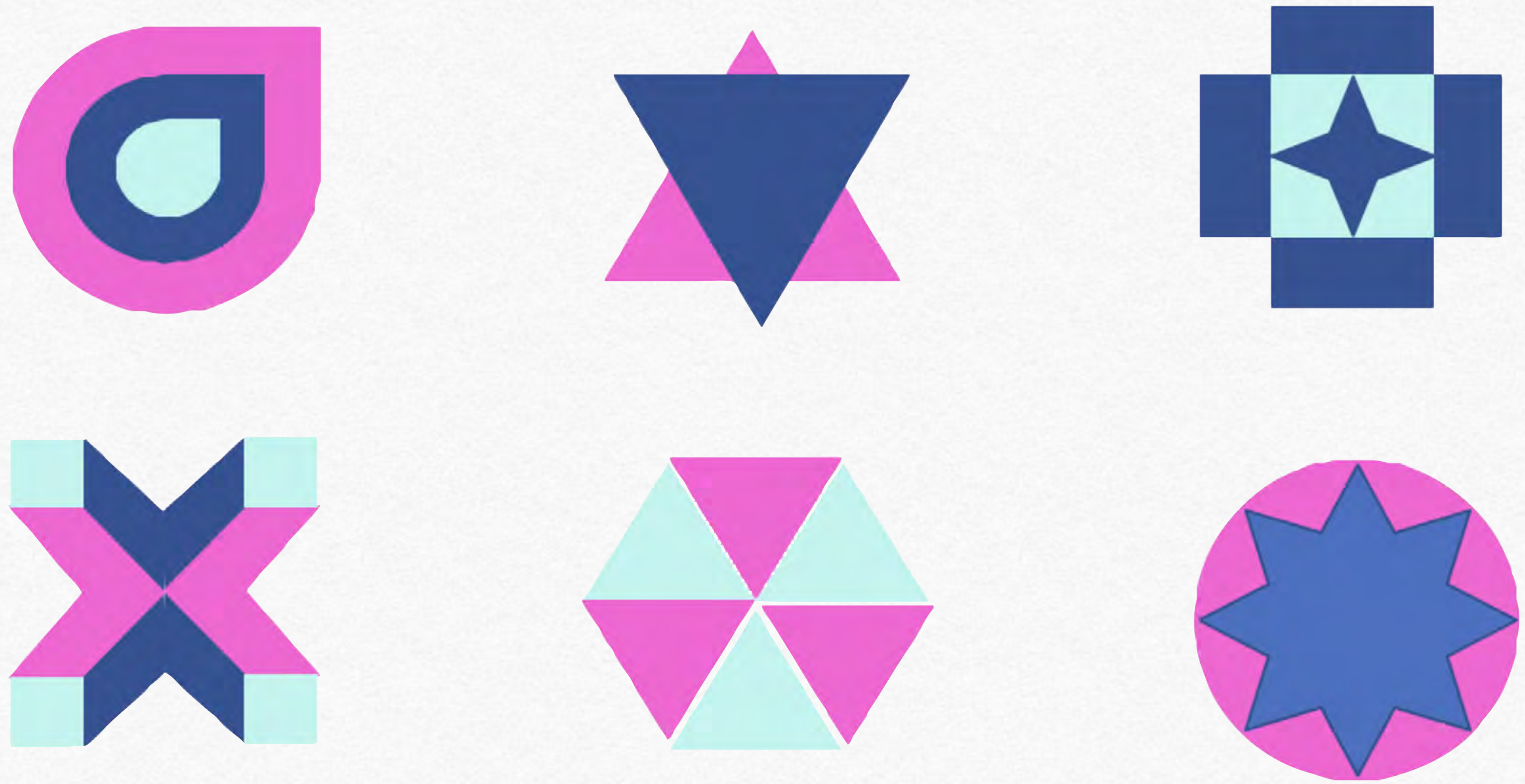
This is the same after 1/3 turn – Order **3**

You can see from those last two shapes that colouring in the pattern can change the order of rotational symmetry.

This piece of jewellery has rotational symmetry of order **9**.

Exercises

16 Do any of the shapes below have rotational symmetry?



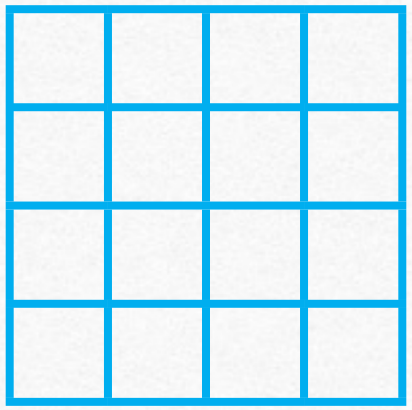
Copy any that have rotational symmetry and write down their order of rotational symmetry.

Food for Thought

You cannot have rotational symmetry of order 1. If it takes a full turn before a shape looks the same, not a part turn, then the shape does not have rotational symmetry. If you are asked to give an order of rotational symmetry for a shape like this one:

You would say: no rotational symmetry.

17 Copy this shape four times:



Colour each of your shapes so that it has:

(i) rotational symmetry of order **2** and no line symmetry

(ii) rotational symmetry of order **4** and no line symmetry

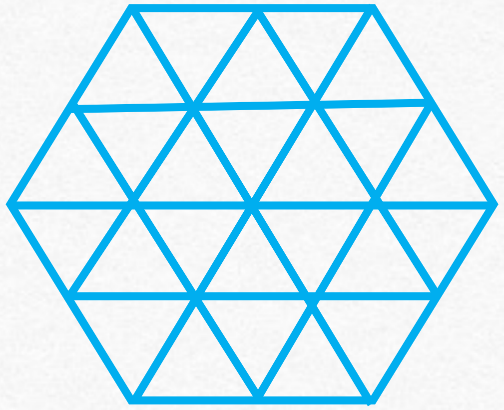
(iii) rotational symmetry of order **2** and **2** lines of symmetry

(iv) rotational symmetry of order **4** and **4** lines of symmetry

Food for Thought

Have you noticed any connection between the order of rotational symmetry and the number of lines of symmetry? Find out after **Q18** in the answer section.

18 Copy this shape five or more times:



Colour each of your shapes so that it has:

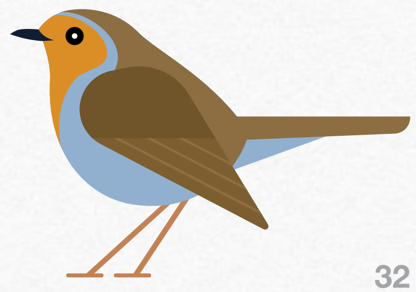
(i) rotational symmetry of order **3** and no line symmetry

(ii) rotational symmetry of order **6** and no line symmetry

(iii) rotational symmetry of order **2** and **2** lines of symmetry

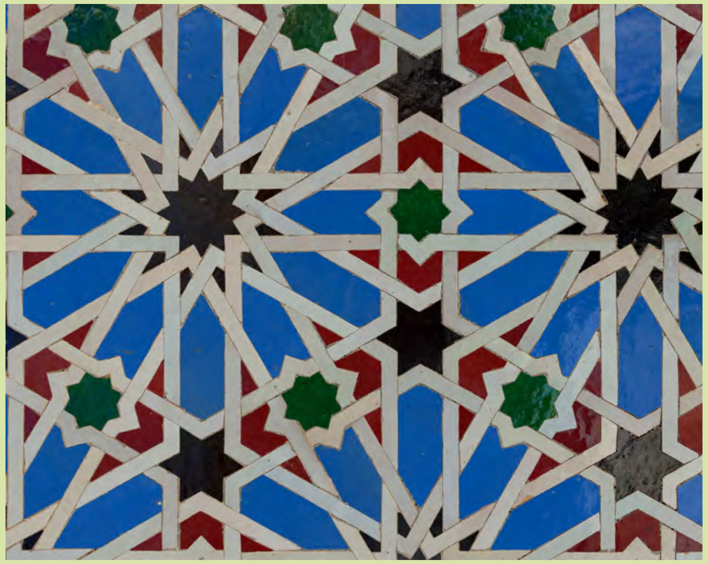
(iv) rotational symmetry of order **3** and **3** lines of symmetry

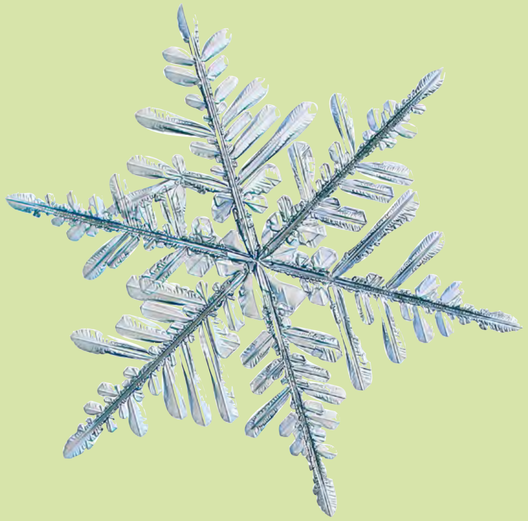
(v) any other ways that you wish, noting the order of rotational symmetry and number of lines of symmetry.



Food for Thought

Open your eyes and look around you. See where you can find symmetry in objects, in views, in buildings, in nature or in patterns. Company logos are often symmetrical. Make a collection of your favourites and try to rate them in terms of being a pleasing design.





Sometimes, the sheer beauty of symmetry
in nature just makes your heart lift with joy.

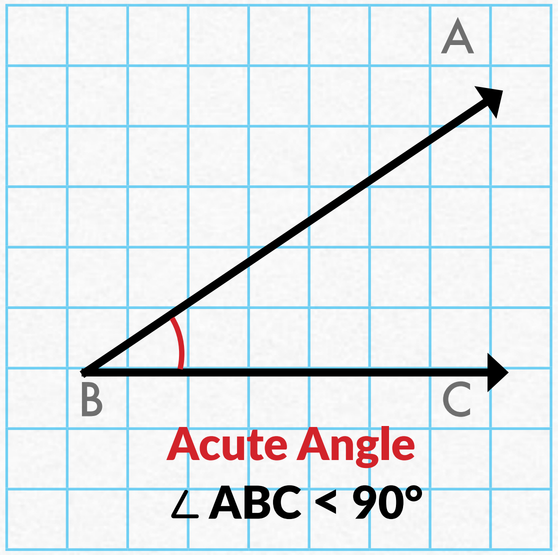
Useful Information about Angles

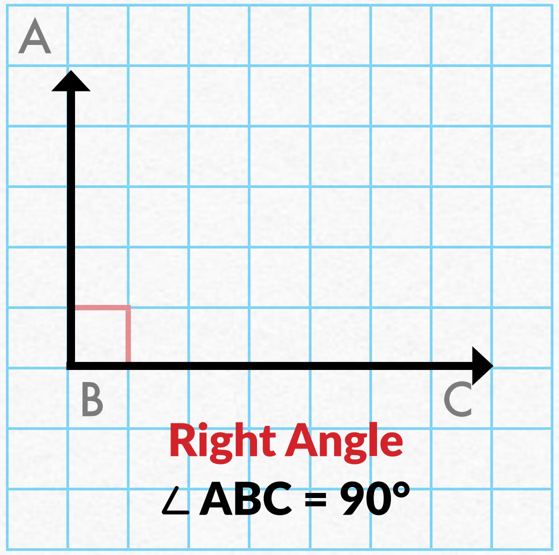
Before we go any further it is sensible to look a bit more at angles. Angles are measured in degrees which are written with a small circle: °

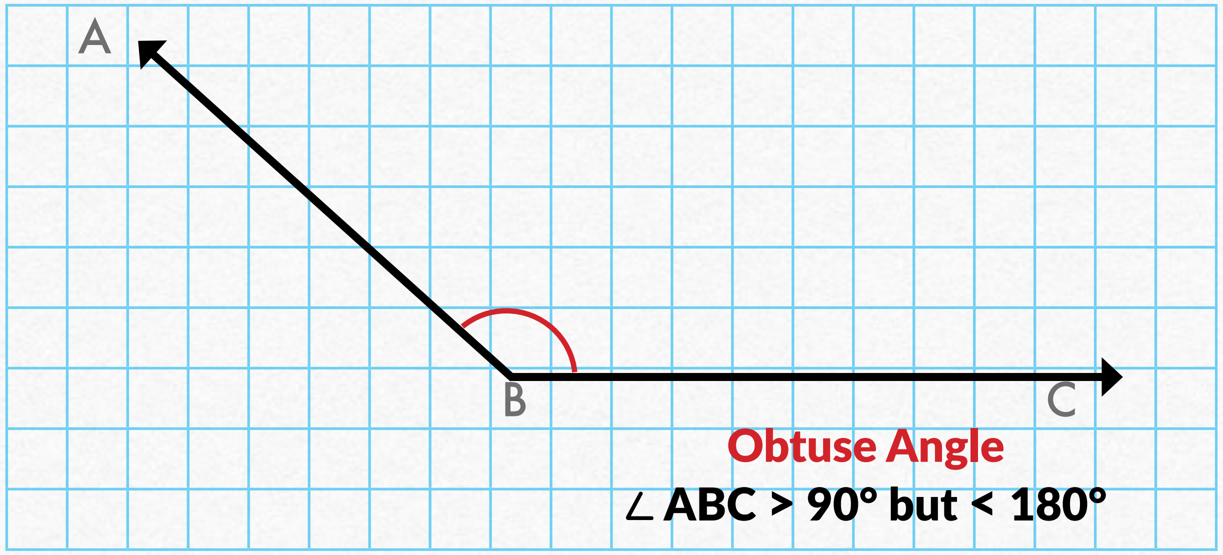
A full circle is **360°**

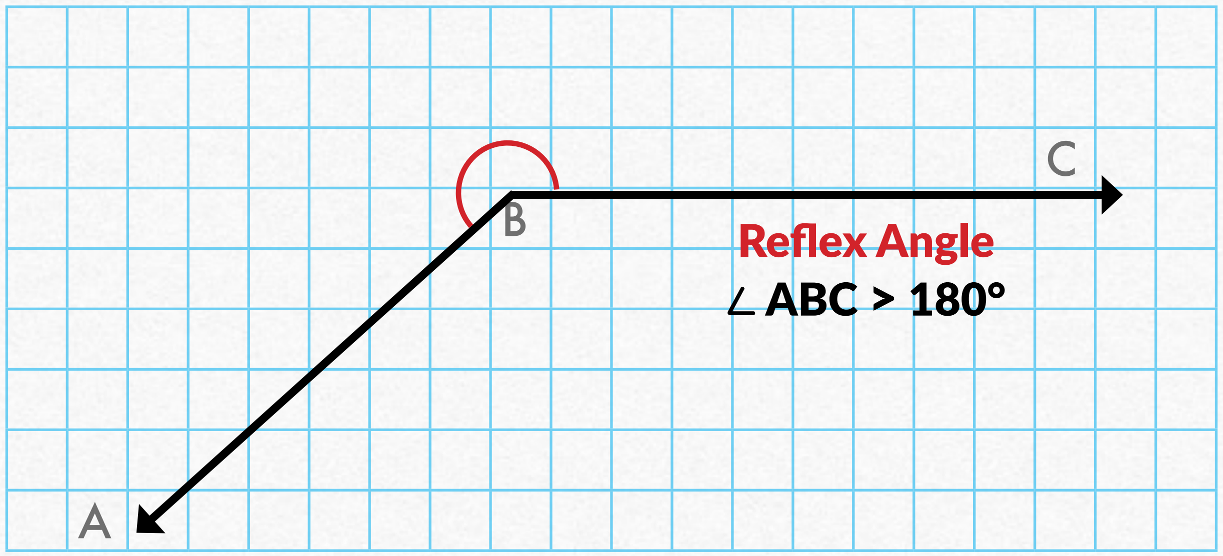
Do you remember that some angles have special names, depending on their size. Note how the angles here are identified by the two lines that meet at a point.

The angle ABC or ∠ABC is formed by two lines AB and BC that meet at B:

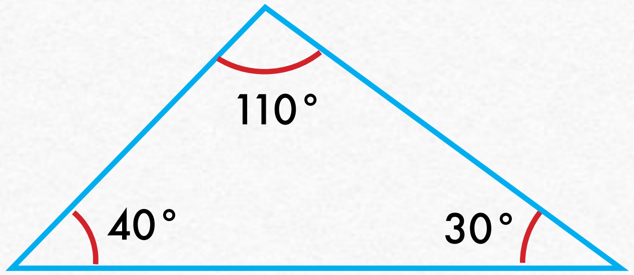




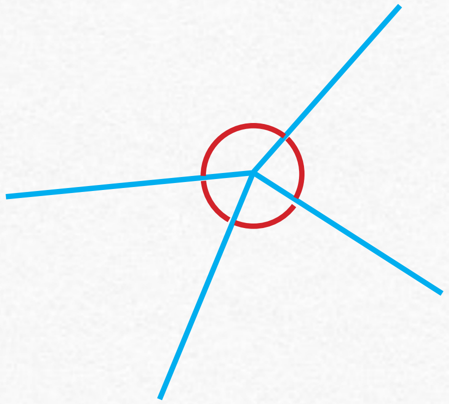




When designing patterns with angles it helps to know their special properties, these facts are important:

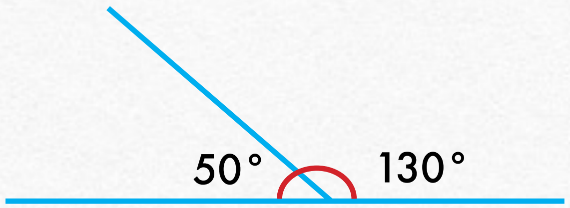


Angles in a triangle
add up to **180°**

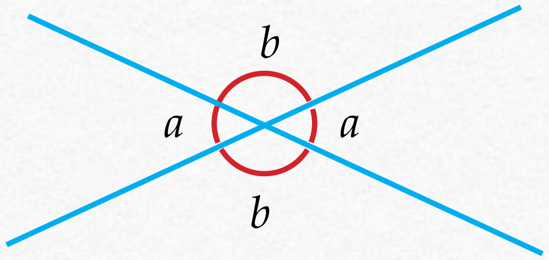


Angles round a point
add up to **360°**

Sometimes you will need to calculate an angle to ensure that your shapes fit on a straight line or round a point. At other times you just need to know the angle fact.



Angles on a straight line
add up to **180°**

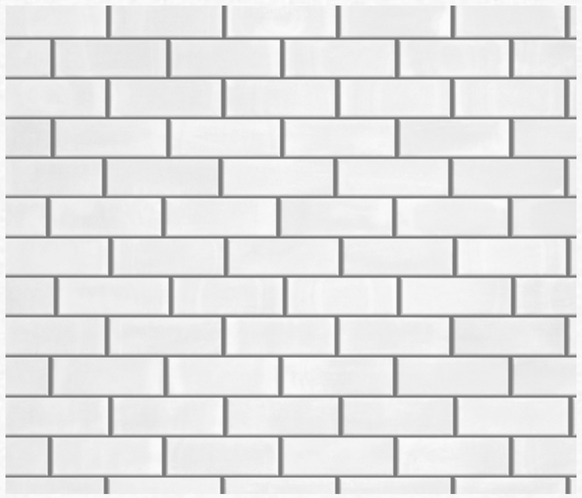


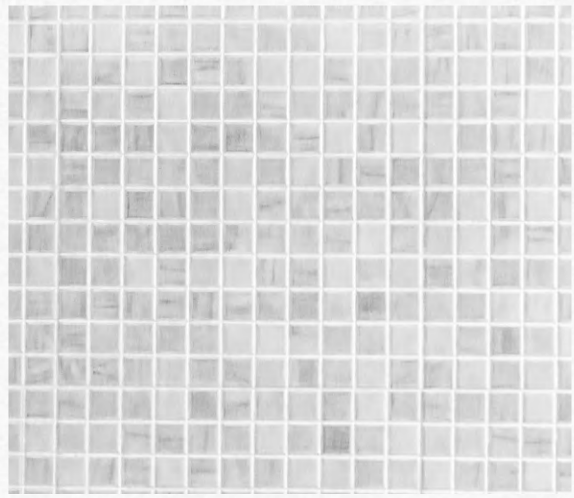
Vertically opposite
angles are equal

You can see both from your work on symmetry and from the angle facts, that **90°** is very important, so important that it has its own name, right angle.

Another important angle in design is **60°**. You could see from your work on symmetry that any shape with **3** or **6** lines of symmetry or rational symmetry of order **3** or **6** is associated with **60°** angles.

When designing complex patterns such as those for fabrics or wallpaper, like the hares you looked at earlier, the overall pattern is built up by putting together smaller patterns. The simplest example of this are bathroom or kitchen tiles.





Tiles are usually either square or rectangular. They can be plain or patterned.

Cleverly designed patterned tiles can be put together to make a vibrant pattern. This is an example from Palermo in Italy:

You can see how the tiles are rotated. If you look at just four together:

Or

You can see that both patterns have both line and rotational symmetry.

Food for Thought

In the Alhambra Palace in Spain, you will find many examples of beautiful tiling patterns as well as remarkable Moorish architecture. There are also plenty of examples of symmetry both in the architecture and the tiling patterns.

Tessellations

A shape like a square or rectangle that can cover a flat surface without any gaps is one that tessellates. The resulting pattern is a tessellation.

Think of all the shapes you know. Will all of the regular polygons tessellate?

Which of the special triangles and quadrilaterals can tessellate?

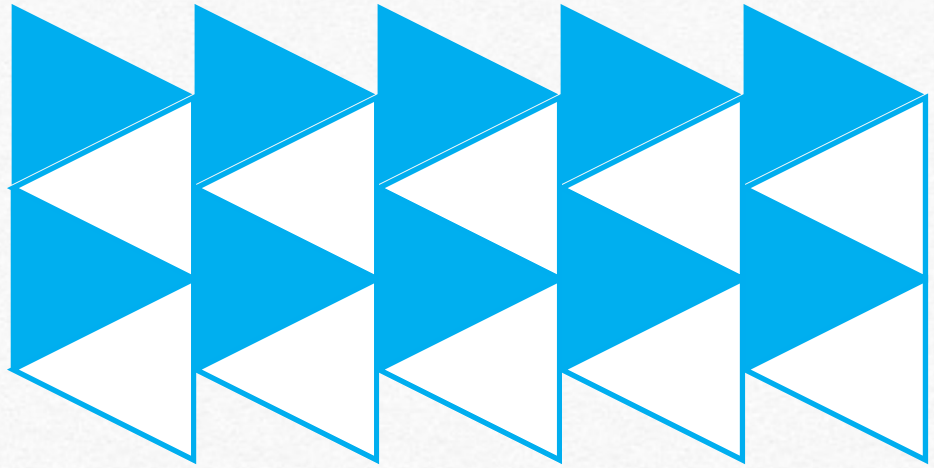
Use the grids at the end of this Part to draw your tessellations.

As you draw, think about the angles.

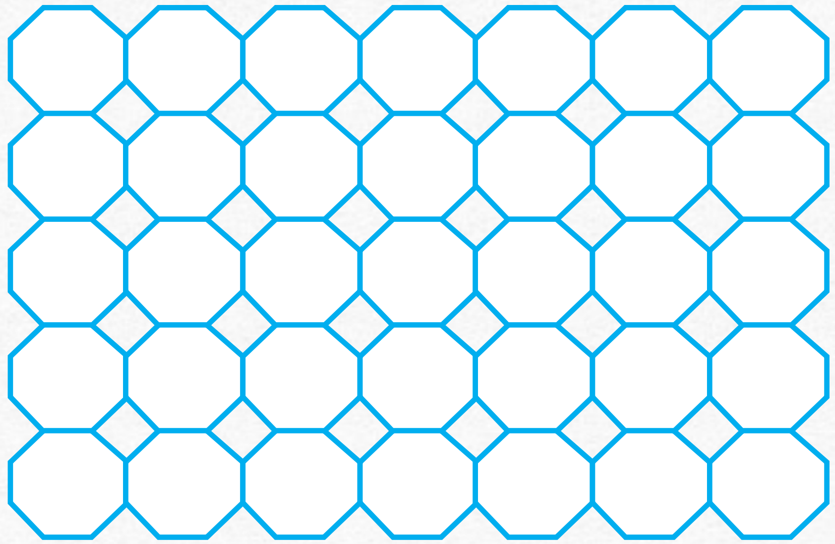
Remember, angles at a point add up to $360°$.

That should help with your design.

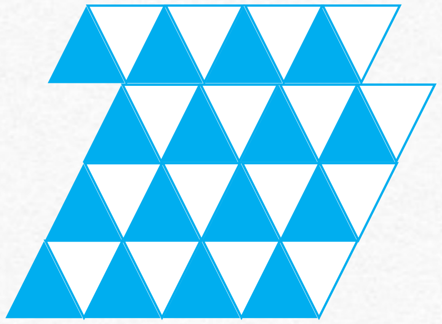
You might want to put some triangles and quadrilaterals together to make a new shape and tessellate that too:

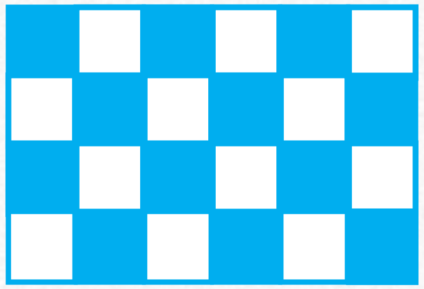


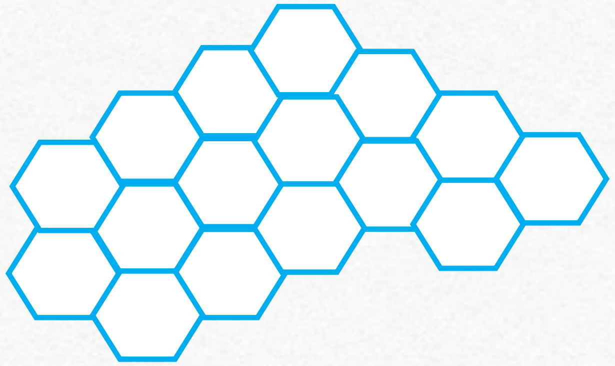
You could also try putting two or more regular polygons together to see if they tessellate:



You should have discovered that the only **regular** polygons that tessellate are equilateral triangles, squares and hexagons:

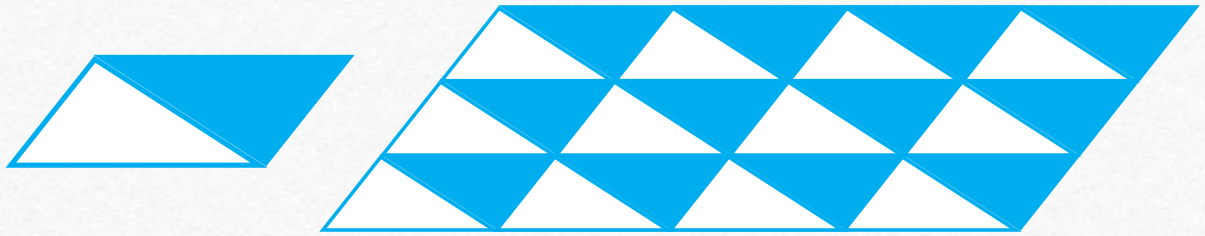






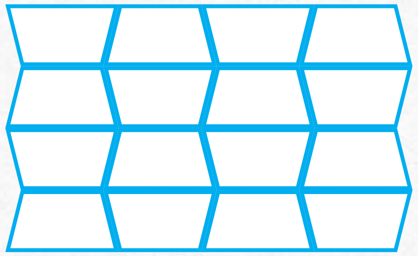
Now think about other polygons that a not regular:

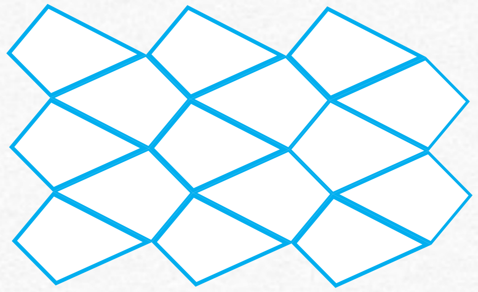
All triangles can tessellate with a bit of help, as you can see from this scalene triangle. A scalene triangle is one which has no equal sides and no equal angles.



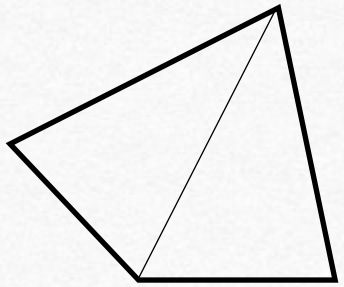
By making a copy of the triangle and rotating it about the midpoint of a side then you will make a parallelogram. A parallelogram (and therefore a rhombus) will always tessellate.

Trapezia and kites will always tessellate:





Which might make you think that all quadrilaterals will tessellate.

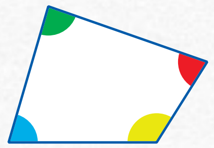


If you consider that a quadrilateral can be divided into two triangles and that the angle sum of a triangle is $180°$, then the angle sum of a quadrilateral must be:

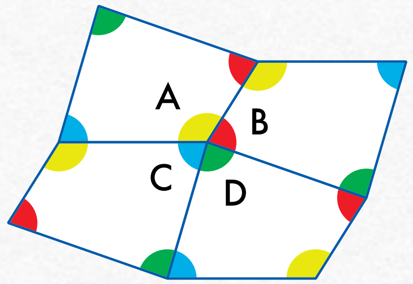
$$2 \times 180° = 360°$$

Consider that angles at a point add up to $360°$.

Here is a quadrilateral whose four angles must add up to $360°$:



Arrange four congruent (maths word for identical) quadrilaterals around a point with all their different angles touching:



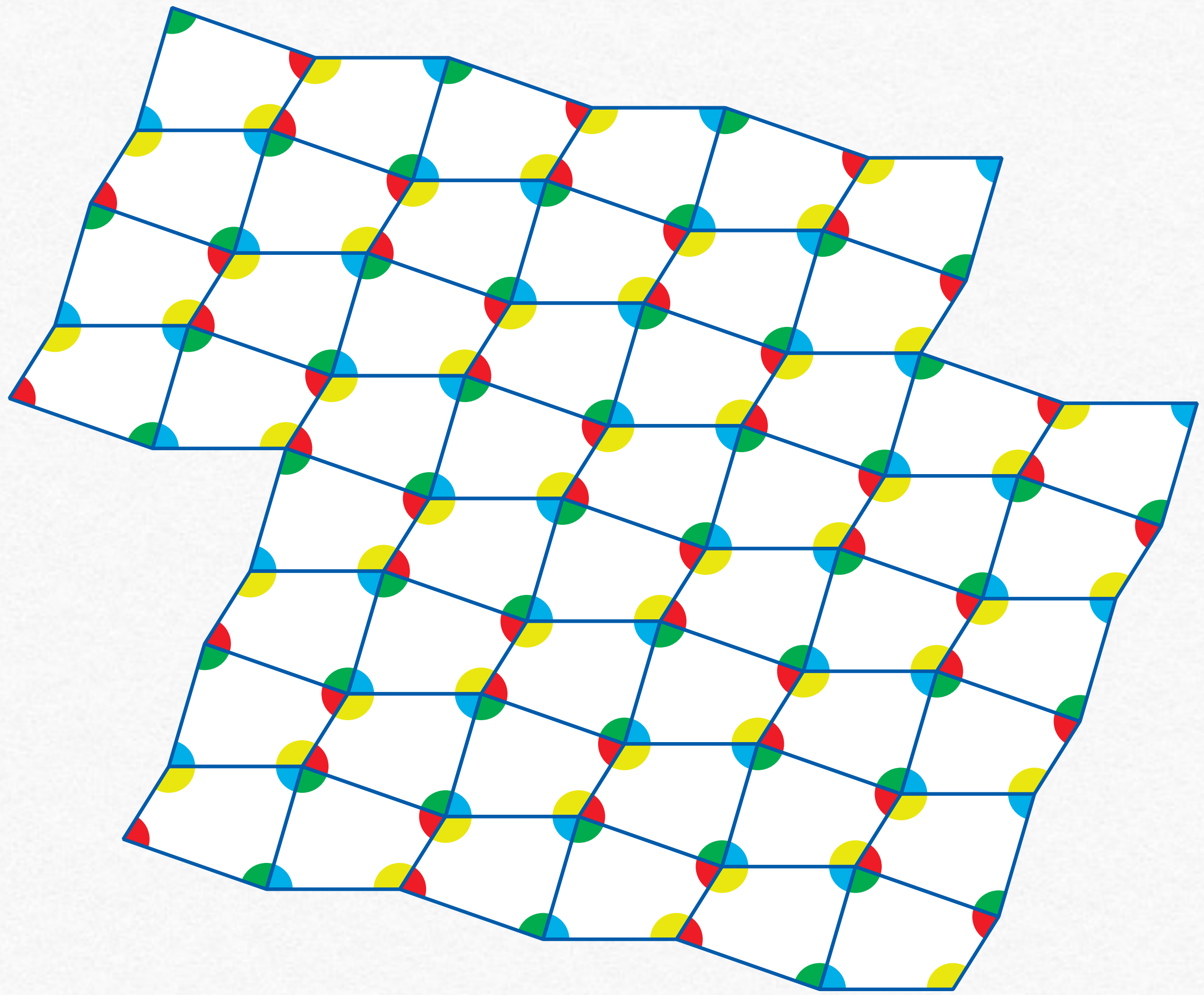
green angle + red angle + yellow angle + blue angle $= 360°$

Can you see that D is a translation of A,

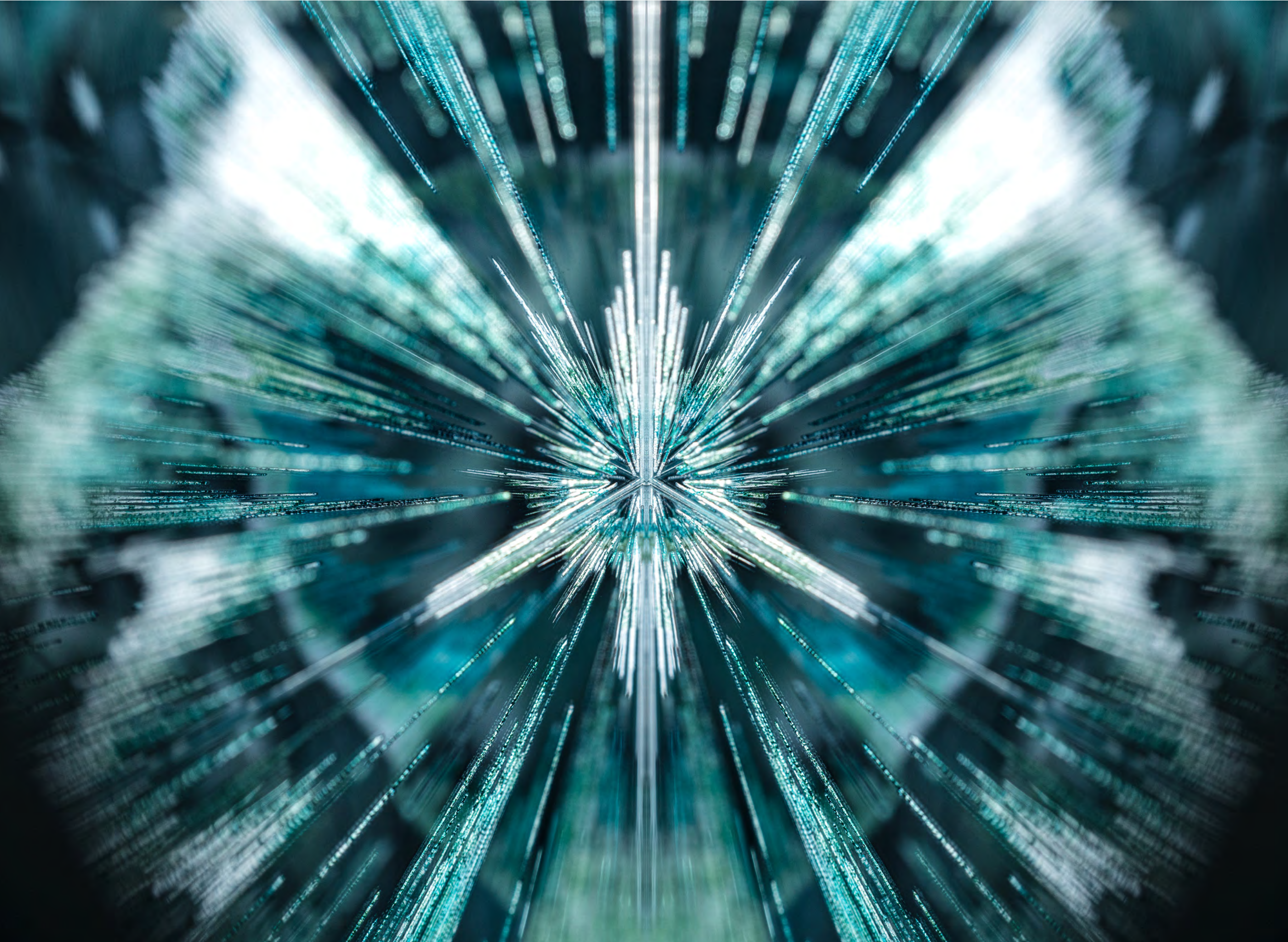
B is a rotation of A by $180°$ about the midpoint of a side and C is a translation of B?

By fitting the quadrilaterals together like this each side sits next to the corresponding side in the adjacent quadrilateral so there are no gaps.

You can then tessellate your quadrilaterals.



Can you see that at every point the four angles of the quadrilateral meet and these add up to $360°$?



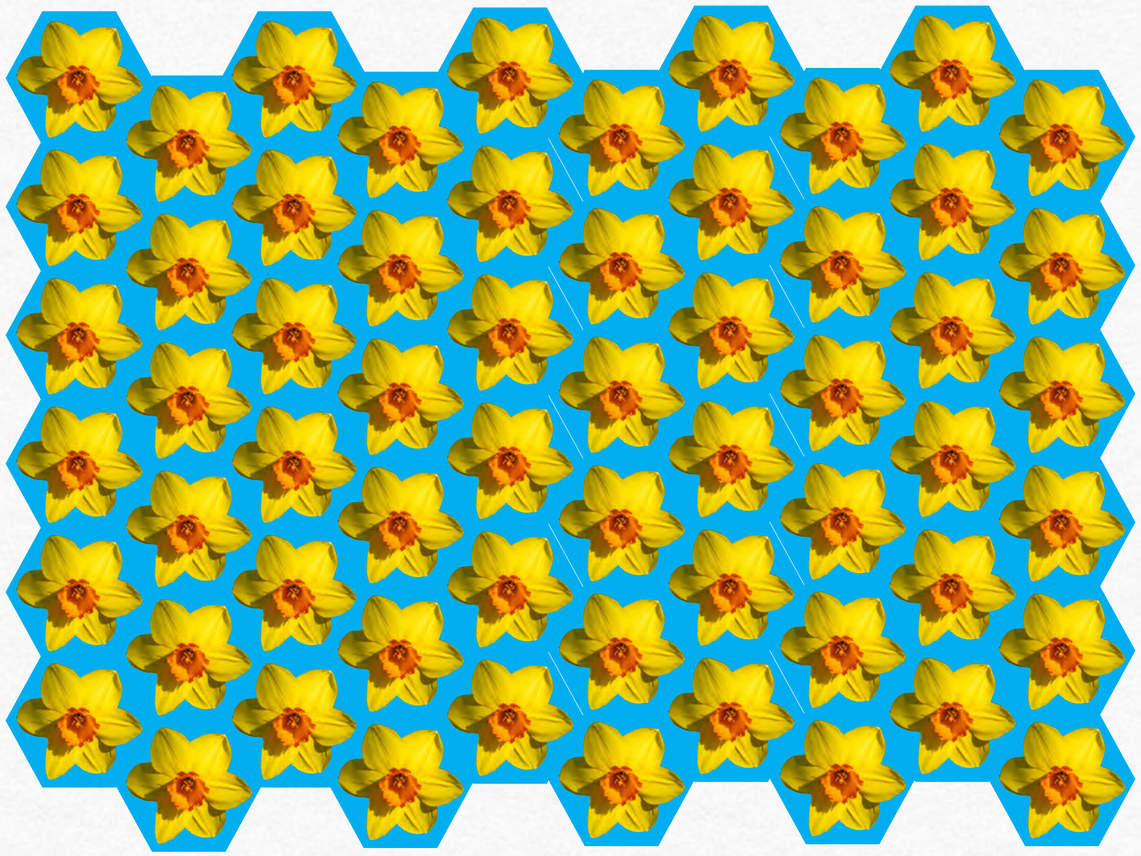
More Design

Look at what a designer did with a photograph of a daffodil:

First, she shaded out the background and filled in with a colour:

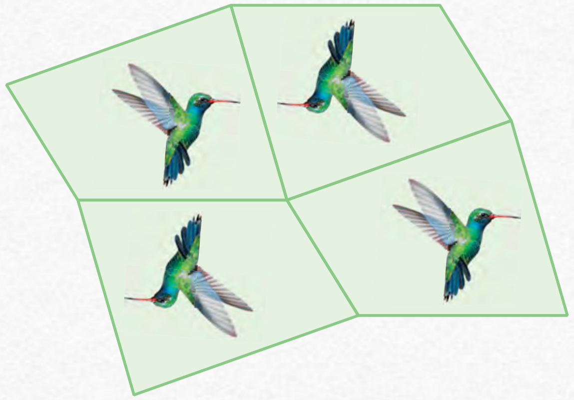
Then she drew a regular hexagon and filled it with the daffodil picture:

Then she tessellated copies of the hexagon. Here is the final pattern:

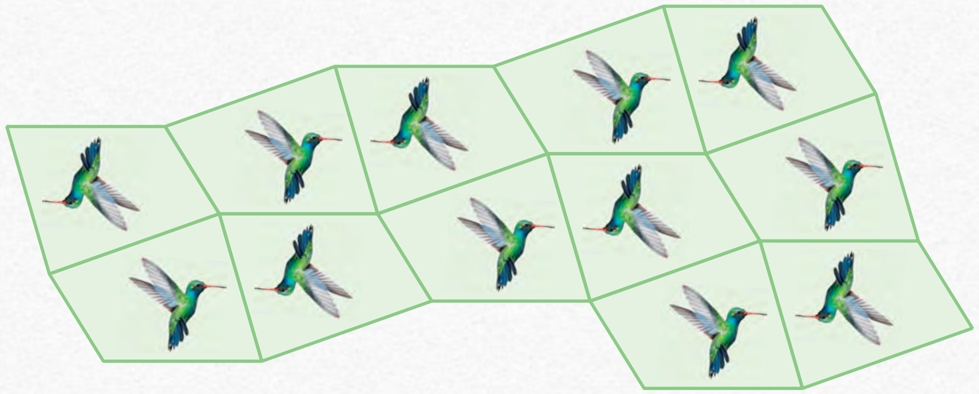


This designer filled a quadrilateral with a hummingbird:

Then she translated, rotated and translated copies of the first quadrilateral to get this:

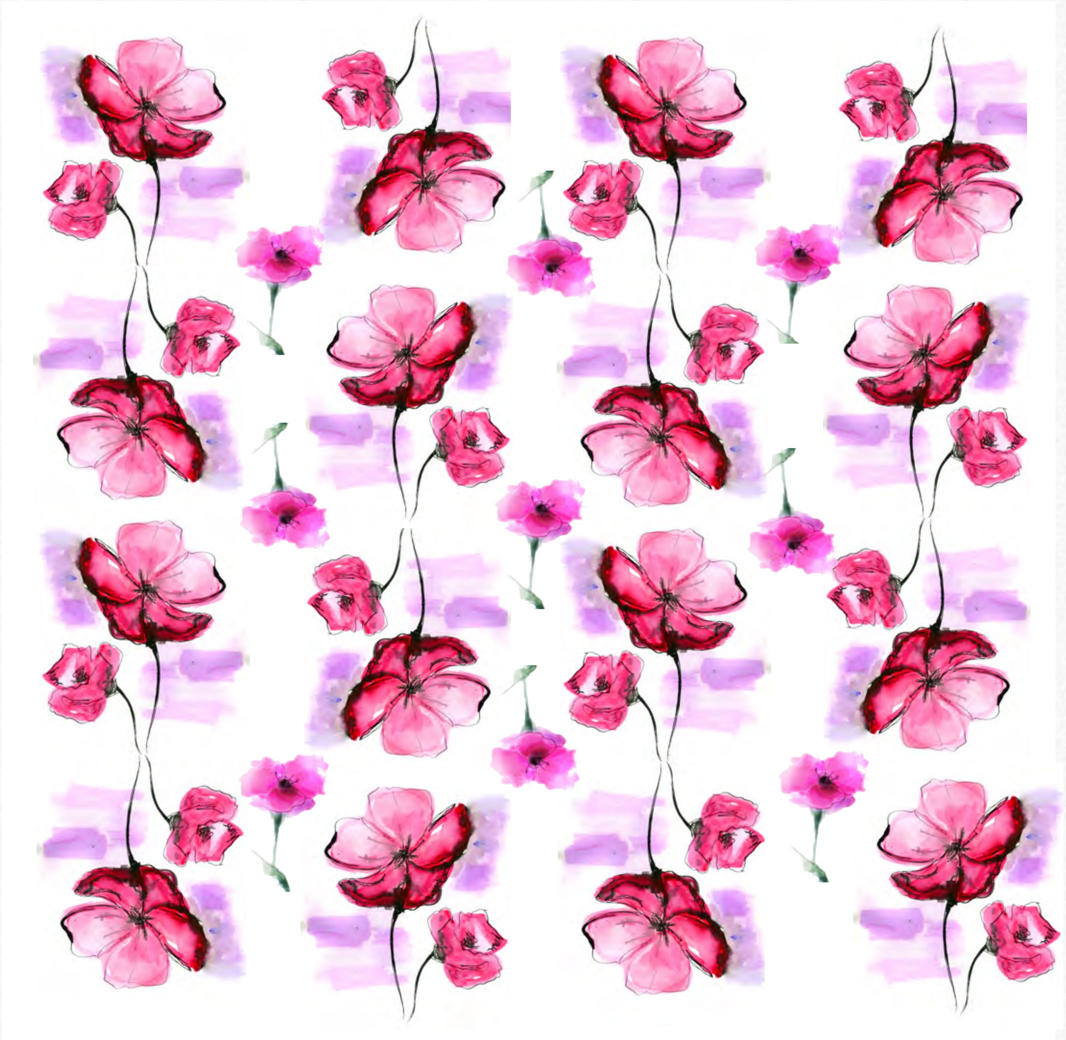


She then tessellated the resulting shape:



Finally, she removed the outlines to the quadrilaterals:

If you look at floral patterns on fabrics or on wall paper, you may see that designers sometimes incorporate a second image on top of the first.



Exercises

19 Draw a scalene triangle, one which has no equal sides and no equal angles, for yourself and then follow the steps to draw a tessellating pattern.

20 Draw a quadrilateral for yourself and then follow the steps to draw a tessellating pattern.

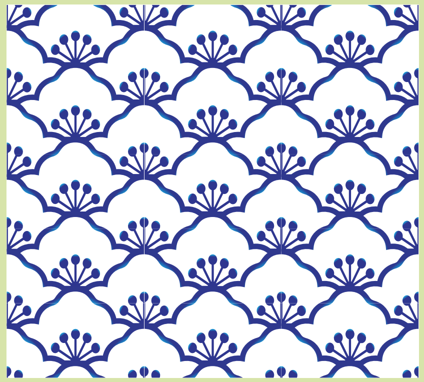
21 See if you can find the reflections, rotations and translations in the floral pattern on the left.

Food for Thought

Wallpaper designs like the flowers opposite are known as 'wallpaper groups'.

Much research, both historical and modern has gone into wallpaper groups. The different ways of combining reflections, rotations and translations have been defined and it has been concluded that there are just 17 possibilities.

This is a simple Chinese pattern made from simple translations:



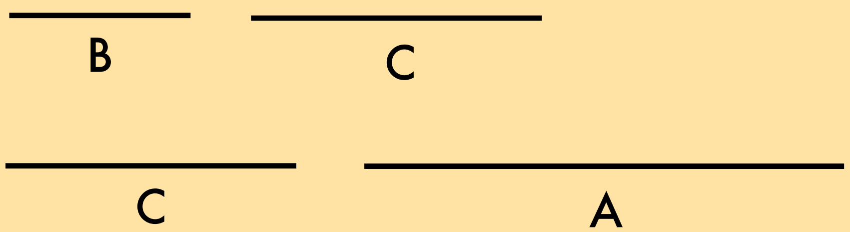
Golden Design

At the end of Step **4** you were introduced to the Golden Ratio.

Many designers use the golden ratio as they develop new designs.

The Golden Ratio

Here's the ancient Greek definition of the golden ratio. The **small** is to the **large**, as the **large** is to the **whole**. So on your drawings, the small (line **B**) is to the large (line **C**) as the large (line **C**) is to the whole (line **A**). The whole (line **A**) is equal to line **B** and **C** added together.

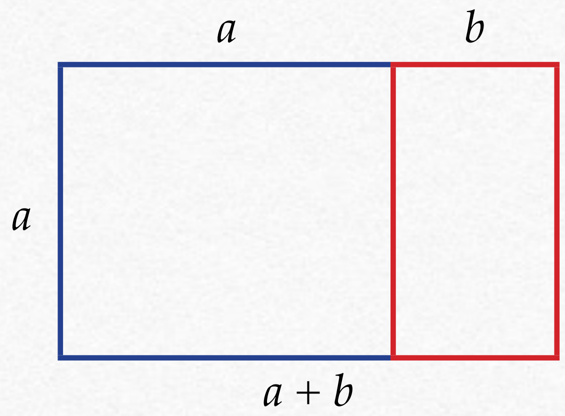


The ratio **B** : **C** is the same as the ratio **C** : **A**.

The Golden Ratio is frequently known by the Greek letter **phi: ϕ**

(Phi is pronounced fy – like the end of fortify)

To remind yourself of what is means numerically look at the rectangle:



To be in the golden ratio then a + b : a = a : b

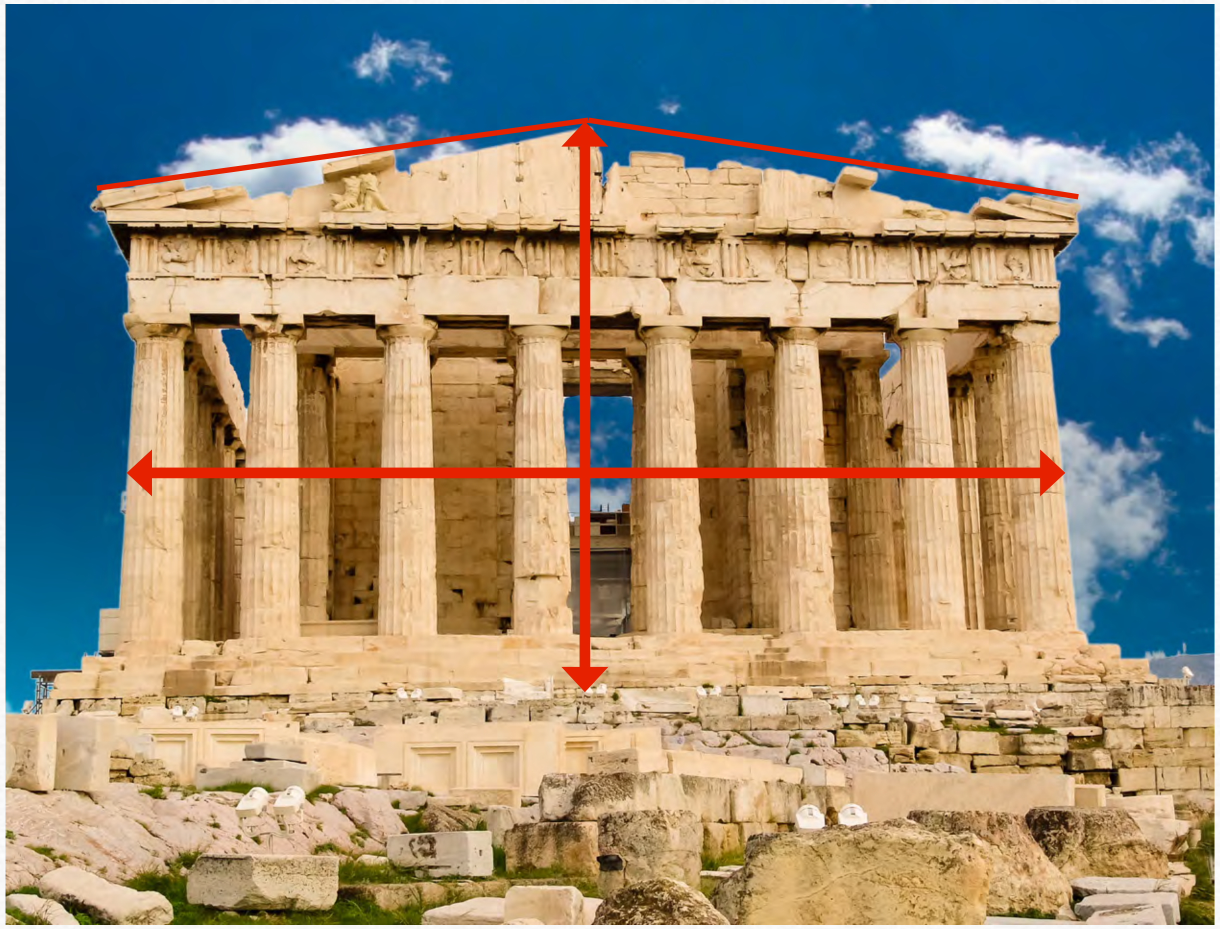
Or as a fraction: ${}^{a+b}/_{a} = {}^{a}/_{b} = \phi$

As you learnt in Step **4**, $\phi \approx 1.6$

It is often stated that many classical and other buildings use the golden ratio in their design. What does this mean?

Example

This is the front façade of the Parthenon in Athens. Show that the facade has been designed using the golden ratio.



First you need to measure the height and width of the façade, having first repaired the triangular pediment.

Because we are going to check the ratio it does not matter what the actual measurements are, it is the proportion of one to the other that is important.

Measuring this photograph:

Width = 7.25 cm

Height = 4.5

$^{Width}/_{Height} = {}^{7.25}/_{4.5} = 1.6111\ldots$

Width divided by height ≈ 1.6 or ϕ

Answer: The façade was designed using the golden ratio

Food for Thought

It is not completely clear if the original Greek architect of the Parthenon used the golden ratio in the design. It was built mostly using curves rather than straight lines to correct the optional illusion that would have been caused by straight lines, a remarkable feat of engineering. The result is visually very pleasing, and that is the philosophy behind the golden ratio – it creates a beautifully balanced image.

How might a designer use this in practice.

It is perfectly acceptable to calculate measurements taking the Golden Ratio, $1 : \phi$ to one decimal place, that is:

$1 : 1.6$

You may notice that if you multiply both parts of the ratio $1 : 1.6$ by 5

then you get the ratio $5 : 8$

Which can be a useful ratio to use for some calculations using the Golden Ratio

Example

Cate has a noticeboard to hang in her dog-grooming salon that she wants to trim so that the ratio of width to height will be in the Golden Ratio.

The noticeboard is 50 cm high by 35 cm wide.

How should Cate trim her noticeboard?

Answer: Cate can trim her noticeboard so that it is 48 cm high by 30 cm wide

Exercises

22 The ancient Greeks did not have calculators. They would make a table of values and read from that. Make a table to calculate dimensions in the golden ratio taking $a/b = 8/5 = 1.6$. Here it is started for you, fill in the missing values. Note the fourth column, which is a and b added together, can be useful when placing an item on a shelf, for example.

a	b	a ÷ b	a + b
1	0.625	1.6	1.625
1.6	1	1.6	2.6
2	1.25	1.6	
3.2	2	1.6	
3		1.6	
	3	1.6	
4		1.6	
	4	1.6	
5			
	5		

Fill in the missing values and then use your table to answer these next two questions.

23 Elisabeth is designing an arch for a theatre.

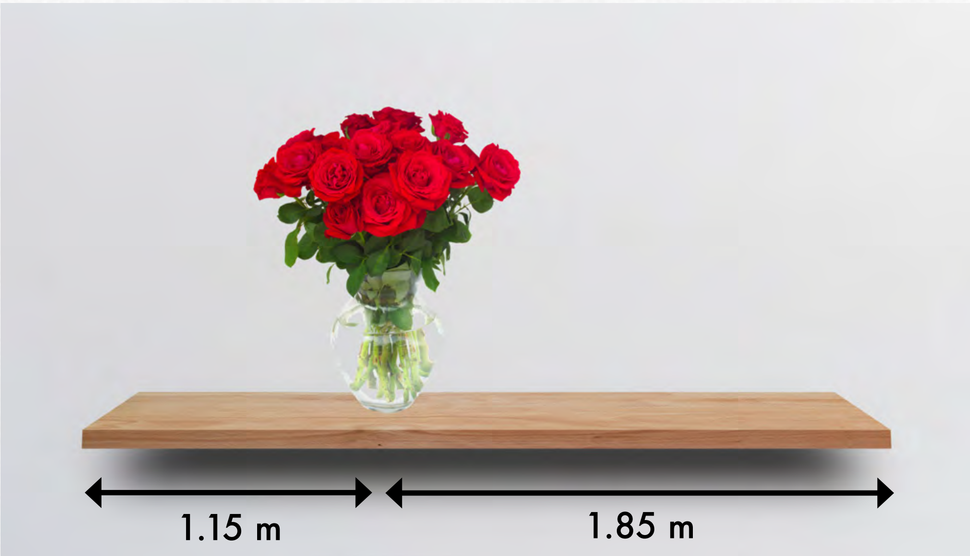
If the arch is **20 m** wide, what should the height be if the height to width ratio is to be $1 : \phi$?

24 Fay keeps the empty bottle of champagne, that her boyfriend gave her on Valentine's day, on top of the cupboard in her bedroom. If the cupboard is **5.2 m** wide, where should she place the bottle so it divides the width in the golden ratio?

Sometimes the dimensions are not exact and then you will have to use your maths skills to calculate.

Example

Phoebe has a shelf **3 m** long. She wants to place a vase of flowers on it. Phoebe places her vase like this:



Ellie tells her that this is an example of the golden ratio. Is Ellie correct?

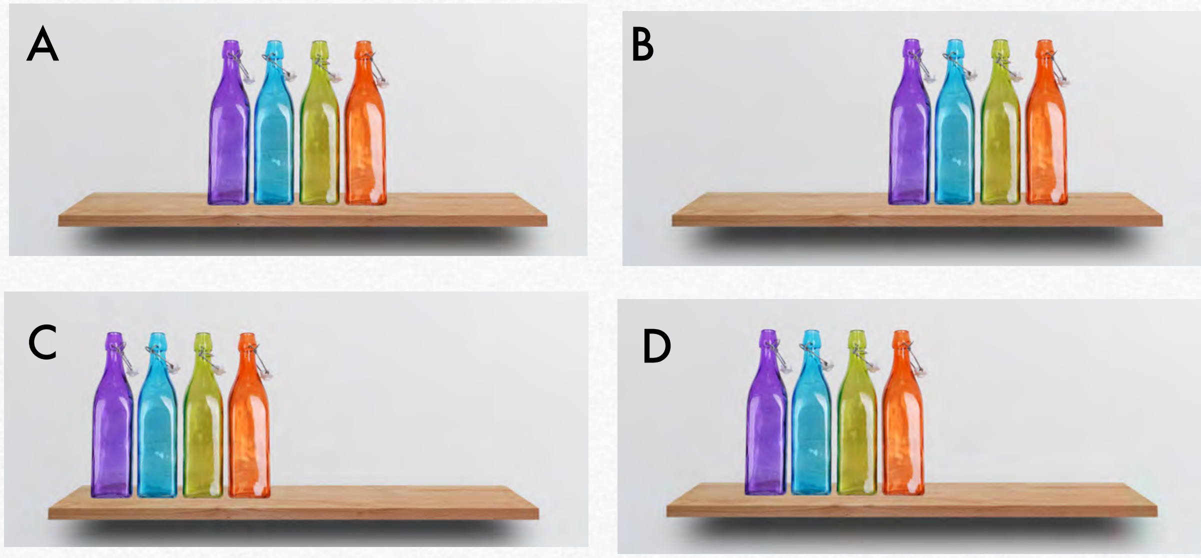
Phoebe checks by dividing **1.85** by **1.15**:

$$1.85 \div 1.15 = 1.6086\ldots$$
$$\approx 1.6$$

Answer: Yes, the vase is placed on the shelf so that the ratio of the space to the right : the space to the left is $\phi : 1$

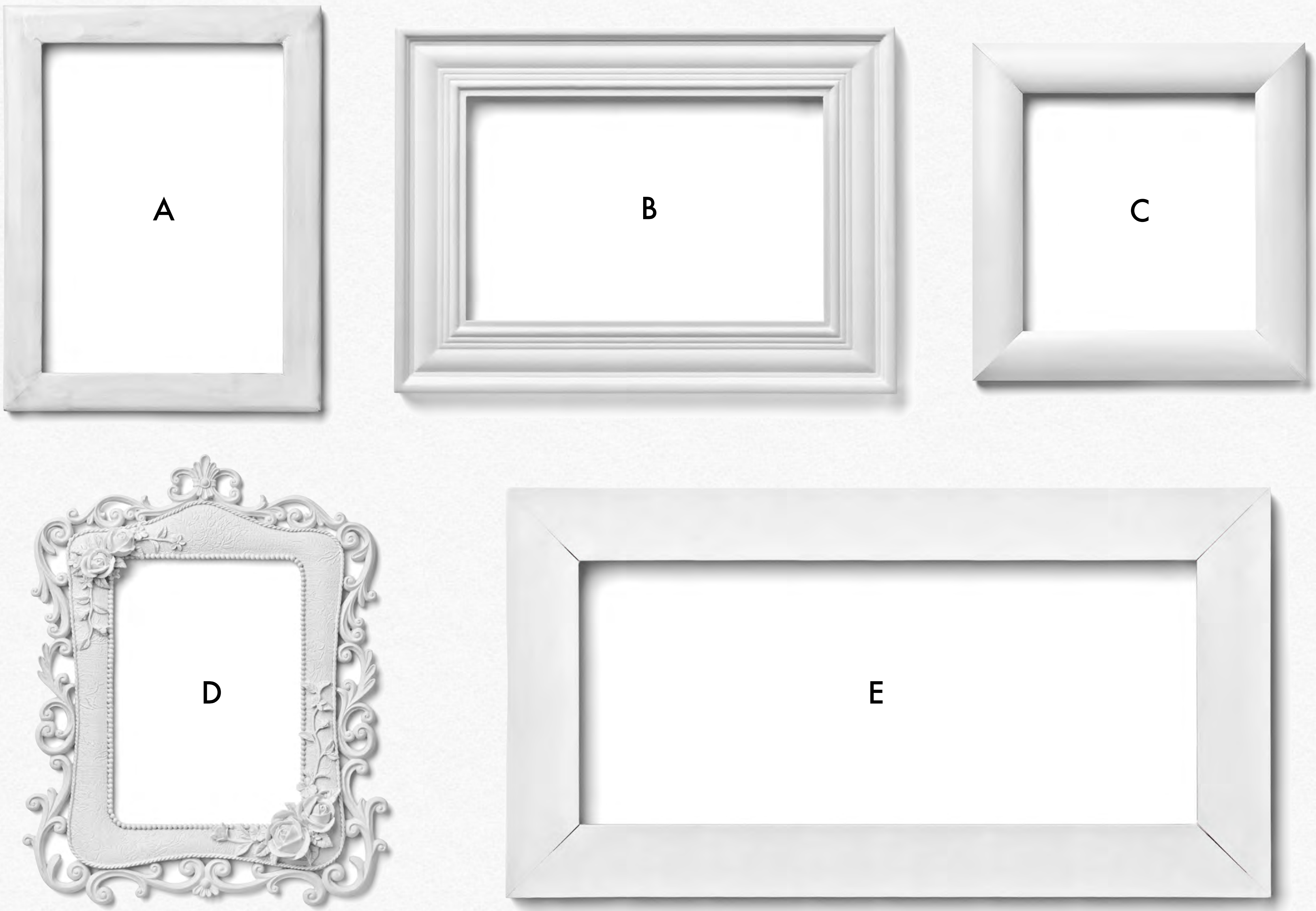
Exercises

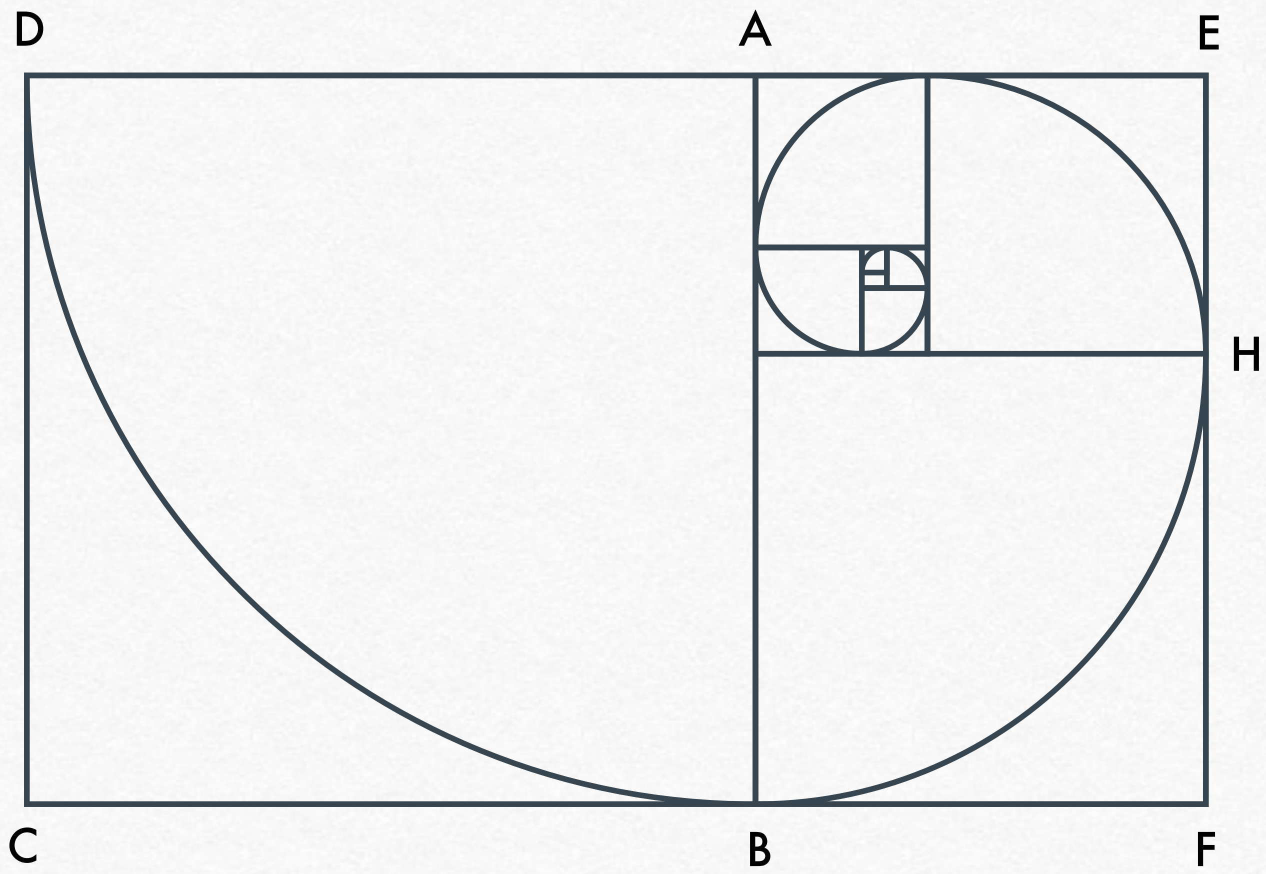
25 Phoebe is experimenting with how to arrange her collection of coloured glass on her mantelpiece. She has narrowed it down to 4 options, A, B, C or D:



In which of her options are the bottles placed so they split the shelf in the golden ratio?

26 Phoebe is framing some photographs. Which of these will frame a photograph that is in the golden ratio?

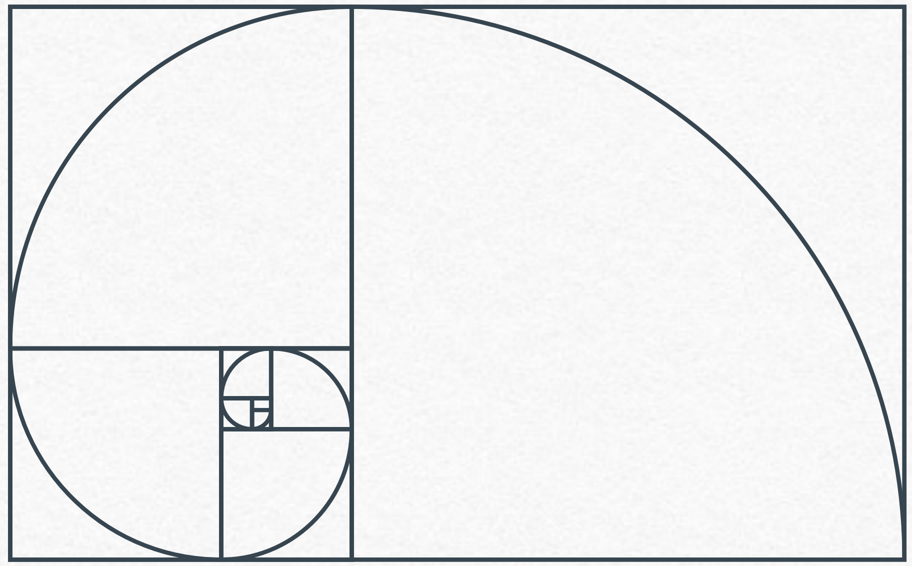




Do you remember the golden spiral that you met in Step **4**?

If you construct this on a computer and make the squares transparent then you can use this as a template to check if designs are in the golden ratio.

You may need to rotate your spiral:



and then move it above your picture or design:

The golden ratio is so called because of the perfect balance that results in a beautiful image.

Food for Thought

The business woman and designer Catherine Isabel Audrey Kidston was born in 1958. She is better known by the name of her design company Cath Kidston. Cath moved to London when she was 18 to work for an interior designer. She opened her own shop near Notting Hill in 1993. Here she sold vintage furniture and furnishings that she had found in car boot sales as well as products that she designed herself. She is particularly known for her vintage style floral patterns. Twenty years later she had almost 200 stores including four in China. She was awarded the MBE in 2009 for services to business.

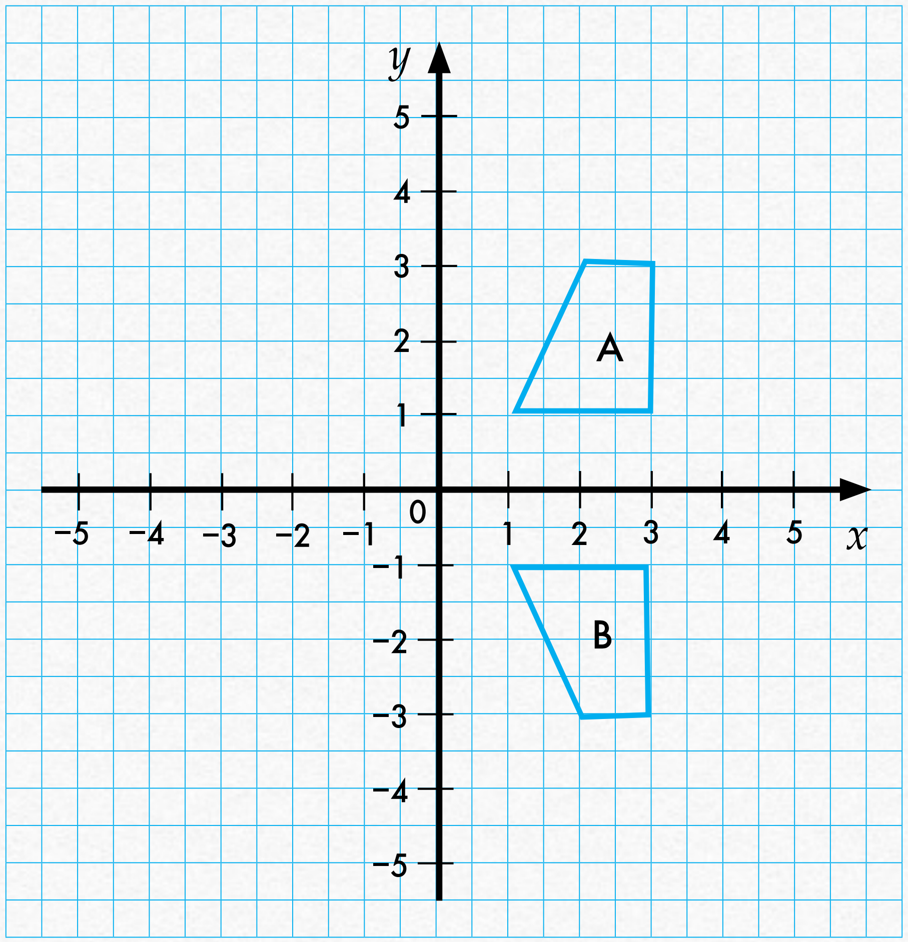
In her 2011 interview for Desert Island Discs, Cath said that despite the company's success hers is still 'a Marmite brand. People either love it or hate it.'

Answers to Part 1

Copy the grid below and then draw the shapes described:

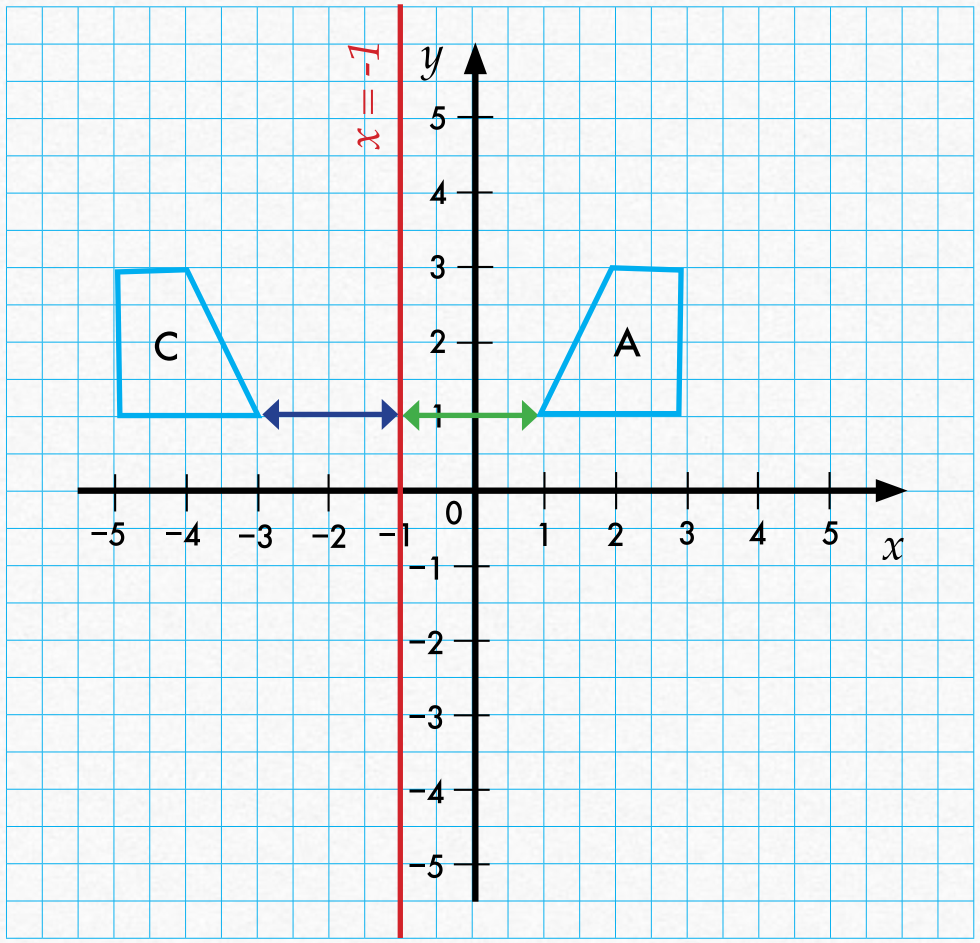
1 Draw B, the image of A after a reflection in the x–axis

This is a simple reflection:



2 Draw C, the image of A after a reflection in the line $x = -1$

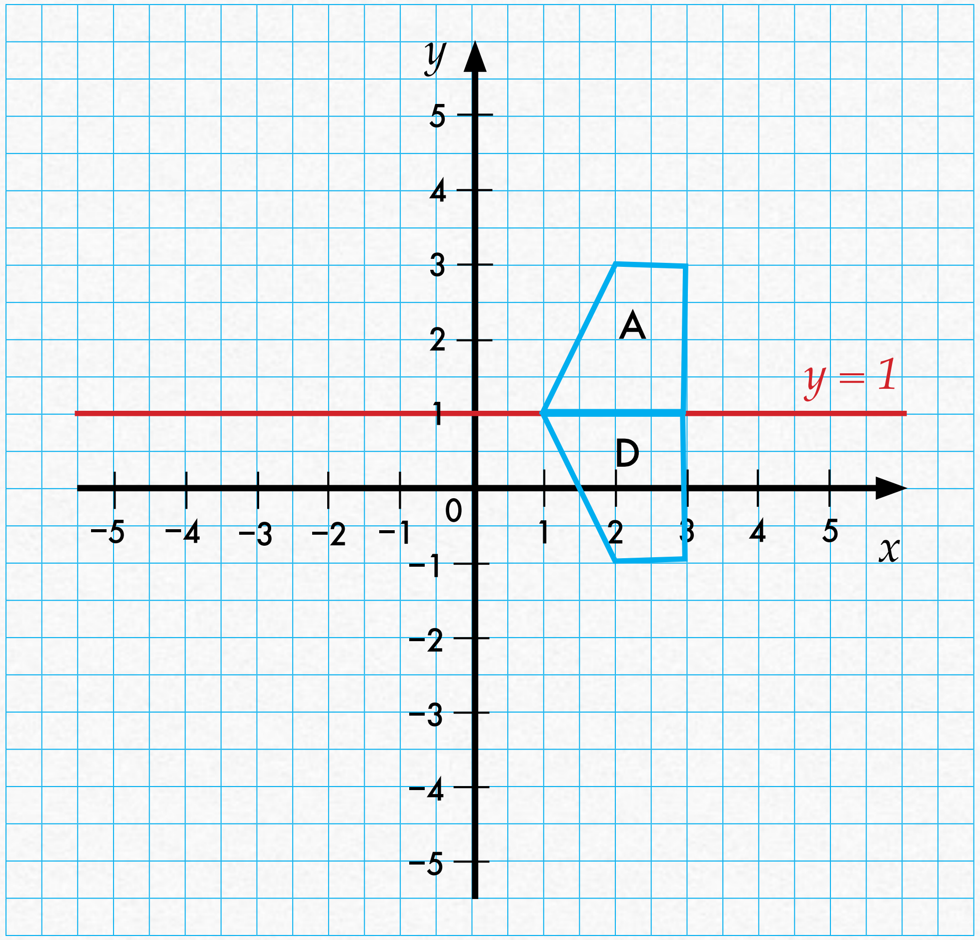
First draw the line **x = –1**, the vertical line that passes through **–1** on the x-axis. Then draw the reflection. You can see the corresponding points are both **2** units away from the line of reflection:



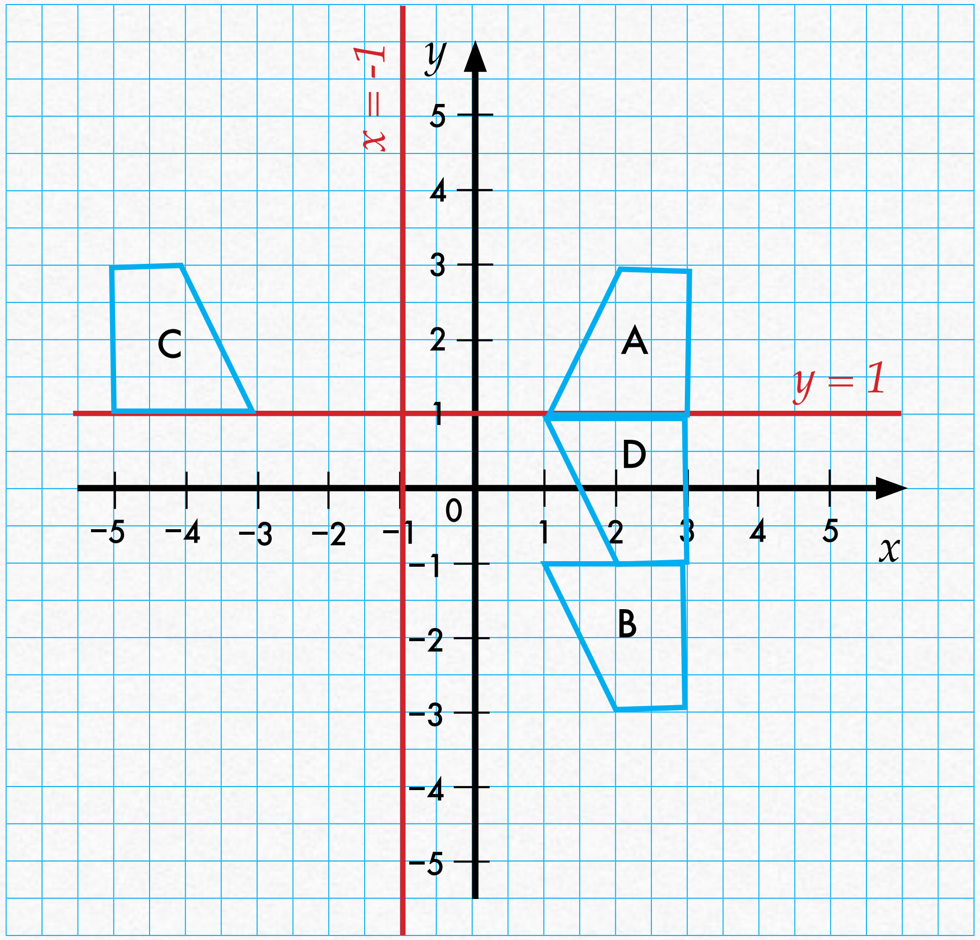
3 Draw D, the image of A after a reflection in the line $y = 1$

First draw the line $y = 1$.

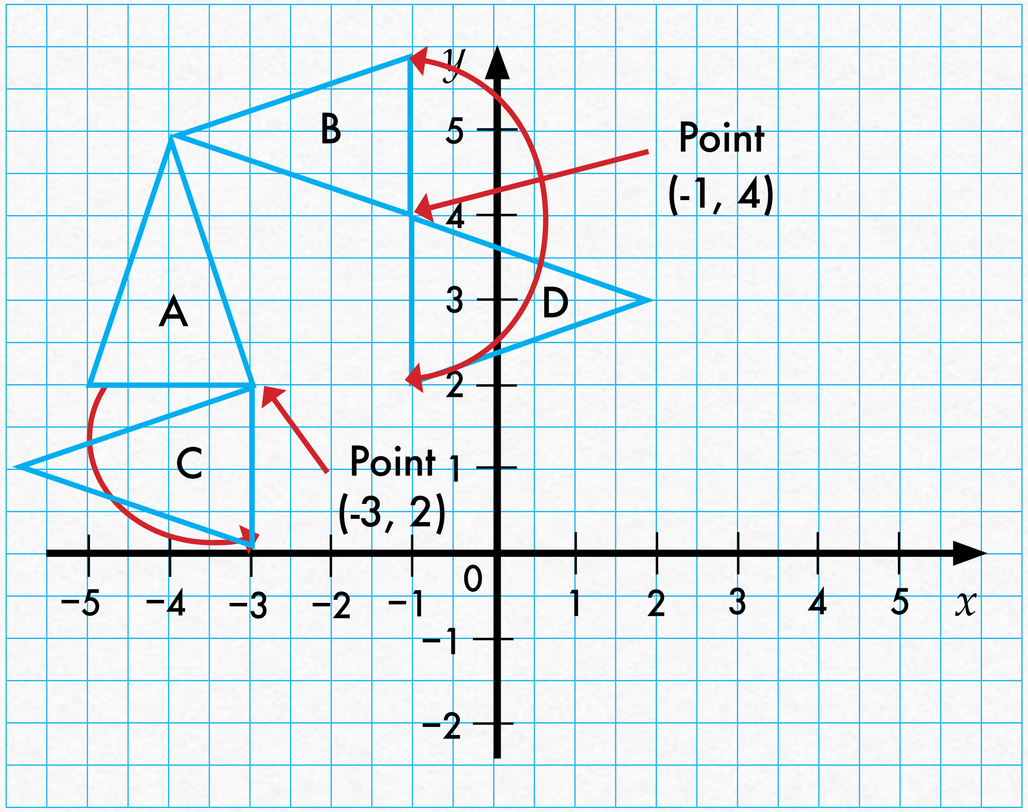
You can see that A lies on the line $y = 1$ and therefore you will reflect the shape about the baseline of Shape A:



Your grid with all three reflections on will look like this:



4 If B is the image of A after a rotation of 90° anticlockwise about the point (−4, 5) describe the image (i) C and (ii) D



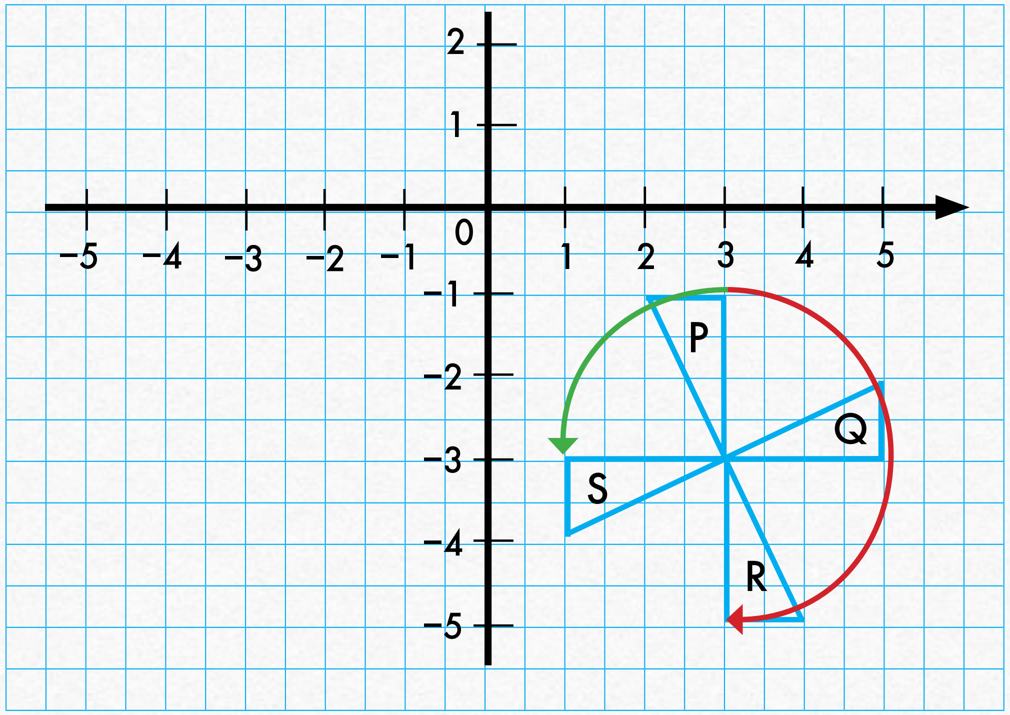
The curved red arrows show the direction and angle of rotation:

Answers:

(i) C is the image of A after a rotation of 90° anticlockwise about the point (−3, 2)

(ii) D is the image of B after a rotation of 180° about the point (−1, 4)

5 If Q is the image of P after a rotation of 90° clockwise about the point (3, −3) describe the rotations of P to give images (i) R and (ii) S



Answers:

(i) R is the image of P after a rotation of 180° about the point (3, −3)

(ii) S is the image of P after a rotation of 90° anticlockwise about the point (3, −3)

or S is the image of P after a rotation of 270° clockwise about the point (3, -3)

6 Copy the coordinate grid opposite and triangle A. Now design a pattern using various rotations of triangle A. (You could use another shape of your own choice if you wish)

When you are pleased with your final pattern, write down the instructions. Pass your instructions to a friend and ask them to follow them.

Has your friend drawn the same pattern that you designed?

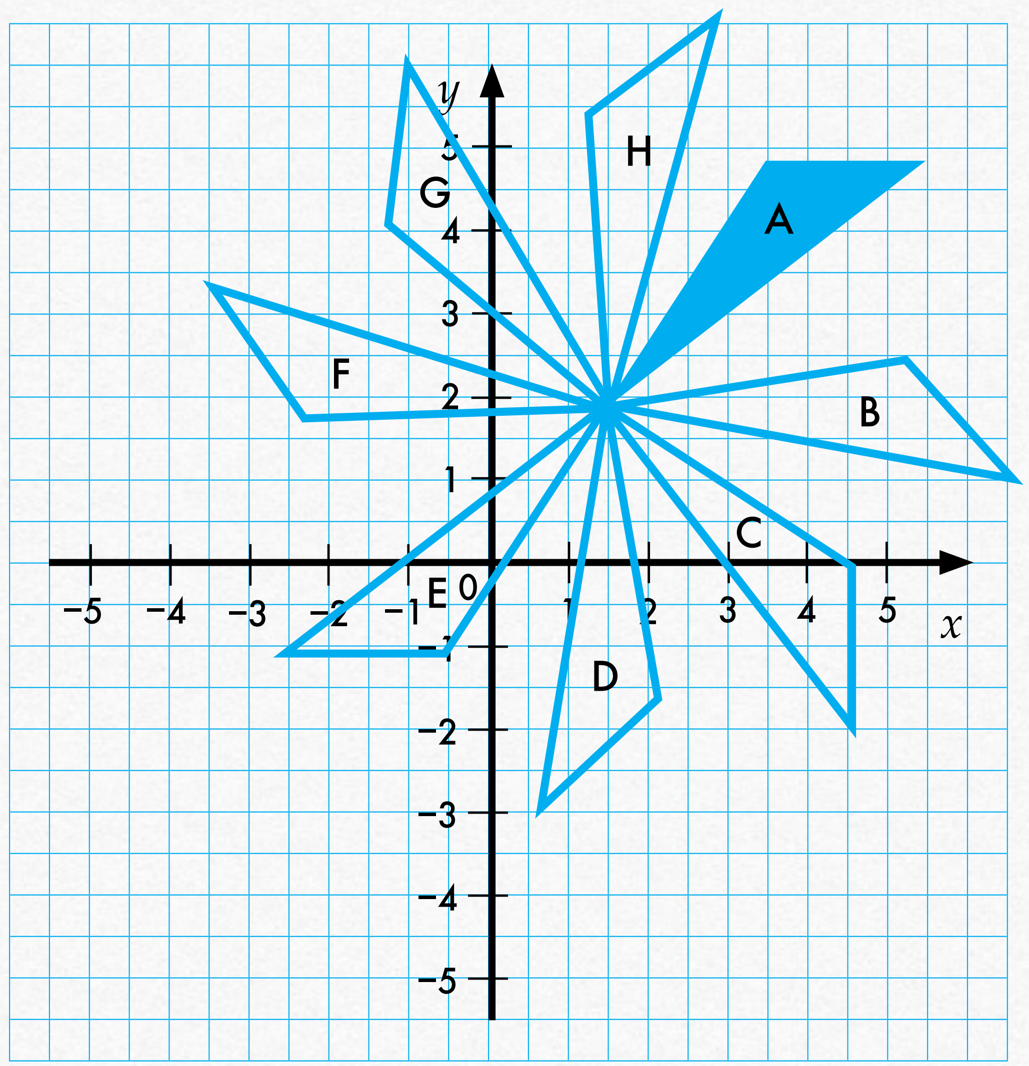
This an example of what you could design:

B is the image of A after a rotation of 45° clockwise about the point (1.5, 2)

C is the image of B after a rotation of 45° clockwise about the point (1.5, 2)

And so on until

H is the image of G after a rotation of 45° clockwise about the point (1.5, 2)



These instructions could be written differently, for example:

C is the image of A after a rotation of 90° clockwise about the point (1.5, 2)

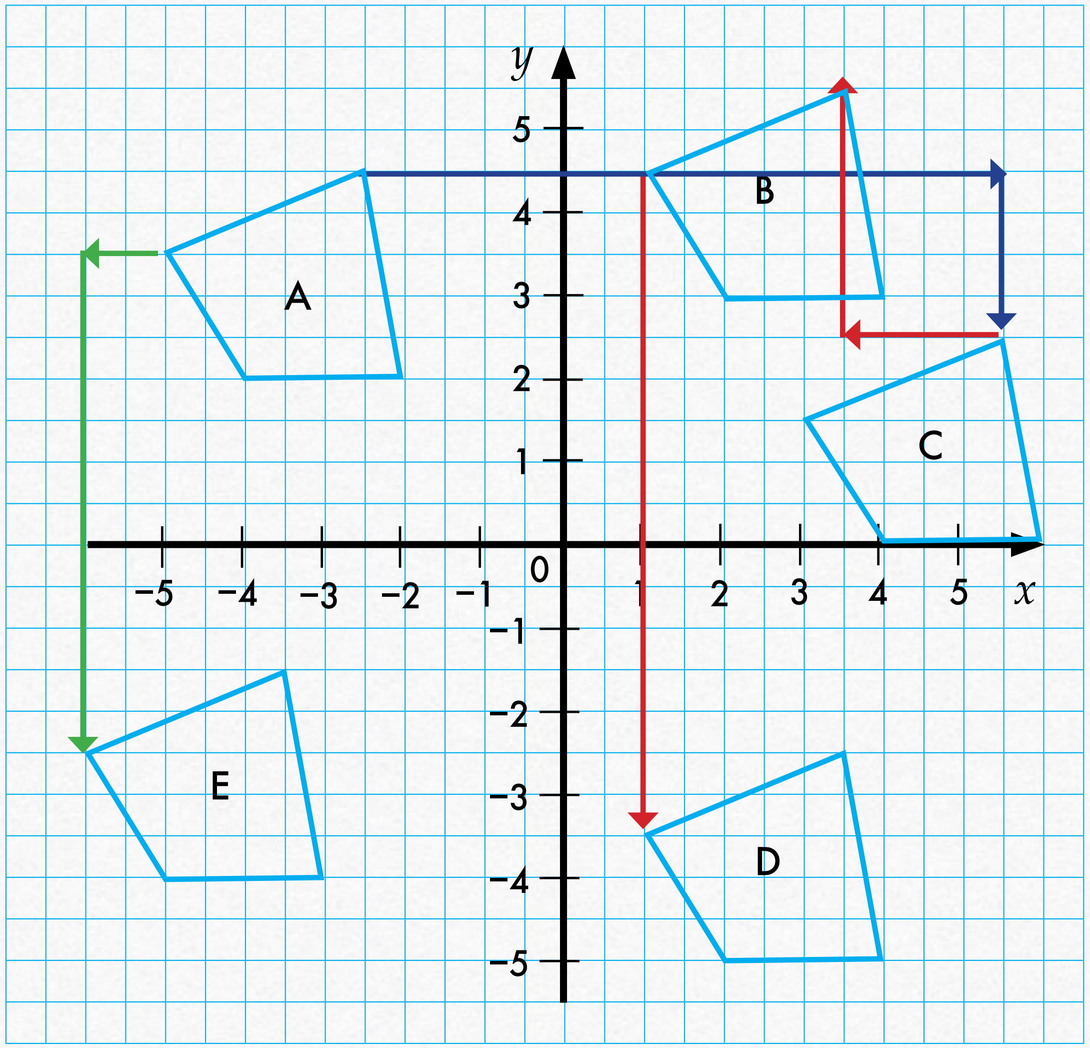
The important thing is that your friend can draw the correct design from your instructions.

The vanes on the Dutch windmill rotate as the windmill is reflected in the waters of Kralingse Plas in the Netherlands.

Q7-10

Describe these translations:

See how the arrows are drawn that show you how to describe the translations. Choose one corner of the starting shape and draw horizontal and vertical arrows to the corresponding corner of the translated shape.



7 From A to C

Answer: C is the image of A after a translation of 8 units right and 2 units down

8 From A to E

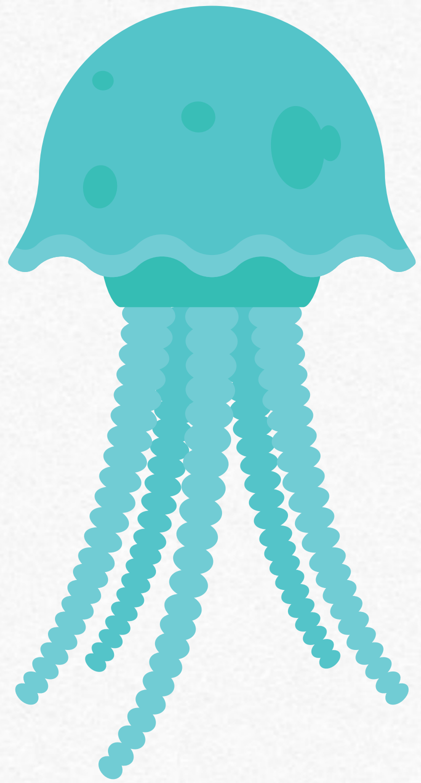
Answer: E is the image of A after a translation of 1 unit left and 6 units down

9 From C to B

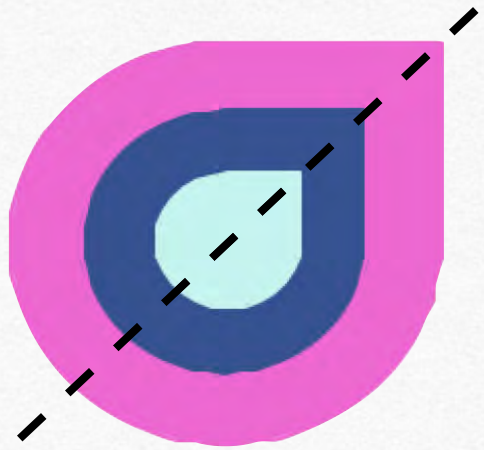
Answer: B is the image of C after a translation of 2 units left and 3 units up

10 From B to D

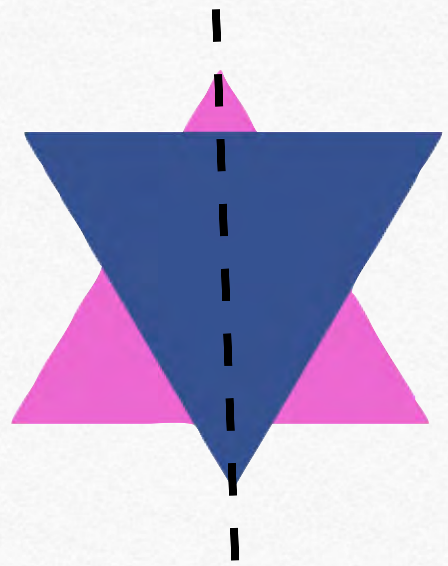
Answer: D is the image of B after a translation of 8 units down



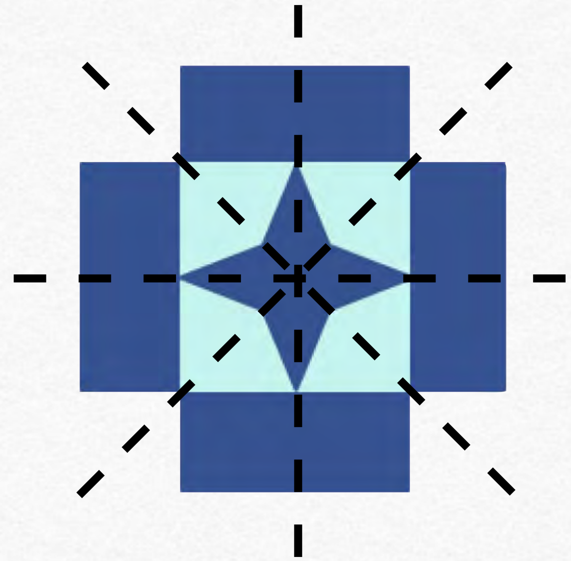
11 On a printout or on the screen, use a mirror or tracing paper to find all the lines of symmetry in these shapes:



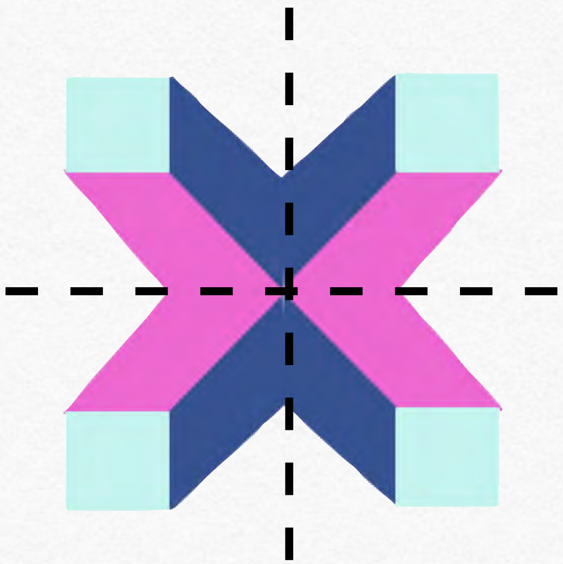
One line of symmetry



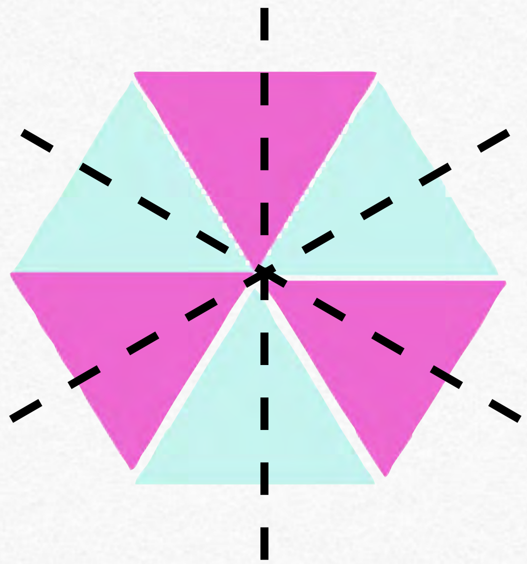
One line of symmetry



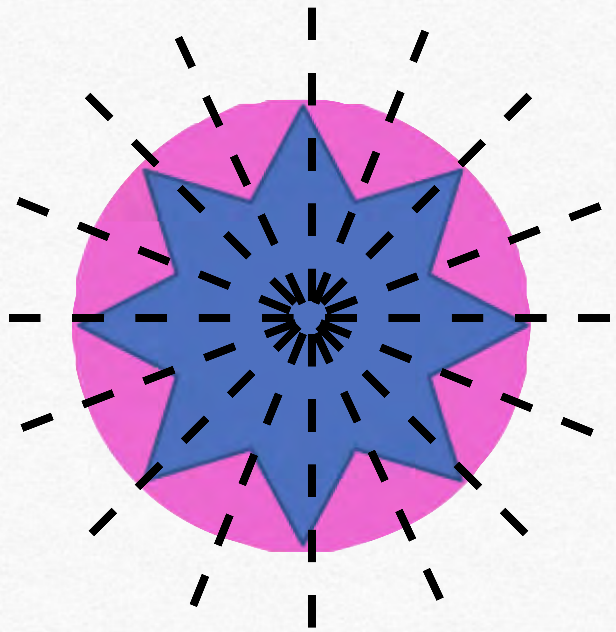
Four lines of symmetry



Two lines of symmetry



Three lines of symmetry

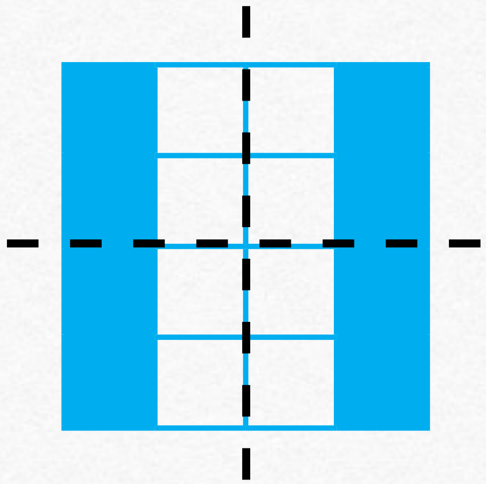


Eight lines of symmetry

Using whichever grid you like, design a pattern to each of these specifications:

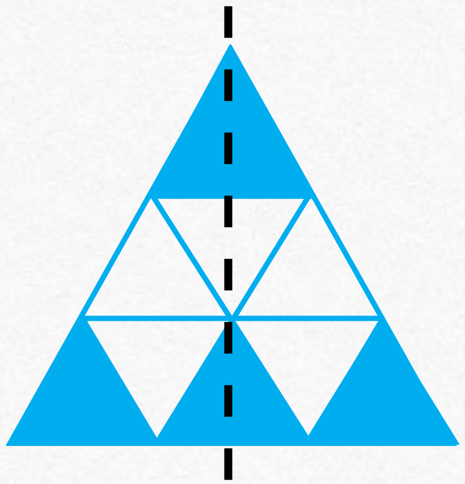
12 A square pattern with two lines of symmetry

A possible design could be:



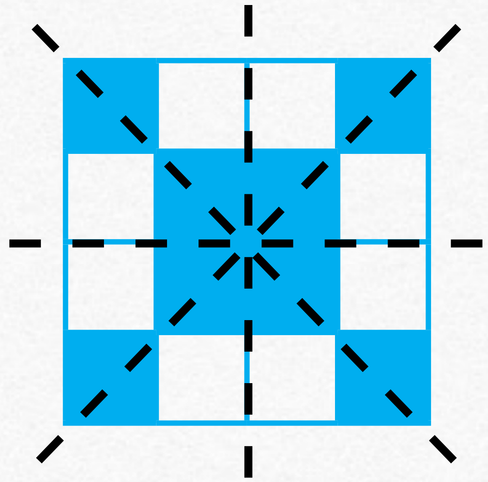
13 A triangle pattern with one line of symmetry

A possible design could be:



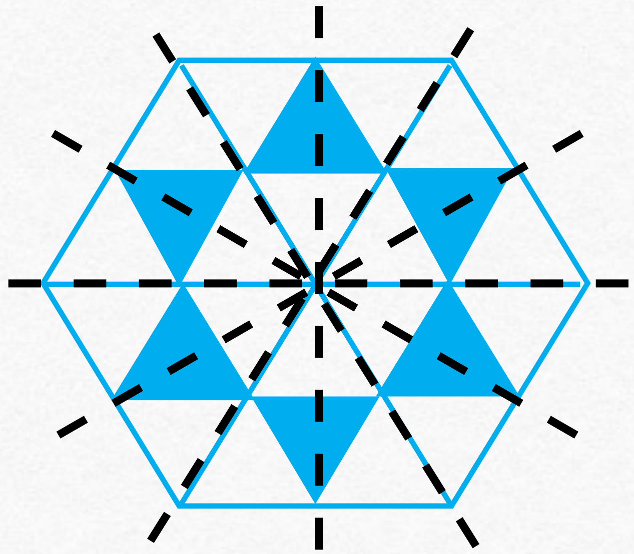
14 A square pattern with four lines of symmetry

A possible design could be:



15 A hexagonal pattern with six lines of symmetry

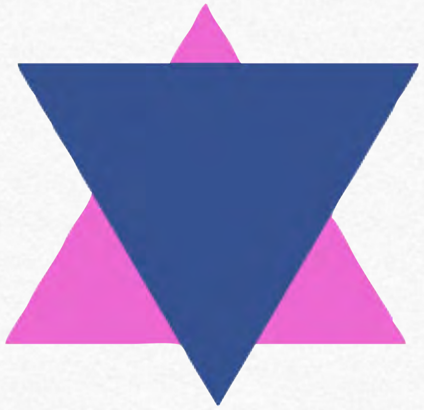
A possible design could be:



16 Do any of the shapes below have rotational symmetry?

Copy any that have rotational symmetry and write down their order of rotational symmetry.

No rotational symmetry



No rotational symmetry

Order **4**

Order **2**

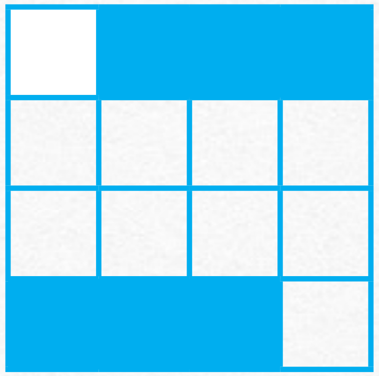
Order **3**

Order **8**

17 Colour each of these shapes so that it has:

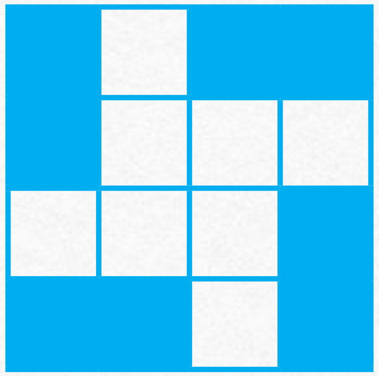
(i) rotational symmetry of order **2** and no line symmetry

A possible design could be:



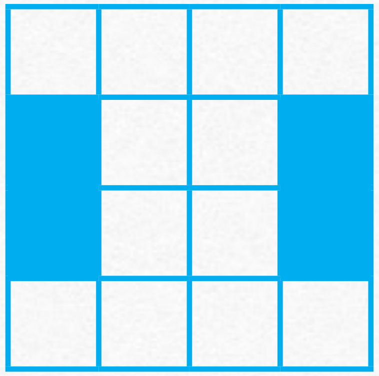
(ii) rotational symmetry of order **4** and no line symmetry

A possible design could be:



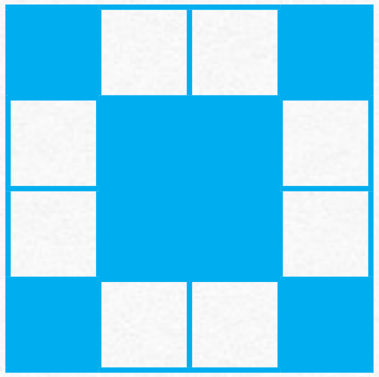
(iii) rotational symmetry of order **2** and **2** lines of symmetry

A possible design could be:



(iv) rotational symmetry of order **4** and **4** lines of symmetry

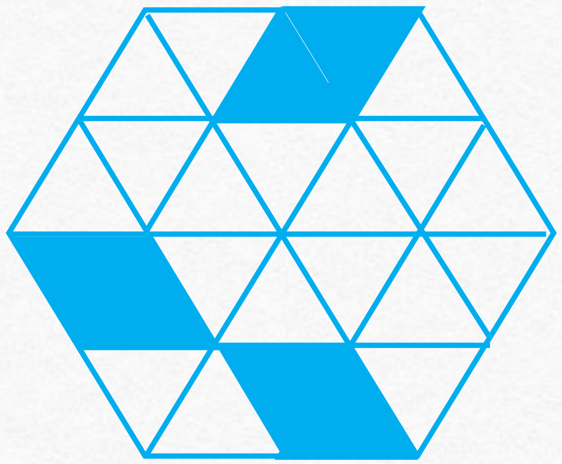
A possible design could be:



18 Colour each of these shapes so that it has:

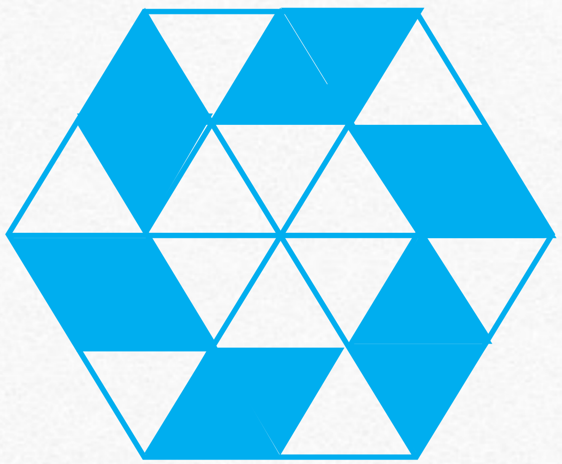
(i) rotational symmetry of order **3** and no line symmetry

A possible design could be:



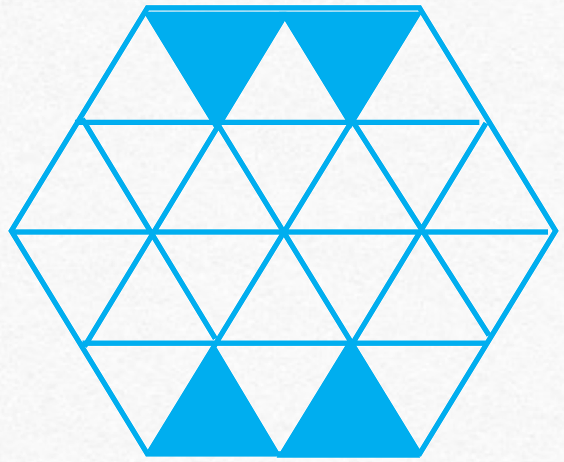
(ii) rotational symmetry of order **6** and no line symmetry

A possible design could be:



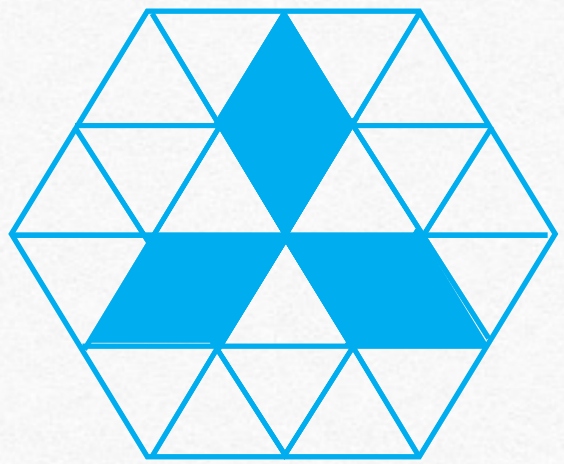
(iii) rotational symmetry of order **2** and **2** lines of symmetry

A possible design could be:



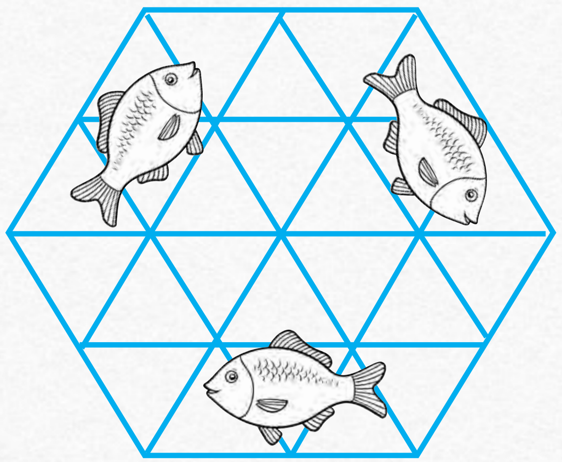
(iv) rotational symmetry of order **3** and **3** lines of symmetry

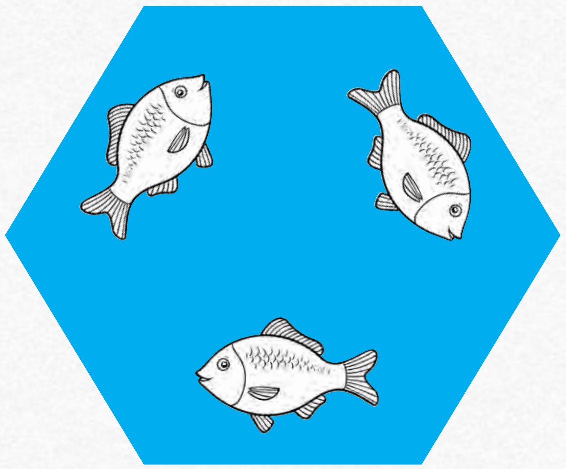
A possible design could be:



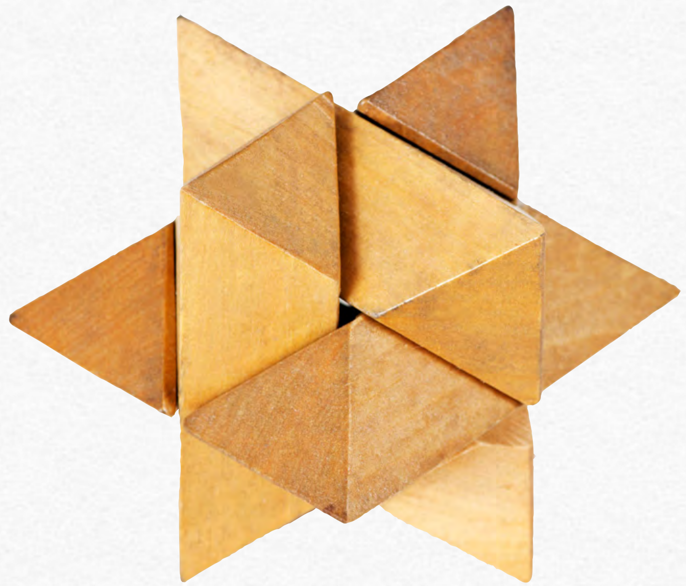
(v) any other ways that you wish, noting the order of rotational symmetry and number of lines of symmetry.

You could also use a grid to reflect or rotate your own sketched images:





Answer: This has rotational symmetry of order 3 but no lines of symmetry.



Food for Thought

Have you noticed any connection between the order of rotational symmetry and the number of lines of symmetry?

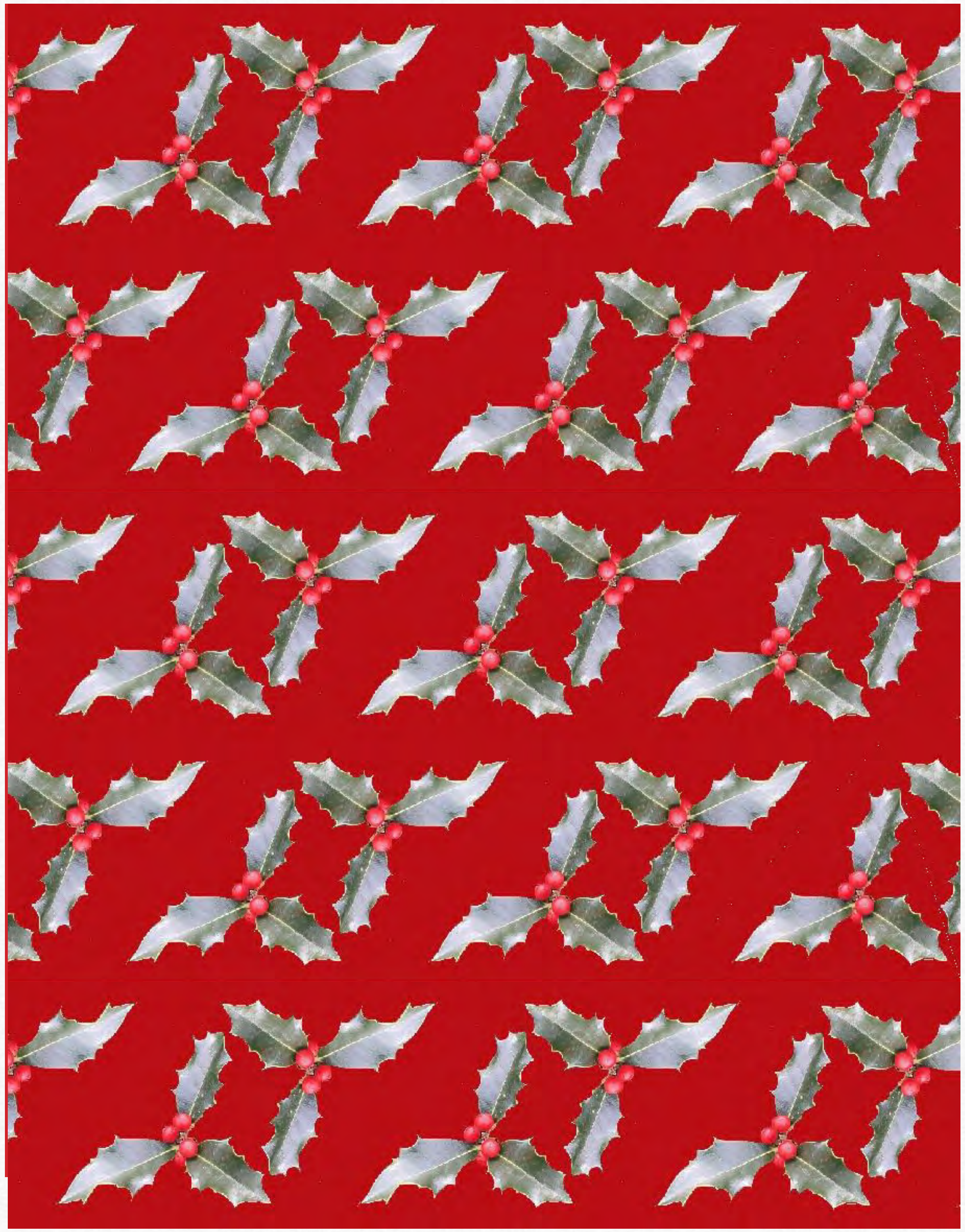
Answer: If a shape has two or more lines of symmetry it has an order of rotational symmetry the same number as the number of lines.

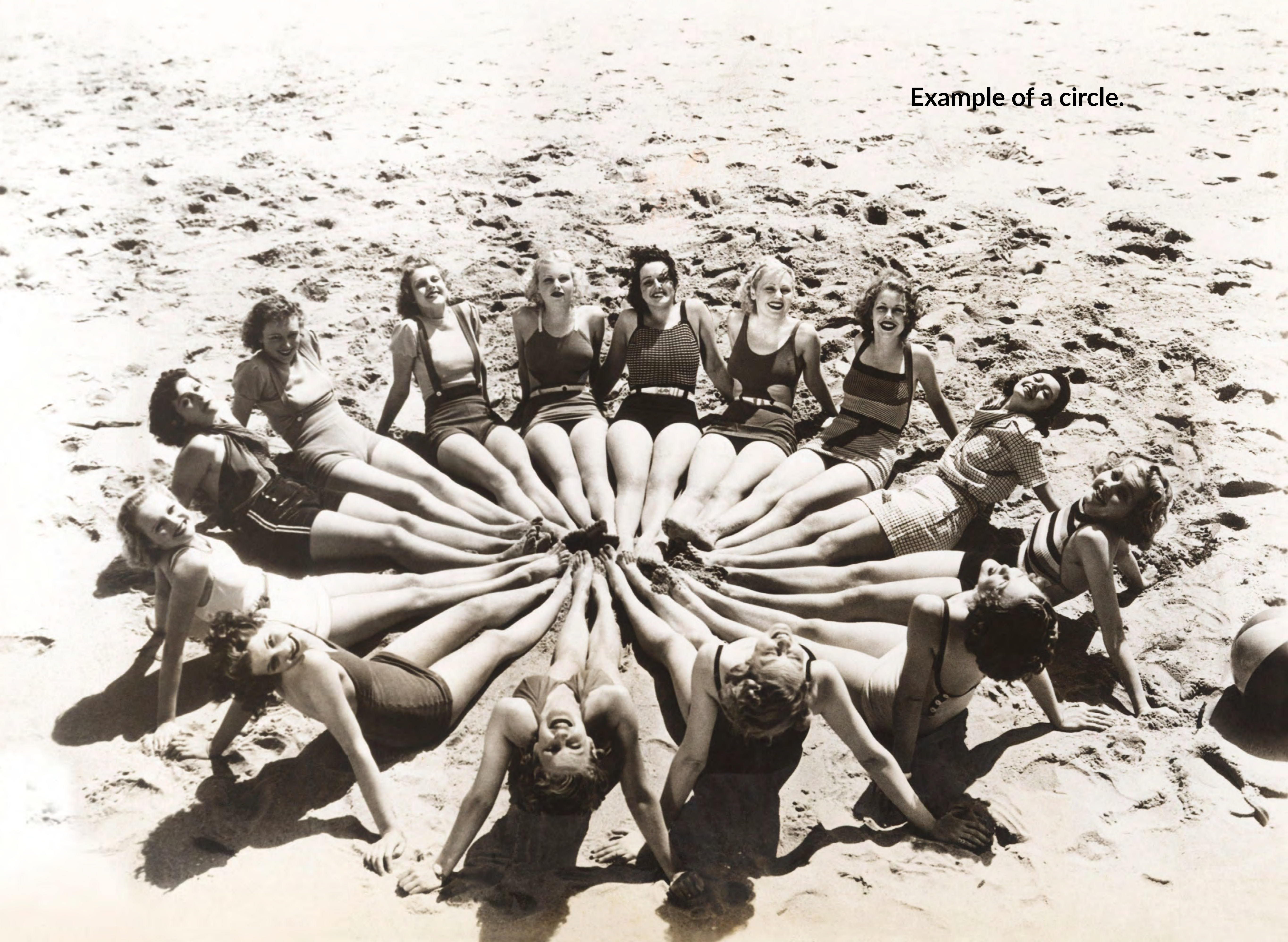
19 Draw a scalene triangle one, which has no equal sides and no equal angles, for yourself and then follow the steps to draw a tessellating pattern.

Here is an example of a possible answer. In this example, a designer has inserted a photograph of a holly leaf into a scalene triangle and then filled around it with a Christmas red:

She then rotates a copy of the triangle and joins them together to make a parallelogram:

She tessellates copies of the parallelogram to make Christmas wrapping paper:



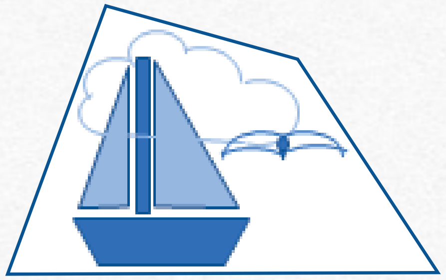


Example of a circle.

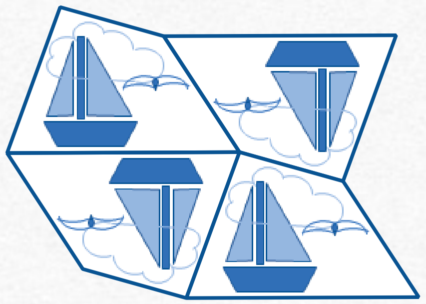
20 Draw a quadrilateral for yourself and then follow the steps to draw a tessellating pattern.

Here's an example of how you could do this:

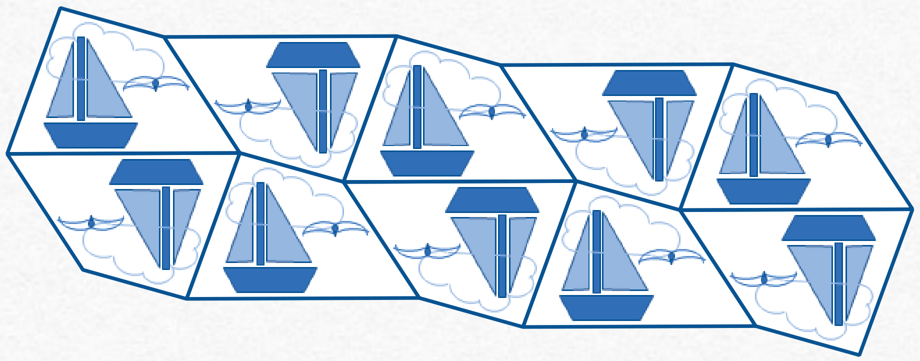
1. Insert a picture or pattern into a quadrilateral



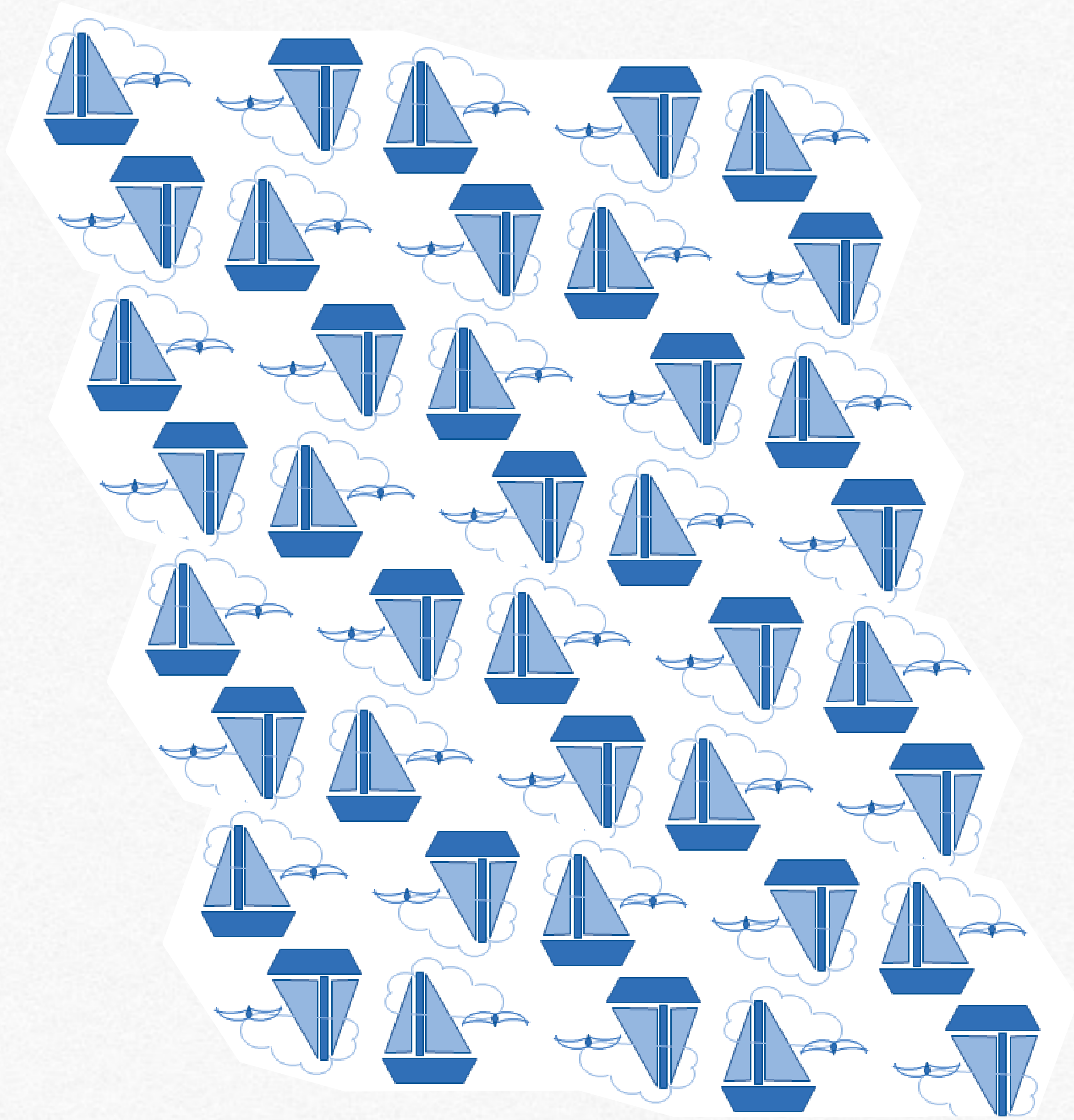
2. Translate, rotate and translate the quadrilateral like this:



3. Tesselate the resulting shape:

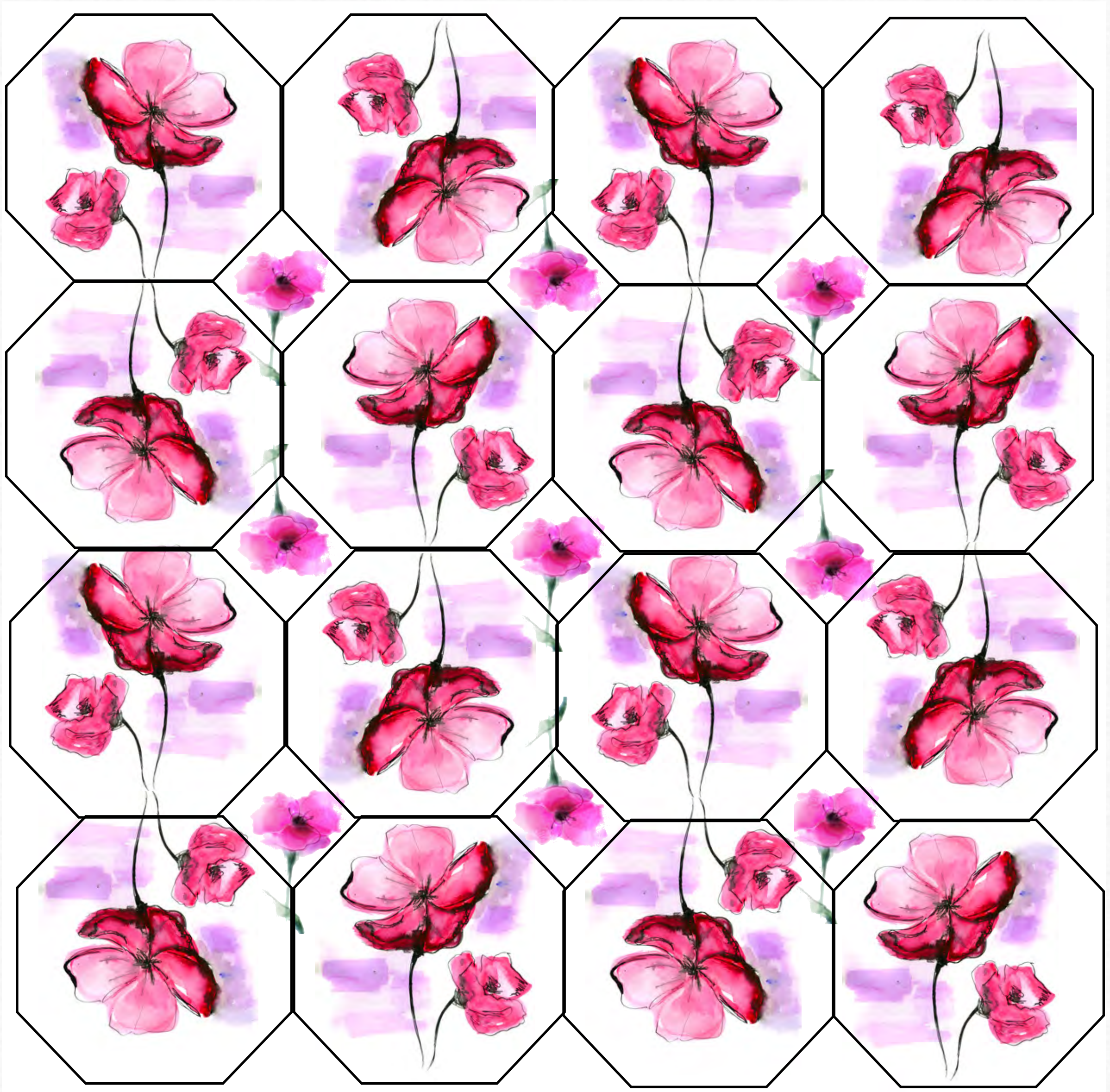


4. Finally, remove the outlines of the quadrilaterals:

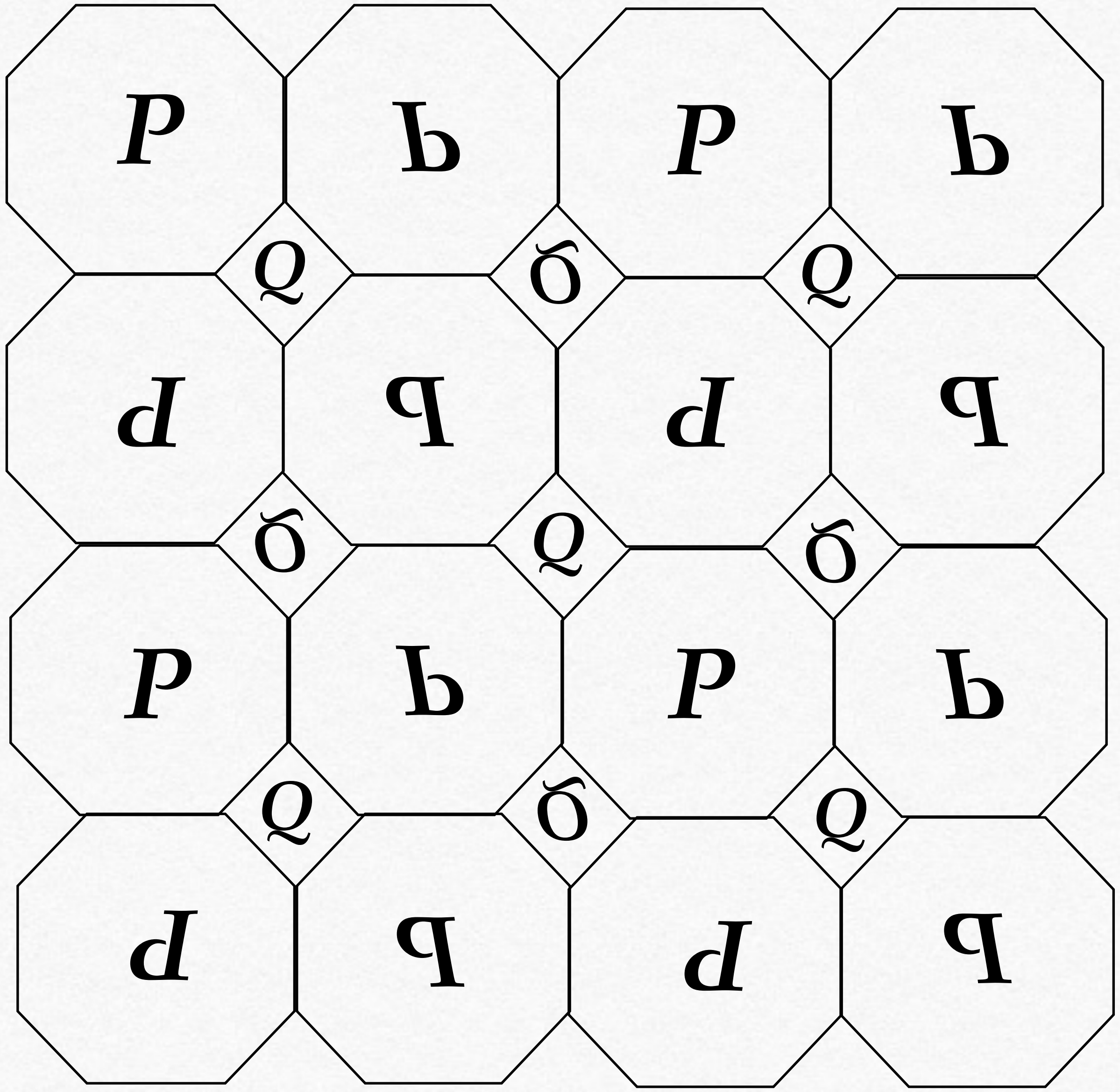


21 See if you can find the reflections, rotations and translations in the floral pattern:

If the borders are put back in the floral pattern you can see it is based on a tessellating pattern of octagons and squares:



Looking at this another way makes the reflections, rotations and translations clearer:



22 The ancient Greeks did not have calculators. They would make a table of values and read from that. Make a table to calculate dimensions in the golden ratio taking $^{a}/_{b} = ^{8}/_{5} = 1.6$. Here it is started for you, fill in the missing values. Note the fourth column, which is *a* and *b* added together, can be useful when placing an item something on a shelf, for example.

Fill in the missing values and then use your table to answer the next two questions.

a	b	a ÷ b	a + b
1	0.625	1.6	1.625
1.6	1	1.6	2.6
2	1.25	1.6	
3.2	2	1.6	
3		1.6	
	3	1.6	
4		1.6	
	4	1.6	
5			
	5		

To work out b in the fifth row: $^{a}/_{b} = ^{3}/_{?} = 1.6$

$$^{3}/_{1.6} = ? = 1.875$$

To work out a in the sixth row: $^{a}/_{b} = ^{?}/_{3} = 1.6$

$$? = 1.6 \times 3$$

$$? = 4.8$$

Do the same for the next rows, but use **4** and **5** instead of **3** in the sums above. When complete your table should look like this:

Answer:

a	b	a ÷ b	a + b
1	0.625	1.6	1.625
1.6	1	1.6	2.6
2	1.25	1.6	3.25
3.2	2	1.6	5.2
3	1.875	1.6	4.875
4.8	3	1.6	7.8
4	2.5	1.6	6.5
6.4	4	1.6	10.4
5	3.125	1.6	8.125
8	5	1.6	13

You do not need to continue the table forever because you can find multiples of these values as in this next question.

23 Elisabeth is designing an arch for a theatre. If the arch is 20 m wide, what should the height be if the height to width ratio is to be $1 : \phi$?

Height to width ratio is $1 : 1.6$

So the width is to be bigger than the height, so width is the a value from the table.

From the table you can see a width to height ratio of $2 : 1.25$ is in the golden ratio,

Scale this up by multiplying by 10:

$20 : 12.5$

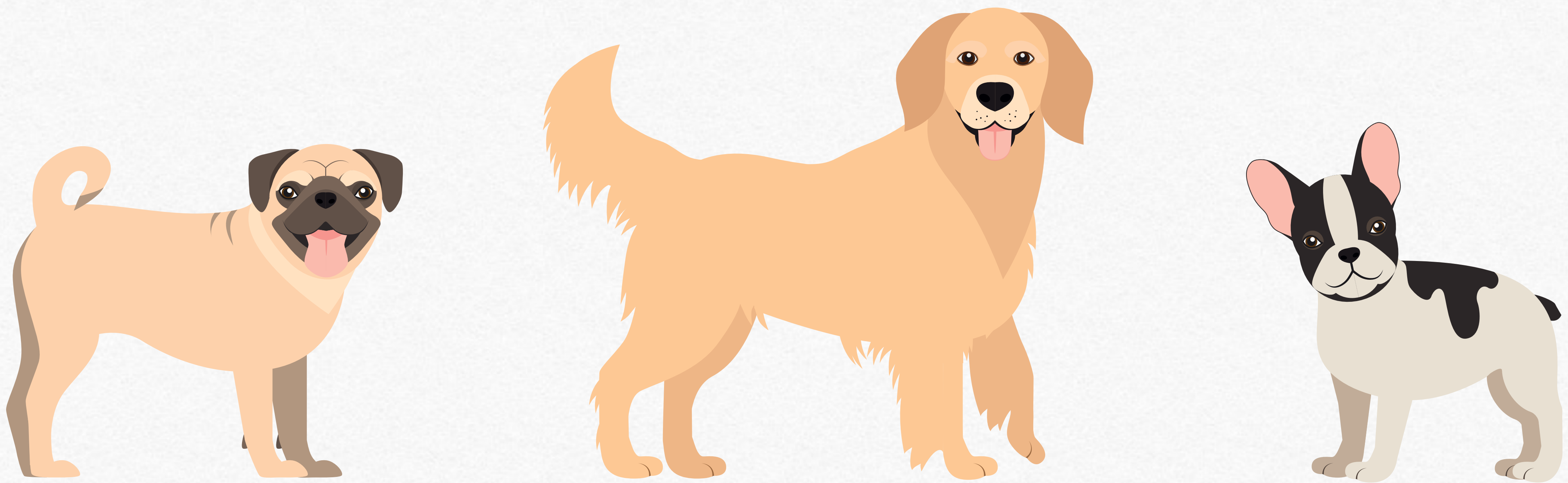
Answer: The height should be 12.5 m

24 Fay keeps the empty bottle of champagne, that her boyfriend gave her on Valentine's day, on top of the cupboard in her bedroom. If the cupboard is 5.2 m wide, where should she place the bottle so it divides the width in the golden ratio?

5.2 m is the length that needs splitting into the two numbers of the golden ratio, so look for 5.2 in the $a + b$ column.

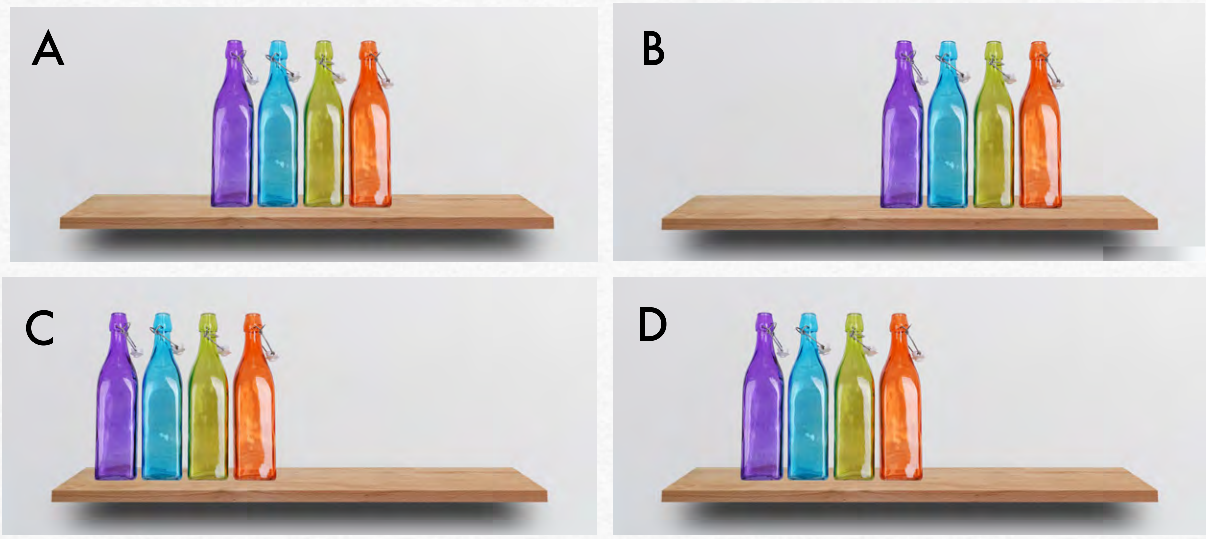
The table shows that 5.2 splits into $3.2 : 2$

Answer: The bottle should be placed 3.2 m from one edge which will then be 2 m from the other.




"Did anybody
mention pattern?"

25 Phoebe is experimenting with how to arrange her collection of coloured glass on her mantelpiece. She has narrowed it down to 4 options, A, B, C or D:



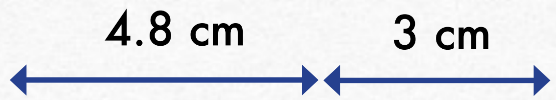
In which of her options are the bottles placed so they split the shelf inthe godlen ratio?

A cannot be in the golden ratio as it is in the middle and C is too close to one end.

As these are scale drawings the exact length of the mantelpiece does not matter, the ratio will be the same as the actual shelf.

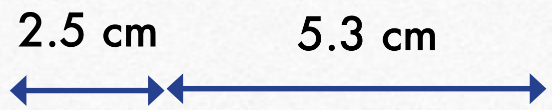
You need to measure B and D. Measure from the edge of the shelf to the middle of the base of the Oscar, then divide the longer length by the shorter:

B



$^{4.8}/_{3} = 1.6$

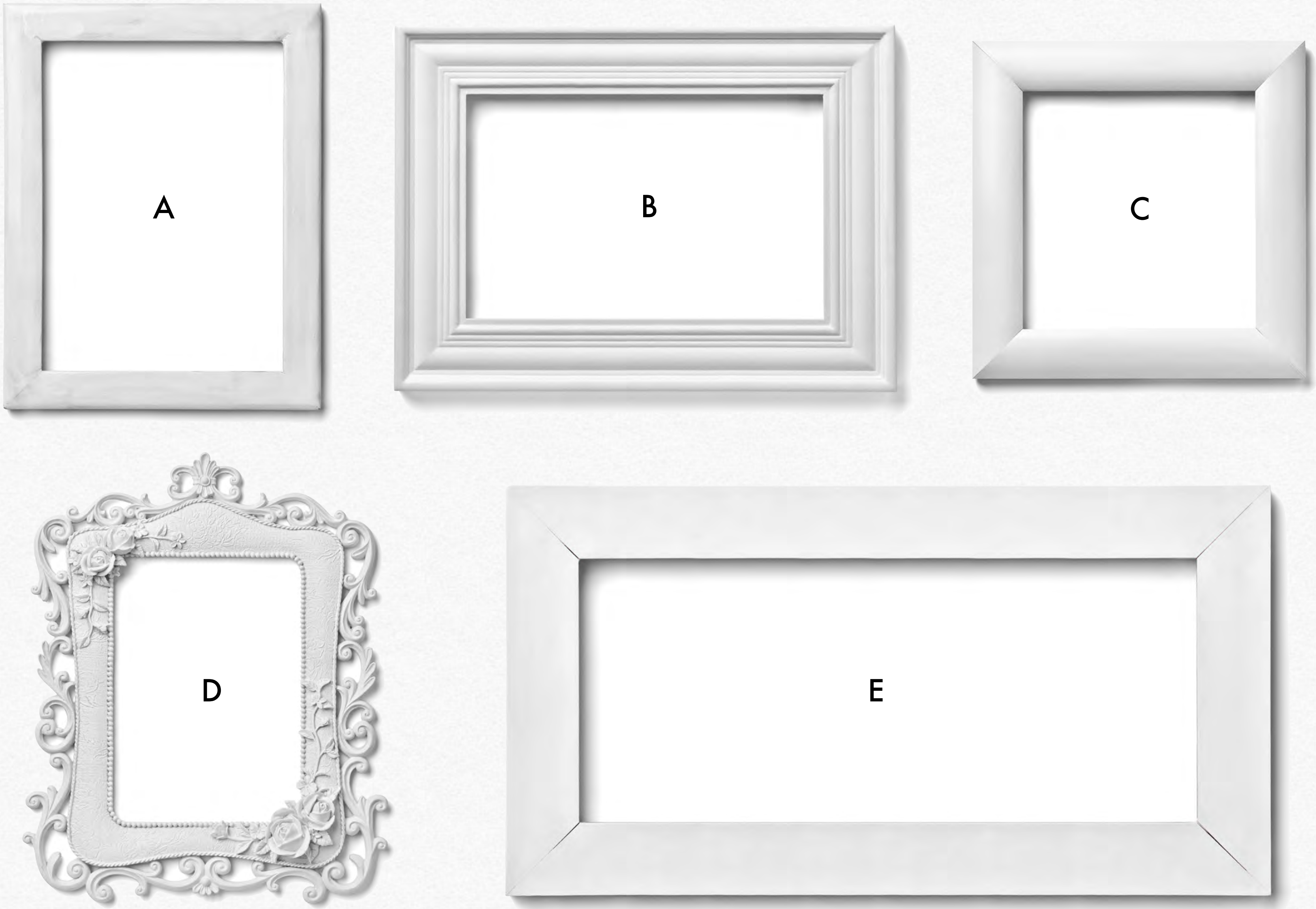
D



$^{5.3}/_{2.5} = 2.12$

Answer: Position B is the one that splits the shelf in the golden ratio.

26 Phoebe is framing some photographs. Which of these will frame a photograph that is in the golden ratio?



You can immediately see that C is almost a square and E is too long and thin. So neither C nor E have the golden ratio.

Measure the inside of frames A, B and D, and work out the length divided by the width, giving your answer to 1 decimal place. Your measurements will be different depending on whether you measure the pictures on the screen or a print out. That shouldn't matter, it is the ratio rather than the exact size that is important:

A measures 3.35 by 2.45: $^{3.4}/_{2.5} = 1.4$

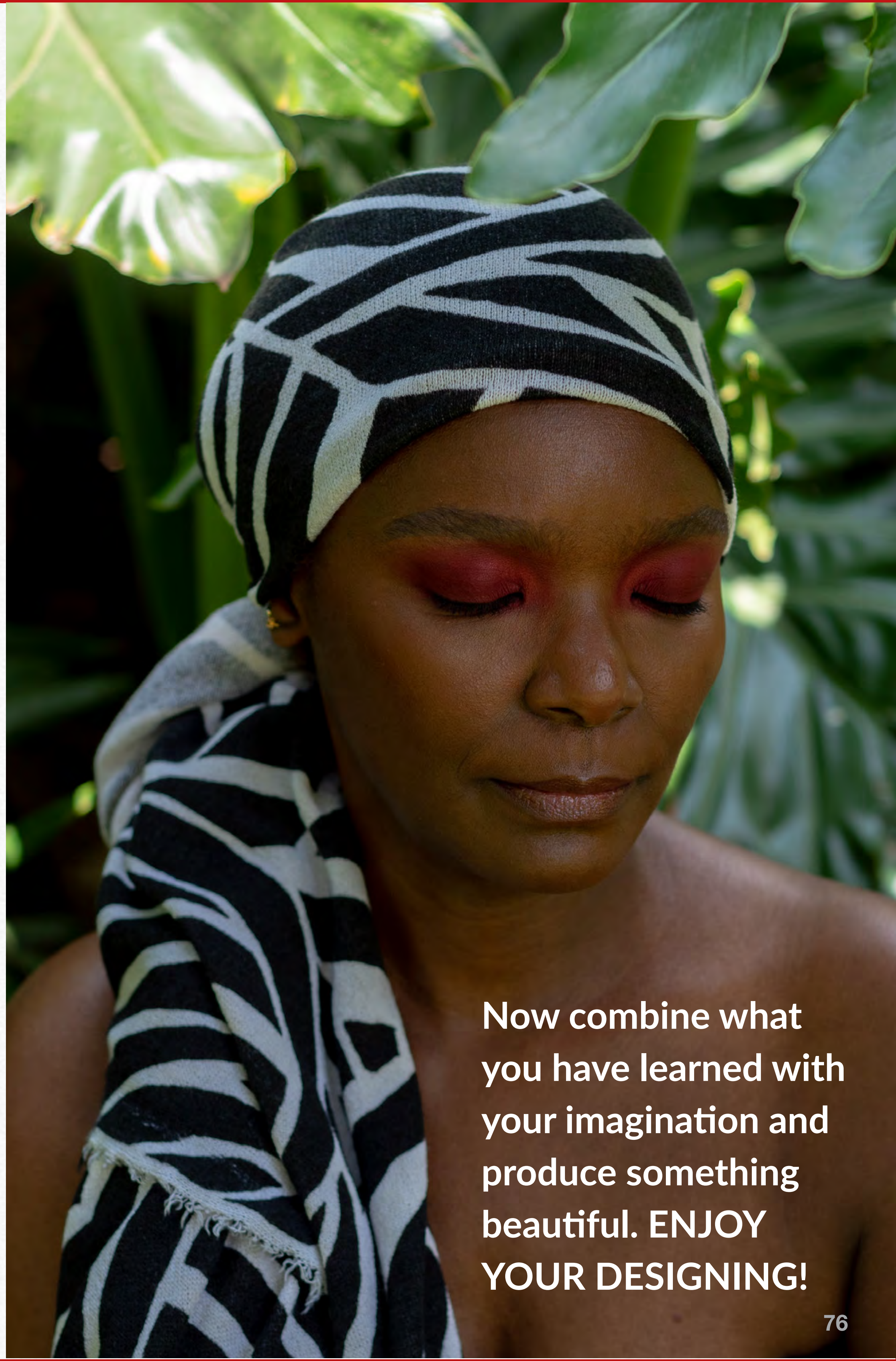
B measures 3.6 by 2.3: $^{3.6}/_{2.3} = 1.6$

C measures 2.85 by 1.9: $^{2.85}/_{1.9} = 1.4$

Answer: Frame B is in the golden ratio.

Food for Thought

You can see it is quite hard to measure these scale drawings, and there could be slight distortion on your screen or from you printer. Whenever possible you should measure the full size drawing or object and then do the division.



YOUR BRAIN WORKOUT

Q1

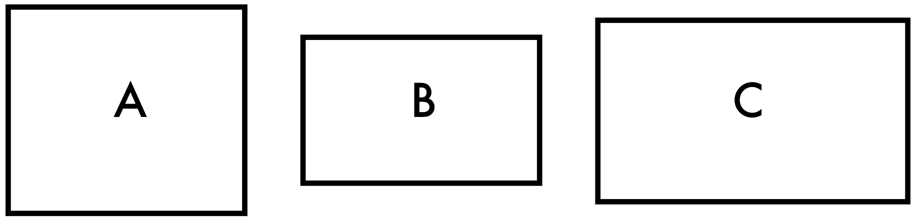
The Golden ratio is when $a + b : a = a : b$

If $b = 1$, what is the approximate value of a?

YOUR BRAIN WORKOUT

Q2

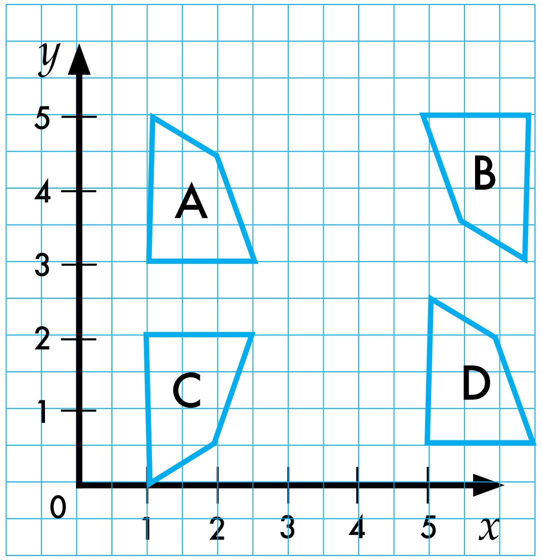
Which of these rectangles has its proportions in the golden ratio?

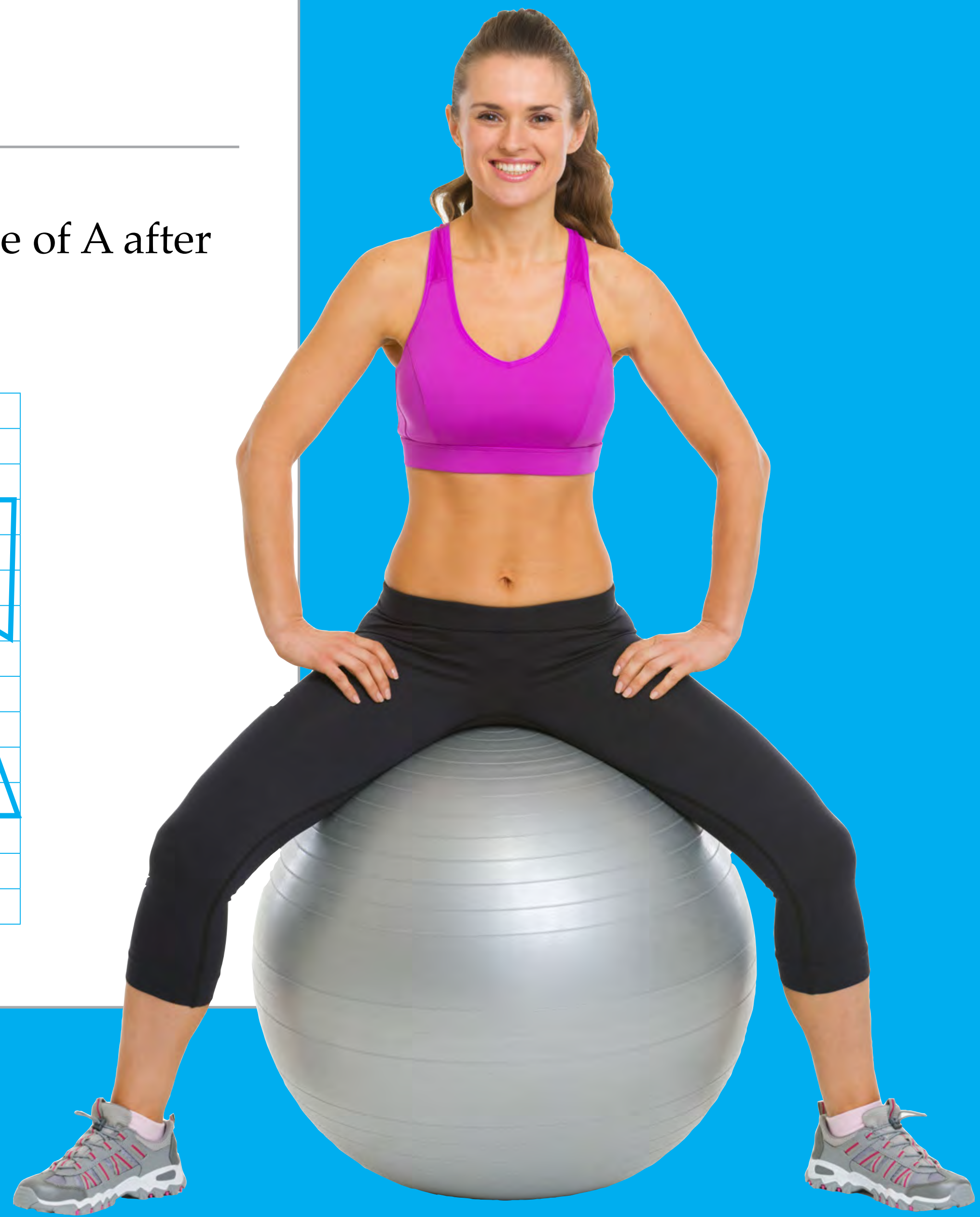


YOUR BRAIN WORKOUT

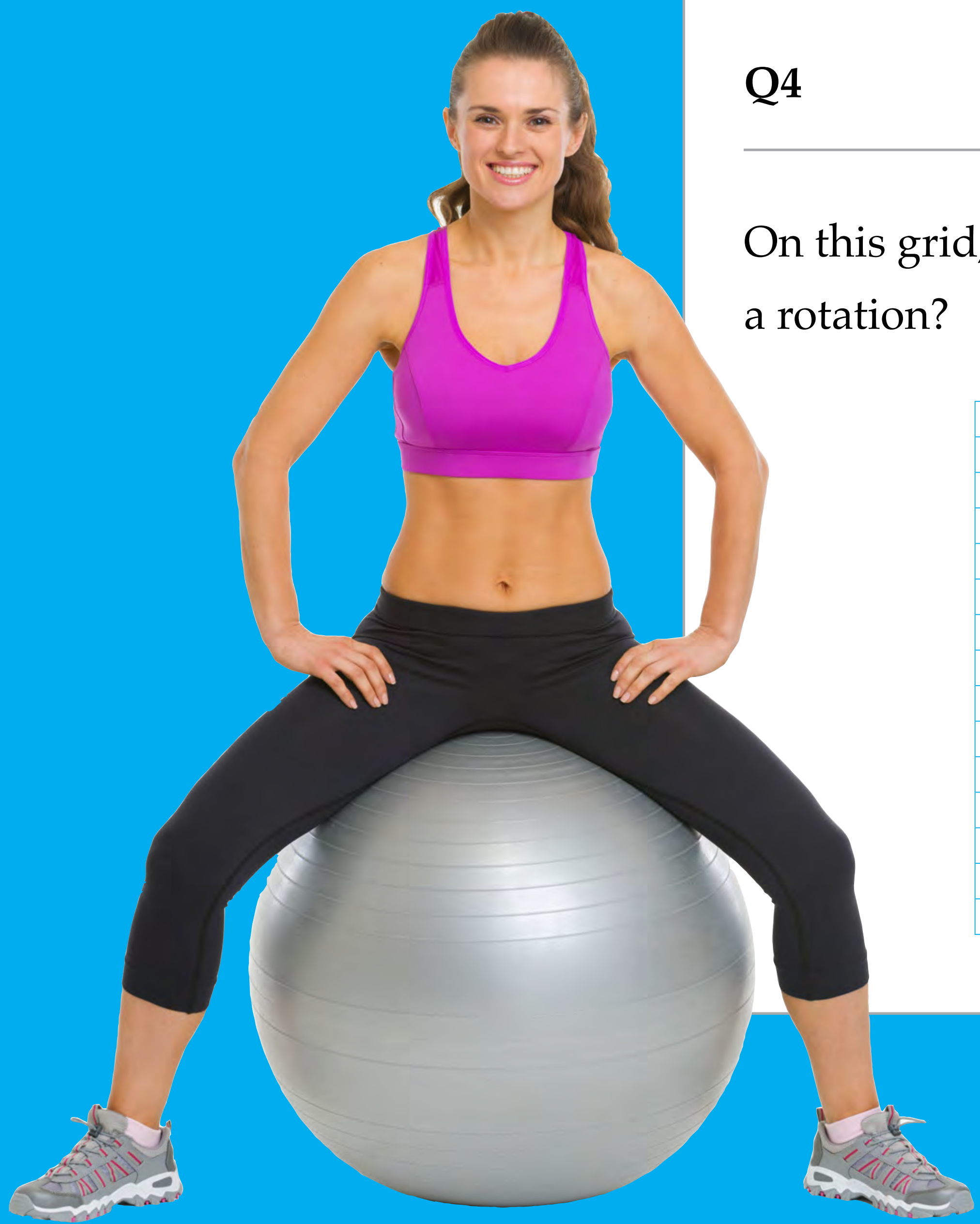
Q3

On this grid, which shape is the image of A after a reflection?



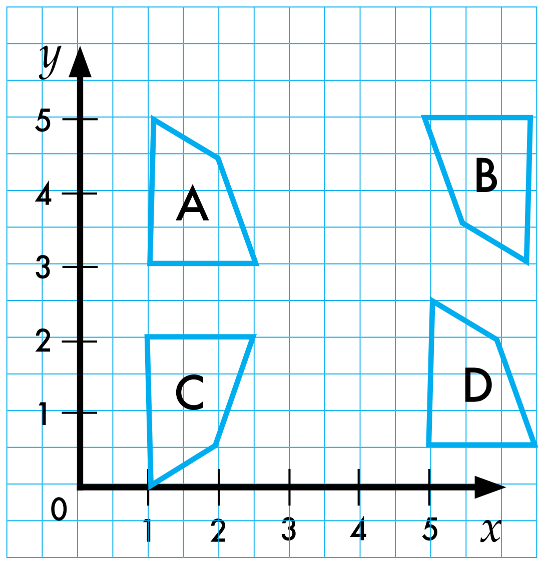


YOUR BRAIN WORKOUT



Q4

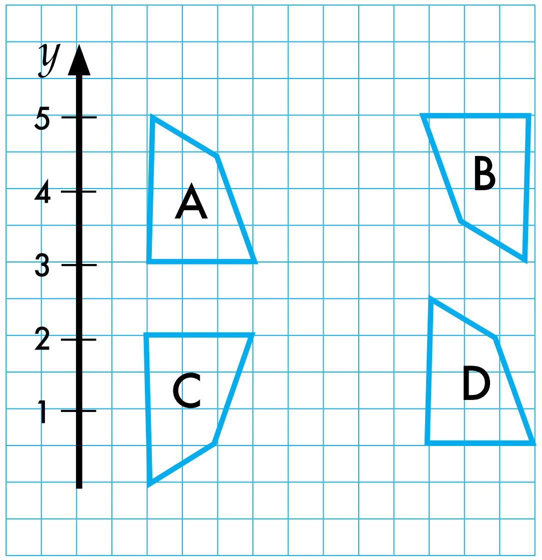
On this grid, which shape is the image of A after a rotation?



YOUR BRAIN WORKOUT

Q5

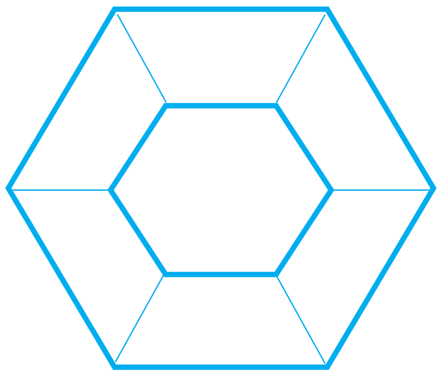
On this grid, which shape is the image of A after a translation?



YOUR BRAIN WORKOUT

Q6

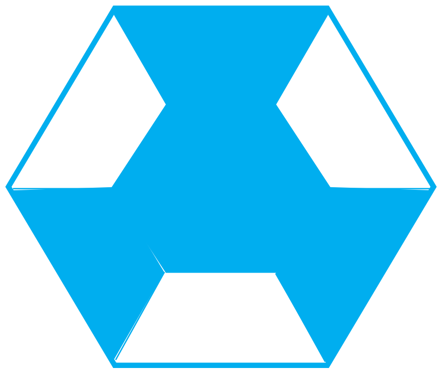
How many lines of symmetry are there on this hexagonal tile:



YOUR BRAIN WORKOUT

Q7

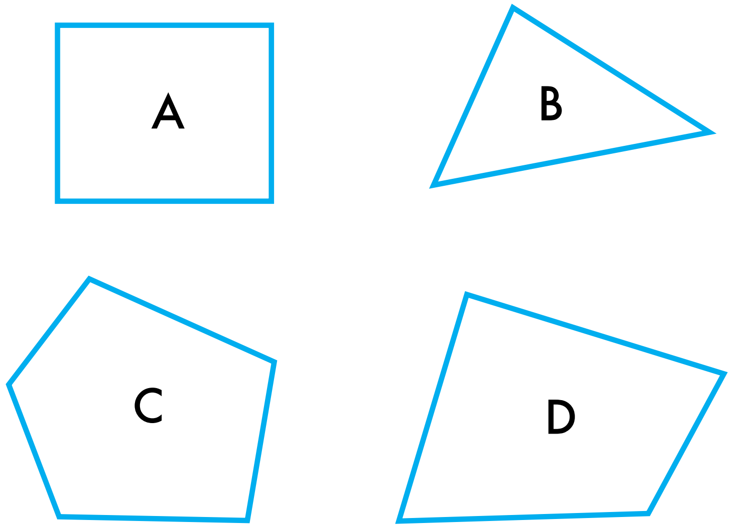
What is the order of rotational symmetry of this hexagonal tile:



YOUR BRAIN WORKOUT

Q8

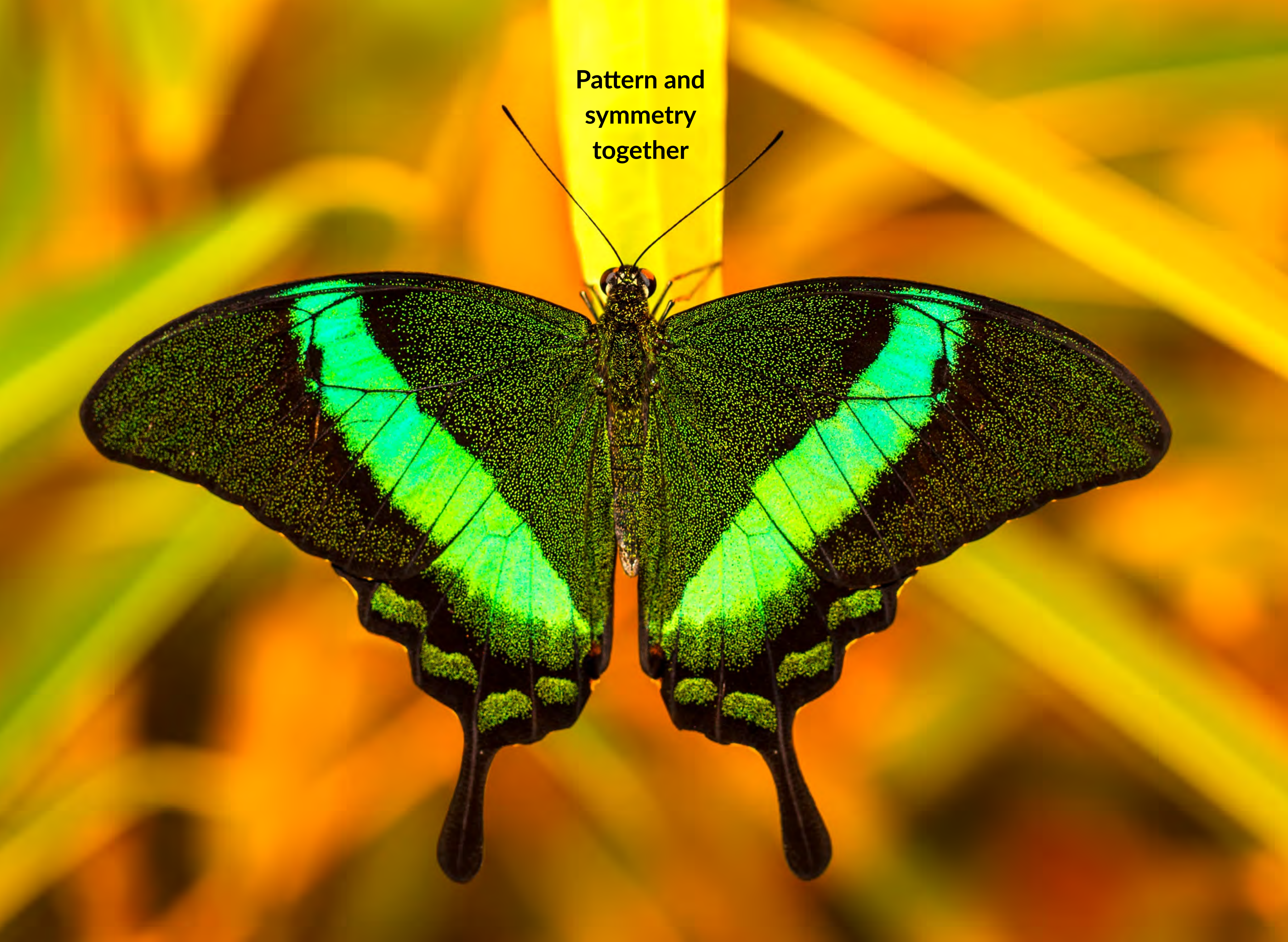
Which of these shapes will tessellate?



YOUR BRAIN WORKOUT

Answers

Q1	1.6
Q2	B
Q3	C
Q4	B
Q5	D
Q6	6 lines of symmetry
Q7	Order 3
Q8	A, B and D will tessellate


Pattern and
symmetry
together

PART 2

MONEY MAKES THE WORLD GO AROUND

Money Makes the World Go Around

As the Hollywood star, Liza Minelli, sang in the film 'Cabaret' having money can certainly help the world go around.

We cannot all be rich, but it important to be able to manage your money and live comfortably.

In this Part, you are going to look in more detail at managing your money including using a spreadsheet and some apps that you may find useful.

Food for Thought

Stress caused by a lack of money is extremely common and obviously very bad for your mental health. Spending time to keep on top of your finances is essential for your overall wellbeing.

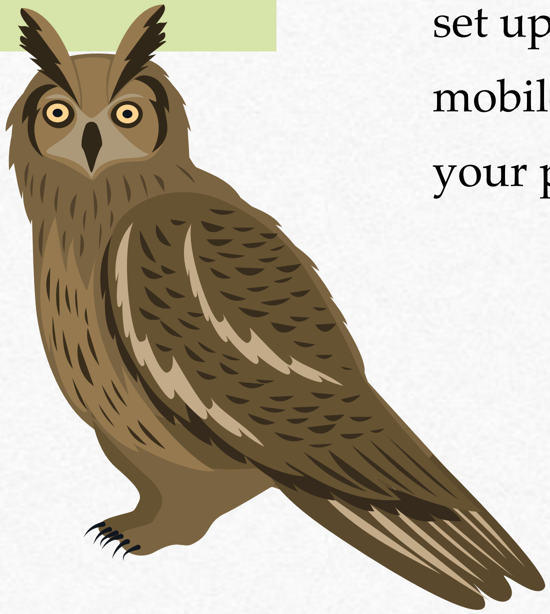
Opening a Bank Account

When you first start receiving more money than you are going to spend immediately, whether it is an allowance from parents, cash gifts from relations, a student loan or earnings from work, then it is time to open a bank account.

For this you will need is a current account.

What is a current account?

A current account is an everyday account where you carry out your regular banking transactions. This is where you will have your allowance, gifts, student loan or salary paid into, and from where you will take out cash. You can also set up regular payments for expenses such as rent and mobile phone costs. It is the first basic step in managing your personal finances.



Food for Thought

Amazing – money just comes out of the wall. Yes, but first you have to have put it into your account.

Young people often find it difficult to manage their finances when they first move away from home. Banks know this. Many banks set high interest rates for any overdraft.

An overdraft is where you have taken more out of your account than you have put in and are therefore in debt to your bank.

How to find the best current account

To find the best current account for your needs, you need to know what you want.

You need to consider the different types of current account providers out there, and the different benefits they offer.

It can be sensible to start by opening an account with the same bank that your parents use. In this way your parents may help by guaranteeing an overdraft up to an agreed amount so that you do not incur high costs when you are still learning to budget.

Traditional high street banks offer a range of benefits for first time account holders. You should be able to find a web site that allows you to compare these. All banks now should also offer an online banking facility.

Some banks are online or app-based only. They may have some benefits, and some of them are well respected and offer an excellent service. However, others are not as well established and could be at risk. If you are tempted by any of these, look up reviews and make your choice carefully.

Food for Thought

If something looks too good to be true then avoid it – it almost certainly **will be** too good to be true. Millions of pounds are lost every year by ordinary people who fall for false claims made by fraudulent operators. Make sure you protect yourself at all times from fraud. Ask parents and informed relatives for their opinions and experiences and follow advice.

Bank or building society?

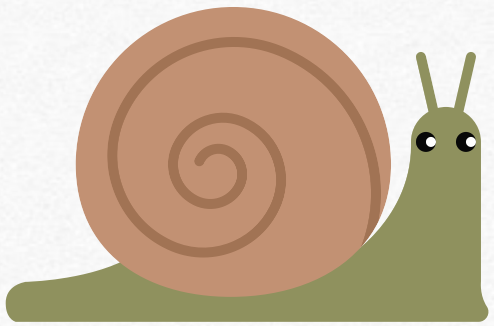
Traditionally, banks offered current accounts, overdraft facilities and loans. Building societies offered good rates on mortgages (loans to cover purchasing a property) and on savings. The distinction is no longer that clear. Some building societies now offer current accounts. Loans from banks and bank savings rates have become more competitive.

When making your choice consider:

- Ease of access: does your choice have branches both close to your home and near your college or place of work? How many branches does it have nationally in case you move?
- Do they allow you to withdraw cash only at their branches or from any other bank without a charge?
- Will they give you a credit and debit card immediately? What is the interest rate on their credit card?
- You may not need a student loan when you open your account, but you probably will if you start

college or university. What additional loan terms for students are being offered?

- What other overdraft facility do they offer and at what interest rate? (If you are a student this should be 0% up to a certain amount but that will not last forever.)
- Is the current account one where there is ‘no fee’ or is there a minimum monthly amount that you must pay in?
- Do they offer you a payment (interest) if your account is in credit?
- How good is their online banking and phone app? Read the reviews!
- Look at any offers made for first time account holders, but do not be swayed by supposed free gifts, all the above points need to be considered first. If not, your free gift may turn out to be an expensive mistake.



Food for Thought

A **debit card** is the modern alternative to using cash. When you buy your weekly shop in the supermarket you may pay for it with your debit card. This immediately takes money out of your current account. (As long as it is there, it is very embarrassing to have your debit card refused at a supermarket checkout.)

You can also pay for your shopping with a **credit card**. This does not take money out of your current account immediately but takes either full or part payment at a later date. In effect, by paying with a credit card you are taking out a loan. You should set up your credit card to be paid off in full every month otherwise the interest charges can grow very high, very quickly (remember how compound interest grows, spiralling quickly out of control if not paid off). The advantages of a credit card can be the protection it gives you when buying high priced goods or holidays, you could get your money back if things go wrong or if a seller won’t refund you. Otherwise, you are generally better off negotiating a loan or overdraft with your bank.


NEW CAR

What is a Student Loan?

A Student Loan allows you to pay for your university or college fees (tuition fee loan) now and to cover your basic living expenses (maintenance loan) while you are studying. You do not have to start paying back the loan until you are earning a reasonable salary.

Some young people are put off from applying to university as they fear that they will run up debts that they will be unable to pay off. This is not the case. Unlike a bank loan, or other loans such as a mortgage, student loans are only paid off when you can afford it. Think of a 'student loan' as a 'graduate repayment scheme'. Some of the examples in this Part make this clear.

Anyone can apply for a loan from their bank, but only a student can apply to Student Finance England for a Student Loan. (Note there are different schemes in Scotland, Wales and Northern Ireland so if you live there, look up your local arrangements)

It is generally the case that you will be able to find a better paid job and earn more money if you have a university degree.

You can find out the current arrangements for a student loan from the government website: https://www.gov.uk/student-finance

If you want to know more about Student Loans then you should download and read this year's guide:

Food for Thought

Some loans may be dependent on your **credit score**.

A **credit score** is a **3** digit number that indicates how reliable you are at borrowing and repaying money. The higher your score the more likely you are to have a loan agreed.

The score is based on a borrower's financial history. Anyone opening their first bank account will have no history and therefore will have a very low credit score.

One way to show you are a responsible borrower would be to use your credit card just enough to be able to pay everything back each month. Another way is for your current account not to become overdrawn.

Some banks offer you interest if your account is in credit – this is called in-credit interest.

Exercises

1 Amy is starting University in September. She wants to open a bank account and has drawn up a shortlist of those banks that have been recommended to her by her friends.

Bank	0% Overdraft	Incentive
ABANK	Up to £3,000*	None
BSOC	Up to £3,000*	Amazon £10 voucher
CBank	Up to £2,000	3yr railcard + up to 1% in-credit interest
D.S.B	Up to £2,000*	Up to 1% in-credit interest

* subject to Credit score

Consider the advantages of each offer and help Amy make her decision.

Using a Spreadsheet to Track Your Finances

Once you have a bank account, then you need to manage your money.

You can use a spreadsheet to check your spending. You can set-up your spreadsheet to do the adding up and other calculations for you. When you first start managing your own money, the first thing to do is to track what you have spent. Once you know where the money is going, then you can plan ahead to make sure that you do not spend more money than you have coming in.

Item Entry date 1-7 Sept 2015	All Costs	Personal Cash	Home	IT	Appearance	Food (inc lunch)	Savings, gifts, charity	Transport	Fun stuff	TOTAL
Personal Cash	20	20								
Home contribution	40		40							
Downpayment for new tablet	38			38						
New apps	2								2	
New shoes	29				29					
Weekly savings	10						10			
Winter vest	2				2					
Smoothies for self + friends	5					5				
Winter scarf	5				5					
New hairdryer	19				19					
5 lunches	28					28				
Contribution to Thelma's leaving gift	2						2			
Various buses	6							6		
Concert ticket	10								10	
TOTAL SPENT	216	20	40	38	55	33	12	6	12	216

This is something that successful organisations do well, it is called a budget.

Food for Thought

In a survey by UCAS in **2020**, almost a fifth of students (**18%**) said that financial concerns had impacted their mental health or wellbeing, while **6%** of university dropouts were finance-related.

This is what a survey found to be the average expenditure for a student living in a hall of residence per month:

Expense	Cost (£) per month
Accommodation	425
Transport	80
Food shop	80
Interests and hobbies	45
Clothing	45
Takeaways and snacks	30
Phone and internet	25
Alcohol and cigarettes	20
Personal care	15
Coffee and tea	10
Other expenses	25
TOTAL	800

The advantage of living in a hall of residence is not only that it is a good way to meet people but also there are no water rates, gas or electricity to pay. If you are renting a house with friends there will be all those costs to consider and you should ask about them before you sign a rental agreement.

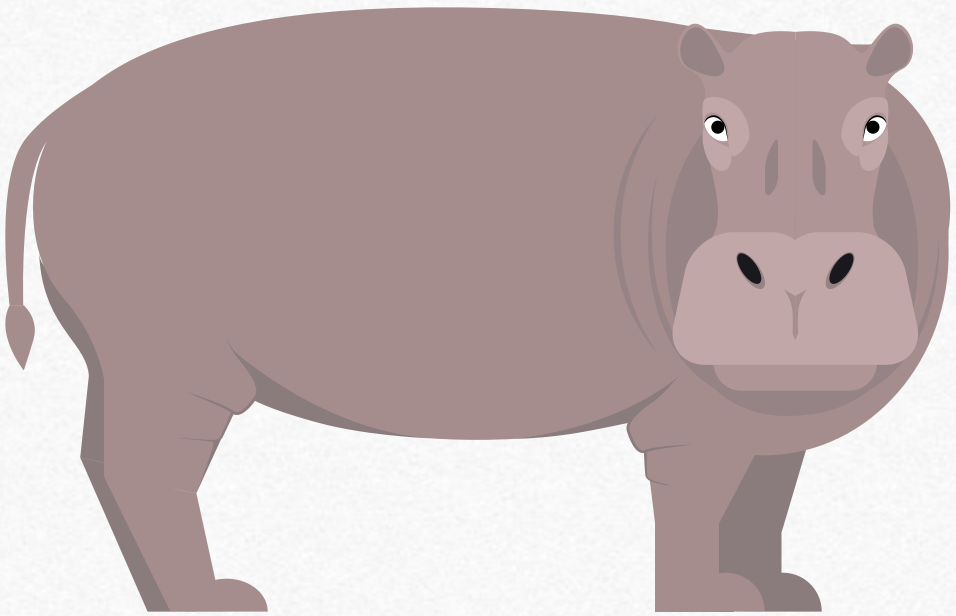
The costs that are highlighted in yellow are fixed costs. You know what they will be in advance. It may be that you can save on transport costs by walking or riding a bicycle, but often cheaper accommodation is further out and then transport costs are higher.

Some of the other costs are up to you but you do have to eat! Make sure that you do not spend too much on eating out or entertainment and then not have anything left to buy food.

You will probably want to budget for a holiday too. If you are a student then your loans will not cover this, that is where a part time job can help.

When setting up a spreadsheet, you need to work out your monthly income and outgoings, and then keep a close eye on them week by week. As you get more experienced at managing your money this will get easier.

Remember that if you have a standard word processing/spreadsheet/presentation package on your laptop or tablet, you should be able to access documents on your mobile phone as well. As your mobile phone screen is not very big, then you may wish to keep your spreadsheet compact.



Example

Bella has set up the following spreadsheet for September:

	A	B	C	D	E
1	September	Budget	Actual	Date	Difference
2	Income	£1,000.00	£1,000.00	01/09/2000	£0.00
3	Expenditure:				
4	Accommodation (Hall)	£425.00			£425.00
5	Transport	£80.00			£80.00
6	Food shop	£80.00			£80.00
7	Interests and hobbies	£45.00			£45.00
8					£0.00
9					£0.00
10	Clothing	£45.00			£45.00
11	Takeaways	£30.00			£30.00
12	Phone and internet	£25.00			£25.00
13	Alcohol and cigarettes	£20.00			£20.00
14	Personal care	£15.00			£15.00
15	Coffee and tea	£10.00			£10.00
16	Other expenses	£25.00			£25.00
17					£0.00
18					£0.00
19					£0.00
20					
21	**Total**	£800.00			**£800.00**
22	Balance:	£200.00			
23					

Note she has left some extra rows for her interests and for additional expenses.

Over the next two weeks Bella spends the following:

Hall fees **£375** on **5**th September, a monthly travel season ticket of **£80** and her monthly mobile fee of **£25** on the **6**th; a rail fare from home to uni of **£35** on the **11**th; her first supermarket bill of **£35** on the **12**th; fees to join a drama club of **£15** and fees to join a gym of **£25** on the **13**th; **£45** on books and **£12.45** buying drinks in the students union on the **14**th.

Enter these on the spreadsheet and see how Bella's budget compares to her actual spending.

When Bella set up her spreadsheet, she used two important formulae:

One is for the total in cell **B21**.

The formula for the total is =SUM(**B4**:**B20**)

She also needed to work out the balance according to her budget.

In cell **B22** the formula is **=B2-B21** and that is how the figure of **£200** is **calculated**. Bella can copy and paste these formulae into other cells if she needs to.

You might note that Bella has spent some money on things that she had not budgeted for. She will put these into her spreadsheet in the additional rows.

Answer: This is what her spreadsheet now looks like:

	A	B	C	D	E
1	September	Budget	Actual	Date	Difference
2	Income:	£1,000.00	£1,000.00	01/09/2000	£0.00
3	Expenditure:				
4	Accommodation (Hall)	£425.00	£375.00	05/09/2000	£50.00
5	Transport	£80.00	£80.00	11/09/2000	£0.00
6	Food shop	£80.00	£35.00	12/09/2000	£45.00
7	Interests and hobbies	£45.00			£45.00
8	Gym		£25.00	13/09/2000	−£25.00
9	drama club		£15.00	13/09/2000	−£15.00
10	Clothing	£45.00			£45.00
11	Takeaways	£30.00			£30.00
12	Phone and internet	£25.00	£25.00	06/09/2000	£0.00
13	Alcohol and cigarettes	£20.00	£12.45	14/09/2000	£7.55
14	Personal care	£15.00			£15.00
15	Coffee and tea	£10.00			£10.00
16	Other expenses	£25.00			£25.00
17	Rail fare		£35.00	11/09/2000	−£35.00
18	books		£45.00	14/09/2000	−£45.00
19					£0.00
20					
21	**Total**	£800.00	£647.45		**£152.55**
22	Balance:	£200.00			
23					

Do not worry too much about the dates but focus on the last column, the difference between the plan and your actual spend. As you can imagine Bella is rather horrified that she has spent so much in just two weeks. However, it is normal to spend more when you first move into a new flat or hall of residence as you need to stock up your kitchen and buy any other basics. It is important to enjoy social activities too and you might not have realised how much clubs or a gym may cost.

Where Bella has probably not budgeted enough is for eating out or drinks with new friends. She will therefore adjust her budget for the next month taking into account her pattern of spending and taking care not to overspend. It is very useful to have some money left over at the end of the month – 'money for a rainy day' or other emergency (or even for your rail fare home!)

Food for Thought

A good habit to get into is to update your budget spreadsheet on a Thursday evening. You can then plan how much you can afford to spend at the weekend. Take out in cash the amount you have allowed yourself and leave your debit and credit cards at home.


2018
January
February
March
April
May
June
July
August
September
October
November
December

Food for Thought

Your bank or other systems may allow you to pay using an app on your mobile phone, which shows how much is in your cash account today.

Exercises

2 In September, Chiara is moving into a new flat that she is to share with two flatmates.

Chiara has agreed that she will pay **£750** a month rent, **£120** a month towards gas and electricity and **£66** a month towards council tax.

Chiara is working as a teacher and earns **£22,400** a year after paying tax and national insurance. She knows that her monthly mobile charge is **£24** and that she will need a monthly travel season ticket costing **£140**. As it is the start of a new school year, Chiara intends to spend **£200** on some new clothes for work.

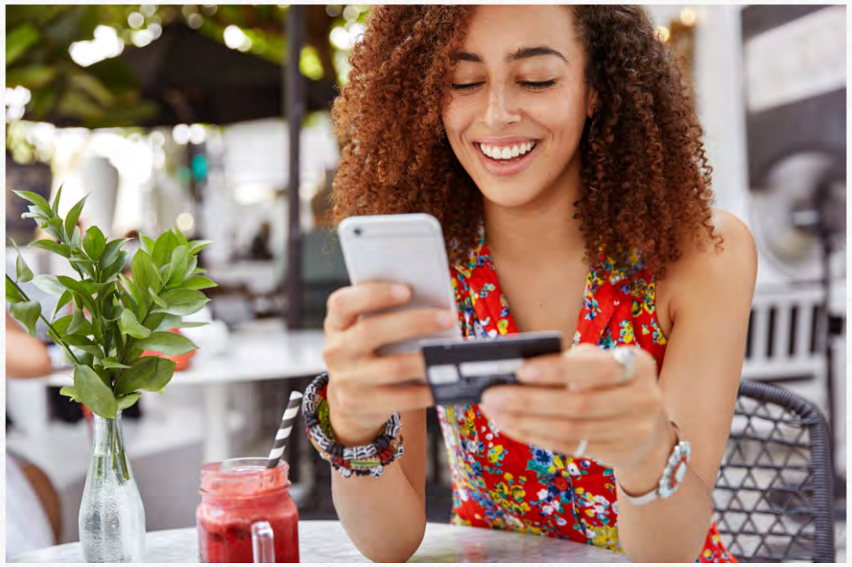
Set-up a spreadsheet for Chiara and help her budget for any other expenditure.

Using a Banking App

Once you have a current account you should be able to access details on an app.

Exactly what the information looks like will depend on your bank.

On the app you should be able to see how much money is in your current account and what the most recent transactions have been. It should also show what transactions are pending. For example, if you go to the supermarket on a Saturday and do your weekly shop, the amount may not come out of your bank account until Monday. However, the app may show this as pending.



Example

Here is an example of what the app may show, this example relates to Chiara in the last question. The top line shows the current balance in Chiara's account. Read this example from the bottom to follow her day to day expenditure:

Today 12.20 PM	
Pending	
Mon 5	£727.88
Myfone	**-£24.00**
September	
Sat 3	£751.88
ASupermkt	**-£46.17**
NandT	**-£145.98**
BestModes	**-£53.47**
Fri 2	£997.50
LT Season T	**-£140.00**
Chez Austin	**-£35.65**
Thur 1	£1,173.15
Mrs L Lady	**-£750.00**
Miss XY ref GandE	**-£120.00**
Miss XY reCT	**-£66.00**
August	
Wed 31	£2,109.15
ABC SCHOOL	**£1,866.67**

When Chiara checks her balance on Sunday she can see that her monthly mobile phone bill will come out of her account on Monday.

On Saturday Chiara has gone shopping for clothes and in the supermarket. She has £751.88 left in her account.

On Friday Chiara buys her monthly travel season ticket and eats out with friends.

Chiara has arranged to pay her rent, gas, electricity and council tax on the first day of each month.

Chiara's monthly salary comes in at the end of each month. She now has £2,109.15 in her account as her salary is added to a £242.48 balance.

You can see that if you use your Budget spreadsheet with your banking app then you can keep a close eye on your finances. There are other useful apps that can help you such as a mobile phone bill app that tells you what you are spending each month on calls, texts and data.

Food for Thought

You can use your mobile phone bill app to set a 'cap'on how much you spend so that you do not suddenly find that you have spend much more than you thought. This is a very good thing to do when you are travelling as some countries have hidden network charges for non-local phone users.

Exercises

3 Update the spreadsheet from your answer to **Q2** with the new info in the banking app.

4 Debs is a student and checks her banking app mid-September after Freshers week at Uni:

Today 12.20 PM		
September		
Sat 17	£............	
Bar X	**-£25.17**	
Supermkt	**-£45.23**	
Tues 13	£............	
Cash	**-£50.00**	
Cash	**-£30.00**	
Mon 12	£............	
Taxi	**-£14.00**	Travel from home
BR	**-£32.15**	
Mon 5	£............	
OofE Hall	**-£1,800.00**	Hall fees
August		
Wed 31	£3,100.00	
SFE ML	**£3,000.00**	Student loan

You can see Debs has received her maintenance grant and paid her self-catering hall fees.

(a) Fill in the missing balances.

(b) What else do you think Debs did in Freshers week?

(c) There are **14** weeks of term left and Debs has no other source of income. Help Debs with her budgeting.

5 Evie starts her summer term at Uni.

(a) How much was Evie overdrawn before she received her maintenance grant?

(b) There are seven weeks of the summer term to go. Evie will need to pay the same amount in June for her gym, telephone and travel card. If each week spends **£35** at the supermarket and takes out **£50** cash, how much will she have left in her account at the end of term?

Today 12.20 PM	
May	
Sat 7	£699.73
Mofone	**-£19.50**
Gym	**-£25.25**
Fri 6	£744.48
Cash	**-£50.00**
Tue 3	£794.48
Travelcard	**-£15.00**
Supermkt	**-£35.63**
Mon 2	£845.11
OofE Hall	**-£1,800.00**
SFE ML	**£3,000.00**

Student Accommodation

Your university will want you to either pay all your hall fees at the start of the academic year or pay in three instalments, at the start of each term. As terms are not the same length you will need to work out what to pay when.

Example

Contract length (weeks)	Payment Term 1 (weeks)	Payment Term 2 (weeks)	Payment Term 3 (weeks)
32	13	12	7

If Ffion's annual hall costs are **£6,000**, what will she need to pay each term?

Ffion works out her accommodation costs as a fraction of the total:

Term 1 = $^{13}/_{32}$ x £6,000 = £2,437.50

Term 2 = $^{12}/_{32}$ x £6,000 = £2,250

Term 3 = $^{7}/_{32}$ x £6,000 = £1,312.50

Answer: Ffion would pay £2,437.50, then £2,250 and £1,312.50 for Terms 1, 2 and 3 respectively.

Ffion then checks her figures by making sure that the three amounts add up to **£6,000** – they do!

Exercises

6 Gina has decided to take up catered accommodation at her university. Although the overall cost is higher, she gets a daily breakfast and evening meal as well as lunch at weekends. The annual cost of her chosen hall is **£7,990** per year but spread over three terms:

Contract length (weeks)	Payment Term 1 (weeks)	Payment Term 2 (weeks)	Payment Term 3 (weeks)
34	14	12	8

What must Gina budget for her accommodation per term?

Loan or Credit Card?

In that last example you can see that if Gina's maintenance loan is **£9,000** then her first term's accommodation costs are going to be higher than the one third of her loan that she will receive in September. She will therefore have to borrow money.

She has various options:

1. To use the overdraft facility at her bank
2. To arrange a loan with her bank
3. To use a credit card.

Using the earlier survey, average university costs outside of accommodation are about **£375** a month.

Therefore, for her first term of **14** weeks Gina estimates that she will need to borrow **4 x £375**, this will cover the extra accommodation fees plus the fact that she will have extra expenses in her first term as she will need to buy books, stationery and any other equipment.

4 x £375 = £1,500.

Option 1

Hopefully, Gina has opened a current account with a bank that allows her to have an overdraft without paying any interest, so this is her best option.

Option 2

If Gina's bank does not offer an overdraft then she will have to arrange a loan. For example, a bank might charge **13%** interest on a short term loans, so if Gina borrowed **£1,500** for a year, then she would pay:

1500 x 13% = £195 in interest.

Option 3

If Gina's bank have offered her a credit card then typically she can only borrow a maximum of **£1,200** at a high interest rate such as **39.9%.** As Gina will not be able to pay off her entire credit card bill at the end of each month this will have a negative impact on her credit score, therefore Gina ignores this option.

Payday Loan

Whether you are in college/university or at work there may be times when you could do with a short-term loan that will just tide you over until your next student loan or pay arrives in your account. These are sometimes referred to as 'pay day loans' as they are designed to tide you over until your net 'pay day'.

Example

Hua wants to borrow **£200** for three months in order to tide her over until her next loan instalment comes in.

For a loan of **£200**, she is told that she should repay **£98.70** a month.

How much interest is she being charged?

Total repayments = 3 x £98.70
= £296.10

Interest = £296.10 – £200
= £96.10

Answer: The interest Hua would pay is £96.10

You can see that although these deals may look attractive in the short term, in reality they are a very expensive way to borrow money.

In the example the interest rate can be calculated by diving the amount of interest to pay by the amount borrowed and multiplying by 100:

$$\text{Interest rate} = \frac{\text{amount of interest}}{\text{amount of loan}} \times 100\%$$

$$= {}^{96.1}/_{200} \times 100\%$$

$$= 48.05\%$$

You can see that this amount of interest is even higher than a credit card and is only for three months.

Food for Thought

Now you can see the importance of budgeting and arranging the best bank facility for you.

Make sure you understand the difference between BUDGET (which is a plan) and BALANCE (which is the actual amount that you have available).


“Money Stuff works for me”

Your First Job

As you will have seen, loans must be paid back and therefore it is best not to have to borrow money if you can avoid it. Therefore, you will need a job.

The jobs that you take up while you are at school, college and university are unlikely to have anything to do with the career that you eventually follow. That is usually because the temporary part time jobs that are available do not require the skills that you will eventually have acquired.

It is never too early to think about what part time jobs might be worth considering. It can be relatively simple to turn your favourite sport, hobby or talent into a qualification. You may be able to ask your school or college to help, for example:

If you like cooking, consider getting a Food Hygiene Certificate Level 1 and 2. You can do this online and that might make it easier for you to get a job in a bar, café or restaurant.

If you are good at a certain sport, you may be able to get an instructor's qualification. You can then teach your sport at summer schools or sports centres.

If you are a musician, you may be able to teach children how to play an instrument. You will need to have a Disclosure and Barring Service (DBS) check to work with children.

If you are good at IT, you may be able to offer IT support to others. Have a look at what other people are offering in your neighbourhood.

If you like working with children, or are planning to take a teaching degree, then you may find part time work in a school, nursery or crèche. You could also find work as a babysitter. Your school or college may be willing to give you a reference.

You may also be able to find work that is linked to your chosen career. For example, if you want to be a vet, then you may be able to find work in an animal sanctuary or veterinary surgery. However, there can be lots of other people also wishing to get valuable 'work experience' and this may not be paid.

There are some things that you need to understand when you get your first job. Do ask your employer if there is anything that you are not sure about.

Tax Year

The tax year runs from the 6th April one year until the 5th April the next. You will see tax years referred to as, for example, 2021/22 or 2023/24.

National Insurance number

National Insurance is the fund that all workers pay into. The fund pays for certain benefits, such as unemployment benefits and maternity benefits, and contributes towards your pension when you eventually retire. You will normally be sent a National Insurance number automatically, in the

3 months before your 16th birthday. If you do not have a National Insurance number, you must apply for one if you plan to work, apply for a student loan or claim benefits.

Keep your National Insurance number in a safe place. You will need it all your life as this is what tracks your National Insurance Contributions (NICs) and eventually determines your state pension. How much National Insurance you will pay each week or month changes over time.

In 2021/22 you would have paid 12% National Insurance on your earnings if they are over £184 a week or £797.00 a month. If you are earning over £4,189 a month the rate changes.

Personal Allowance / Tax Code

Everyone can earn a certain amount of money every year before they pay tax. This is called your personal allowance and you are given a tax code that records this. Your tax code will normally start with a number and end with a letter.

1257L is the current tax code used for most people who have one job and this means that the associated personal allowance is £12,570. The code is important but is likely to change over time.

When you start you first job you should be asked to fill in a ‘Starter Checklist’ to help determine your tax code.

Income Tax

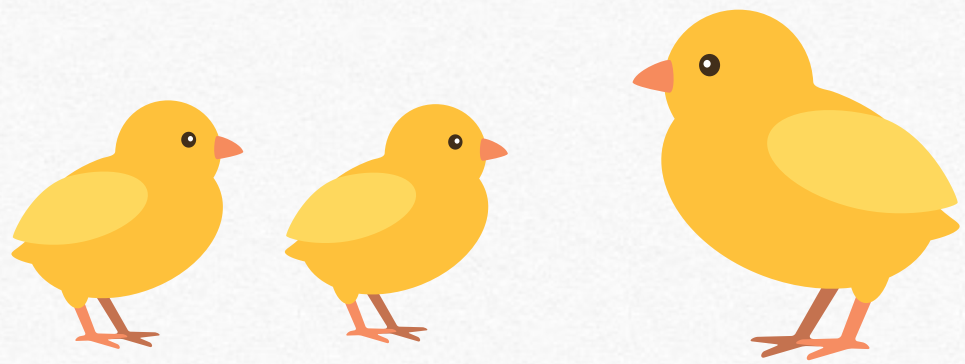
If you follow the news you will know there is something called a ‘Budget’ that the Government announces once or twice a year. This is rather like your personal budget but for the whole country. Governments need income too and this comes from National Insurance and income tax. The rate of tax that you pay and your personal allowance will be determined at a Budget.

Government also collects tax from other sources, such a Value Added Tax (VAT) and taxes on items such as fuel, alcohol and cigarettes.

HMRC

His Majesty's Revenue and Customs (sometimes called Inland Revenue) are the UK's tax, payments and customs authority, and collect the money that pays for the UK's public services. They also help families and individuals with targeted financial support.

HMRC say 'We do this by being impartial and increasingly effective and efficient in our administration. We help the honest majority to get their tax right and make it hard for the dishonest minority to cheat the system.'



PAYE

PAYE stands for pay as you earn. This is the system for most employed workers in the UK. Tax and NICs are deducted by the employer and sent directly to the Inland Revenue.

This keeps things very simple as it means that you as an employee do not have to worry about paying the right amount of tax.

Self Employed

However, you may find that you find work, such as a sports instructor, for an organisation that expects you to operate as a 'self-employed' contractor. This means that you must fill in a tax return and pay your own tax. This is more complicated and you should first make sure that the organisation is correct and should not be treating you as an employee. If you are to be self-employed, then you should seek advice on how to pay tax. It is not complicated but it is important to get it right.

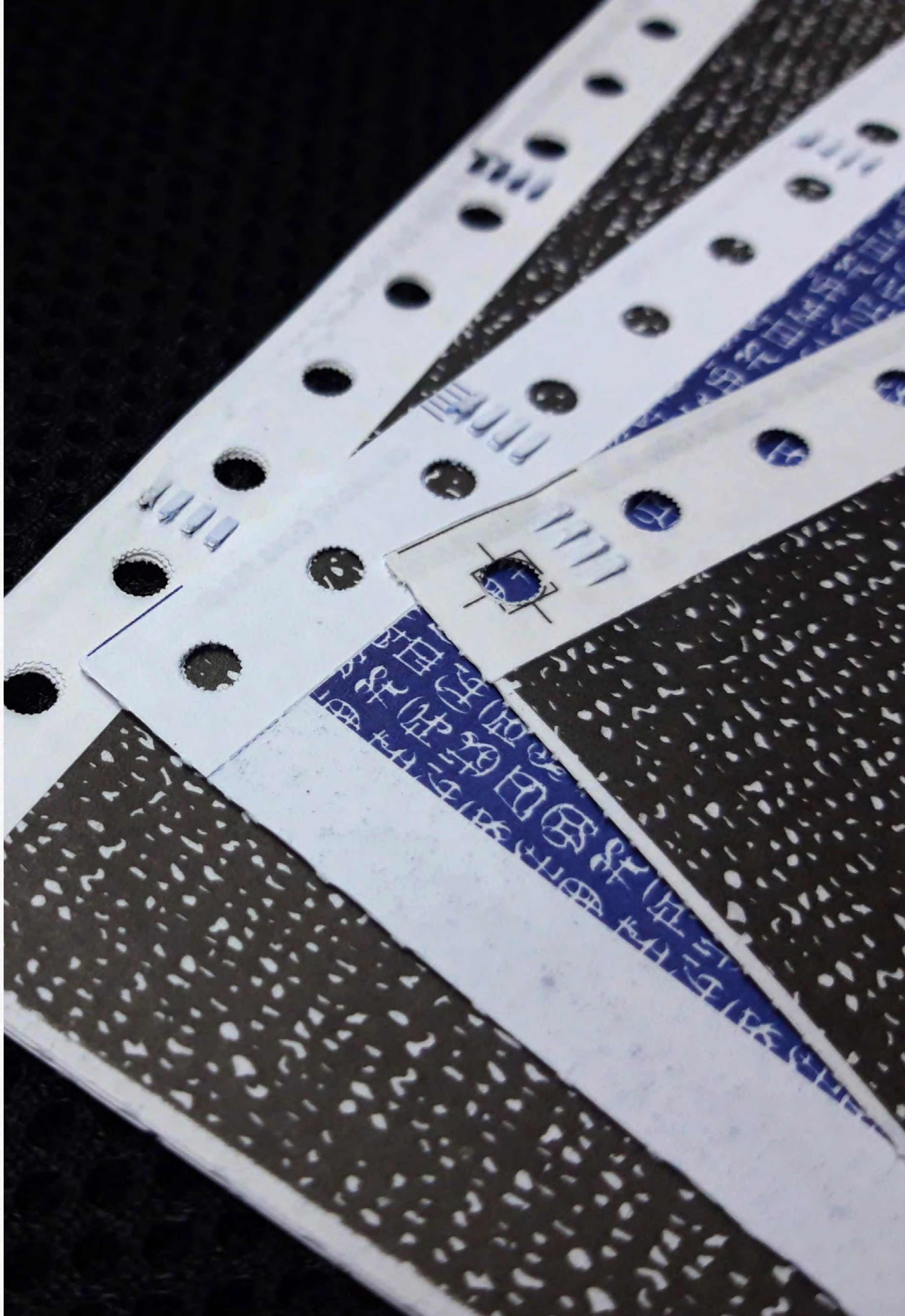
Minimum Wage

Every worker is entitled to be paid a minimum hourly rate. The rate depends on your age. This is called the National Minimum Wage (NLW) for those of at least school leaving age and the National Living Wage for those aged 23 and over. There is also a scale of hourly rates for apprentices. The rates change every April.

Age	Rates from April 2021
23 and over (NLW)	**£10.42**
21 to 22	**£10.18**
18 to 20	**£7.49**
Under 18 and apprentices	**£5.28**

P45

When you leave a job you should be given a **P45** form that states the amount you have been paid in that tax year, the tax and National Insurance that has been deducted. If you are not given one, then ask.



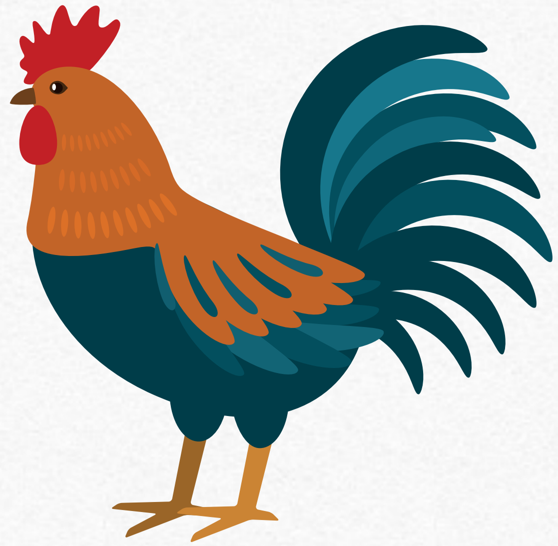
Checking your payslip

When you receive your payslip, you should always:

- Check your personal details, name, and NI number are correct.
- Check that your tax code is correct.
- Check that the hours that you have worked are correct.
- Roughly check that the amount you are paying in tax is correct by:
    1. Dividing your personal allowance by **12** (**12,570 ÷ 12 = 1,047.50**)
    2. Subtracting that from your monthly earnings
    3. Multiplying the result by **20%** (or **0.2**). That is what your tax payment should be while you are earning less than **£50,000** per year.
- Roughly check that the amount you are paying in NI is correct by:
    1. Subtracting **£790** from your monthly earnings.
    2. Multiplying the result by **12%** (or **0.12**). That is what you NI payment should be until your monthly earnings are **£4,000** or more.

These checks are 'rough' because they change at times and there may be other small amounts to take into account. If you want to check the exact amounts then you may wish to use the free HMRC app:

You will need to apply on-line for a log in and password in order to get started.



Understanding your payslip

Getting your first payslip is very exciting, sometimes followed with disappointment when you see how much has been deducted. It is important that you know how to check the deductions and can query them if you think they are incorrect. A pay slip for Lydia, who is 17 and on the basic national minimum wage, but with more for overtime, might look like this (Lydia is a temporary employee and has opted out of pension contributions):

Payroll ID	Employee Name		Payment date		NI Number
NEO/A123	Miss Lydia Bennet		31/04/2022		YW123456L
Payment	Hours	Rate	Amount	Deductions	
Salary	105	£6.56	£688.80	PAYE TAX	£0
Overtime	20	£8.20	£164.00	NI	£7.30
Holiday pay				Pension	£0
Address		This Period		Year to date	
Miss Lydia Bennet		Total Gross Pay	£852.80	Total Gross PAY TD	£852.80
Longbourne		Gross for tax	£0	Taxable Pay TD	£0
Meryton		Earnings for NI	£60.80	Tax Paid TD	£0
Hertfordshire				Earnings for NI TD	£60.80
MN12 34X				National Insurance Paid	£7.30
				Pension Paid TD	£0
Netherfield Estate Office				NET PAY	
Tax Code 1257L	Tax Period 1	Payment Method: BACS		**£845.50**	

Your unique Payroll ID first has a code for the employer and then for the employee.

This section shows how Lydia earned her money this month.

This section shows the deductions this month. Lydia has not earned enough for any tax to be payable but she does pay National Insurance.

TD is 'to date'. As this is for April, this is the first month of the tax year so this is just the amounts for the one month

This is the net pay Lydia received at the end of April 2022

Once you have made sure that you understand what the payslip is telling you, next look at Lizzie Bennett's payslip.

Lizzie is **21** and is working as a admin assistant. This is her first full-time job and her salary is **£21,600** per annum. (Lizzie has opted out of pension contributions). The company pays her salary on the **5**th day of each month to tie in to the tax year. Her payslip for **6**th March to **5**th April might look like this:

Payroll ID	Employee Name		Payment date		NI Number
DEL/B235	Miss Elizabeth Bennet		**05/04/2022**		YW456789L
Payment			**Amount**	**Deductions**	
Salary			£1,800	PAYE TAX NI Pension	£150.50 £120.36 £0
Address		**This Period**		**Year to date**	
Miss Elizabeth Bennet Longbourne Meryton Hertfordshire MN12 34X		Total Gross Pay Gross for tax Earnings for NI	£1,800.00 £752.50 £1,003.00	Total Gross PAY TD Taxable Pay TD Tax Paid TD Earnings for NI TD National Insurance Paid Pension Paid TD	£21,600 £9,030.00 £1,806.00 £12,036.00 £1,444.32 £0
Darcy Enterprises Ltd				**NET PAY**	
Tax Code 1257L	Tax period 12	Payment Method: BACS		**£1,529.14**	

This section shows Lizzie's monthly salary: £21,600 ÷ 12

This section shows the deductions this month. Lizzie pays both tax and NI but she is not earning enough to have to pay back her student loans

TD is 'to date'. As this is for 5th April, this is the last full month of the tax year so this is the full amount for that tax year

This is the net pay Lizzie received on 5th April

Exercises

7 Do a rough check to make sure that Lizzie's deductions for tax and NI for 6th March – 5th April 2022 are correct (assuming basic rate tax is 20%).

8 During March 2022 Lizzie had an excellent employment review and successfully applied for a better paid job in the same company. From 6th April 2022, her annual salary will be £27,000.

Lizzie wants to work out how much money she will be paid on her next pay slip on 5th May with her new salary.

(a) What will Lizzie's gross monthly salary be?

(b) If Lizzie's tax code is unchanged and the basic tax rate is still 20%, how much tax will she pay monthly?

(c) If the NI thresholds and rates are unchanged how much NI will she pay monthly?

(d) How much will Lizzie's take home pay be at the end of April?

Repaying your student loan

You will have to repay everything you borrowed, if you can afford to, but how much you repay each month depends on your income, not how much you borrowed.

You will not have to repay anything until you are earning over a certain amount.

By the time you start repaying, the amounts will be different. Have a look at the table to see some examples of what you might repay based on the 2021 threshold of £27,000 a year:

Annual salary	Monthly repayment
£22,000	£0
£27,000	£3
£31,000	£33
£33,000	£48

If your income changes for any reason, your repayments will change automatically, but it is sensible to have factored your loan repayments into your budgeting.

When you calculate your net pay, you should deduct your student loan repayments from your gross pay first.

Pension

In the examples above both Lydia and Lizzie had decided to opt out of a pension scheme. Although that probably seems sensible when you are in your early twenties and really need to take home the maximum wage possible, once you are settled in your career then paying into a pension scheme is prudent.

In **2021/22** the basic state pension was **£9,337.80** per year. If you had not paid into a pension scheme, then that is all you would have to live on once you are past retirement age.

Your employer should explain to you how the pension scheme works. You do not pay tax on your pension contributions; they are taken from your salary before tax. In addition, do note that your employer will generally match a proportion of your own pension contributions. It may seem a very long time until you actually retire but you will eventually be really glad that you took advantage of all these tax breaks and contributions.

Example

An employee earning **£24,000** per annum decides to contribute **£100** a month towards their pension.

(a) If their tax code is **1275 L**, on what amount will they be paying tax each month?

(b) How much do they save in tax compared to if they had not decided to pay into the pension scheme?

(a) First you need to calculate the pay on which the tax calculation will be based, so you deduct your monthly personal allowance and pension contribution from your gross monthly pay:

Monthly gross pay = 24,000 ÷ 12
= £2,000

Monthly personal allowance = 12,570 ÷ 12
= £1,047.50

Gross pay for tax = £2,000 − £1,047.50 − £100
= £852.50

Tax payable = 20% of £852.50
= £170.50

Answer: Monthly tax to pay will be £170.50

(b) If you had not deducted the **£100**, the calculations would have been:

Gross pay for tax = £2,000 − £1,047.50
= £952.50

Tax payable = 20% of £952.50
= £190.50

£190.50 − £170.50 = £20

Answer: You have saved £20 in tax.

Before you tackle these next questions, remember that although pension contributions are deducted before the tax calculation, student loan repayments are not.
NI contributions are unchanged.

Exercises

9 Lizzie Bennett decides to pay **£150** a month into the company pension scheme. She also has to start paying back her student loan. Using the figures from **Q8** and the student loan figures above, work out all the deductions and then Lizzie's net monthly salary after her pension and loan contributions have been made.

10 **25**-year-old Jane Bennett is employed as an engineer on a salary of **£32,000** per annum. What is her net monthly salary if she contributes **£200** per month into the company pension scheme and pays **£33** a month to repay her student loan?

Food for Thought

Women Engineers of the Year 2020

Ella Podmore was Woman Engineer of the Year 2020, crowned in March 2021. Ella is a Materials Engineer for luxury British supercar maker McLaren Automotive. She is responsible for all the material investigations in the business across all development phases of the company's supercars; from concept drawings, all the way to customers in the field. Balancing her time between experiments and leading technical meetings, Ella created this department from the ground-up and plans to demonstrate the importance of materials in the automotive industry even further. Ella helped launch the recent competition McLaren Automotive competition run with BBC Blue Peter asking children to design their 'supercar of the future'and was one of Autocar's Top 10 'Rising Star' in 2019.
https://www.theiet.org/membership/member-news/member-news-2021/member-news-january-to-march-2021/iet-young-woman-engineer-of-the-year-winners-revealed/

Shrouk El-Attar, an Electronic Engineer at Elvie is the 2020 Woman Engineering Society's woman of the year. She engineers smart tech that improves the lives of women and trans men, whilst breaking down barriers and smashing taboos. Shrouk previously worked with surgeons operating on the eye, on IoT Tech at Intel and at Fujitsu in Kawasaki and did her master's research in Electron Spin Resonance (ESR). Shrouk has been a STEM Ambassador since 2011, teaching children about engineering solutions and most recently headed up a project, teaching maths to children of refugees.


“Sometimes you
need to take a break”

Time to Splash Out?

You saw earlier how not managing your money well can lead to expensive borrowing.

However, there are other times, such as a student loan, when you can only afford something by arranging to borrow money. You need to decide whether the borrowing is essential, e.g. for education, or for buying a home or for gratification such as an expensive holiday.

Food for Thought

All loans and other borrowings must be repaid – there is no such thing as a free lunch!

WARNING: At purchase point, many opportunities to pay by instalments may be offered to you: they may tempt and sweep you into debt.

For purchases such as an expensive bed – many shops may allow you to spread payments over a few years without charging interest.

Example

Indigo wants to buy a sofa costing **£400**. The sofa company does not charge interest if she pays the cost back over four years in monthly instalments. What is the cost of her monthly instalments?

Four years is **12 x 4 = 48** monthly instalments.

So Indigo divides the total cost by **48**:

£400 ÷ 48 = £8.33

Answer: Indigo will pay £8.33 a month for four years.

A car – this would generally be bought by paying an initial deposit and then monthly payments for about five years. There is normally interest charged on these finance deals. You should look around and work out the numbers carefully to make sure that you get the best value.

Example

Mina has decided on her first car. The price is **£13,000** and she has saved up **£3,000** which she pays as a deposit.

She then agrees to pay **£195** a month for five years.

(a) How much does Mina pay in total for the car?

(b) How much of what she pays is interest?

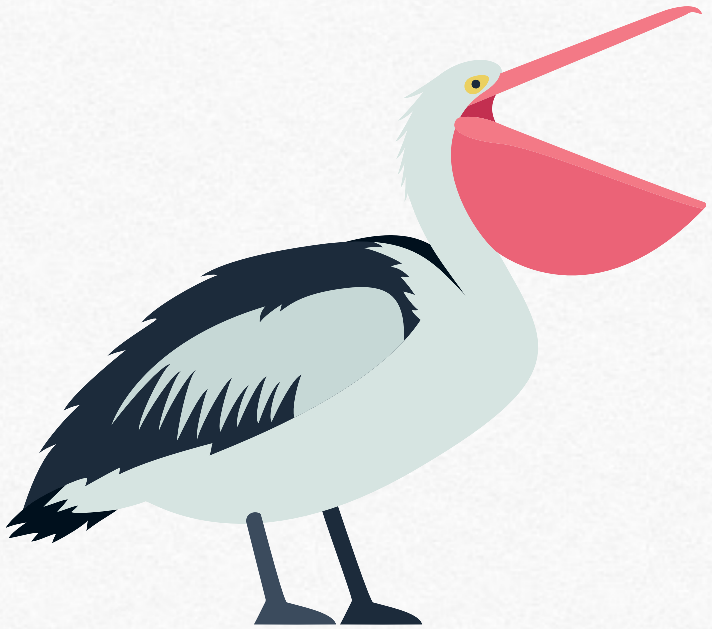
(a) Over five years Mina will pay:

5 x 12 x 195 = £11,700

Total price paid = deposit plus monthly payments

= £3,000 + £11,700
= £14,700

Answer: Mina will pay a total of £14,700



(b) Interest is the difference between the total paid and the actual price:

Interest = £14,700 – £13,000
= £1,700

Answer: Mina will have paid £1,700 interest over 5 years.

Food for Thought

Buying your first car is an important moment in your life. Hopefully, you will remember your first car with love and affection. Make sure you buy the car that is right for you. Do not be pressurised into making a purchase that might not be the one that suits you for the sake of saving a few pounds a month. You should also consider user reviews, insurance rates and fuel consumption before you make your final choice. There is plenty of information on consumer web sites to help you.

A flat or house – unless you are super rich you will need to take out a mortgage to buy a property.

This is similar to buying a car except that the numbers are larger and the length of time to repay is longer, and therefore over this time you will pay more interest.

This book is not going to cover the mechanics of arranging a mortgage but will just remind you of two things:

You will need to have saved a deposit, usually at least 10% but sometimes 5% of the purchase price.

You will need to have a good credit history.

Bear those two facts in mind as you manage your money and when the time is right, you should have no problem with getting a mortgage.

Make sure that you consult a reputable bank or building society. At times there are helpful government funded schemes to make it easier to buy your first home so look out for those.

Exercises

11 Nina is buying a new bicycle costing **£449**. She pays **£50** deposit and agrees to pay the balance off over **2** years. No interest is charged. How much does Nina pay each month?

12 Olivia has saved **£2,500** which she puts down as a deposit for her new car priced at **£14,500**. She arranges to pay **£229** a month for **5** years.

(a) What is the total amount she will have paid for her car?

(b) How much interest will she have paid?

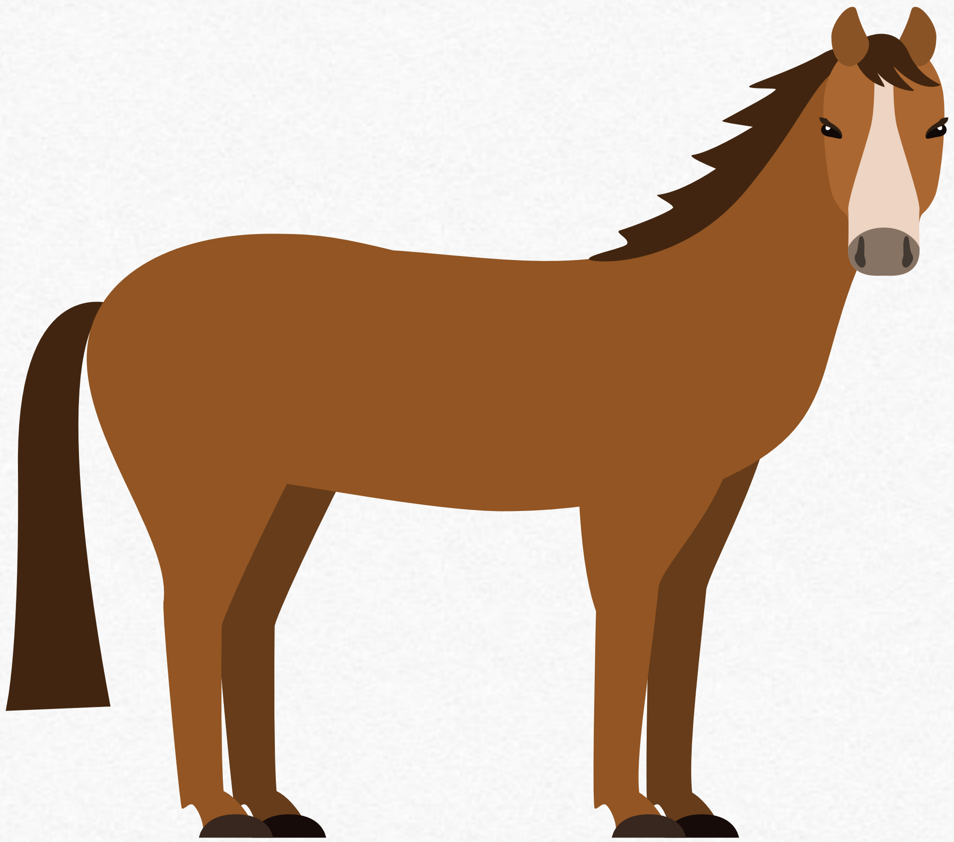
Food for Thought

When making expensive purchases be wary of other services that you may be offered. An extended guarantee or warranty is an example. These may look like a small extra amount every month but over time they can add a large amount to the total you are spending.

13 Phil is looking at buying a new laptop. She can either pay **£899** up front or spread the cost over three years by making monthly payments of **£31.85**?

(a) What is the total amount Phil will pay for her laptop if she takes the three-year monthly payment option.

(b) How much interest will she have paid?



Interest as a Percentage

When looking at credit cards you will see the term Annual Percentage Rate or APR. For a credit card, the actual interest is calculated daily as money in and money out changes frequently. For a long-term loan, the APR is slightly different as the amount and the time frame is fixed.

For long term loans the APR is sometimes quoted as 'variable' as it can be based on the national base rate, and that changes. Also, other charges are sometimes added to interest and all are quoted as APR.

APR can be useful for comparison, for example a monthly interest rate of **2%** is actually an APR of **24%** but otherwise rather than being concerned with APR, it can be simplest to work out just what percentage interest you will be paying for the loan and divide that by the duration of the loan.

When calculating the percentage, you divide the interest paid by the original value and multiply by 100:

$$\frac{\text{Interest paid}}{\text{Original loan}} \times 100$$

Then divide by the number of years and give your answer to one decimal place.

Food for Thought

Another extra charge that may be offered is called payment protection. This is supposed to help you pay the monthly charges in the event that you lose your job. However, the small print may rule out any claims and these schemes are best avoided.

Example

What percentage interest has Mina paid on her car purchase? After paying her deposit she had **£10,000** to pay back which cost her a total of **£14,700** over **4** years.

From the previous example we see that the interest in total was £1,700.

$$\text{Percentage interest} = \frac{\text{Interest paid}}{\text{Original loan}} \times 100$$

$$= {}^{1,700}/_{10,000} \times 100\ \%$$

$$= 17\%$$

$17 \div 4 = 4.25\%$ per year

Answer: Mina pays 17% interest overall or 4.3% per year.

Exercises

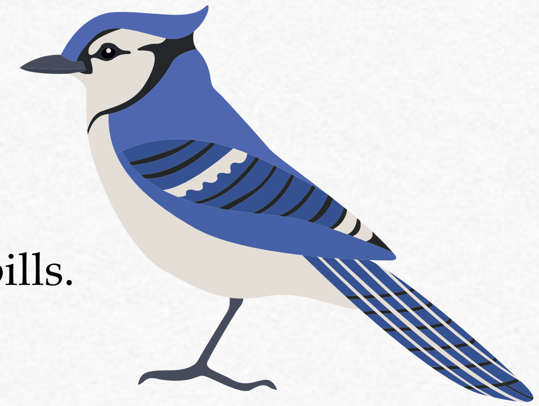
14 Calculate the percentage interest that Olivia in Q12 pays.

15 Calculate the percentage interest that Phil in Q13 pays.

Other Useful Apps and Tips

As well as your banking app and the HMRC app there are some others that you may find useful when managing money.

Always remember to check the reviews and the cost before downloading!



Mobile Phone

Download the app for your mobile network and keep a check on your bills.

Splitting the bill

When sharing a flat or going on holiday or going out for a meal, you will want to split some spending between several people.

You will probably find that some people have spent some money on joint expenses, whilst others have not.

Search for: ‘best apps for splitting bills’

Budgeting

Instead of your own spreadsheet you could use an app that does the work for you.

Search for: best budgeting apps

Saving on insurance and energy suppliers

There are many comparison web sites and it is a good idea to check that you are getting the best rate before you renew any contract.

Remember to check reviews and make sure that there are no hidden costs. With insurance, make sure that the terms of your new policy suit your needs.

Grants and other assistance

From time to time assistance may be offered to help people to make responsible purchases or to switch to more environmentally friendly options. See if you can find a Cycle to Work scheme to help you but a suitable bicycle. Look up any grants that may be available to help you insulate your home.

Food for Thought

Cycling is not only an economical way of travelling it is also very good for your fitness and overall health. Could your new best friend be your bike? Don't forget to budget for a safety helmet too!

Food for Thought

Being in control of your finances will be an enormous boost to your happiness and wellbeing.

Food for Thought

Whitney Wolfe Herd was born in **1989** and launched her first business when she was **20** by selling tote bags made from bamboo in order to raise money to help the areas affected by a BP oil spill. While still in college, she also started a clothing line to raise awareness about human trafficking and fair trade. After she graduated, she travelled to South East Asia to work with orphanages.

On her return, she joined the development team of a dating app, but left after she received online hate messages. Whitney then founded a new dating app, Bumble, that is designed to protect women from online abuse. Women have to start any conversation. Men must then respond to that message within **24** hours.

In **2021**, Whitney Wolfe Herd was named the world's youngest female self-made billionaire.

Answers to Part 2

1 Amy is starting University in September. She wants to open a bank account and has drawn up a shortlist of those banks that have been recommended to her by her friends.

Bank	0% Overdraft	Incentive
ABANK	Up to £3,000*	None
BSOC	Up to £3,000*	Amazon £10 voucher
CBank	Up to £2,000	3yr railcard + up to 1% in-credit interest
D.S.B	Up to £2,000*	Up to 1% in-credit interest

* subject to Credit score

Consider the advantages of each offer and help Amy make her decision.

Amy notes the asterisk carefully and then draws up a table:

Bank	Advantages	Disadvantages
ABANK	Higher overdraft limit	Subject to credit score No incentive
BSOC	Higher overdraft limit £10 voucher	Subject to credit score
CBank	Not subject to credit score 1% in-credit interest 3 year rail card is worth £70	Lower overdraft limit
D.S.B	1% in credit interest	Lower overdraft limit Subject to credit score

Answer: Amy decides to open an account with bank C.

It is worth looking at reviews of these banking offers. For some banks with the 'subject to credit score' noted, reviewers say that it is almost impossible to negotiate an overdraft at all.

2 In September, Chiara is moving into a new flat that she is to share with two flatmates.

Chiara has agreed that she will pay **£750** a month rent, **£120** a month towards gas and electricity and **£66** a month towards council tax.

Chiara is working as a teacher and earns **£22,400** a year after paying tax and national insurance. She knows that her monthly mobile charge if **£24** and that she will need a monthly travel season ticket costing **£140**. As it is the start of a new school year, Chiara intends to spend **£200** on some new clothes for work.

Set-up a spreadsheet for Chiara and help her budget for any other expenditure.

Answer: There is no single correct answer, but Chiara's spreadsheet could look something like this:

	A	B	C	D	E
1	**September**	Budget	Actual	Date	Difference
2	Income:	£1,866.67	£1,866.67		£0.00
3	Expenditure: Fixed				
4	Rent	£750.00	£750.00		£0.00
5	Gas and electrics	£120.00	£120.00		£0.00
6	Council tax	£66.00	£66.00		£0.00
7	Mobile	£24.00	£24.00		£0.00
8	Season Ticket	£140.00	£140.00		£0.00
9	Expenditure: Variable				
10	Supermarket	£120.00			£120.00
11	Clothes	£200.00			£200.00
12	Personal care	£25.00			£25.00
13	Coffee and snacks	£25.00			£25.00
14	Stationary	£50.00			£50.00
15	Eating out	£70.00			£70.00
16	Extra flat expenses	£25.00			£25.00
17	Books	£40.00			£40.00
18	Other	£200.00			£200.00
19	**Total**	**£1,855.00**	**£1,100.00**		**£755.00**

3 Update the spreadsheet from your answer to **Q2** with the info in the banking app.

Chiara's spreadsheet might now look like this:

	A	B	C	D	E
1	**September**	Budget	Actual	Date	Difference
2	From previous balance		£242.48		
3	Income:	£1,866.67	£1,866.67	31/8/2022	
4	In account at start of month		£2,109.15		
5	Expenditure: Fixed				
6	Rent	£750.00	£750.00	1/9/2022	£0.00
7	Gas and electrics	£120.00	£120.00	1/9/2022	£0.00
8	Council tax	£66.00	£66.00	1/9/2022	£0.00
9	Mobile	£24.00	£24.00	5/9/2022	£0.00
10	Season Ticket	£140.00	£140.00	2/9/2022	£0.00
11	Expenditure: Variable				
12	Supermarket	£120.00	£46.17	3/9/2022	£73.83
13	Clothes	£200.00	£199.45	see below	£0.55
14			£145.98	3/9/2022	
15			£53.47	3/9/2022	
16	Personal care	£25.00			£25.00
17	Coffee and snacks	£25.00			£25.00
18	Stationary	£50.00			£50.00
19	Eating out	£70.00	£35.65	2/9/2022	£34.35
20	Extra flat expenses	£25.00			£25.00
21	Books	£40.00			£40.00
22	Other	£200.00			£200.00
23	**Total**	**£1,855.00**	**£1,381.27**		**£473.73**
24	In account:		£727.88		

Chiara has added rows to tell her the amount that was in her account before and after she received her income at the end of last month and therefore the balance in the orange box is the same as in her banking app.

The column on the right, headed difference, shows how much left she has to spend according to her budget.

Chiara can see that she has already spent what she budgeted for on clothes and has already spent more than a quarter of her monthly budget at the supermarket. However, she has already paid all her essential bills. She can see that she can still meet her budget at the end of the month if she is careful not to overspend on optional items like eating out – **SUCCESS!**

“Learn where you feel like it.”

4 Debs is a student and checks her banking app mid-September after Freshers week at Uni.

You can see Debs has received her maintenance grant and paid her self-catering hall fees.

(a) Fill in the missing balances.

Today 12.20 PM	
September	
Sat 17	£1,103.45
Bar X	**-£25.17**
Supermkt	**-£45.23**
Tues 13	£1,173.85
Cash	**-£50.00**
Cash	**-£30.00**
Mon 12	£1,253.85
Taxi	**-£14.00**
BR	**-£32.15**
Mon 5	£1,300.00
OofE Hall	**-£1,800.00**
August	
Wed 31	£3,100.00
SFE ML	**£3,000.00**

b) What else do you think Debs did in Freshers week?

Debs paid for her train ticket and for a taxi as she would have had lots of luggage.

She took out cash to spend on other expenses such as stationery for her new study/bedroom. It was not enough so she took out some more (that usually means she was tempted out to go eating / drinking with some new friends)

She bought some things from the supermarket and then went out to a bar.

(c) There are **14** weeks of term left and Debs has no other source of income. Help Debs with her budgeting.

Debs should work out that she will have to pay regular amounts, probably about **£25** a month for mobile phone costs and perhaps **£35** for transport. She can then set-up a spreadsheet.

Again, there is no single correct answer but she is likely to set her spreadsheet out month by month so that she can see what she can spend each week:

	A	B	C	D	E	F
1		September	October	November	December	Remainder
2	Known					£1,103.45
3	Mobile	£25.00	£25.00	£25.00	£25.00	
4	Transport	£30.00	£30.00	£30.00	£30.00	
5	Fare home				£32.15	
6		£55.00	£55.00	£55.00	£87.15	£851.30
7	Weeks	2	5	5	2	14
8	Per month	£121.61	£304.04	£304.04	£121.61	
9	Per week	£60.81	£60.81	£60.81	£60.81	

After accounting for her known future expenses for the rest of term, Debs has divided the balance of **£851.30** by the number of weeks – **14**, and to work the amount that she has available to spend each week.

5 Evie starts her summer term at Uni.

(a) How much was Evie overdrawn before she received her maintenance grant?

On the **2**nd May, Evie should have had:

£3,000 – £1,800 = £1,200 in her account

Today 12.20 PM	
May	
Sat 7	£699.73
Mofone	**-£19.50**
Gym	**-£25.25**
Fri 6	£744.48
Cash	**-£50.00**
Tue 3	£794.48
Travelcard	**-£15.00**
Supermkt	**-£35.63**
Mon 2	£845.11
OofE Hall	**-£1,800.00**
SFE ML	**£3,000.00**

As she had only **£845.11** she must have been overdrawn by:

£1,200 – £845.11 = £354.89

Answer: Evie was £354.89 overdrawn

(b) There are seven weeks of the summer term to go. Evie will need to pay the same amount in June for her gym, telephone and travel card. If each week she spends **£35** at the supermarket and takes out **£50** cash, how much will she have left in her account at the end of term?

Evie turns to her budget spreadsheet:

	A	B	C	D	E
1		May	June	M + J	Remainder
2	Known				£699.73
3	Mobile		£19.50		
4	Transport		£15.00		
5	Gym		£25.25		
6		£0.00	£59.75		£639.98
7	Weeks	3	4		
8	Budget	£274.28	£365.70	£639.98	
9	Cash	150	£200.00	£350.00	£289.98
10	Supermarket	105	£140.00	£245.00	£44.98

Answer: If Evie sticks to spending £35 at the supermarket and £50 cash per week, then she will have just under £45 left in her account. Evie hopes there will be enough to have a bit of fun but let us hope that she remembers to keep enough back so that she can return home at the end of term!

6 Gina has decided to take up catered accommodation at her university. Although the overall cost is higher, she gets a daily breakfast and evening meal as well as lunch at weekends. The annual cost of her chosen hall is **£7,990** per year but spread over three terms:

Contract length (weeks)	Payment Term 1 (weeks)	Payment Term 2 (weeks)	Payment Term 3 (weeks)
34	14	12	8

What must Gina budget for her accommodation per term?

Term 1 = $^{14}/_{34}$ x £7,990 = £3,290

Term 2 = $^{12}/_{34}$ x £7,990 = £2,820

Term 3 = $^{8}/_{34}$ x £7,990 = £1,880

Answer: Gina will need £3,290, £2,820 and £1,880 for Term 1, 2 and 3 respectively.

7 Do a rough check to make sure that Lizzie's deductions for tax and NI for 6th March – 5th April 2022 are correct (assuming basic rate tax is 20%).

For the tax:

Monthly earnings less personal allowance:

£1,800 − £1,047.50 = £752.50

Tax = £752.50 x 20%

= £150.50 (this is the same as on payslip)

For the NI:

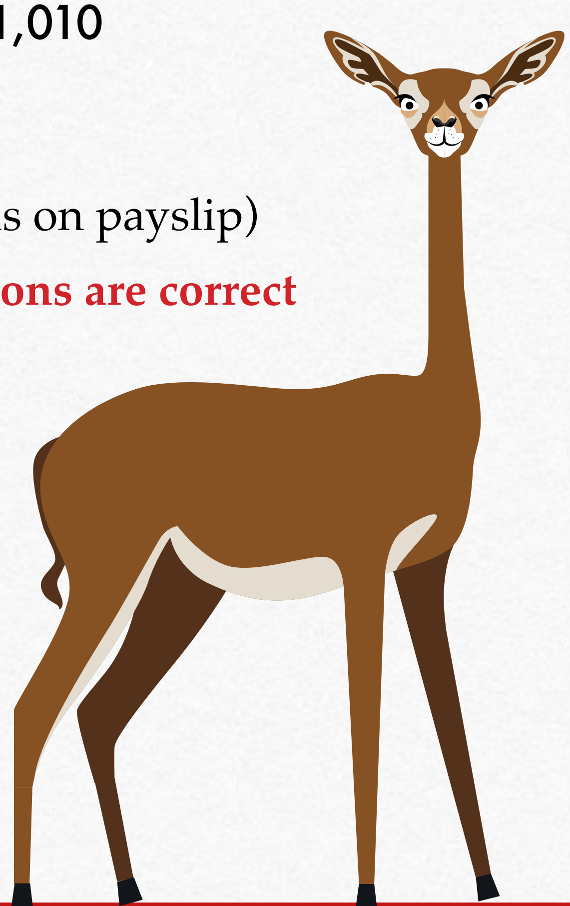
Monthly earnings less £790 = £1,800 − £790

= £1,010

NI = £1,010 x 12%

= £121.20 (almost the same as on payslip)

Answer: The tax and NI deductions are correct



8 During March 2022 Lizzie had an excellent employment review and successfully applied for a better paid job in the same company. From 6th April 2022, her annual salary will be £27,000.

Lizzie wants to work out how much money she will be paid on her next pay slip on 5th May with her new salary.

(a) What will Lizzie's gross monthly salary be?

(b) If Lizzie's tax code is unchanged and the basic tax rate is still 20%, how much tax will she pay monthly?

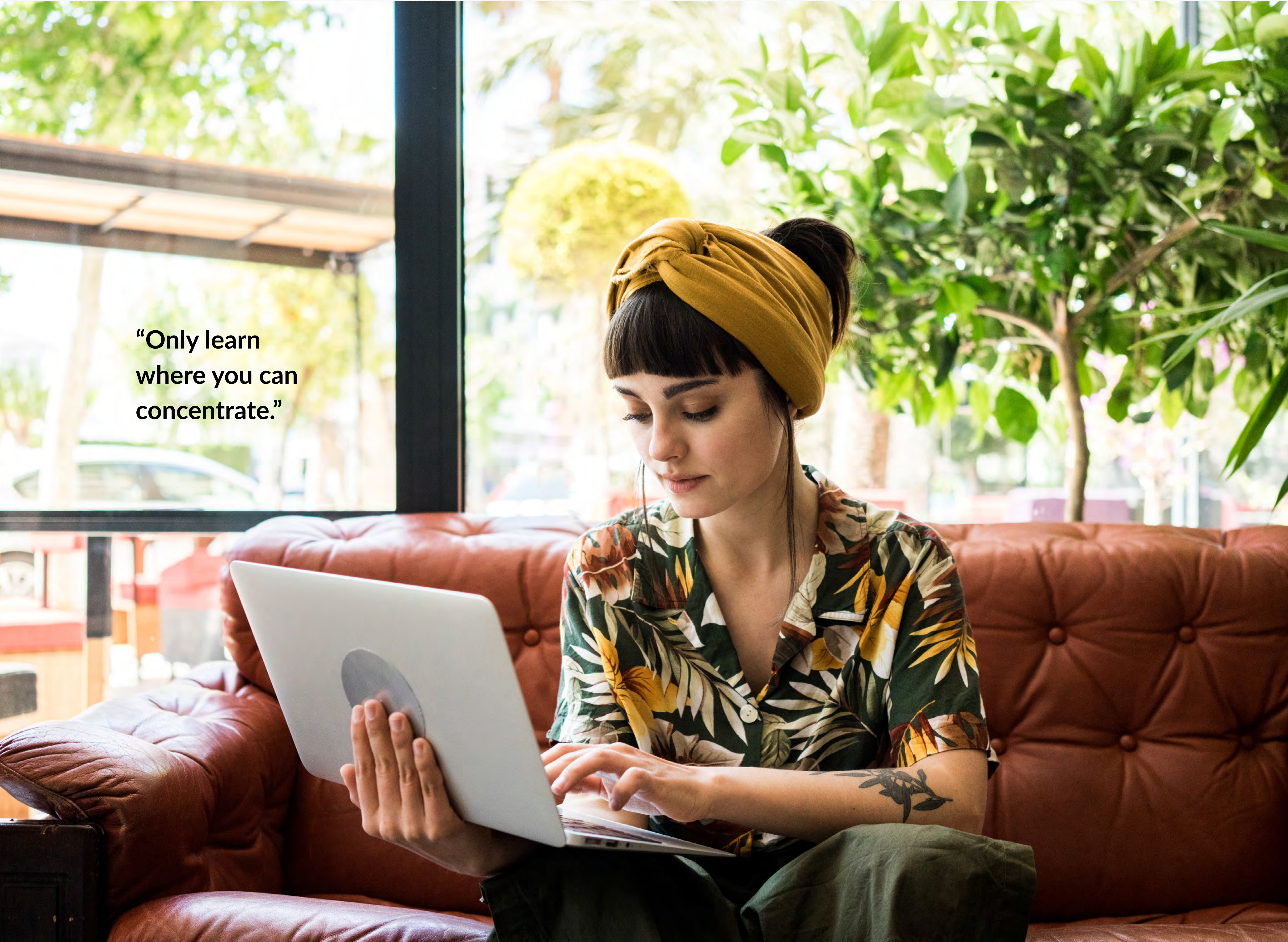
(c) If the NI thresholds and rates are unchanged how much NI will she pay monthly?

(d) How much will Lizzie's take home pay be at the end of April?

(a) Monthly gross salary = £27,000 ÷ 12

= £2,250

Answer: Lizzie's gross monthly salary will be £2,250



“Only learn where you can concentrate.”

(b) Monthly earnings less personal allowance
£2,250 – £1,047.50 = £1,202.50

Tax = £1,202.50 x 20%
= £240.50

Answer: Lizzie will pay £240.50 in tax every month

(c) Monthly earnings less £790 = £2,250 – £790
= £1,460

N.I.= £1,460 x 12%
= £175.20

Answer: Lizzie's monthly NI contributions will be £175.20

(d) Lizzie will be paid £2,250 – £240.50 – £175.20
= £1,834.30

Answer: £1,834.30

9 Lizzie Bennett decides to pay **£150** a month into the company pension scheme. She also has to start paying back her student loan. Using the figures from **Q8** and the student loan figures above, work out all the deductions and then Lizzie's net monthly salary after her pension and loan contributions have been made.

Lizzie will now contribute **£3** a month to pay off her student loan.

Lizzie's tax calculation can be based on a figure reduced by **£150**

Tax = (£1,202.50 – £150) x 20%
= £210.50

Lizzie's net pay will be based on:

Gross pay:	£2,250
PAYE tax:	£210.50
NIC:	£175.20
Pension:	£150
Student loan:	£3

Net pay = £2,250 – (£210.50 + £175.20 + £150 + £3)
= £1,711.30

Answer: Lizzie's net pay will be £1,711.30

10 **25**-year-old Jane Bennett is employed as an engineer on a salary of **£32,000** per annum. What is her net monthly salary if she contributes **£200** per month into the company pension scheme and pays **£33** a month to repay her student loan?

Being a financial whizz, Jane sets this up on a spreadsheet.

	A	B	C	D	
1	Salary	£32,000.00	Monthly	£2,666.67	Formula in D1 =B1/12
2	Personal Allowance	£12,570.00		£1,047.50	Formula in D2 =B1/12
3	Pension			£200.00	
4	Student loan			£33.00	
5					
6			Tax base	£1,419.17	Formula in D6 =D1-D2-D3
7			Tax	£283.83	Formula in D7 =D6*0.2
8					
9			NI base	£1,876.67	Formula in D9 =D1 - 790
10			NI	£225.20	Formula in D10 =D9*0.12
11					
12			Total deductions	£742.03	Formula in D12 = D3+D4+D7+D10
13			Net pay	£1,924.64	Formula in D13 =D1-D12

Answer: Jane's net monthly pay is £1,924.64

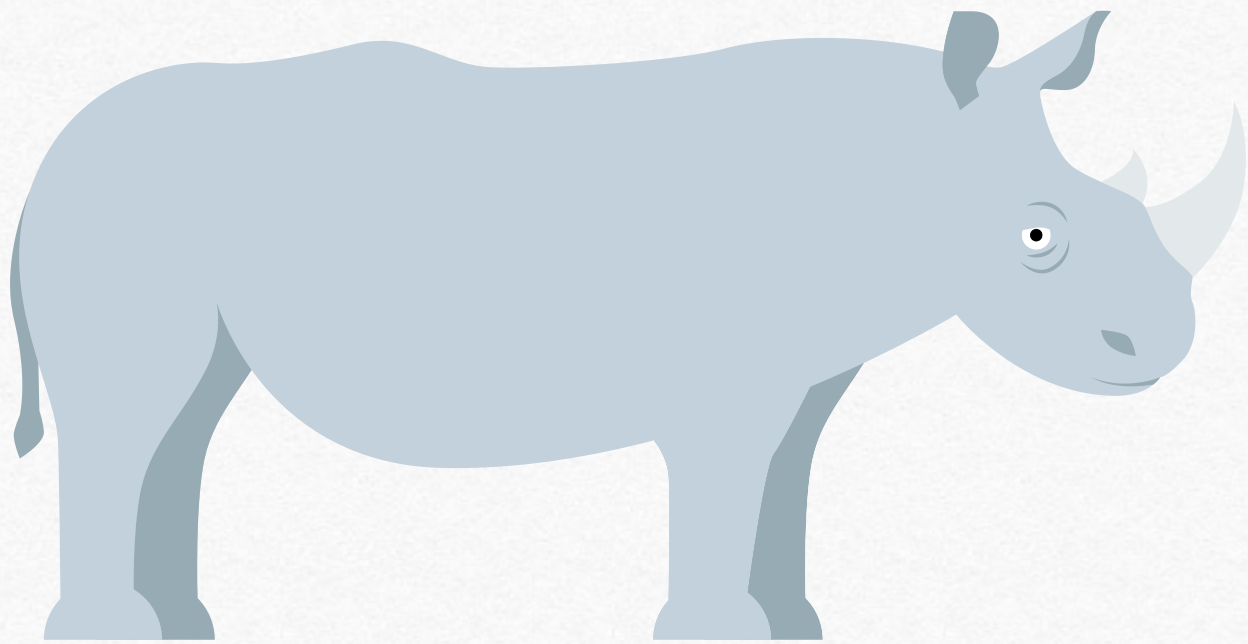
11 Nina is buying a new bicycle costing **£449**. She pays **£50** deposit and agrees to pay the balance off over **2** years. No interest is charged. How much does Nina pay each month?

Amount to pay = £449 − £50
= £399

Monthly payments = £399 ÷ 24
= £16.625
= £16.63

Remember, money is always rounded to **2** decimal places.

Answer: Nina's monthly payments will be £16.63



12 Olivia has saved **£2,500** which she puts down as a deposit for her new car priced at **£14,500**. She arranges to pay **£229** a month for **5** years.

(a) What is the total amount she will have paid for her car?

5 years at £229 a month = 60 x £229
= £13,740

Plus deposit = £2,500 + £13,740
= £16,240

Answer: Olivia will pay £16,240 in total

(b) How much interest will she have paid?

Amount of loan = £14,500 − £2,500
= £12,000

Interest paid = £16, 240 − £12,000
= £4,240

Answer: Olivia will have paid £4,240 interest

13 Phil is looking at buying a new laptop. She can either pay **£899** up front or spread the cost over three years by making monthly payments of **£31.85**.

(a) What is the total amount Phil will pay for her laptop if she takes the three-year monthly payment option?

3 years at £31.85 = 36 x £31.85
= £1,146.60

Answer: Phil will pay £1,146.60 in total

(b) How much interest will she have paid?

Interest paid = £1,146.60 − £899
= £247.60

Answer: She will have paid £247.60 in interest

14 Calculate the percentage interest that Olivia in **Q12** pays.

$$\text{Percentage interest} = \frac{\text{Interest paid}}{\text{Original loan}} \times 100\ \%$$

$$= {}^{4,240}/_{12,000} \times 100\%$$

$$= 35.33\%$$

$$\text{Interest per year} = 35.33\% \div 5 \text{ years}$$

$$= 7.1\%$$

Answer: 35.3% overall interest paid or 7.1% per year

15 Calculate the percentage interest that Phil in **Q13** pays.

$$\text{Percentage interest} = \frac{\text{Interest paid}}{\text{Original loan}} \times 100\ \%$$

$$= {}^{247.60}/_{899} \times 100\ \%$$

$$= 27.541\ldots\%$$

$$= 27.5\%$$

$$\text{Interest per year} = 27.5\% \div 3 = 9.2\%$$

Answer: 27.5% interest paid overall or 9.2% per year

Food for Thought

You will generally pay a much higher interest rate when you take out a loan that is paid back over a short number of years. That is why arranging a bank loan can be a better way of managing your money.

What will you buy when your finances are stable enough to arrange a loan?

YOUR BRAIN WORKOUT

Q1

If I earn **£21,000** a year after paying tax and National Insurance, what will I receive each month?

YOUR BRAIN WORKOUT

Q2

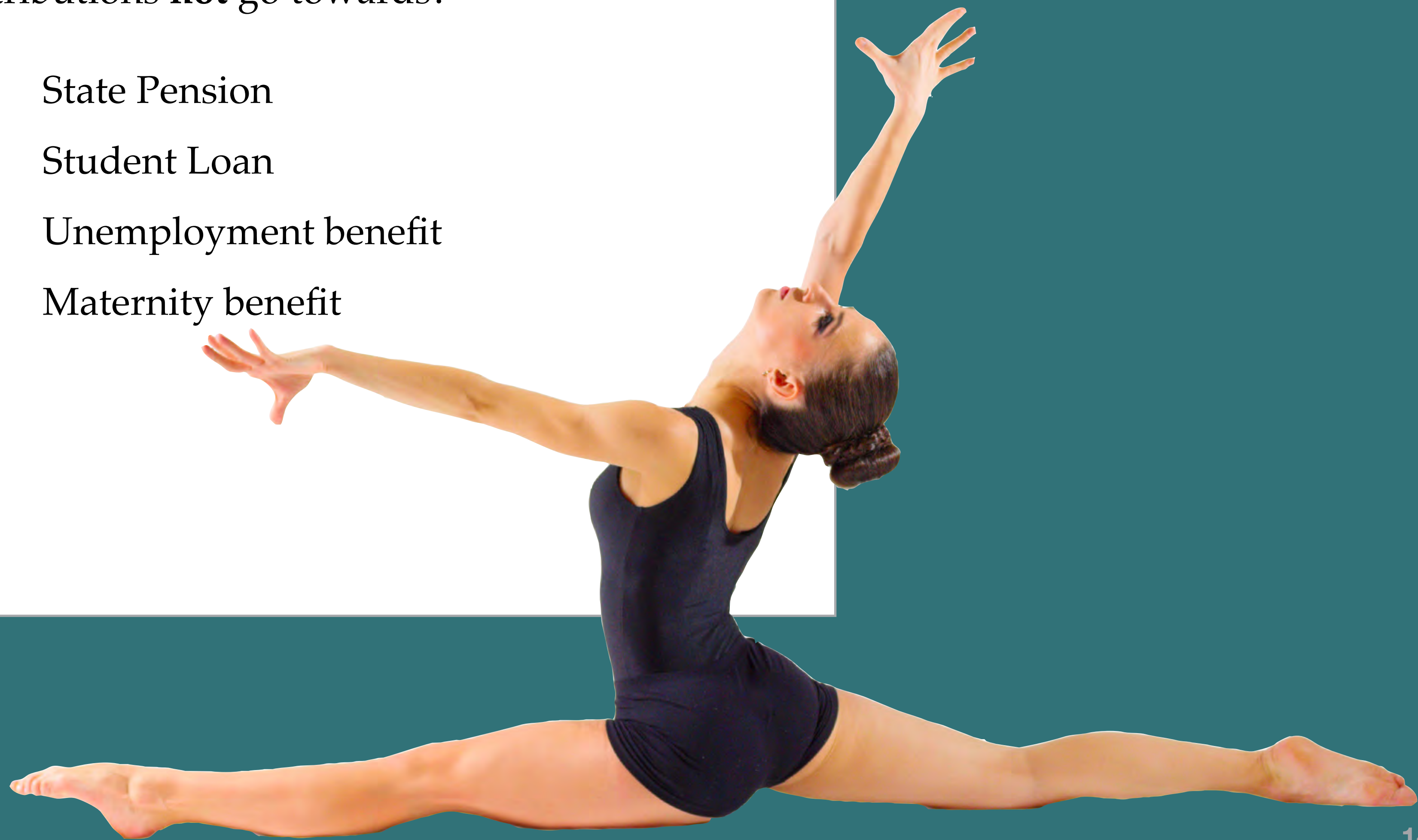
Which of these do National Insurance contributions **not** go towards?

A: State Pension

B: Student Loan

C: Unemployment benefit

D: Maternity benefit



YOUR BRAIN WORKOUT

Q3

Roughly, how much must you earn per week before you start paying National Insurance?

YOUR BRAIN WORKOUT

Q4

What is the basic rate of income tax?

YOUR BRAIN WORKOUT

Q5

My termly student loan of **£3,000** is paid into my account. My banking app tells me I now have **£2,765** in my account. What was my bank balance before my loan was paid in?

YOUR BRAIN WORKOUT

Q6

My pay slip at the end of June tells me that my pay TD is **£5,400**. What am I paid each month?

YOUR BRAIN WORKOUT

Q7

I am buying a bicycle that costs **£600**. I am going to spread the cost by monthly payments over two years. If the bike company is not charging any interest, what are my monthly payments?

YOUR BRAIN WORKOUT

Q8

I am buying a new laptop, a printer and good quality headphones. The total cost is **£1,260**. I have agreed to pay the computer store **£45** a month for three years. How much interest am I paying? Roughly, what is this as an annual percentage?

YOUR BRAIN WORKOUT

Answers

Q1	£21,000 ÷ 12 = £1,750
Q2	B: Student Loan
Q3	Over £184 a week in 2021
Q4	20%
Q5	£3,000 − £2,765 = £235 overdrawn
Q6	June is the third month of the tax year, £5,400 ÷ 3 = £1,800
Q7	£600 ÷ 24 = £25 per month
Q8	£45 x 12 x 3 = £1,620 £1,620 − £1,260 = £320 interest paid £320 ÷ £1,260 x 100 ≈ 30% 30 ÷ 3 = 10% per year

PART 3

A FORMULA FOR FORMULAE

What is a Formula?

The word formula can mean different things depending on the context.

In mathematics, a formula is a mathematical rule that links variables and is always true. (A variable can be a quantity like time or just a number, often called x.) Into the rule you can substitute values for the variables that you know to calculate one that you do not. An example is that in a circle the diameter is equal to twice the radius or:

$$D = 2r$$

A formula can also be expressed in words as in the last chapter when you saw that:

$$\text{Percentage interest} = \frac{\text{Interest paid}}{\text{Original loan}} \times 100\,\%$$

In science, a chemical formula explains the ratio of the atoms that link together to make a compound. You probably know that the chemical formula for water is H_2O because two hydrogen atoms link to each oxygen atom.

A formula can also be used to describe the proportion in which ingredients should be mixed.

You have probably heard the term ‘baby formula’ known in the US just as ‘formula’.

Baby formula is a mix of dried milk and other ingredients that is mixed with water in a precise ratio and fed to babies who are not breast fed. Babies of different ages require a different proportion of the various ingredients and so the products are age related. As you know, it is best, if possible, for babies to be breast fed.

Food for Thought

The plural of formula is not formulas – although many people do say that - but formulae as the origin of the word is Latin.

There is also the term Formula One that you may be familiar with. A Formula One car has to be built to a specific formula, it should be a single-seat, open-cockpit, open-wheel racing car to be used in competition at Formula One racing events.

You may wonder why you need to know about formulae if you are not going to be a scientist.

There are many everyday uses of formulae and also many jobs that will require you to apply formulae. The purpose of this chapter is to help you to understand how to use a formula.

Have you ever thought of becoming a Formula One driver?

Desiré Randall Wilson (born 1953) is, to date, the only woman to win a Formula One race of any kind. She won at Brands Hatch in 1980. As a result of this achievement, she has a grandstand named after her.

When Might You Need to Use a Formula?

Many professions use technology to give them data. This could be medical data such as temperature or blood pressure, it could be data about the weather or mechanical data, such as that associated with a car or piece of machinery.

Those using technology need to understand the data that they receive. Firstly, they need to be able to trust that it is accurate and then to apply the data to whatever is relevant. Applying data will often need a formula.

People that use technology are often referred to as technicians. Traditionally, a technician was someone who worked in a laboratory or who looked after technical equipment. Nowadays the word has much greater range of applications and includes people who are skilled at a technique associated with an art or a craft, such as a textile technician, environmental technician or a beauty technician.

Jobs may include the supervising or the running of machinery, but not the design of it. They may involve recording readings and entering records in a maintenance book.

You may find work in these industries really rewarding when they reflect your own interests. For example, if you are passionate about the environment, you may find yourself working in an environmental company.

If you are interested in cosmetics, then you may consider working as a technician in a bio-medical company.

This chapter is not going to teach you any technical skills, but will revise some mathematical terms and topics that might be useful.

Food for Thought

Do you remember your algebra?

A formula uses basic rules of algebra:

Letters, or variables, are used to represent numerical values. There are no multiplication or division signs.

$3ab$ means $3 \times a \times b$

$\frac{pq}{s}$ or $^{pq}/_{s}$ both mean $p \times q \div s$

You could also consider some formulae that you already know. Formulae for the area of different shapes, or formulae for calculating interest.

A Formula for Formulae

In the last Part, you looked at a formula for calculating percentage interest:

$$\text{Percentage interest} = \frac{\text{Interest paid}}{\text{Original loan}} \times 100\ \%$$

You then substituted in the amount of interest paid and the original price and calculated the percentage rate.

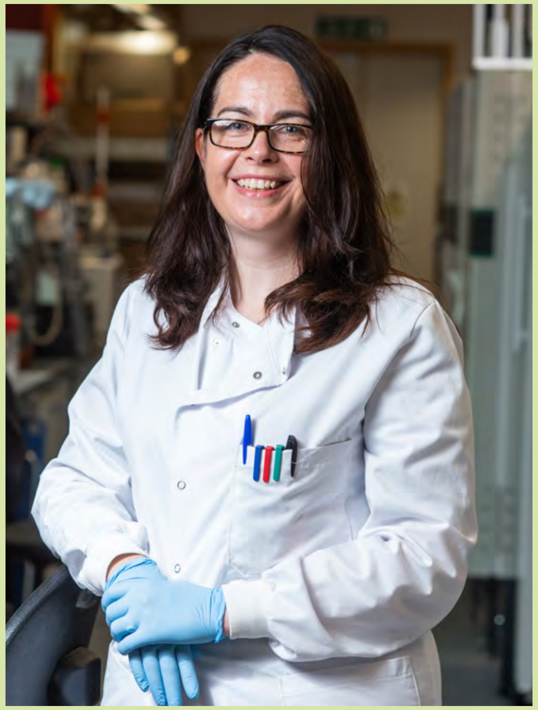
That is it! The formula for formulae is simply to substitute numbers and then calculate.

Do not be put off if a formula includes complicated arrays of letters and mathematical terms like square roots. You just need to follow the steps.

Food for Thought

When a vaccine is developed, the ingredients that go into it are made up to a formula. As the vaccine is tested and improved, the proportion of the ingredients, or the formula, may change. In 2020, Professor Sarah Gilbert and her colleague, Professor Teresa Lambe were quick to create a vaccine that was successful in preventing people suffering from the severe symptoms of the Covid-19 virus.

Professor Sarah Gilbert



Professor Teresa Lambe

The professors were assisted by a team of laboratory and other technicians.

When working with a formula you just follow these five simple steps.

1. Write down the **formula**

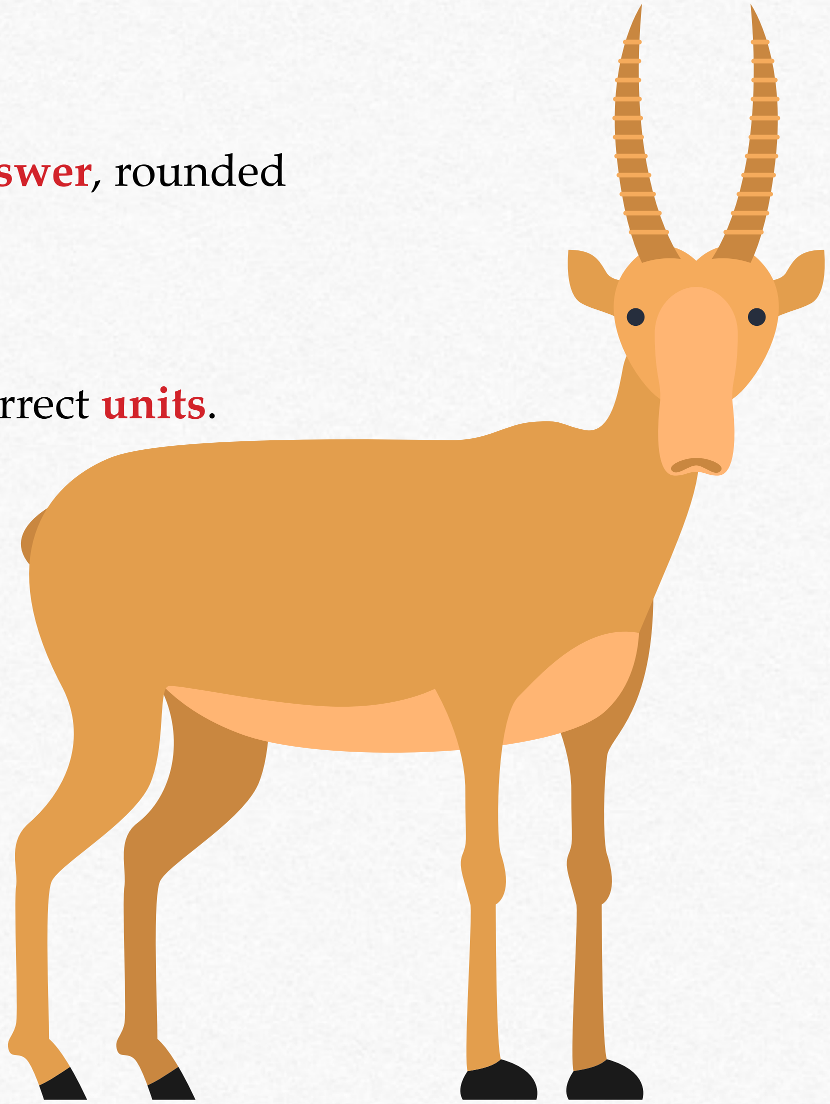
2. **Substitute** in all the numerical valued you have

3. **Calculate**

4. Write your **answer**, rounded if necessary

5. Include the correct **units**.

And that is it!



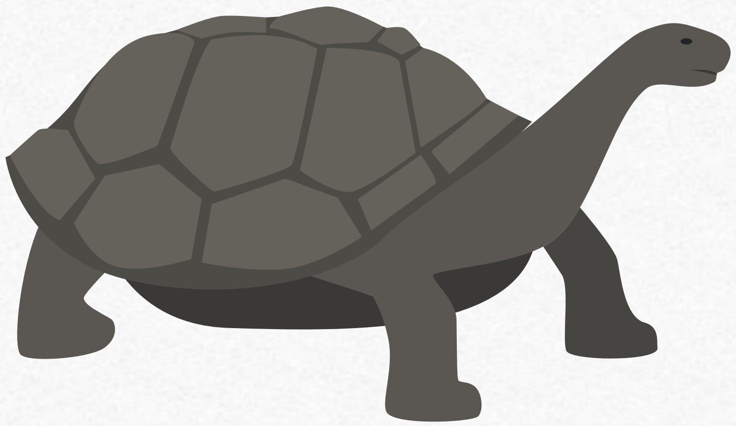
You may notice that sometimes the letters in a formula use a 'squiggly' font (italics). This is so that the letters or variables are not confused with symbols such as the **x** that means multiply.

There are other formulae that you may come across that do not use letters at all but words or phrases, like the one for percentage interest.

Example

If $S = {}^{d}/_{t}$ what is the value of S if $d = 60$ and $t = 5$?

$S = {}^{d}/_{t}$	Formula
$= {}^{60}/_{5}$	Substitute
$= 60 \div 5$	Calculate
$= 12$	Answer



Food for Thought

Do not try and skip a stage such as calculating before you have substituted, that leads to mistakes. Think of substitution like a team sport, send a player off and then substitute another. Only when the substitution is complete do you play the game.

In this next example the formula is in words:

To roast beef, cook the joint for 20 minutes per 500 g plus 20 minutes. How long should I cook a joint of beef weighing 3 kg?

3 kg is 6 x 500 g

Time = 20 mins per 500 g plus 20 mins	Formula.
= 20 x 6 + 20	Substitute
= 120 + 20	Calculate
= 140 mins	Answer
= 2 hours 20 mins	Units

Answer: 2 hours 20 minutes cooking time for a 3 kg joint

The formula in this next example may look more complicated but just follow the steps, you may need another line of calculation if you are not using a calculator.

If you are using a calculator app you will probably need to turn your device sideways to find the right functions:

To find $\sqrt{25}$, try entering:

$^2\sqrt{x}$ | 2 | 5 | =

You should have the answer **5** because **5** is the square root of **25**.

Example

If $P = \sqrt{a^2 + b^2}$, find the value of P when $a = 3$ and $b = 4$.

$P = \sqrt{a^2 + b^2}$	Formula
$= \sqrt{3^2 + 2^2}$	Substitute
$= \sqrt{9 + 16}$	Calculate
$= \sqrt{25}$	Calculate
$= 5$	Answer and Units (no units were used in this example)

The calculation you will do will be slightly different if you are using a scientific calculator rather than a calculator app.

If the app then you would enter:

3 | X^2 | + | 4 | X^2 | = | $^2\sqrt{x}$ | =

That may also work on some scientific calculators. Test your scientific calculator and see how you get to the correct answer, you may need to type in:

$^2\sqrt{x}$ | 3 | x^2 | + | 4 | x^2 | =

Exercises

1. Amy is an archaeologist. She estimates the heights of the inhabitants of an ancient site using the formula:

Height = 7 x length of foot

If Amy measured 3 footprints: 8.5 cm, 17.2 cm, 24 cm, what did she calculate the heights of the three inhabitants of the site to be? What conclusions about the owners of the footprints might she draw?

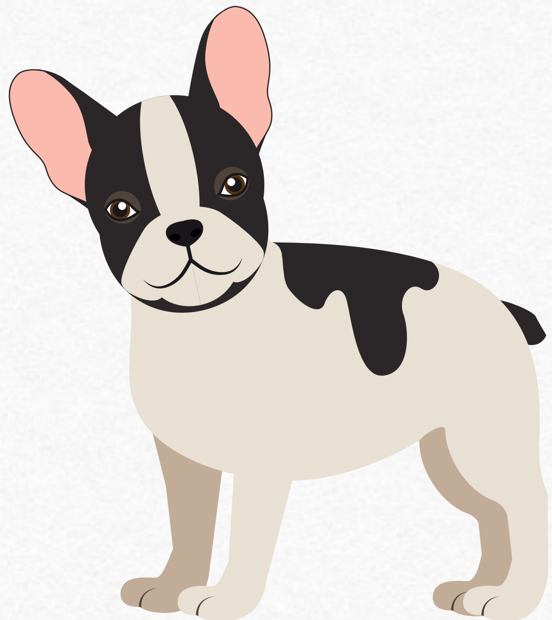
2. The cooking time for a chicken is determined by the formula:

Cooking time in minutes = 45 x weight in kg + 20

For how long should Bea roast a chicken weighing 2 kg?

3. If $a = 5$, $b = 12$ and $c = 15$, find the value of:

(a) P when $P = ab$
(b) Q when $Q = {}^{c}/_{a}$
(c) R when $R = {}^{bc}/_{2a}$
(d) S when $S = a^2+b^2$
(e) T when $T = \sqrt{a^2+b^2}$



"This isn't a dog biscuit, it's an ancient footprint."

Food for Thought

Mary Anning was born in Lyme Regis, Dorset, in May 1799. Her father was a cabinet maker but ran a sideline selling fossils. As children, Mary and her brother Joseph would look for fossils but Mary took over the whole fossil business once her brother became an apprentice to an upholsterer. Amongst Mary's discoveries were the first correctly identified ichthyosaur skeleton; the first two nearly complete plesiosaur skeletons; a pterosaur skeleton and fish fossils. Mary became an expert in the anatomy of these extinct creatures that she dug up.

A picture of Mary Anning and a pliosaur at the Natural History Museum, London

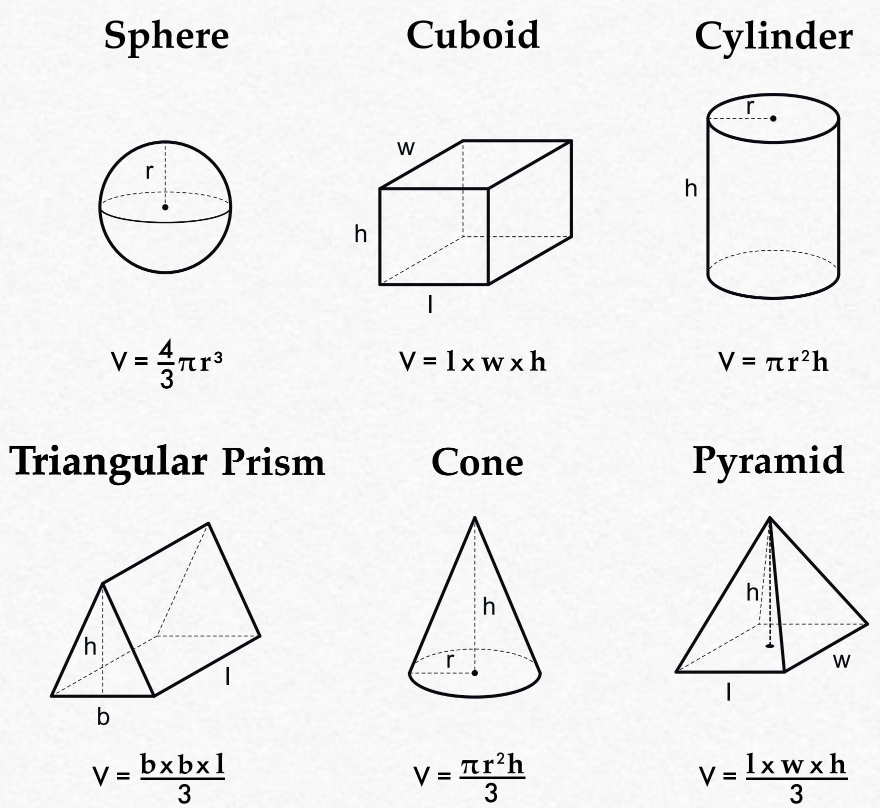
Although, as a woman, Mary Anning was not eligible to join the Geological Society of London, she became well known in geological circles in Britain, Europe, and America. Her discoveries changed scientific thinking about prehistoric life and the history of the Earth. The 2020 film 'Ammonite' is loosely based in her life and work.

Formulae for Volume

You probably know that the formula for the volume of a cube of side a:

$$V = a^3$$

Here are some other formulae for the volume of solid shapes:



You use volume formula just like any other. In 3-D shapes with circular cross sections you have the symbol π. Do you remember that π = 3.1415926535897932... but for everyday purposes two decimal places are sufficient, so 3.14. For estimates and rough calculations you can take π to be 3.

When using a calculator or calculator app, you should find and use the button marked π.

Example

How much medicine will fill a syringe of radius 5 mm and length 6 cm?

Note that a syringe is a cylinder. First change the radius to cm:

5 mm = 0.5 cm

Volume	$= \pi r^2 h$	Formula
	$= \pi \times 0.5^2 \times 6$	Substitute
	$= 4.712...$	Calculate
	$= 4.7$	Answer
	$= 4.7\ \text{cm}^3$	Units

Answer: The syringe holds 4.7 cm^3 or 4.7 ml

On your calculator app you will have entered the following sequence:

3 x 0 . 5 X^2 x 6 =

Check that you know how to get the correct answer before continuing.

Exercises

Follow the **5** steps for using a formula and the formulae on the previous page to calculate these. Give your answers to the nearest **cm³** (TIP: Change all dimensions to **cm** before you start.)

4. Find the amount of air needed to fill an inflatable ball of radius **15 cm.**

5. Find amount of water needed to fill a cylindrical bucket of radius **15 cm** and height **0.5 m**. What is this in litres? (Note: **1** litre **= 1,000 cm³**).

6. Calculate the amount of glass needed to make a triangular prism with b **= 35 mm**, h **= 40 mm** and l **= 25 cm.**

7. (a) Calculate the amount of ice cream that will fill a cone of radius **25 mm** at the widest point and height **12 cm**.

(b) Now calculate the volume of a spherical scoop of ice cream of radius **25 mm.**

(c) Add your two volumes together to find the total volume of ice cream.

8. Calculate the volume of a marble paperweight in the shape of a pyramid with l **=** w **=** h **= 6.5 cm**.

9. Find the volume in **m³**, of a tent in the shape of a triangular prism of height **1.5 m**, base **1.8 m** and length **3 m.**

10. The great pyramid at Giza, Egypt has sides of 230 m and a height of 146 m. What is its volume to the nearest 100 cubic meters?

Formulae and Rates

There are some professions that use a formula to work out their charges based on an hourly rate.

There are other times you will be charged using a formula, even though you may not realise it.

Time as a fraction

When calculating charges based on an hourly rate, the time may be less than an hour. You will then need to think of time as a fraction. Some fractions you know:

30 minutes = ½ hour

15 minutes = ¼ hour

45 minutes = ¾ hour

If you need to work out other fractions of an hour, do remember that there are 60 minutes in an hour.

To simplify fractions, you divide the top and bottom numbers by a common factor:

Example

What is 10 minutes as a fraction of an hour:

10 minutes = $^{10}/_{60}$ hour = $^{1}/_{6}$ hour

You can see that 10 and 60 were both divided by 10.

Exercise

11. Write these minutes as fractions of an hour:

(a) 20 minutes (b) 40 minutes (c) 25 minutes

Now, see how rates are used in a formula. You will see the same steps are used:

Formula, substitute, calculate, answer, units

Example

A plumber works out his charge for a job using the formula:

Charge = call out fee plus number of hours **x £60**

If his weekday callout fee is **£50** what will he charge for a job that takes **45** minutes?

Charge = call out fee plus number of hours **x £60**

= £50 + $\frac{3}{4}$ x £60

= £50 + 3 x £15

= £50 + £45

= £95

Answer: The plumber charges £95

Have you ever seen an electricity bill?

This uses rates too.

Example:

This is Chrystelle's electricity bill. She cannot read the last figures as she has spilt her coffee over them:

Current period		1st Dec–28th Feb		90 days
Previous reading	**Current reading**	**Usage**	**Rate**	**Tota**[illegible]
18,395	19,151	756	17.13 p	[illegible]
Days	90	Daily rate	16.80 p	[illegible]
			Total	[illegible]

Calculate the total charge for the **3** months. What she should budget per month in future?

Chrystelle could work out each line and then add them up. Instead she chooses instead to use the formula:

Total = (usage x unit rate) + (days x daily rate)

After substituting the values from her bill, Chrystelle calculates:

Total = 756 x 17.13 + 90 x 16.80
= 14,462.28p
= £144.62

Answer: The total charge for the three months is £144.62

Chrystelle then divides by 3 to work out the price per month:

£144.62 ÷ 3 = £48.206. . .

Answer: Chrystelle should budget £48.21 per month

Food for Thought

In fact, Chrystelle is unlikely to use the same amount of electricity every month throughout the year. She will almost certainly use more in the winter, as she will need to heat her flat, and less in the summer.

Exercises

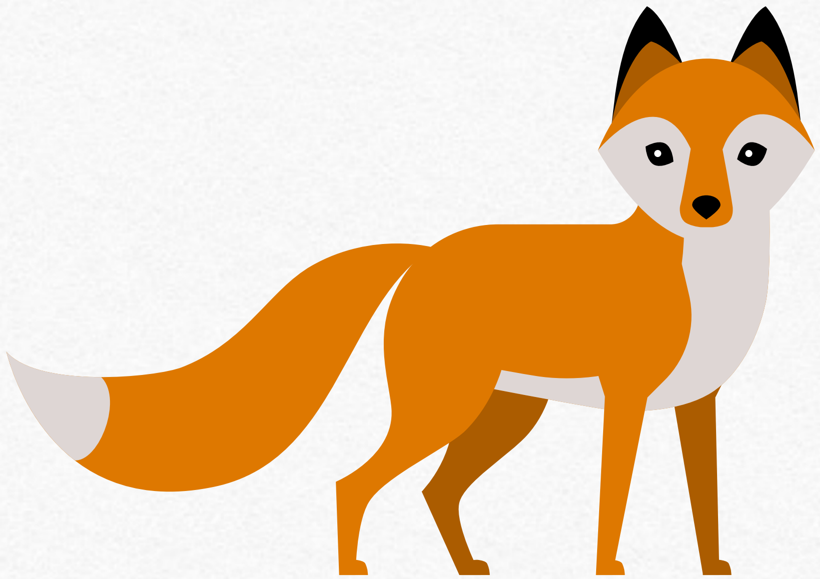
12. The plumber in the above example charges a callout fee of £75 at weekends. What will he charge for a job on a Saturday that takes one and a half hours?

13. Della is a hairdresser who visits customers in their own homes. She charges her customers using the formula:

Charge = £25 for the visit plus £45 an hour.

(a) What will she charge a customer who wants a simple haircut that took 20 minutes?

(b) What will she charge a customer who wanted a cut and colour that took an hour and 40 minutes?



14. This is another electricity bill for Chrystelle:

Current period		1st Jun–31st Aug		..... days
Previous reading	Current reading	Usage	Rate	Total
19,561	19,836	756	17.13p	....
Days	....	Daily rate	16.80p	....
			Total	....

Calculate the missing values and work out her total bill for that quarter. (Note: Three months is often called a quarter since it is a quarter of a year.)

15. A mobile telephone company charges using the formula:

Total = £20 per month for up to 300 minutes + 40p per minute for additional calls.

(a) What will Emma's monthly bill be if her total call time is 240 minutes?

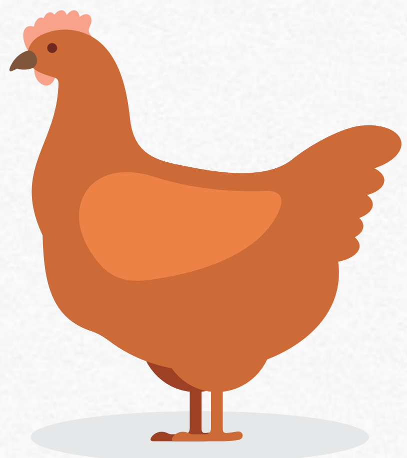
(b) What will Fran's monthly bill be if her total call time is 380 minutes?

16. Gemma is a freelance writer. She charges £0.10 per word.

(a) What will Gemma earn for a 1,500 word article?

(b) If it takes Gemma 3 hours to write, edit and deliver her article, what is her rate of pay per hour?

(c) Gemma is asked to quote a fixed fee for an article that will involve some research, not just writing. Gemma estimates that the whole project will take her 3 working days or 24 hours in total. What should she quote based on her hourly rate?



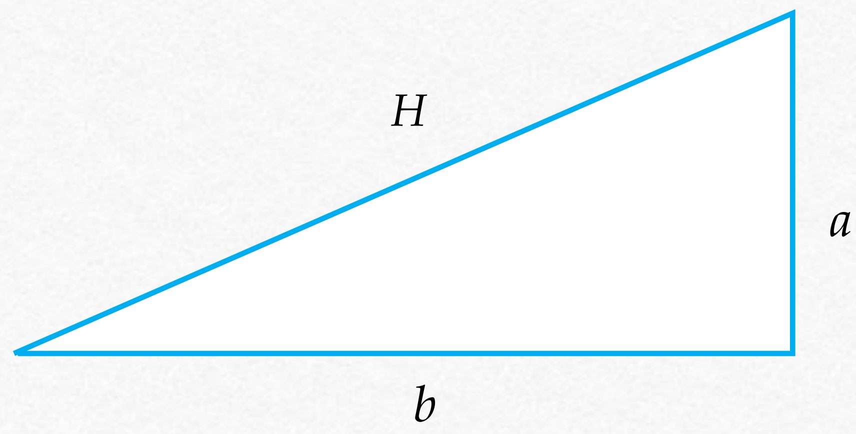

“I can afford the clothes I want”

Pythagoras's Theorem

(or can I get it through the door)

Although the ancient Greek mathematician Pythagoras is credited with the theorem that describes a formula, it was known at least a thousand years earlier both in ancient Babylon and also in China.

The formula relates to a right-angled triangle and states that the square of the longest side, the hypotenuse, is equal to the sum of the squares of the other two sides:



Therefore $H^2 = a^2 + b^2$

You do not need to know WHY this is true or how to PROVE it (although you can find out very easily if you are interested) but as a right angled triangle is half a rectangle, this is a very useful formula to know. You will probably find it quickest to use it in the form:

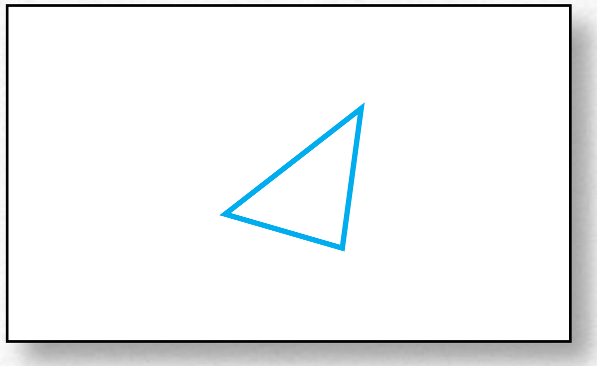
$$H = \sqrt{a^2 + b^2}$$

as you can put the calculation very easily into your calculator.

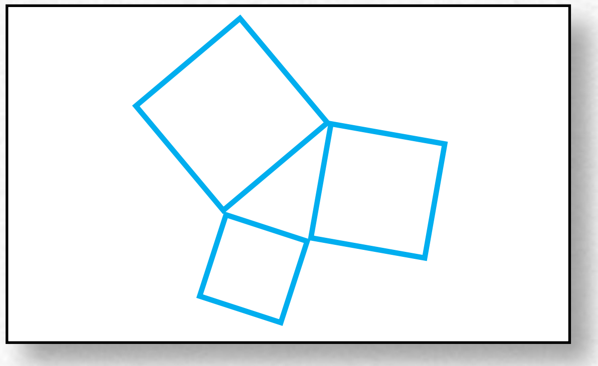
Perigals's Dissection

This is an activity to help you understand the formula.

First, draw a right-angled scalene triangle near the middle of a large sheet of paper. It does not matter exactly what its dimensions are. Draw your triangle askew like this.

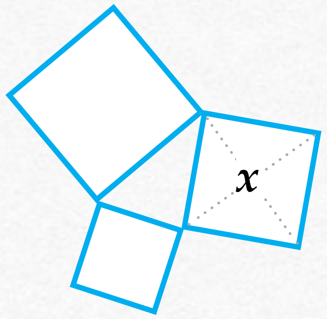


Next draw a square on each side of the triangle. Your paper will look like this.

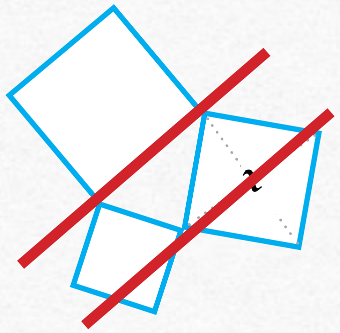


Your right angle triangle is now surrounded by three squares, one large, one small and one middle-sized.

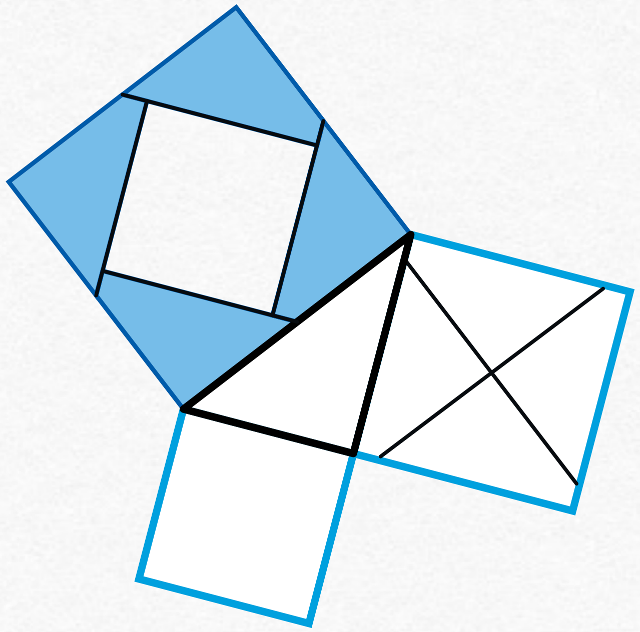
Take the middle-sized square, draw two diagonals very lightly to find its centre.



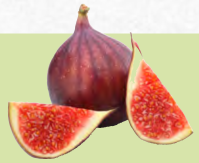
You now want to draw a line through the centre parallel to the hypotenuse. Line up a ruler on the hypotenuse, carefully slide it towards the centre of the middle sized square and draw the line. Now draw another line at right angles to that one:



Now trace over the smallest square and the middle sized square with its two lines. Cut out your tracings out so you have five pieces. Now arrange your **5** pieces in the big square:



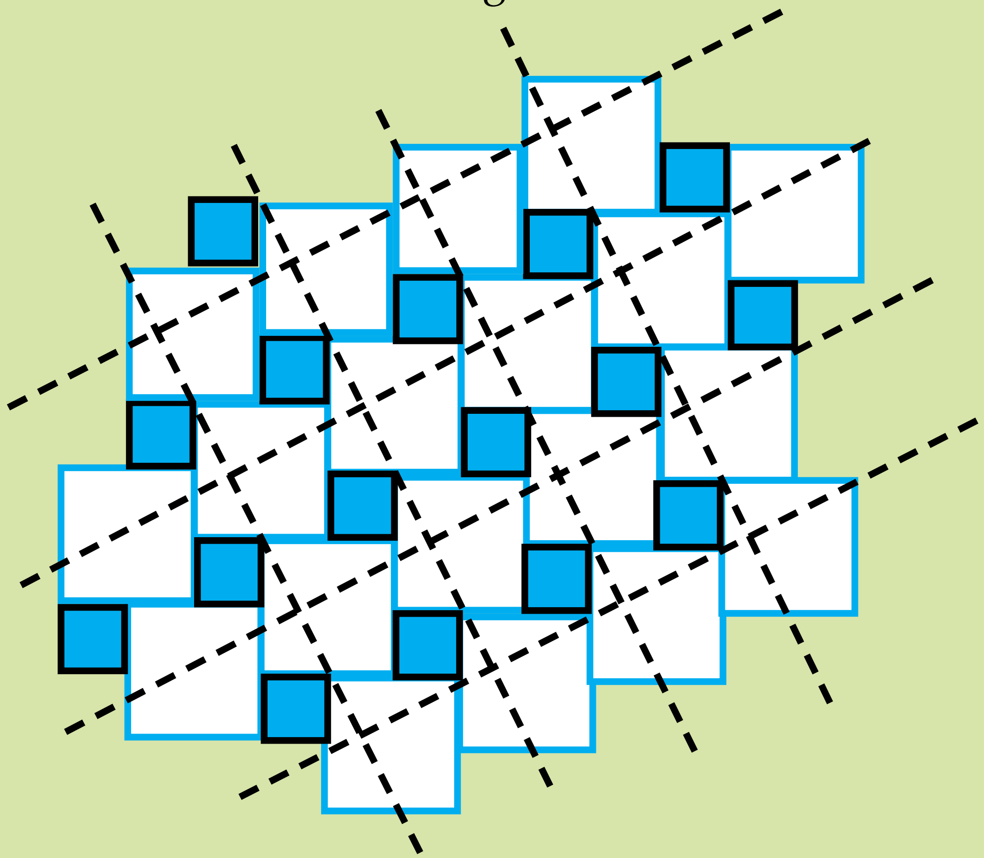
You can now see how the square on the hypotenuse is equal to the sum of the squares on the other two sides!



Food for Thought

This jigsaw puzzle to demonstrate the relationship between the triangles of a right-angled triangle was designed by Henry Perigal (1801 –1898). Henry was a British stockbroker who was also an amateur astronomer and a mathematician.

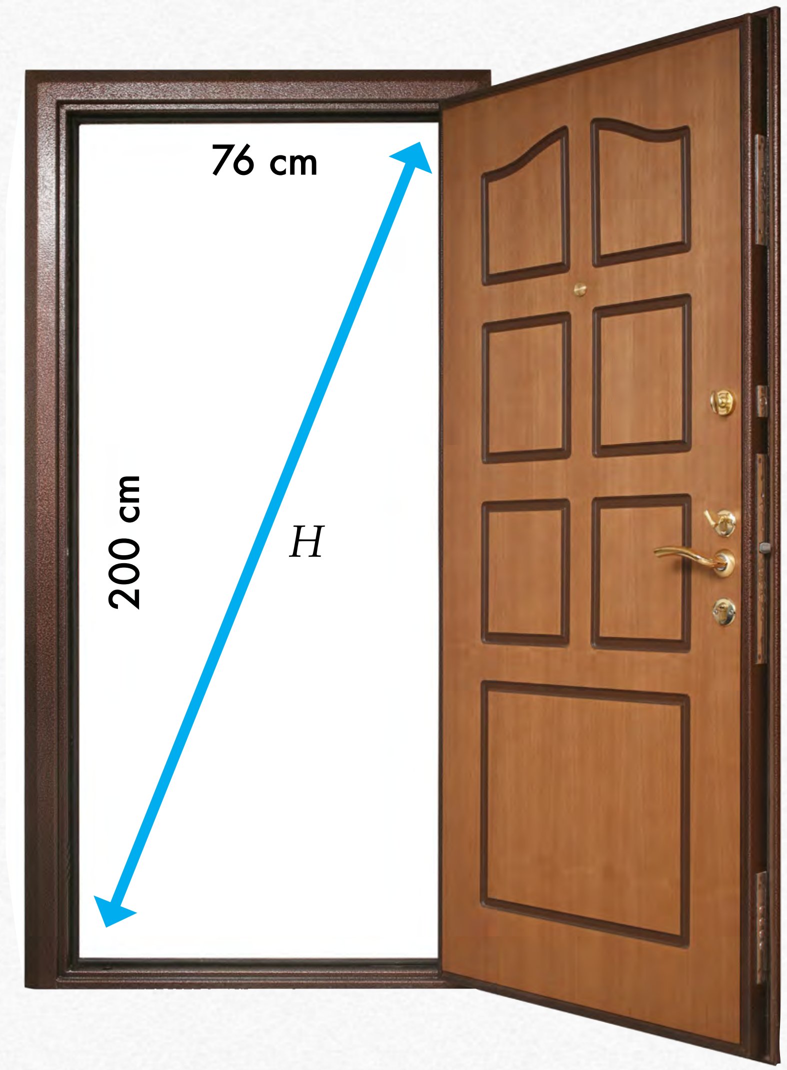
You may find examples of Perigal's clever proof of the theorem $H^2 = a^2 + b^2$ in designs:



Example

My front door is **2 m** tall and **76 cm** wide. Can I get a picture measuring **210 cm** by **210 cm** through the door?

Clearly, the picture is bigger than the height of the door, but suppose I tilt the picture, will it fit through the diagonal?



Change the **m** to **cm** and use the formula to calculate the length of the diagonal:

$$\begin{aligned} H &= \sqrt{a^2 + b^2} \\ &= \sqrt{200^2 + 76^2} \\ &= 213.95\ldots \\ &= 214 \text{ cm} \end{aligned}$$

Answer: As long as the picture is not too thick, I should be able to get it through the front door.

Food for Thought

Look around you. Just how many rectangles can you see?

Doors, windows, tables, books, computer screen . . .

Look outside too: Houses, shop fronts, playing fields, car parking spaces, shopping trolleys . . .

Food for Thought

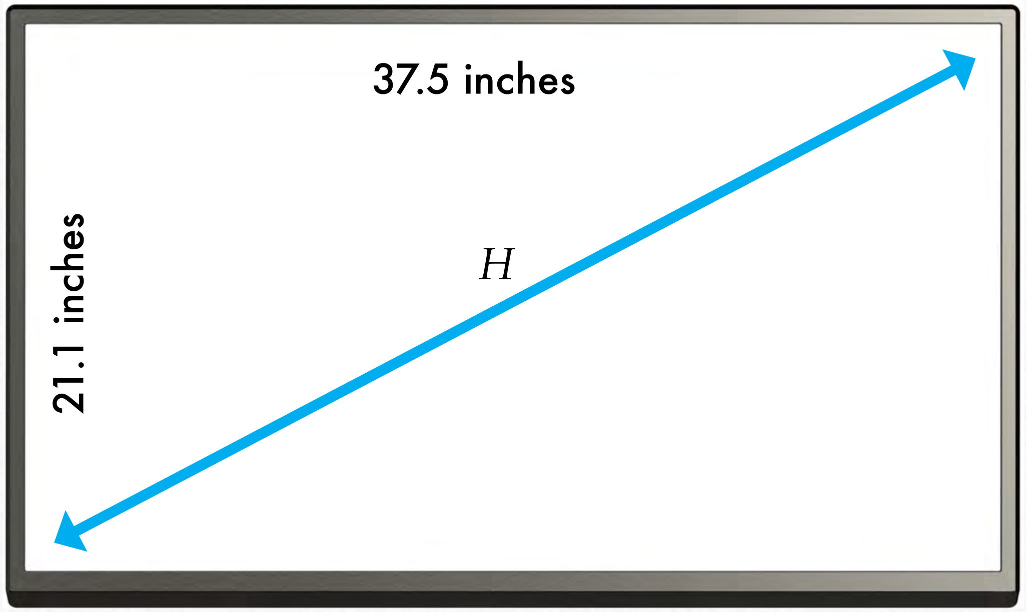
Have you ever bought a television? Although the metric system is in common use throughout the UK, TV screens are still quoted in inches. Just to add to the confusion, a 43 inch TV is not one that is 43 inches wide, but one with a diagonal of 43 inches. When shopping you will see this is written 43″, the symbol ″ stands for inches.

Quick Tip

You should always draw a sketch before you answer any questions about shapes and label it with any given measurements. This ensures that you get the correct dimensions in the right places.

Another example

What is the advertised size of a TV that is 37.5″ wide and 21.1″ high?



$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{37.5^2 + 21.1^2}$$
$$= 43.028\ldots$$
$$= 43 \text{ inches}$$

Answer: This is a 43 inch TV

Exercises

For each of the following questions, start by sketching a right angled triangle and identifying which side is H.

17. Work out the length of the diagonals in these rectangles:

(a) 3 m wide and 4 m high

(b) 120 cm wide and 50 cm high

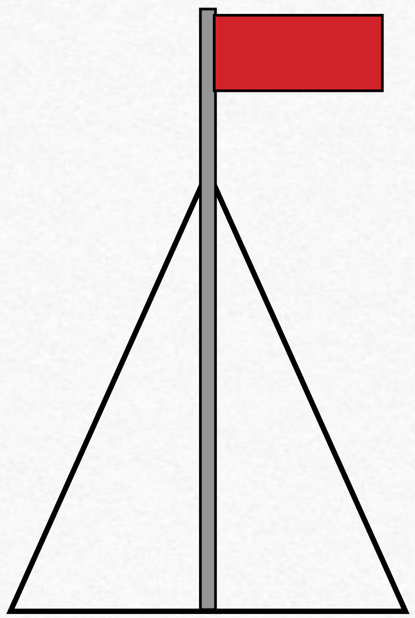
(c) a square measuring 20 cm by 20 cm

18. A playing field is 105 m by 68 m. Rather than walking round it, Holly takes a short cut along the diagonal.

(a) How far is Holly's walk along the diagonal?

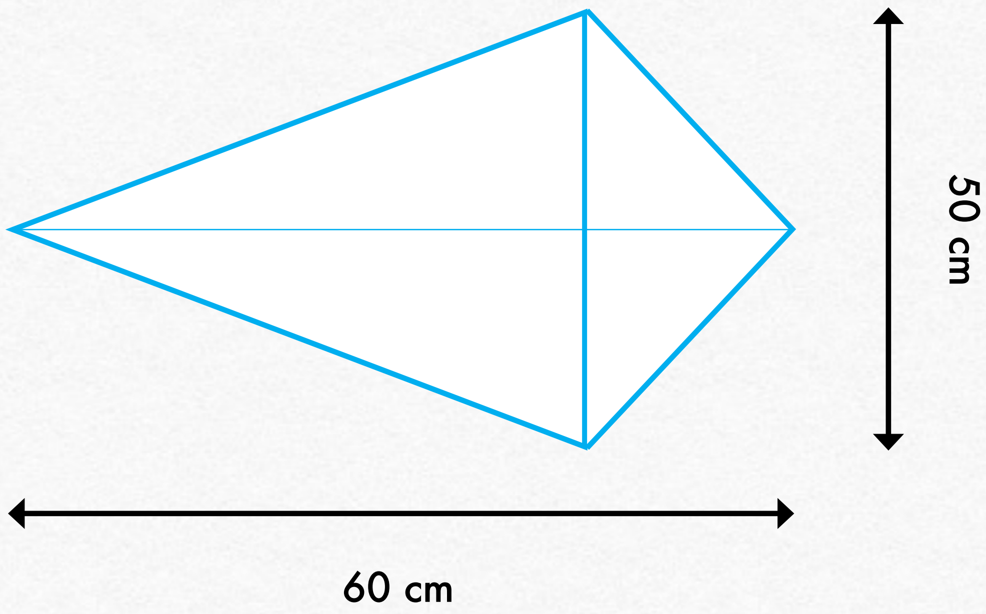
(b) How much shorter is the distance Holly has walked than if she had walked around two sides of the field?

19. A flagpole is held up by ropes that run from a point 4 m up the pole to a point on the ground 1.5 m from the base of the pole. How long are the ropes?



20. Irene has folded a silk square of sides **40 cm** in half to make a triangular scarf. She now needs to sew some trim round three sides of the scarf. What is the length of the trim?

21. Jilly is making a kite. Her kite has two pieces of wood as its diagonals, one **50 cm** long and the other **60 cm** long. The shorter stick is placed a quarter of the way down the longer stick. What are the lengths of the sides of the kite?



Finding a Shorter Side

So far you have calculated the longer side of a right-angled triangle. To find the length of one of the shorter sides of the triangle, you need to rearrange the formula:

$$H^2 = a^2 + b^2$$

Now subtract b^2 from both sides of the formula:

$$H^2 - b^2 = a^2$$

Swap the sides:

$$a^2 = H^2 - b^2$$

Take the square root:

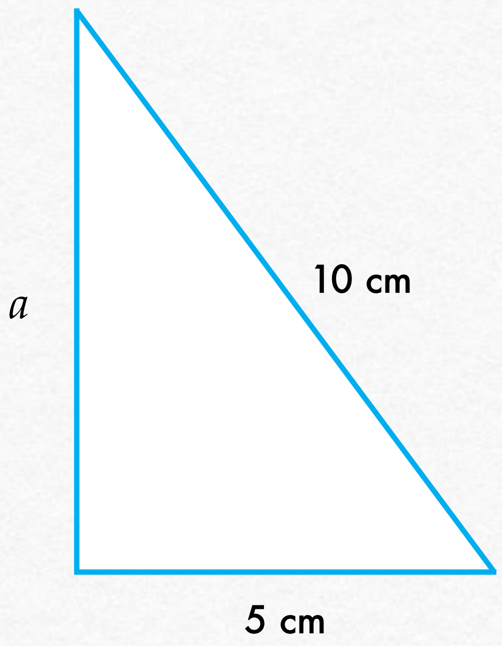
$$a = \sqrt{H^2 - b^2}$$

It does not matter which of the shorter sides you call a or b, just use the formula by subtracting the square of one side from the square of the longer side before pressing the square root button.

Example

A right-angled triangle has a base of **5 cm** and a hypotenuse of **10 cm**. What is its height to one decimal place?

Let the unknown value (height) be a



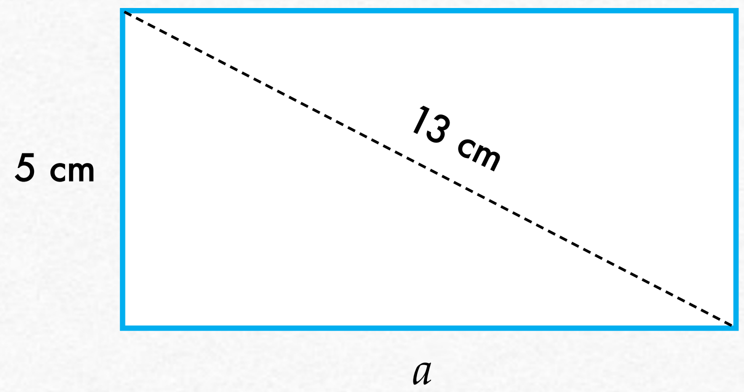
$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{10^2 - 5^2}$$
$$= 8.660...$$
$$= 8.7 \text{ cm}$$

Answer: The height of the triangle is 8.7 cm

Another Example

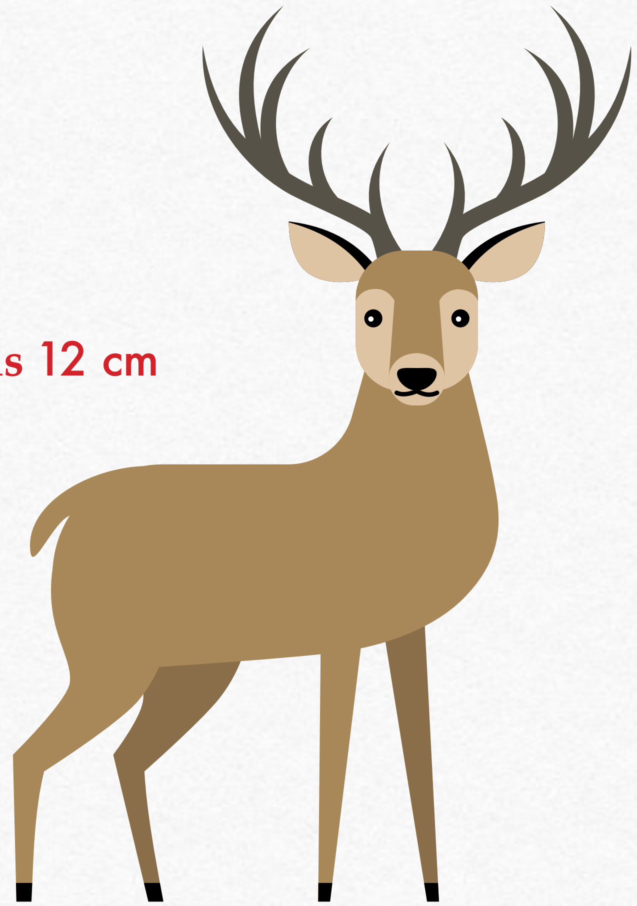
A rectangle has height **5 cm** and its diagonals are **13 m** long. What is its width?

Let the unknown value (the width this time) be a



$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{13^2 - 5^2}$$
$$= 12 \text{ cm}$$

Answer: The width is 12 cm



Food for Thought

You may have spotted that twice now the answer has been an exact number. The first was when

$5 = \sqrt{3^2 + 4^2}$ or $3^2 + 4^2 = 5^2$ and now you have
$13 = \sqrt{5^2 + 12^2}$ or $5^2 + 12^2 = 13^2$

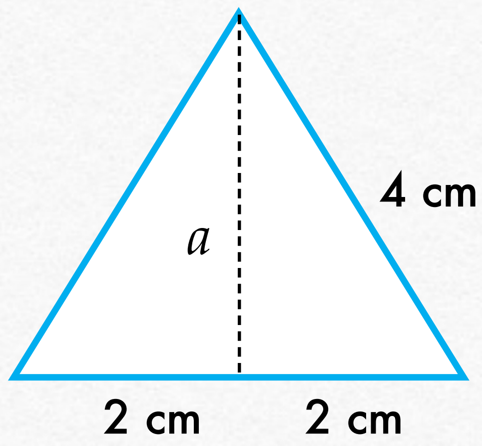
Groups of three whole numbers where the sum of the smaller squares equal the larger square are called Pythagorean triplets. You may enjoy investigating these. See answer section for more on this.

<u>Exercises</u>

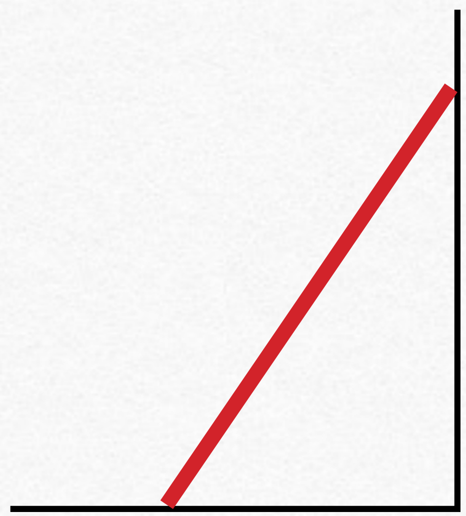
22. Find the height of these right-angled triangles, giving answers to 1 decimal place:

(a) base of 8 cm and hypotenuse 12 cm
(b) base of 5 m and hypotenuse 8 m
(c) base of 150 m and hypotenuse 200 m

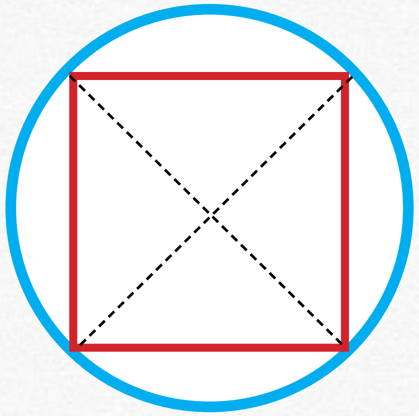
23. Kate is using a computer programme to draw an equilateral triangle of side 4 cm. What will be the height of the triangle?



24. Lulu needs to place a ladder 8 m long against a wall so that it reaches a window 6 m above the ground. How far from the base of the wall should she put the foot of the ladder?



25. Manu is designing a pattern based on a square drawn inside a circle like this:



a) If the radius of the circle is 5 cm, what is the length of a side of the square?

(b) It the square has sides of 5 cm, what is the radius of the circle?

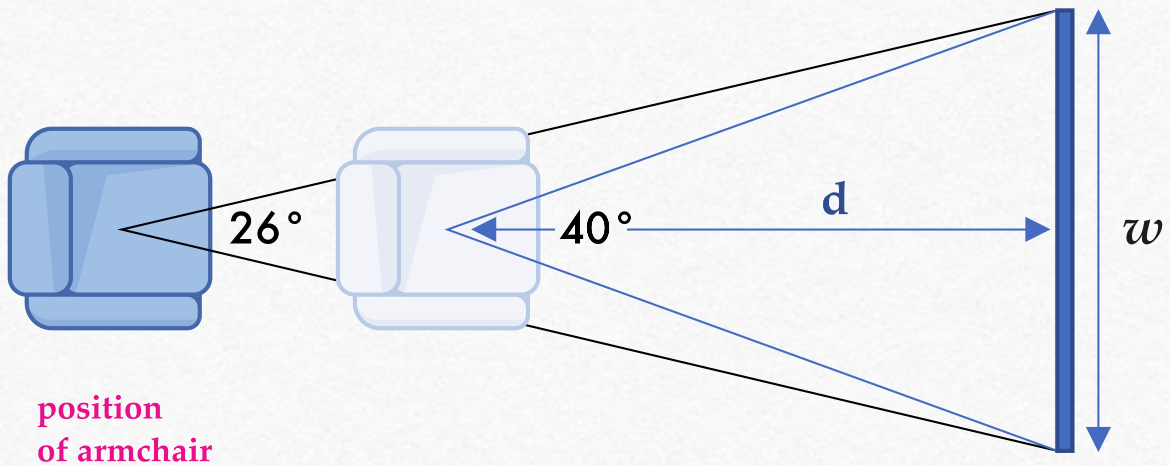
A Formula for Your TV

When buying a TV and deciding on its size you might think that it is all about how the screen looks in the room.

That is important but what you do need to consider first is how far you will sit from the screen and what is called the viewing angle.

If you sit too far from a TV then the picture is not good enough and if you sit too close then you cannot see the edges of the screen without turning your head.

Looking from above, the optimum angle is therefore said to be between 26° and 40°:



There are standard ratios between the length and width of right-angled triangles for known angles and these tells us that for a viewing angle of 26°:

Distance of viewer from screen = 2 x width of screen

or $d = 2w$

You can use this formula to find out how far you should place your chair from your TV.

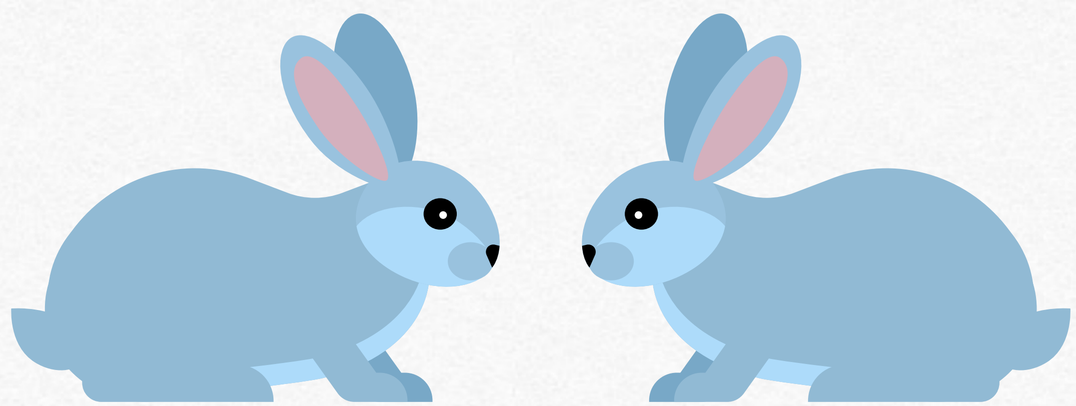
Example

If my television has width of 1.2 m, what is the furthest away from it that I should sit?

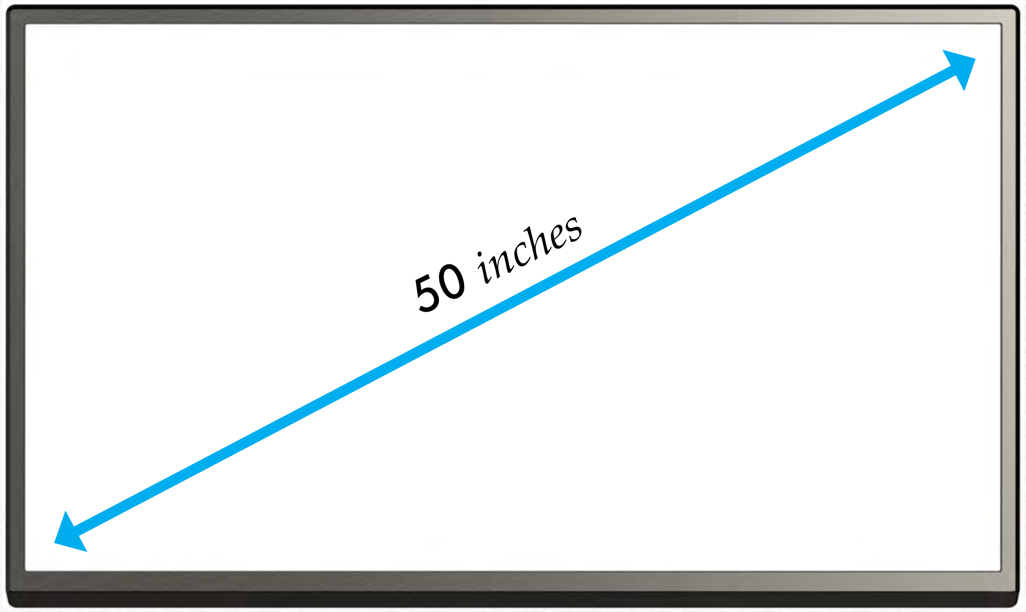
$$d = 2w$$
$$= 2 \times 1.2$$
$$= 2.4 \text{ m}$$

Answer: I should sit no more than 2.4 m away from the TV

That seems quite straightforward as the television is already in place, but suppose that you want to buy a television? You will see that television sizes, and computer screen sizes, are quoted in inches and the standard sizes for a television are 32, 43, 50, 55, 65 and 75″. Note the “ symbol stands for inches.



To complicate matters, the size quoted is not the width but the diagonal:

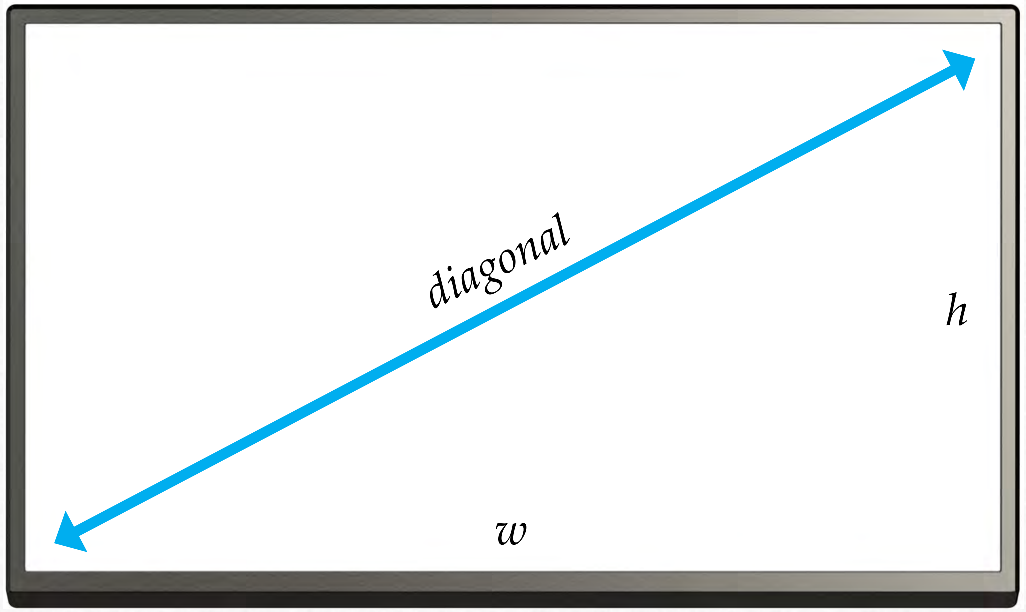


The width and height are then set by the ratio:

width : height = 16 : 9

This is the same ratio as used for films when viewed in a cinema. You can now watch a film on your TV set and see the whole screen. If you watch old films, you may see that they are narrower and that is because the former ratio of width: height was 4 : 3.

Here are two more formulae for your TV screen, note the formula using the ratio can be written in two ways:



$$\text{diagonal} = \sqrt{w^2 + h^2}$$

$$h/w = 9/16 \quad \text{or} \quad w/h = 16/9$$

Example

What will be the height of a television that is about 37″ wide? What standard TV size is this?

First use the ratio to find the height:

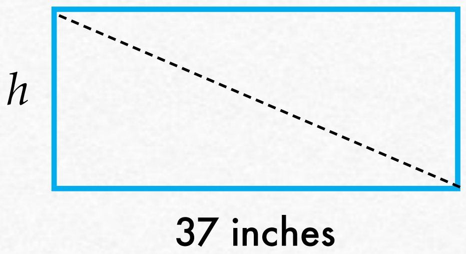
$$h/w = 9/16$$

$$h/37 = 9/16$$

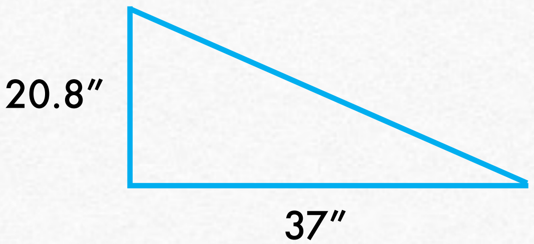
$$h = \frac{9 \times 37}{16}$$

$$= 20.81\ldots$$

$$= 20.8''$$



Now use Pythagoras' formula to find the diagonal and hence the size:



$$\text{diagonal} = \sqrt{w^2 + h^2}$$

$$= \sqrt{37^2 + 20.8^2}$$

$$= 42.445\ldots$$

$$= 42.4 \text{ inches}$$

Looking back at the standard screen sizes, 43" is the closest.

Answer: The screen size will be 43 inches

In that last example, you will see that answers were not exact. That is to be expected as the measurements that you start with will not be exact either.

Now try these:

Exercises

26. Nollie is arranging the furniture in her new flat. If her television is 80 cm wide, how far away should she put her sofa so that she can watch TV comfortably?

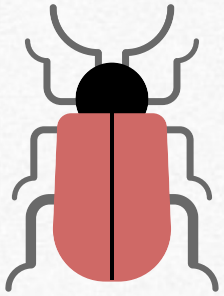
27. Olla has a television that is 90 cm high. How wide will this be?

28. Polly needs to replace her television. The new TV will need to sit neatly in the same place as her old one, which was 44 inches (or 110 cm) wide. What standard television size should she buy?

You may find that you need to work out the dimensions of a screen when all you know is its diagonal size.

FOLLOW THIS WORKING ONLY IF YOU FIND IT INTERESTING. IF NOT, SKIP TO THE FORMULAE AT THE BOTTOM OF THIS COLUMN

As $w : h = 16 : 9$



Because h and w are at right angles and

$$\sqrt{16^2 + 9^2} = \sqrt{337}$$
$$= 18.357\ldots$$
$$= 18.4$$

Then $w : h : d = 16 : 9 : 18.4$ (where d is the diagonal)

$$^{w}/_{d} = {}^{16}/_{18.4} \text{ or } w = \frac{16 \times d}{18.4}$$

And similarly $h = \dfrac{9 \times d}{18.4}$

DO NOT WORRY IF YOU DID NOT FOLLOW THAT. ALL YOU NOW NEED TO DO IS USE THE FORMULAE.

$$w = \frac{16 \times d}{18.4} \text{ or } h = \frac{9 \times d}{18.4}$$

Example

What is the height of a 21.5″ laptop screen?

You are looking for the height, so use the formula $h =$:

$$h = \frac{9 \times d}{18.4}$$
$$= \frac{9 \times 21.5}{18.4}$$
$$= 10.5 \text{ inches}$$

Answer: The height is 10.5″

Exercises

29. What will be the width of a 14″ laptop screen?

30. Qia is choosing a new laptop. How much wider will a 16″ screen be than a 14″ screen? If 1 inch is 2.54 cm, what is the answer in cm?

Food for Thought

Now you have your new television, will it fit through the living room door which is **2 m** tall and **76 cm** wide? What is the largest television you could buy that would fit through the door?

Answers to Part 3

1. Amy is an archaeologist. She estimates the heights of the inhabitants of an ancient site using the formula:

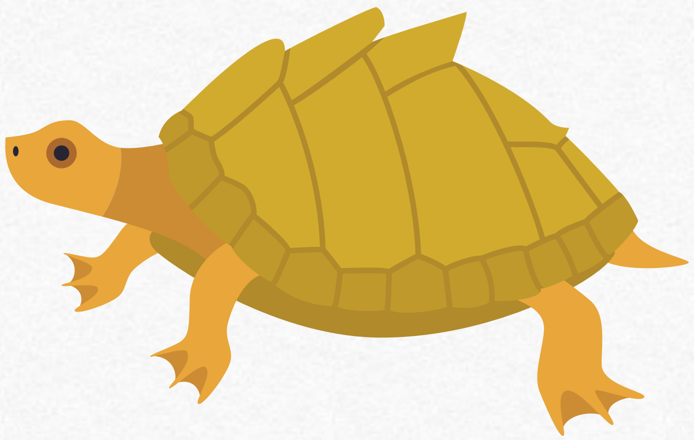
Height = 7 x length of foot

If Amy measured 3 footprints: 8.5 cm, 17.2 cm, 24 cm, what did she calculate the heights of the three inhabitants of the site to be? What conclusions about the owners of the footprints might she draw?

Height 1 = 7 x 8.5
= 59.5 cm

Height 2 = 7 x 17.2
= 120.4 cm

Height 3 = 7 x 24
= 168 cm



Answer: Amy calculates the heights to be 59.5 cm, 120.4 cm and 168 cm.

Only Height 3 is similar to modern day adult height so Amy might conclude that these are the prints of one adult and two children.

2. The cooking time for a chicken is determined by the formula:

Cooking time in minutes = 45 x weight in kg + 20

For how long should Bea roast a chicken weighing 2 kg?

Cooking Time = 45 x weight + 20
= 45 x 2 + 20
= 90 + 20
= 110 minutes or 1 hour 50 mins

Answer: Bea roasts her chicken for 1 hour and 50 mins

3. If $a = 5$, $b = 12$ and $c = 15$, find the value of:

(a) P when $P = ab$

$$P = ab$$
$$= 5 \times 12$$
$$= 60$$

Answer: $P = 60$

(b) Q when $Q = {}^{c}/_{a}$

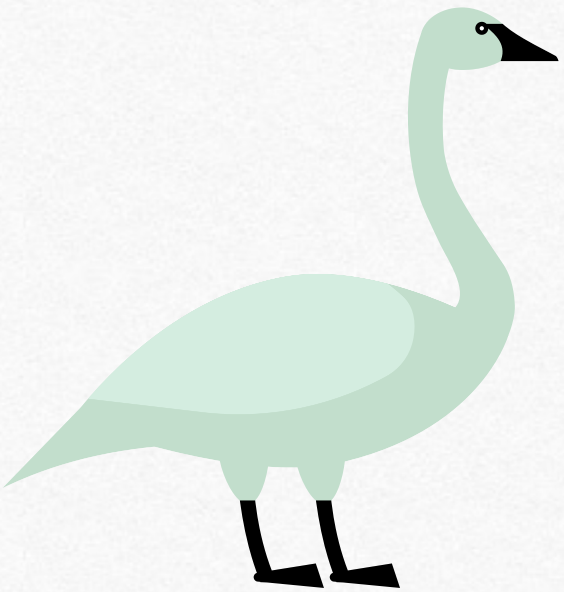
$$Q = {}^{c}/_{a}$$
$$= {}^{15}/_{5}$$
$$= 3$$

Answer: $Q = 3$

(c) R when $R = {}^{bc}/_{2a}$

$$R = {}^{bc}/_{2a}$$
$$= \frac{12 \times 15}{2 \times 5}$$
$$= {}^{180}/_{10}$$
$$= 18$$

Answer: $R = 18$



Note that you could cancel the fraction down like this:

$$\frac{{}^{6}\cancel{12} \times \cancel{15}^{3}}{\cancel{2} \times \cancel{5}} = 18$$

(d) S when $S = a^2 + b^2$

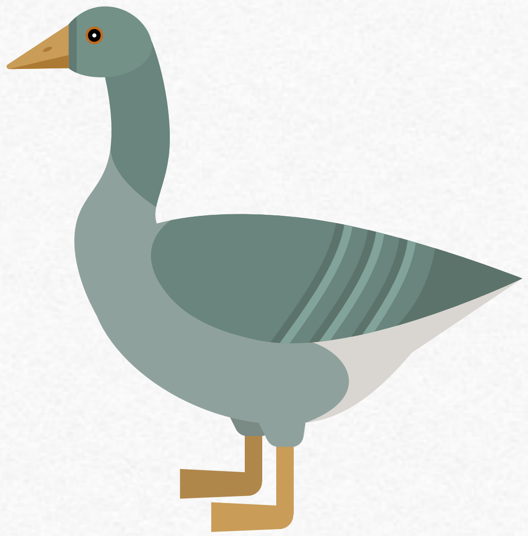
$$S = a^2 + b^2$$
$$= 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$

Answer: $S = 169$

(e) T when $T = \sqrt{a^2 + b^2}$

$$T = \sqrt{a^2 + b^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13$$

Answer: $T = 13$



Follow the **5** steps for using a formula and the formulae on the previous page to calculate these. Give your answers to the nearest cm^3 (TIP: Change all dimensions to **cm** before you start):

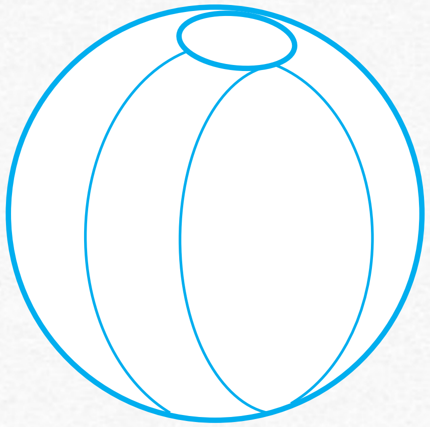
4. Find the amount of air needed to fill an inflatable ball of radius **15 cm**

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 15^3$

$= 14{,}137.16\ldots \text{ cm}^3$

Answer: Volume of the ball is 14,137cm^3



5. Find the amount of water needed to fill a cylindrical bucket of radius **15 cm** and height **0.5 m**. What is this in litres? (Note: **1** litre = **1000** cm^3)

Convert to **cm**: $h = 0.5 \text{ m} = 50 \text{ cm}$

Volume of cylinder $= \pi r^2 h$

$= \pi \times 15^2 \times 50$

$= 35{,}342.91\ldots \text{ cm}^3$

Divide the answer by **1,000** to turn this into litres

Answer: 35,342 cm^3 or 35.3 l of water is needed to fill the bucket

6. Calculate the amount of glass needed to make a triangular prism with $b = 35$ **mm**, $h = 40$ **mm** and $l = 25$ **cm**

Convert **mm** to **cm**: $b = 35 \text{ mm} = 3.5 \text{ cm}$, $h = 40 \text{ mm} = 4 \text{ cm}$

Volume of triangular prism $= \dfrac{b \times h \times l}{2}$

$= \dfrac{3.5 \times 4 \times 25}{2}$

$= 175 \text{ cm}^3$

Answer: 175 cm^3 of glass is needed

7. (a) Calculate the amount of ice cream that will fill a cone of radius 25 mm at the widest point and height 12 cm

Convert mm to cm: $r = 25$ mm $= 2.5$ cm

Volume of cone $= \dfrac{\pi r^2 h}{3}$

$= \dfrac{\pi \times 2.5^2 \times 12}{3}$

$= 78.539\ldots \text{ cm}^3$

Answer: 79 cm^3 of ice cream will fill the cone

(b) Now calculate the volume of a spherical scoop of ice cream of radius 25 mm.

$r = 25$ mm $= 2.5$ cm

Volume of sphere $= {}^4/_3\, \pi r^3$

$= {}^4/_3 \times \pi \times 2.5^3$

$= 65.449\ldots \text{ cm}^3$

Answer: The volume of ice cream in the scoop is 65 cm^3

(c) Add your two volumes together to find the total volume of ice cream.

$79 + 65 = 144 \text{ cm}^3$

Answer: The total amount of ice cream = 144 cm^3

Food for Thought

For technical calculations you would not add two rounded answers together but use the full values. But in this context, as the measure of ice cream cannot be exact, it is fine to work with inexact answers.

8. Calculate the volume of a marble paperweight in the shape of a pyramid with $l = w = h = 6.5$ cm.

Volume of pyramid $= \dfrac{l \times w \times h}{3}$

$= \dfrac{6.5 \times 6.5 \times 6.5}{3}$

$= 91.541\ldots \text{ cm}^3$

Answer: The volume of the paperweight is 92 cm^3


“Interior decorators need maths.”

9. Find the volume , in m^3, of a tent in the shape of triangular prism of height 1.5 m, base 1.8 m and length 3 m.

$$\text{Volume of prism} = \frac{b \times h \times l}{2}$$

$$= \frac{1.8 \times 1.5 \times 3}{2}$$

$$= 4.05 \text{ m}^3$$

Answer: The volume of the tent is 4.05 m^3

10. The great pyramid at Giza, Egypt has sides of 230 m and a height of 146 m. What is its volume to the nearest 100 cubic metres?

$$\text{Volume of pyramid} = \frac{l \times w \times h}{3}$$

$$= \frac{230 \times 230 \times 146}{3}$$

$$= 2{,}574{,}466.667\ldots\ldots \text{ m}^3$$

Answer: The volume of the great pyramid at Giza is 2,574,500 m^3

Food for Thought

The Pyramids of Giza were built around 4,500 years ago.

Reflect for a moment on the skill of those ancient civilisations that built such incredible monuments without any of our technology.

11. Write these minutes as fractions of an hour:

(a) 20 minutes (b) 40 minutes (c) 25 minutes

(a) 20 minutes = $^{20}/_{60}$ = hour = $^{1}/_{3}$ hour

Answer: $^{1}/_{3}$ hour

(b) 40 minutes = $^{40}/_{60}$ hour = $^{2}/_{3}$ hour

Answer: $^{2}/_{3}$ hour

(c) 25 minutes = = $^{25}/_{60}$ hour = $^{5}/_{12}$ hour

Answer: $^{5}/_{12}$ hour

12. The plumber in the above example charges a callout fee of £75 at weekends. What will he charge for a job on a Saturday that takes one and a half hours?

Charge = call out fee plus number of hours x £60

= £75 + 1.5 x £60

= £75 + £90

= £165

Answer: The plumber will charge £165

13. Della is a hairdresser who visits customers in their own homes. She charges her customers using the formula:

Charge = £25 for the visit plus £45 an hour.

(a) What will she charge a customer who wants a simple haircut that took 20 minutes?

Charge = £25 for the visit plus £45 an hour

= £25 + £45 x $^{1}/_{3}$

= £40

Answer: Della will charge £40

(b) What will she charge a customer who wanted a cut and colour that took an hour and 40 minutes?

Charge = £25 for the visit plus £45 an hour

= £25 + £45 x $1^2/_3$

= £25 + £75

= £100

Answer: Della will charge £100

14. This is another electricity bill for Chrystelle:

Current period		1st Jun–31st Aug		..... days
Previous reading	Current reading	Usage	Rate	Total
19,561	19,836	....	17.13 p	....
Days	....	Daily rate	16.80 p	....
			Total	....

Calculate the missing values and work out her total bill for that quarter. (Note: Three months is often called a quarter since it is a quarter of a year.)

Usage = 19,836 − 19,561

= 275

Cost 275 @ 17.13 = 4,710.75 p

= £47.11

Days = June + July + August

= 30 + 31 + 31 = 92

Cost = 92 x 16.80

= 1,545.6 p

= £15.46

Total cost = £47.11 + £15.46

= £62.57

Answer: Chrystelle's bill will look like this:

Current period		1st Jun–31st Aug		92 days
Previous reading	Current reading	Usage	Rate	Total
19,561	19,836	275	17.13 p	£47.11
Days	92	Daily rate	16.80 p	£15.46
			Total	£62.57

15. A mobile telephone company charges using the formula:

Total = £20 per month for up to 300 minutes + 40p per minute for additional calls.

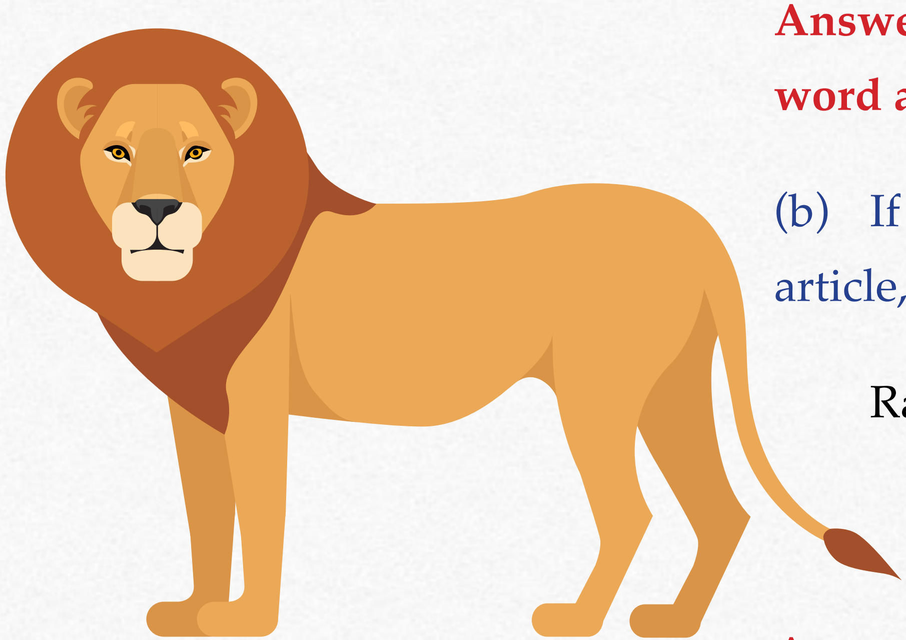
(a) What will Emma's monthly bill be if her total call time is 240 minutes?

As Emma has used less than 300 minutes she will pay the £20 charge for up to 300 minutes and nothing for additional minutes

Answer: Emma's bill will be £20

(b) What will Fran's monthly bill be if her total call time is 380 minutes?

Extra minutes = 380 − 300
= 80 mins



Charge = £20 + 40p x extra minutes
= £20 + 40p x 80
= £20 + 3,200p
= £20 + £32
= £52

Answer: Fran's bill will be £52

16. Gemma is a freelance writer. She charges £0.10 per word.

(a) What will Gemma earn for a 1,500 word article?

Charge = words x £0.10
= 1,500 x £0.10
= £150

Answer: Gemma will earn £150 for the 1,500 word article

(b) If it takes Gemma 3 hours to write, edit and deliver her article, what is her rate of pay per hour?

Rate per hour = Total ÷ hours
= £150 ÷ 3
= £50

Answer: Gemma's rate of pay would be £50 per hour


"I can afford my dog"

(c) Gemma is asked to quote a fixed fee for an article that will involve some research, not just writing. Gemma estimates that the whole project will take her **3** working days or **24** hours in total. What should she quote based on her hourly rate?

Quote = hourly rate x no of hours
= £50 x 24
= £1,200

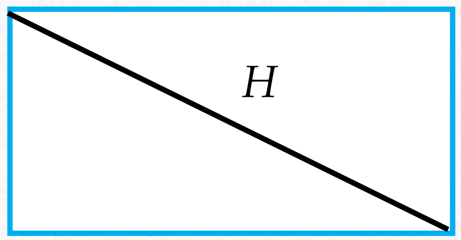
Answer: Gemma should quote £1,200

For each of the following questions, start by sketching a right angled triangle and identifying which side is H.

17. Work out the length of the diagonals in these rectangles:

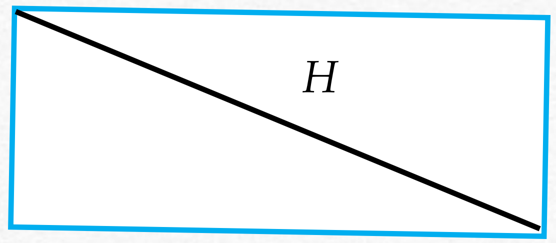
(a) **3 m** wide and **4 m** high

$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ cm}$$



Answer: 5 cm

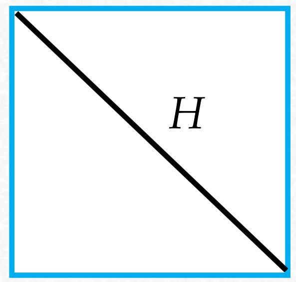
(b) **120 cm** wide and **50 cm** high



$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{120^2 + 50^2}$$
$$= \sqrt{16{,}900}$$

Answer: 130 cm

(c) a square measuring **20 cm** by **20 cm**

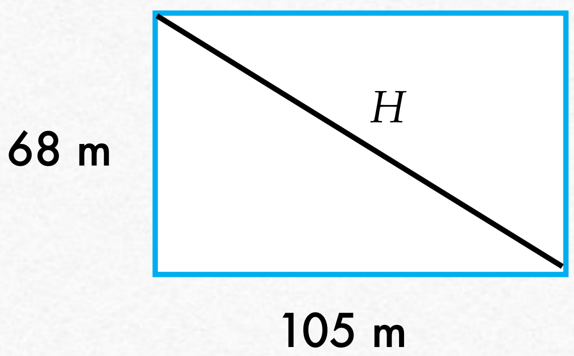


$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{20^2 + 20^2}$$
$$= \sqrt{800}$$
$$= 28.284\ldots \text{ cm}$$

Answer: 28 cm

18. A playing field is **105 m** by **68 m**. Rather than walking round it, Holly takes a short cut along the diagonal.

(a) How far is Holly's walk along the diagonal?



$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{105^2 + 68^2}$$
$$= \sqrt{125.095...\text{ m}}$$
$$= 125 \text{ m}$$

Answer: Holly's walk along the diagonal is 125 m

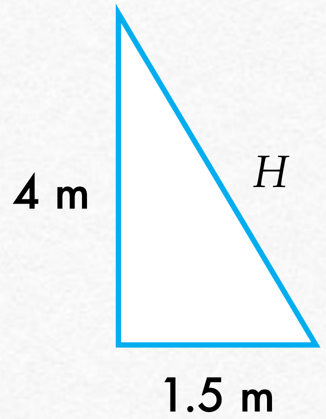
(b) How much shorter is the distance Holly has walked than if she had walked around two sides of the field?

The two sides = 105 + 68
= 173 m

Holly has saved 173 − 125 = 48 m

Answer: Holly walked 48 m less than she would have around the edge.

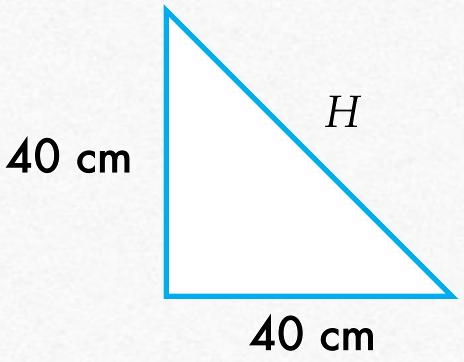
19. A flagpole is held up by ropes that run from a point **4 m** up the pole to a point on the ground **1.5 m** from the base of the pole. How long is each rope?



$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{4^2 + 1.5^2}$$
$$= 4.272......\text{ m}$$
$$= 4.27 \text{ m (to the nearest cm)}$$

Answer: Each rope is 4.27 m each

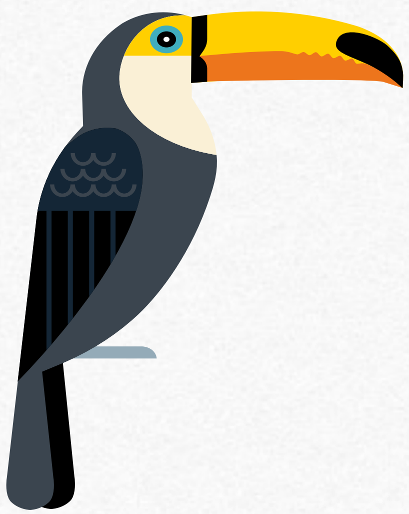
20. Irene has folded a silk square of sides **40 cm** in half to make a triangular scarf. She now needs to sew some trim round three sides of the scarf. What is the length of the trim?



$$H = \sqrt{a^2 + b^2}$$
$$= \sqrt{40^2 + 40^2}$$
$$= 56.568\ldots \text{ cm}$$

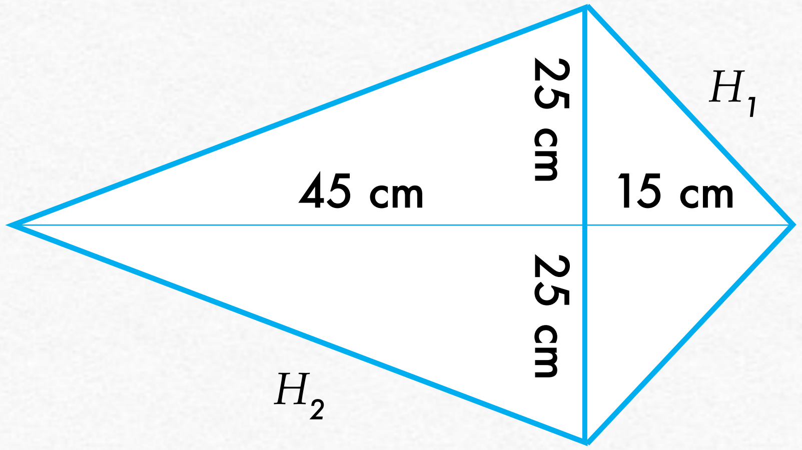
Length of trim $= 40 + 40 + 56.568\ldots$ cm
$= 136.568\ldots$ cm

Answer: Irene needs 137 cm of trim

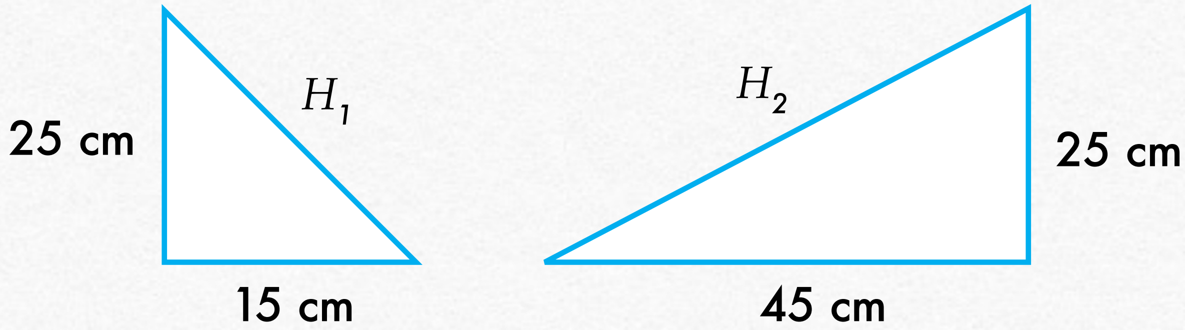


21. Jilly is making a kite. Her kite has two pieces of wood as its diagonals, one **50 cm** long and the other **60 cm** long. The shorter stick is placed a quarter of the way down the longer stick. What are the lengths of the sides of the kite?

First Jilly puts the dimensions on a sketch:



Then she sketches the two right angled triangles:



$$H_1 = \sqrt{a^2 + b^2}$$
$$= \sqrt{25^2 + 15^2}$$
$$= 29.154\ldots \text{ cm}$$
$$= 29.2 \text{ cm (to the nearest mm)}$$

$$H_2 = \sqrt{a^2 + b^2}$$
$$= \sqrt{45^2 + 25^2}$$
$$= 51.478\ldots \text{ cm}$$
$$= 51.5 \text{ cm (to the nearest mm)}$$

Answer: The sides of the kite are 29.2 and 51.5 cm

Food for Thought

You may have spotted that twice the answer has been an exact number. The first was when

$5 = \sqrt{3^2 + 4^2}$ or $3^2 + 4^2 = 5^2$ and now you have

$13 = \sqrt{5^2 + 12^2}$ or $5^2 + 12^2 = 13^2$

Groups of three whole numbers where the sum of the smaller squares equal the larger square are called Pythagorean triplets. You may enjoy investigating these. If you do investigate Pythagorean Triplets you will find that there are some sets.

For example:
You have seen that 3, 4 and 5 are a triplet

And then so are:
6, 8 and 10
9, 12 and 15

As well as
1.5, 2 and 2.5

Because the sides are all in the same ratio
3 : 4 : 5

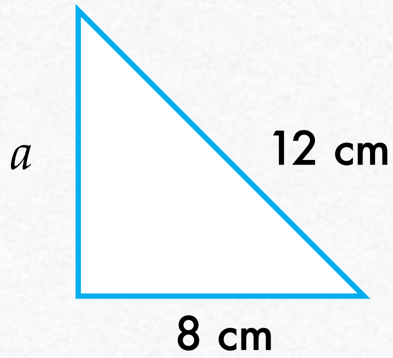
The other ratio from the example was:
5 : 12 : 13

And you may discover some more, such as
8 : 15 : 17,
7 : 24 : 25
and 20 : 21 :29

22. Find the height of these right-angled triangles, giving answers to 1 decimal place:

(a) base of **8 cm** and hypotenuse **12 cm**

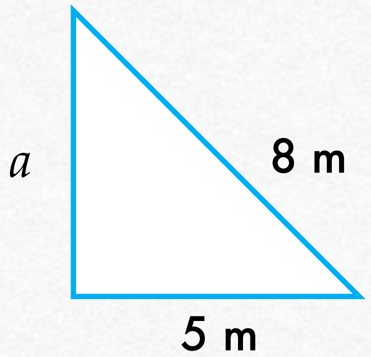
$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{12^2 - 8^2}$$
$$= 8.944\ldots$$
$$= 8.9 \text{ cm}$$



Answer: The height of the triangle is 8.9 cm

(b) base of **5 m** and hypotenuse **8 m**

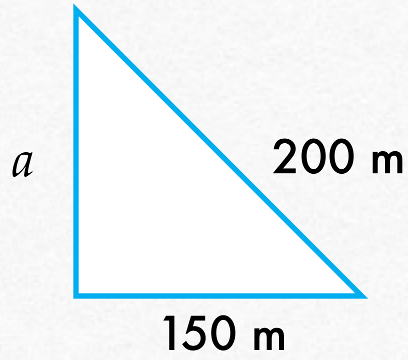
$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{8^2 - 5^2}$$
$$= 6.244\ldots$$
$$= 6.2 \text{ m}$$



Answer: The height of the triangle is 6.2 m

(c) base of **150 m** and hypotenuse **200 m**

$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{200^2 - 150^2}$$
$$= 132.28\ldots$$
$$= 132.3 \text{ m}$$

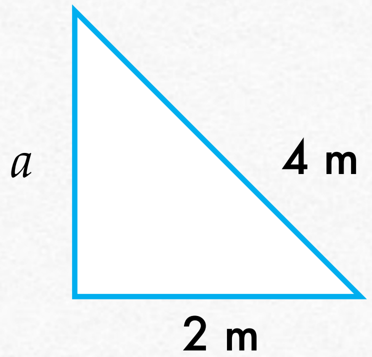


Answer: The height of the triangle is 132.3 m

23. Kate is using a computer programme to draw an equilateral triangle of side **4 cm**. What will be the height of the triangle?

You can see from the sketch that the height divides the base of the equilateral triangle in two.

$$a = \sqrt{H^2 - b^2}$$
$$= \sqrt{4^2 - 2^2}$$
$$= 3.464\ldots \text{ cm}$$



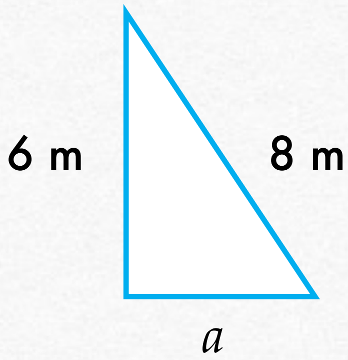
Answer: The height of the triangle will be 3.46 cm

Food for Thought

Designers frequently need to use this formula to calculate the exact dimensions of various shapes.

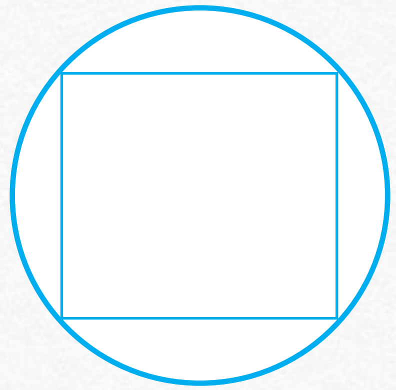
24. Lulu needs to place a ladder **8 m** long against a wall so that it reaches a window **6 m** above the ground. How far from the base of the wall should she put the foot of the ladder?

$$\begin{aligned} a &= \sqrt{H^2 - b^2} \\ &= \sqrt{8^2 - 6^2} \\ &= 5.291\ldots \\ &= 5.3 \text{ m} \end{aligned}$$



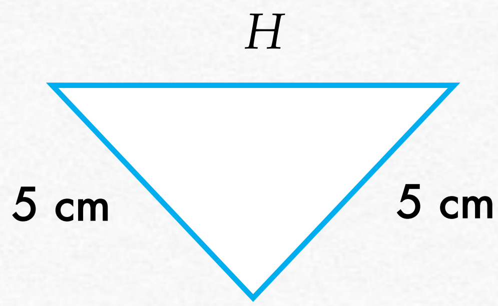
Answer: The ladder will be 5.3 m from the wall

25. Manu is designing a pattern based on a square drawn inside in a circle like this:

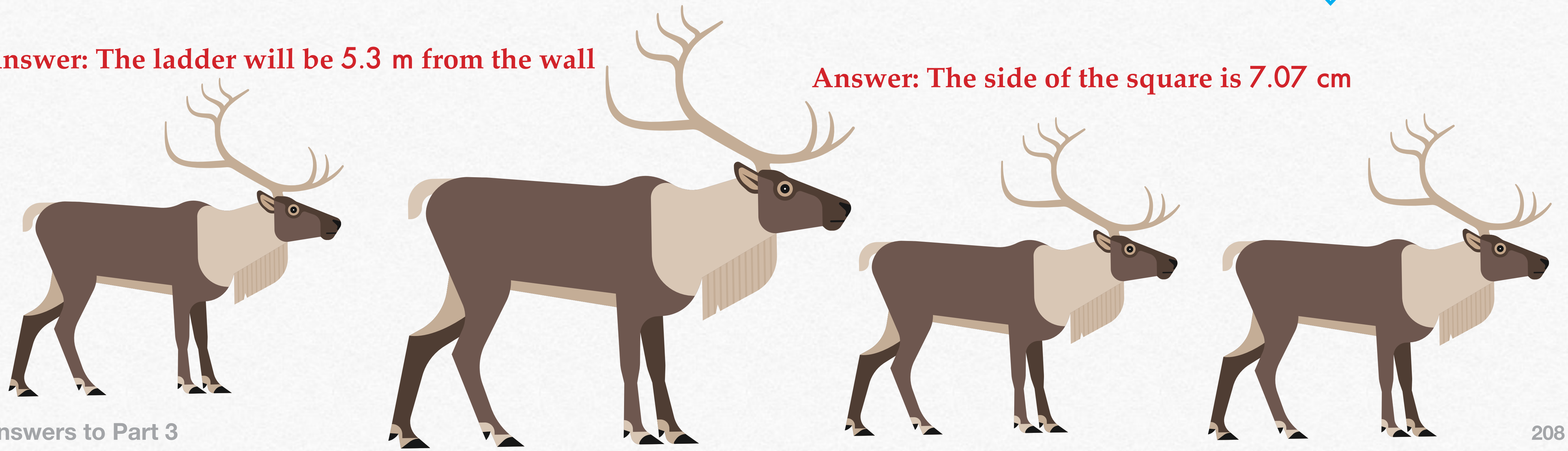


(a) If the radius of the circle is **5 cm**, what is the length of a side of the square?

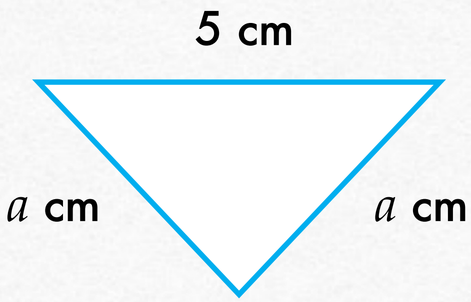
$$\begin{aligned} H &= \sqrt{a^2 + b^2} \\ &= \sqrt{5^2 + 5^2} \\ &= 7.071\ldots \text{ cm} \end{aligned}$$



Answer: The side of the square is 7.07 cm



(b) It the square has sides of **5 cm**, what is the radius of the circle?



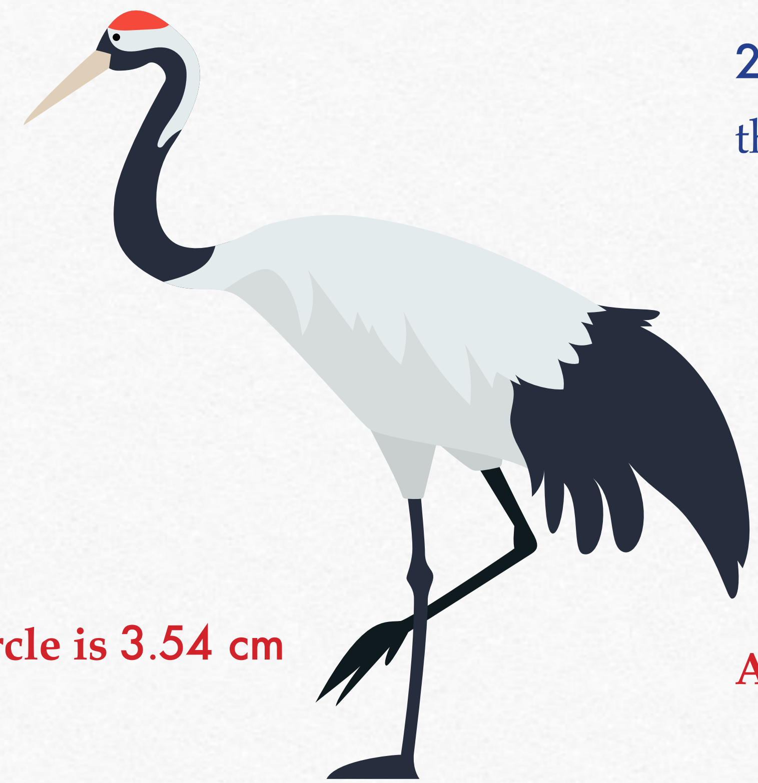
This time you will need to use the very first formula we looked at for Pythagoras' theorem:

$$H^2 = a^2 + b^2$$
$$5^2 = a^2 + a^2$$
$$25 = 2a^2$$

Therefore $a^2 = 25 \div 2 = 12.5$

$$a = \sqrt{12.5}$$
$$= 3.5355...$$

Answer: The radius of the circle is 3.54 cm



26. Nollie is arranging the furniture in her new flat. If her television is **80 cm** wide, how far away should she put her sofa so that she can watch TV comfortably?

$$d = 2w$$
$$= 2 \times 80$$
$$= 160 \text{ cm or } 1.6 \text{ m}$$

Answer: Nollie should put her sofa 1.6 m away from her TV

27. Olla has a television that is **90 cm** high. How wide will this be?

$$w/h = 16/9$$
$$w/90 = 16/9$$
$$w = \frac{16 \times 90}{9}$$
$$= 160 \text{ cm or } 1.6 \text{ m}$$

Answer: Olla's television is 1.6 m wide

28. Polly needs to replace her television. The new TV will need to sit neatly in the same place as her old one, which was **44** inches (or **110 cm**) wide. What standard television size should she buy?

First calculate the height using the ratio formula with h on top

$$h/w = 9/16$$

$$h/44 = 9/16$$

$$h = \frac{9 \times 44}{16}$$

$$= 24.75''$$

Then calculate the diagonal using Pythagoras:

$$diagonal = \sqrt{w^2 + h^2}$$

$$= \sqrt{44^2 + 24.75^2}$$

$$= 50.48...$$

$$= 50 \text{ inches}$$

Answer: Polly should buy a 50″ television

29. What will be the width of a **14″** laptop screen?

You are looking for the width, so use the formula beginning with w =

$$w = \frac{16 \times d}{18.4}$$

$$= \frac{16 \times 14}{18.4}$$

$$= 12.173... \text{ inches}$$

Answer: 12.2″

30. Qia is choosing a new laptop. How much wider will a 16″ screen be than a 14″ screen?

For the 16″ screen:

$$w = \frac{16 \times d}{18.4}$$

$$= \frac{16 \times 16}{18.4}$$

$= 13.913...$ inches

$= 13.9$ inches

The difference $= 13.9 - 12.2$

$= 1.7$ inches

Answer: The 16″ screen is 1.7″ wider than the 14" screen

If 1 inch is 2.54 cm, what is the answer in cm?

1 inch ≈ 2.54 cm

Then 1.7 inches ≈ 2.54 cm x 1.7

≈ 4.318 cm

≈ 4.3 cm

Answer: The 16″ screen is 4.3 cm wider than the 14" screen

Food for Thought

Now you have your new television, will it fit through the living room door which is **2 m** tall and **76 cm** wide? What is the largest television you could buy that would fit through the door?

As you calculated in an earlier example the diagonal of standard door is **214 cm**, therefore all the televisions that we have been considering will easily fit through the door.

What is the largest television that would fit?

As televisions have some depth to them it would be sensible to estimate that the largest TV to fit through the door would be **200 cm** high.

If that is the height of the television, then the width would be:

$$\begin{aligned} w/h &= 16/9 \\ w/200 &= 16/9 \\ w &= \frac{16 \times 200}{9} \\ &= 355.55\ldots \\ &= 356 \text{ cm or } 3.56 \text{ m} \end{aligned}$$

Use the formula for distance away from the screen:

$$\begin{aligned} d &= 2w \\ &= 2 \times 3.56 \\ &= 7.12 \text{ m} \end{aligned}$$

You would sit **7.12 m** away from the screen.

Which all means that you will need a very large room for your home cinema!

Food for Thought

The optimum angle for viewing a television depends very much on individual preferences, but there is also said to be a difference between the best angle for men and for women, which may be why this is the cause of so much argument! You can now reason the case for your chosen distance from the TV using mathematics!

Remember the art of estimating plus make sure that you have control of the remote

YOUR BRAIN WORKOUT

Q1

If $a = 3$, $b = 4$ and $c = 14$, find the value of:

P when $P = \dfrac{b + c}{2a}$

YOUR BRAIN WORKOUT

Q2

The volume of a triangular prism is given by the formula:

$$V = \frac{\mathit{base} \times \mathit{height} \times \mathit{length}}{2}$$

How much chocolate is in a solid triangular prism chocolate bar of base 3 cm, height 3 cm and length 4 cm?

YOUR BRAIN WORKOUT

Q3

What is **5** minutes as a fraction of an hour?

YOUR BRAIN WORKOUT

Q4

A taxi company charges a basic fee of £2.60 plus 50 p per minute for a taxi ride. What is the cost of a 15 minute ride?

YOUR BRAIN WORKOUT

Q5

Use the formula $H = \sqrt{a^2 + b^2}$

to calculate H when $a = 6$ and $b = 8$.

YOUR BRAIN WORKOUT

Q6

Use the formula $a = \sqrt{H^2 - b^2}$ to calculate a when $H = 20$ and $b = 12$

YOUR BRAIN WORKOUT

Q7

If the formula for the volume of a sphere is $V = {}^{4}/_{3}\, \pi r^3$, what is the volume of a table-tennis ball of radius 2 cm, taking π as 3.14?

YOUR BRAIN WORKOUT

Q8

The formula for the cooking time of a turkey is **45** minutes per kilogram, plus **20** minutes. How long will it take to cook a **4 kg** turkey?

YOUR BRAIN WORKOUT

Answers

Q1 $\dfrac{4 + 14}{2 \times 3} = {}^{18}/_{6} = 3$

Q2 $V = \dfrac{3 \times 3 \times 4}{2} = {}^{36}/_{2} = 18\ \text{cm}^3$

Q3 ${}^{5}/_{60} = {}^{1}/_{12}$ hour

Q4 £2.60 + £0.50 x 15 = £2.60 + £7.50 = £10.10

Q5 $H = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

Q6 $a = \sqrt{20^2 - 12^2} = \sqrt{400 - 144} = \sqrt{256} = 16$

Q7 $V = \dfrac{4 \times 3.14 \times 2^3}{3} = 33.493\ldots = 33.5\ \text{cm}^3$

Q8 45 x 4 + 20 = 180 + 20 = 200 mins

= 3 h 20 m

PART 4

MATHS FOR EXPLORERS

See the World

The opportunity to travel is one of the most wonderful gifts available to you. So much can be learned from other countries and other cultures. It is fascinating to see how their history weaves into that of your own, how various religions have so many similarities as well as their differences and, perhaps most exciting of all, is to experience local customs and the associated food and drink from around the world, discovering new flavours and taste sensations.

However, travel needs planning. You need to understand how to get to where you want to be, to be aware of time zones, to know and understand the local currency and keep track of your spending.

You also need to keep yourself safe. That means doing some research and making sure that you are booked into reputable accommodation, avoiding any locations that have a bad reputation and being particularly careful about anywhere you go alone.

Beautiful souvenirs are always there to tempt you when you explore the shops and bazaars overseas. Sadly, they too often lose their appeal once you get them home. Unless you actually want that fridge magnet, keep your money firmly in your pocket and rely on tons of photographs to remind you of your travels.

Time, Distance and Speed

When travelling, you need to be able to work out the time it takes you to get to your destination. This is not always as straightforward as it sounds because most travel is in several parts. You need to get from your home to the airport, you need to arrive at the airport some time before your flight, on arrival you have to get to where you are to stay, and by then you may be in a different time zone.

When you are in a different time zone you will sleep and eat according to local time, but it is useful to be aware of the time back at home. You do not want to wake your sleeping boyfriend up at two in the morning, especially if it is just to say what a lovely time you are having.

The time zone in Britain we know as Greenwich Mean Time (GMT) is also called Co-ordinated Universal Time (UTC) by scientists, pilots and weather forecasters. It is the time at 0° longitude, which happens to run through Greenwich near London, England.

In Britain, clocks are changed in Spring and Autumn in order to give longer evenings in summer. This is known as British Summer Time (BST).

In the US, autumn is known as 'fall'so if you can remember **Spring Forward Fall Back**, you will know what to do with your clocks.

In Britain, clocks go forward on the last Sunday of March and go back on the last Sunday of October. These dates are the same for most of Europe, but can be different in the United States and Canada. Not all countries change their clocks in this way, so you do need to check if you are travelling.

Example

Asta goes to bed on the last Saturday of March and sets her alarm clock for 9 am. Asta has forgotten that the clocks go forward overnight. When her alarm rings in the morning, what is the actual time on Sunday?

As the clocks go forward, add one hour:
9 am becomes 10 am

Answer: Asta's alarm eventually goes off at 10 am, let us hope that she was not planning anything special that morning!

Food for Thought

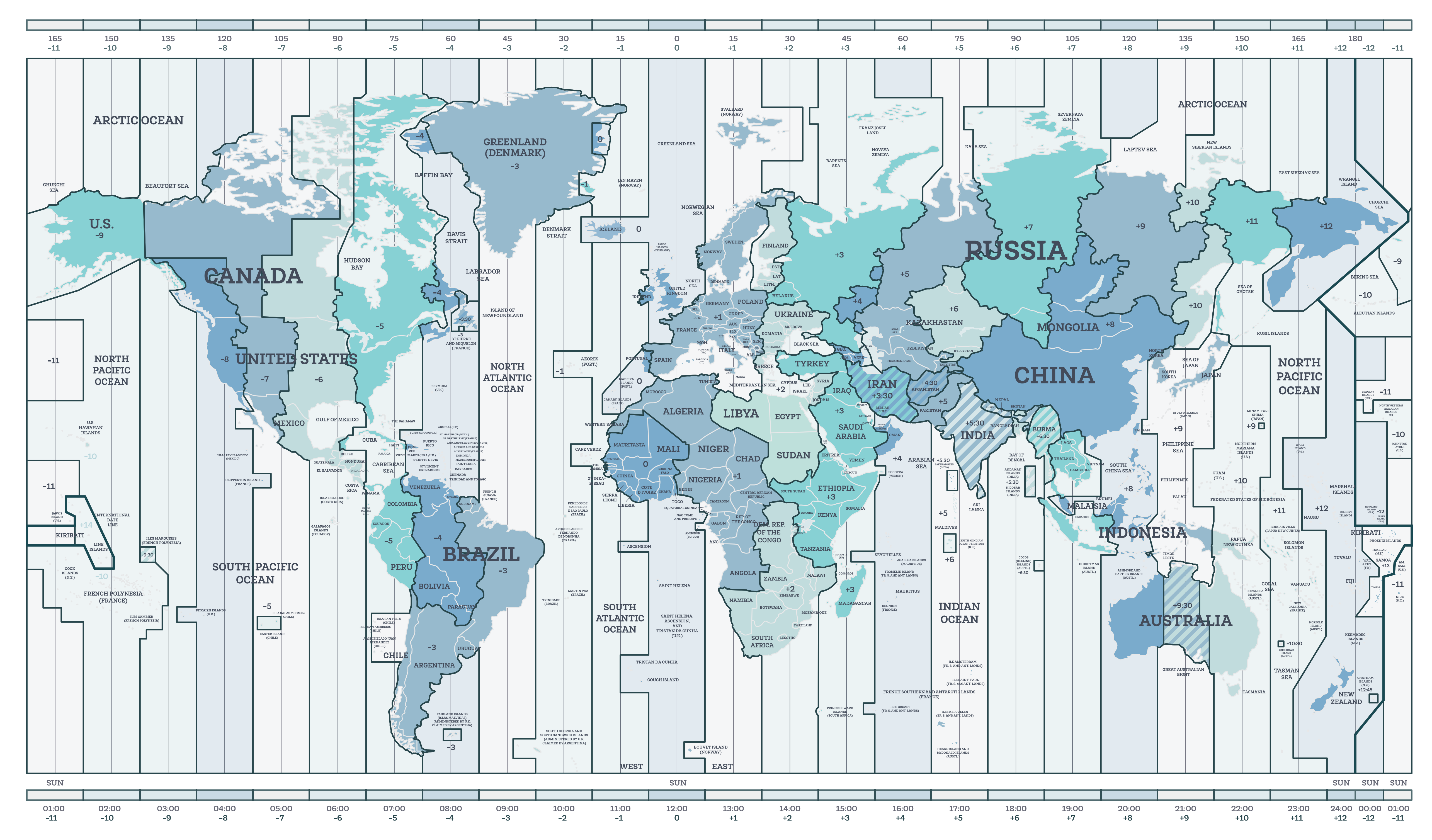
Modern smart watches and mobile phones change the time forwards or back automatically. Other clocks do not and so you may need to reset you central heating clock, the clock on your car and on your oven, as well as a traditional alarm clock.

Exercises

1. Betty has forgotten that the clocks should have changed overnight. Her traditional alarm clock tells her that the time is a quarter past eight on the last Sunday in October, what is the actual time?

2. Cherie had a really relaxed weekend. She got up as usual on Monday morning but to her surprise when she arrived at work she was told that she was an hour late. Why do you think that was and what was the date?

Here is a map showing the various time zones around the world. You can see that if you travel east, to Europe, for example, time is ahead, and if you travel west, to America for example, time is behind.



Food for Thought

You can see that China has the same time zone for the whole country, whilst the USA has several different time zones as they change with the longitude.

See also the thicker black line at the far right. This is the international date line. If you cross it, going east from New Zealand then you will put your calendar back by one day.

Example

If it is **2 pm** in London, what is the time in (a) Paris, (b) Beijing (c) New York?

It can be best to work with the **24** hour clock. From the map you can see that:

(a) Paris is one hour ahead, Zone **+1**

14:00 + 1 = 15:00

Answer: The time in Paris is 3 pm

(b) Beijing is in China which is **8** hours ahead or Zone **+8**

14:00 + 8 = 22:00

Answer: The time in Beijing is 10 pm

(c) New York is **5** hours behind, Zone **−5**

14:00 − 5 = 09:00

Answer: The time in New York is 9 am

Exercises

3. If it is **5 pm** in London, what is the time in:

(a) Los Angeles (UTC **−8**), (b) South Africa (UTC **+2**), (c) Thailand (UTC **+7**)?

4. Della's boy friend is working in Miami (UTC −**5**). She wants to talk to him at **6 pm** his time. What time will that be in the U.K.?

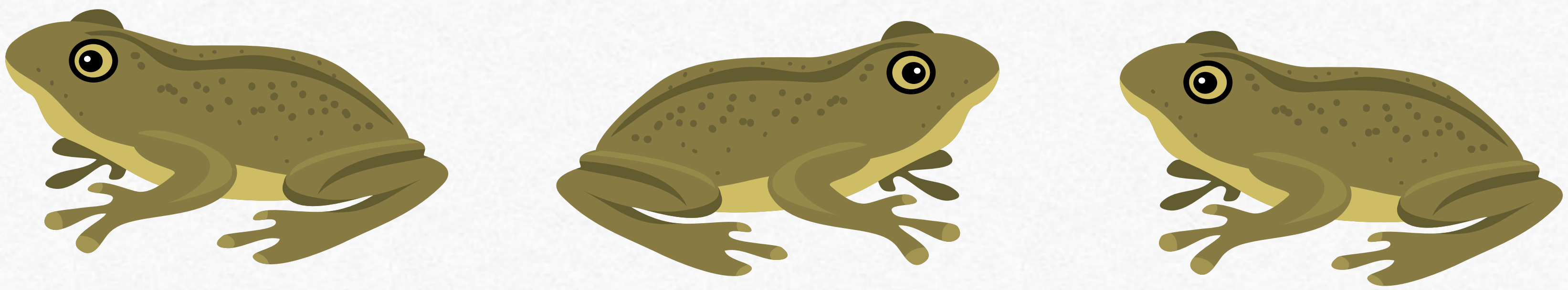
5. Eithne is travelling in New Zealand (UTC +**12**). She wants to call her mum in the UK on her birthday. If she decides to call her mum at **8 pm** UK time, what will be the time in New Zealand?

6. In March, Freya travels to Dubai (UTC+**4**). Her parents live in Manchester, England (UTC). and her boyfriend is travelling in South America (UTC−**3**). Freya has set up a card that reminds of the times in each country so that she can call at a convenient time of day:

Dubai	UK	Brazil
	8 am	
	12 noon	
	7 pm	

(a) Fill in the missing times on the card.

(b) In the last Sunday in March, UK clocks go forward to British Summer Time, but they do not in Dubai or Brazil. If Freya wants to call her parents on Sunday evening at **7 pm** UK time, what time will it be in Dubai?



Distance

Although most lengths in Britain are measured using the metric system, non-metric miles are still used for long distances. Therefore, the distances you see on road signs are in miles and speed limits are in miles per hour.

Miles are also used in the United States but in most other countries long distances are measured in kilometres.

The most useful conversion is:

5 miles = 8 km

Food for Thought

Do you know what various road signs mean? Before you can drive you will have to pass a driving test and will be tested on your knowledge of road signs. It is never too early to start learning their meanings.

Speed

When you are travelling, you go a certain distance in a time. The exact time that you spend travelling that certain distance depends on your speed. If you travel very fast, such as by car, then the time taken is less. If you travel slowly, such as when you are walking, then the time taken is more.

Time, distance and speed are connected by the formula:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

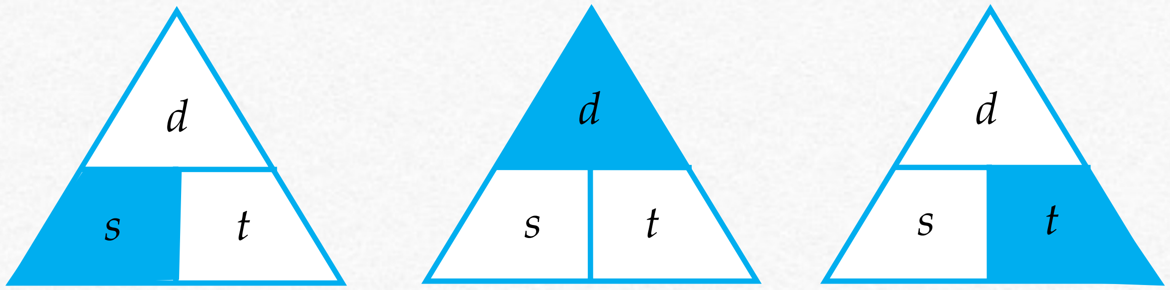
Therefore, if you travel 100 miles in two hours you can calculate your speed by following the 5 steps to use a formula (formula, substitute, calculate, answer and units):

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= {}^{100}/_{2}$$

$$= 50 \text{ miles per hour}$$

The formula can be rearranged to find time and distance. Some people use a triangle to help them remember the three versions of the formula.



$$\text{Speed} = \frac{\text{distance}}{\text{time}} \qquad \text{distance} = \text{speed} \times \text{time} \qquad \text{time} = \frac{\text{distance}}{\text{speed}}$$

or

$$s = {}^{d}/_{t} \qquad d = s \times t \qquad t = {}^{d}/_{s}$$

In real life, you rarely need to calculate speed or distance, but you do need to calculate time so that you know when to set out in order to arrive on time.

Do you know how fast you walk?

Exercises

7. Georgie wants to find out how fast she walks. She timed herself as she walked for 1 mile. It took her 20 minutes, how fast did she walk?

8. Hattie knows that she can walk the 2 miles to the gym in 30 minutes. At what speed does she walk?

Once you know your walking speed, you can calculate the time it takes you to walk to a destination. This will always be an estimate because you do not walk at the same speed all the time as you have to stop to cross roads, slow down to look at a shop window or speed up to overtake someone.

You can find the distance to your destination from a road sign, or on a map or on your phone.

Example

Isha walks at 4 miles an hour. She is walking to Helmsley and reaches this road sign at 11 am. What time should she get to Helmsley?

$t = {}^{d}/_{s}$

$= {}^{3}/_{4}$ hour

$= 45$ minutes

Answer: Isha should reach Hemsley at 11:45 am

When the numbers are more complicated, you may need to use your calculator app:

Another Example

Jasmine walks at 3.5 miles an hour. How long will it take her to walk from the road sign to Sproxton?

$t = {}^{d}/_{s}$

$= {}^{1.25}/_{3.5}$

$= 0.357...$ hours

$= 21.42$ minutes

Answer: It will take Jasmine about 22 minutes.

Note that Jasmine has multiplied the awkward decimal answer by 60 to change the time in hours to minutes and then rounds up – as this is only an estimate.

Exercises

9. Kailie walks at 3 miles an hour. She has arrived at the Exeter by bus and wants to visit the museum. Her guide book tells her that it is 2 miles from the central bus station to the museum. How long does it take her to walk to the museum?

10. Lisa is camping in the New Forest. She can see form her map that it is $2\ ^1/_2$ miles from the camp site to the village of Boldre where she can buy supplies. If Lisa walks at 4 miles an hour, how long will it take her to get to the village and back, allowing 15 minutes to do her shopping?

11. From the signpost on the previous page work out at what time Mina will arrive at Sproxton if she left Oswalkirk at 9:15 am and her walking speed is $3\ ^1/_2$ miles an hour.

12. If Nina is not going to break the speed limit, how long will it take her to drive the $1\ ^3/_4$ miles through Half Moon village?

Food for Thought

Does your school or college offer you the opportunity to take part in the Duke of Edinburgh Award scheme, or DofE?

Any young person from 14 to 24 years old can do their DofE – regardless of ability, gender, background or location. The programme leads to a Bronze, Silver or Gold Duke of Edinburgh's Award.

There are four sections to complete at Bronze and Silver level and five at Gold. These include completing an expedition, so map skills and time calculations will be important.

As the DofE is not a competition but about setting personal challenges and pushing personal boundaries, it should prove to be worthwhile whatever your eventual ambitions may be.

Food for Thought

Lady Hester Stanhope was born in 1776 and became an adventurer, archaeologist and one of the most famous travellers of her age, a remarkable achievement for a woman at that time.

After an early career managing the household of her uncle, the Prime Minister, William Pitt, she left England for good in 1810 with her physician, her maid and her boyfriend. Her party travelled first to Athens, from there they went to Constantinople, nowadays known as Istanbul. Here Lady Hester chose to dress as a Turkish male rather than wear tradition women's clothing and veil. She then travelled to Cairo, surviving a shipwreck, and went on to explore the Mediterranean and the Middle East.

In 1815, she embarked on an archaeological excavation in Palestine. She was determined that their archaeological heritage would not be looted for treasures sold in Europe but rather benefit the Ottoman government.

She lived for a while in Lebanon before settling in Syria where she held great authority over the surrounding districts and became the de facto ruler of the region before her death in 1839.

Foreign Exchange

When travelling abroad, one of the most important things that you need to do is to get to grips with the local currency. You need to do this before you travel so that you can budget how much your travelling will cost and how much money you will either need to take with you or to have access to from your UK bank account.

Think about a trip to Thailand. The first thing to do is to look up the exchange rate:

1 Pound sterling (£1 GBP) equals 43.15 Thai Baht (THB ฿)

And then find out the cost of a hostel, the price of meals and any bus or train tickets that you will need:

Daily costs:

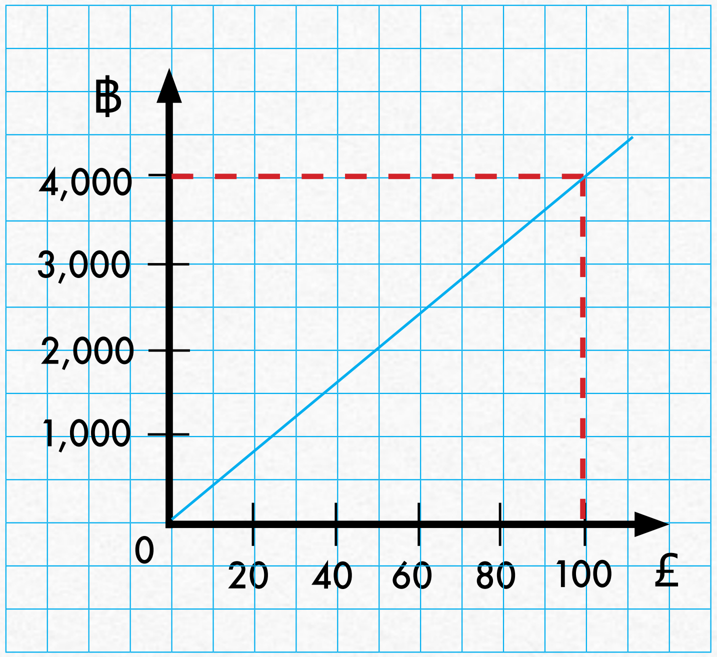
- **Basic guesthouse room: 500–1,000฿**
- **Midrange hotel room: 1,000–4,000฿**
- **Market/street stall meal: 40–100฿**
- **Small bottle of beer: 100฿**
- **Restaurant meal: 150–350฿**
- **Organised tour or activity: 1,000–1,500฿**

You can revise how to plan the budget for your travelling from the end of Step **3** – How to spend money. This section looks at converting currency.

Firstly, remember that the conversion rate will always be an estimate as it costs a fee to exchange or take out foreign currency, also the rate of exchange will change every day. Therefore, when you do your budgeting, round the exchange rate **down**.

£1 GBP equals 40฿ THB

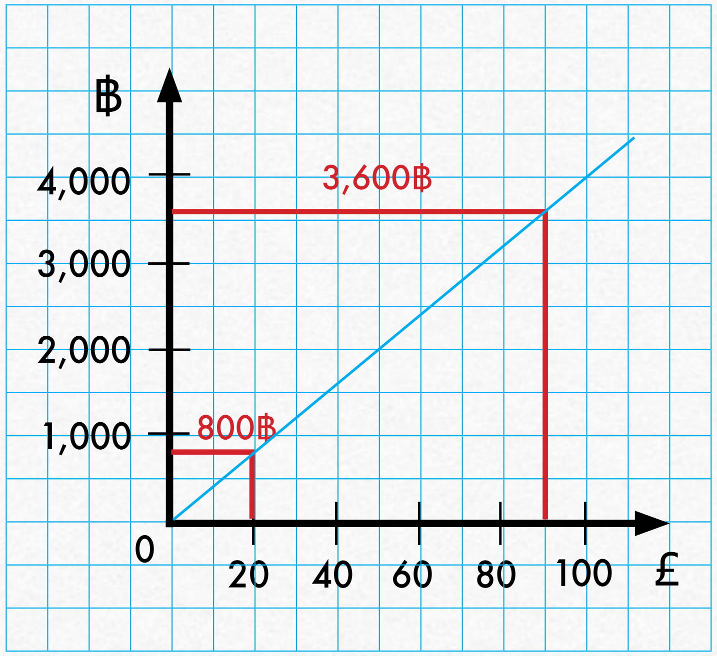
You can use a calculator to work out the pound sterling equivalent of your travel costs, but it can also be useful to make a conversion graph and take it with you:



Example

a) Oona has booked into a guesthouse that costs **800฿** per night. What is that in pounds?

b) Oona and her two friends have booked a tour that costs **1,200฿** each. What is the total cost in pounds of the three tours?



Oona works out her costs by drawing a line from the cost in bahts to the sloping line and then a line down to the amount in pounds.

Answers

a) 800B is approximately £20

The tour for 3 friends will be **3 x 1,200฿ = 3,600฿**

b) On the graph 3,600฿ ≈ £90

Exercises

13. Using the bahts / **£** graph work out the cost of:

(a) One night at a midrange hotel at **2,500฿**

(b) Two market stall meals at **80฿** each and two bottles of beer at **100฿** each

(c) A restaurant meal for **4** people costing **250฿** each

(d) Seven days motorbike hire at **200฿** per day

14. Draw a Turkish lire to pound sterling graph using the exchange rate:

£1 equals 11.5 Turkish Lire (TL)

Use your graph to work out the cost in pounds of:

(a) a mid range hotel room costing **220 TL**

(b) a local meal for two costing **80 TL** each

(c) four day trips to Gallipoli costing **90 TL** each

(a) a four day Gulet boat trip costing **650 TL**

15. Draw an Australian dollar to pound sterling graph using the exchange rate:

£1 equals 1.79 Australian dollars (AUD $)

Use your graph to work out the cost in pounds of:

(a) a hostel dorm bed costing AUD **$40**

(b) a café meal for three costing AUD **$30** each

(c) four tickets to a gig costing AUD **$30** each

(d) a round Sydney bike tour costing AUD **$145**

If you prefer, you can use a calculator.

For example, using

1 pound sterling equals 43.15 Thai baht

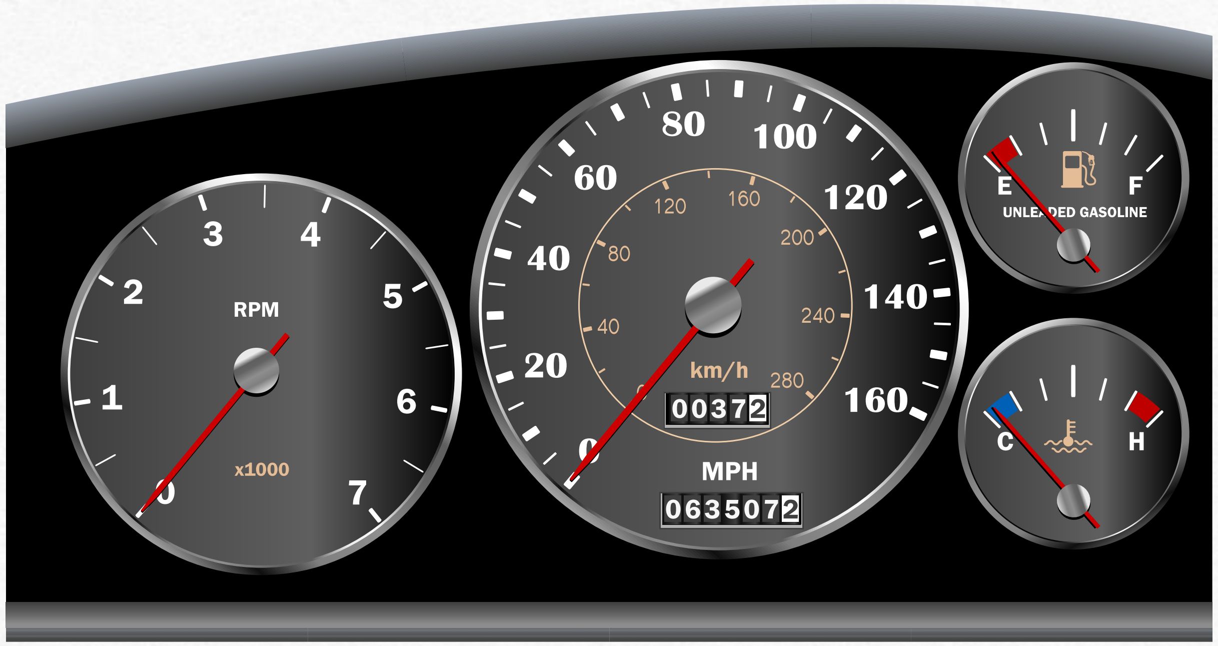
Then to find the value of **800฿** in pounds you would divide **800** by **43.15** to get the answer: **£18.54**.

Exercise

16. Now answer the previous exercises **Q13 – Q15** by using a calculator and without rounding the exchange rate down.

Understanding Your Car

When you travel by car, there are plenty of numbers on display and it is important to know what these are telling you.



Basic information is shown as the example above. The large central dial is a speedometer which tells you the speed the car is going in miles per hour (**mph**), with an inner scale giving the same speed in kilometres per hour (**km/h**).

The fastest a car can legally be driven in the UK is **70 mph** although the limit is higher in some other countries.

The dial to the left shows revolutions per minute (RPM). In general, the lower the RPM the more efficiently the engine is running.

To the right, the dials show the amount of fuel in the tank and the temperature of the engine. Most cars will sound an alarm when the temperature is too high or the fuel too low.

Fuel is important when you are considering the cost of running a car. There is not a simple formula that tells you that for a certain distance then the cost per mile will be **£***x*. Different models of cars will quote a figure MPG, that stands for miles per gallon. (That is not a very helpful figure as nowadays petrol is sold in litres.) Actual fuel consumption will depend on the speed that you are driving at, the RPM and the amount of times you go up and down the gears. Basically, driving at various speeds in a town is less economical than driving at a steady speed on a motorway, but not too fast.

Calculating miles per litre and MPG

To work out the cost of running your car look at these two figures on the speedometer:

0 0 3 7 2

0 6 3 5 7 2

The lower tells you the number of miles that your car has travelled in its lifetime and the one above is a 'trip recorder' that can be reset to tell you the number of miles on one trip.

To find the miles per litre, first fill your car up and then change the trip recorder to zero.

0 0 0 0 0

Then drive the car as normal. Next time you fill the tank right up take a note of the amount of litres that you put in, it should tell you on your receipt:

38.45 litre @ **129.7** P/L = **£49.87**

And take a note of your trip record, the last figure on the white background is tenths of a mile:

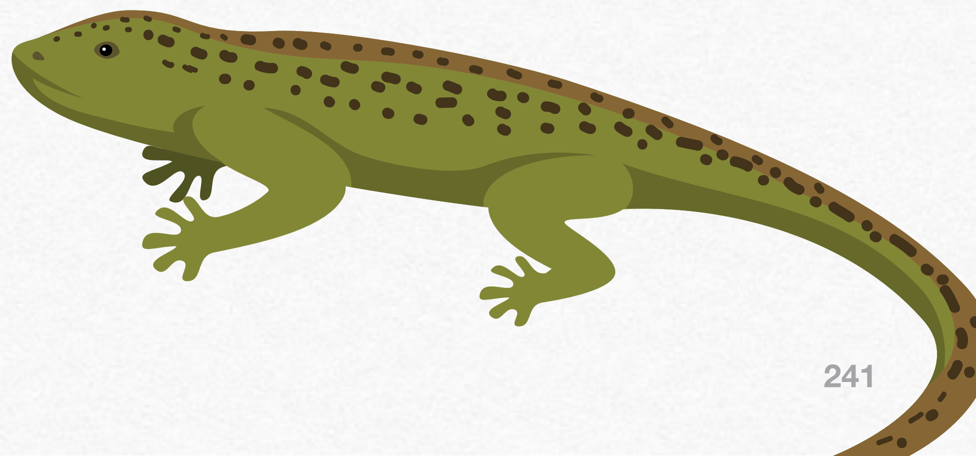
0 3 8 9 7

To find the number of miles per litre, divide the number of miles by the number of litres:

$$\text{Miles per litre} = \frac{\text{miles}}{\text{litres}} = \frac{389.7}{38.45} = 10.1 \text{ MPL}$$

If you want to compare this to the miles per gallon quoted for your model of car then multiply by **4.55** (the number of litres in a gallon):

10.1 MPL = **45.955** MPG



Calculating cost per mile

To find the associated cost per mile, divide the cost of filling your tank by the number of miles you drove:

$$\text{Cost per mile} = \frac{\text{cost}}{\text{miles}}$$

$$= \frac{49.87}{389.7}$$

$$= £0.127\ldots \text{ or } 13\text{p per mile}$$

Food for Thought

However much you might fall in love with a certain model of car, make sure that you do your calculations about the cost of running it carefully before you make your purchase. You should also look at user reviews and articles in car magazines. It is also important to have a good long test drive to make sure that its handles as well as it looks.

You should also read the manual that comes with your new car.

Many cars have a trip computer that tells you the current fuel consumption:

Exercises

Give your answers to 1 d.p. (decimal place) unless the answer is money.

17. Peta had put her trip recorder to zero last time she filled up. It now reads:

0	4	2	1	3

Peta now puts 35.7 litres into her tank. How many miles per litre has she done? What is that in MPG?

18. If the cost of a litre of petrol is 125.8p, what is the cost per mile for Peta's car?

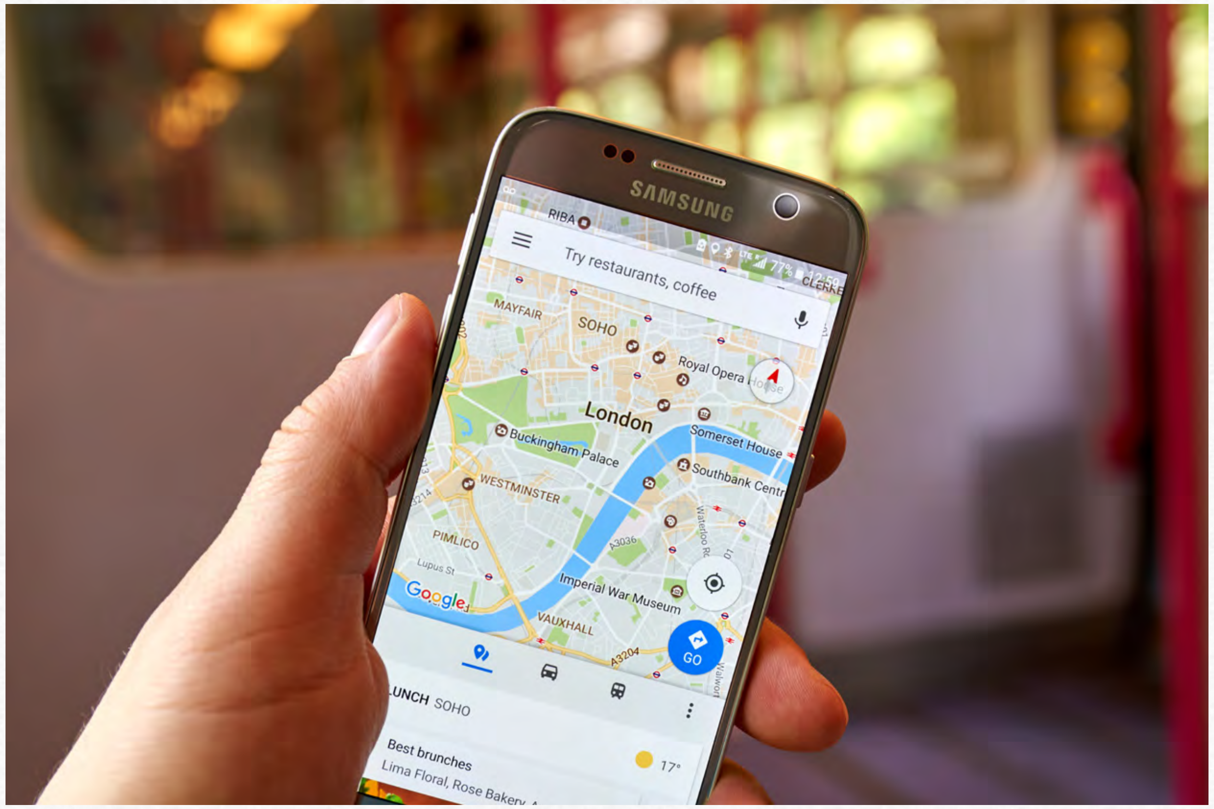
19. Peta is driving to London. She passes a sign saying that she has 145 miles to go and her petrol gauge shows the tank is a quarter full. If her tank holds 50 litres, has she got enough fuel to complete the trip?

20. Rana has driven herself and two friends from London to Liverpool. The distance is 220 miles and her car does 50 MPG. If the cost of a litre of petrol is 125.8p. What should she ask each friend to pay as their share of the cost?

How to Read a Map

There are various types of map available to explorers.

The one that you are most familiar with is likely to be the map app on a mobile telephone:



You can put in a place name or a post code and the app will tell you how far away the place is and how to get there.

You can also ask the app to show you how to get somewhere by public transport. Many apps also can be used to find useful places nearby such as banks or supermarkets.

If you want to use your phone app for exploring there are a few things to remember:

- The app can use a lot of phone battery so do make sure that you have your phone fully charged.
- It can be a good idea to take screen shot of the route, print it out and use that. Then you will not be at risk of having your phone snatched from your hand.
- If walking, make sure that you know if the route and the time assumes you are on foot and not in a car.
- When abroad, using the map app needs roaming to be on, and this can be very expensive. Check with your phone provider what they charge when overseas.

This girl is at risk of having her phone stolen!

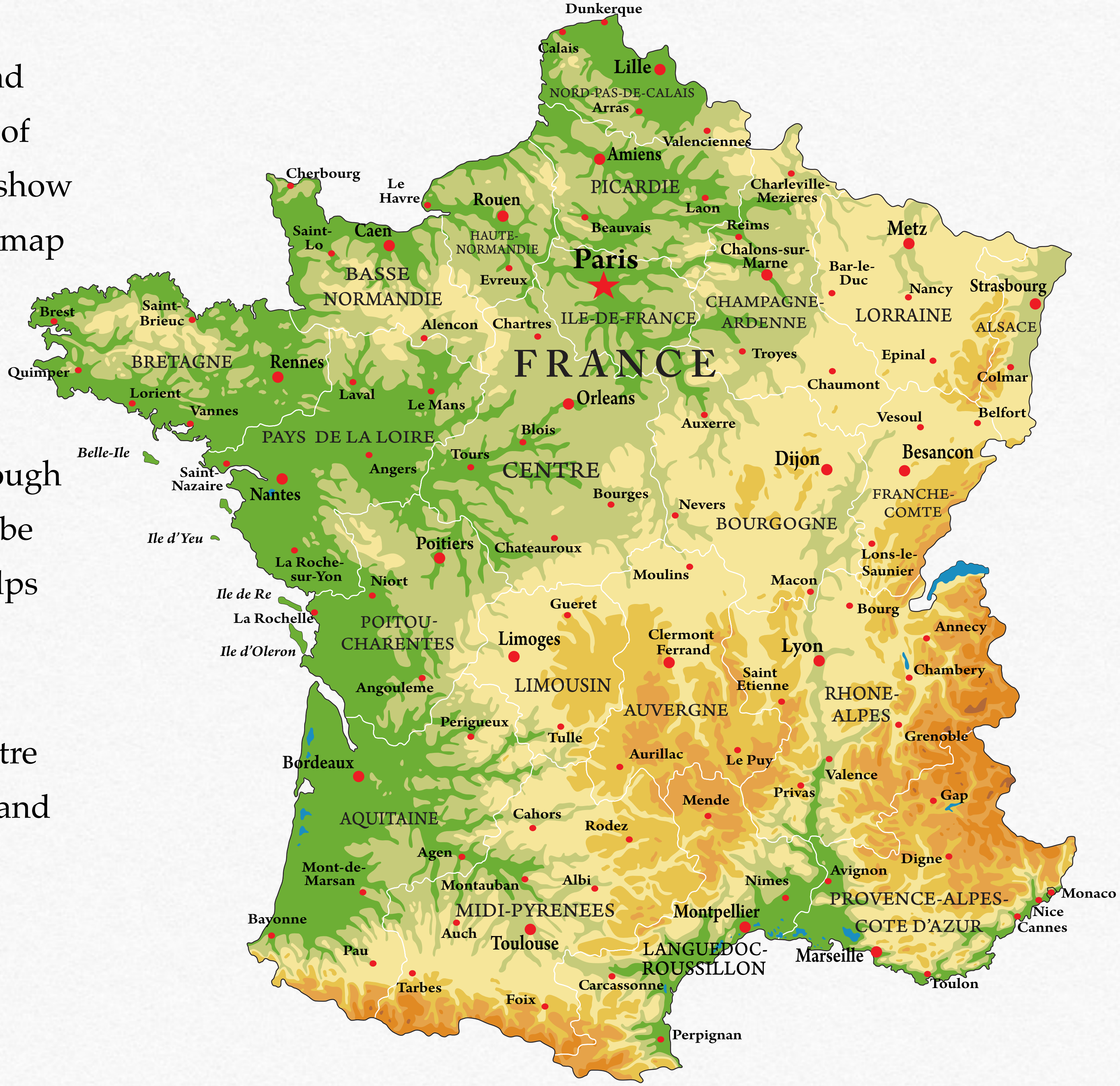
There are other types of map that can be useful for explorers.

An **atlas** is useful when learning about a country and its neighbours. An atlas will often have appendices of useful information. In an atlas, a political map will show the layout of countries, cities and states. A physical map will show mountains and rivers:

A Physical Map of France

The colour of the land changes from pale green through yellow to brown as the height increase. You should be able to identify the Pyrenees to the south and the Alps to the south east.

Did you know that there is that high area in the centre of southern France? It is called the 'Massif Central' and covers about 15 % of the country.



A **road map** is what you need for driving. It gives you the names of the roads, the number of each motorway junction and distances. This map shows that to drive from Winchester to Emsworth you will join the **M3** at junction **9**, then join the **M27**. You will leave the **M27** at junction **12** and continue of the **A27**. The total distance will be **8 + 19 + 11 = 38** miles.

You should be able to find an app that will give this information too. It will show you exactly where you are on your route and give directions. However, you should always have a physical road map in your car as you may not always have an internet connection, particularly if you are exploring a remote location.

For exploring in Britain, you will need an **Ordnance Survey** map. There are two types.

The **OS Explorer** maps are best for walking, running and hiking. They are also good for off-road cycling, running, horse-riding, climbing and even kayaking. The scale is **1 : 25,000** (**4 cm** on the map **= 1 km** in the real world). These maps clearly display footpaths, rights of way, open access land, as well as the vegetation on the land as well as car parks, contour lines, campsites, pubs etc.

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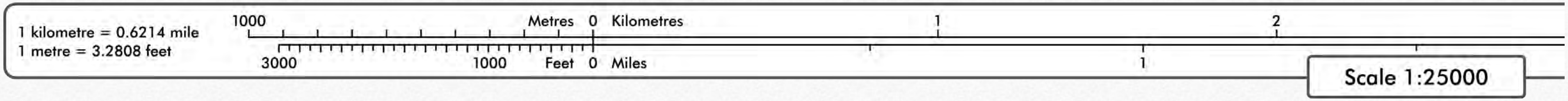
The **OS Landranger** maps are better for cycling and longer routes. They are good for driving holidays and finding the best tourist attractions. The scale is **1 : 50 000** (**2 cm** on the map **= 1 km** in real world). The maps display roads and tourist features as well as footpaths and rights of way. They cover a larger area than Explorer maps but with less detail.

You can get both types of map as an app on a smart phone.

The next examples and exercises are all based on OS Explorer map **OL22** for the New Forest.

When using a map, you need to have a look at the scale. This is the scale for an OS Explorer map:



You can see that it gives the scale as a ratio **1 : 25,000** as well as a drawing of what represents **1** mile.

When you are exploring in Britain, as road signs give distances in miles, it makes sense to work in miles. In other countries it would make more sense to work in kilometres.

The scale **1 : 25,000** means that **4 cm** on the map **= 4 x 25,000 cm** or **100,000 cm** in real life which is equal to **1 km**.

If you are working in miles, as **5** miles **= 8 km**, then **1 km =** $^5/_8$ mile. Therefore **4 cm** on the map is $^5/_8$ mile.

Example

Sienna is on an expedition for her DofE. From her OS Explorer map she has measured that the distance from her camp site to the rendezvous point is **27 cm**. What is this in miles?

Sienna uses scale as a ratio by drawing a table and applies the unitary method:

Map	Real
4 cm	$^5/_8$ mile
1 cm	$^5/_8 \div 4 = {}^5/_{32}$ miles
27 cm	$27 \times {}^5/_{32}$ mile = 4.2 miles

Answer: 27 cm on the map is 4.2 miles (to 1 d.p.)

Another Example

Sienna walks at **4 mph**. How long does it take her to walk from her camp site to the rendezvous?

$$t = {}^{d}/_{s}$$

$$= {}^{4.2}/_{4}$$

$$= 1.05 \text{ hours}$$

Answer: it will take Sienna just over an hour, but as 4 mph is quite fast she will probably get tired and then walk a bit slower, she allows 1 hour and 15 minutes.

When you are walking or cycling the paths and tracks that you follow are rarely a straight line, therefore a more practical way of working with a map is to use a piece of string. Use the visual scale and mark off every half mile on your string, up to say **5** miles. You can then wind your piece of string over the tracks you are going to follow. When you have worked out your route then you can straighten out your string and count the number of miles.

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Walking at **3 mph** would take an hour but cycling at **12 mph** would take ${}^{3}/_{12} = \frac{1}{4}$ hour or **15** minutes.

Exercises

Use this section of OS Explorer map **OL22** to answer the questions on the next page.

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21. On the OS Explorer map of the New Forest find the shortest route to walk from the car park at Clayhill Heath to Stag Park. Calculate the real distance from the car park at Clayhill Heath to Stag Park if the distance on the map measures **21 cm**. How long would it take to walk this distance at **3 mph**?

22. On the OS Explorer map of the new Forest, calculate the real distance using the various paths and tracks from the Lime Wood Hotel to the start of Bishops Dyke. How long would it take to cycle this distance at **12 mph**?

23. Sienna is in charge of catering. At **5 pm** she realises that she needs some extra potatoes and heads off from the campsite next to Clayhill to the farm shop at Denny Cottage. If she walks at **3.5 mph**, takes **10** minutes over her shopping and then will need **30** minutes to prepare the potatoes, what is the earliest time that she can call the rest of her group to eat supper?

Food for Thought

What other tasks do you think might have been allocated to members of Sienna's group? Someone will have had to make the fire, others to put up tents. Are they any other jobs that you can identify?

What kit do you think members of the expedition will have to bring with them? Is there any prior experience that would be of use?

Food for Thought

Rosie Stancer (born 1960) is a mother of one and a Polar explorer. She is a cousin of Queen Elizabeth II.

In 1997, Rosie was one of 20 women on the first all-women's expedition to the North Pole. A relay of five teams crossed 500 miles of pack ice in temperatures down to minus 40°C. In 1999, Rosie and four colleagues from the first expedition organised and managed their own expedition to the South Pole, without guides. They also gathered valuable meteorological data en route.

Rosie has skied solo to the South Pole. She attempted to become the first woman to trek solo to the geographic North Pole. Over 84 days, Rosie travelled significantly further than any previous attempts by a woman and created a new World Record.

The expedition finally had to be abandoned due to treacherous conditions that are now recognised as the worst on record. Rosie terms her attempt as the "World's Last".

Using a Compass

One vital piece of equipment is a compass.

The map will show you the area around where you are, but you have to use your own knowledge to find out exactly where you are on the map.

You can see how maps show houses, places of interest and landmarks and this can help you to orientate yourself. Maps are drawn with north upwards, and this is why you need a compass.

This is a typical compass used for orienteering and expeditions:

You can see how you can hang it round your neck. It has an arrow that you can point in the direction that you are walking. The magnetic needle in the central casing swivels to always point to the magnetic north pole. The black dial with points of the compass marked on it can be rotated.

Points of the compass

Turning a full circle is turning an angle of 360°. Points of the compass are measured from north, which is 0°. Thus east is 90°, south is 180° and west is 270°.

As all bearings have three figures, we say that east is 090° (oh-nine-oh).

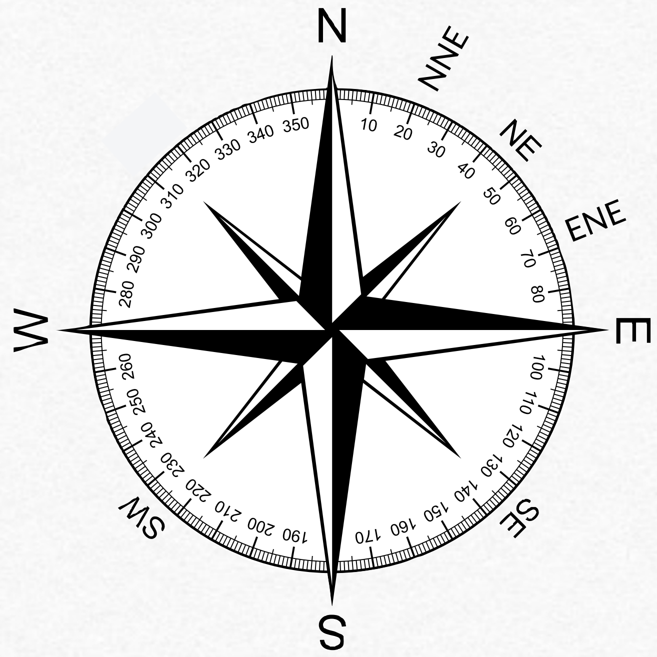
The point midway between N and E is NE, northeast.

The point midway between north and northeast is NNE, north-northeast.

The point midway between NE and east is ENE, east-northeast.

Exercises

24. Copy this compass, and mark on it all the missing bearings.



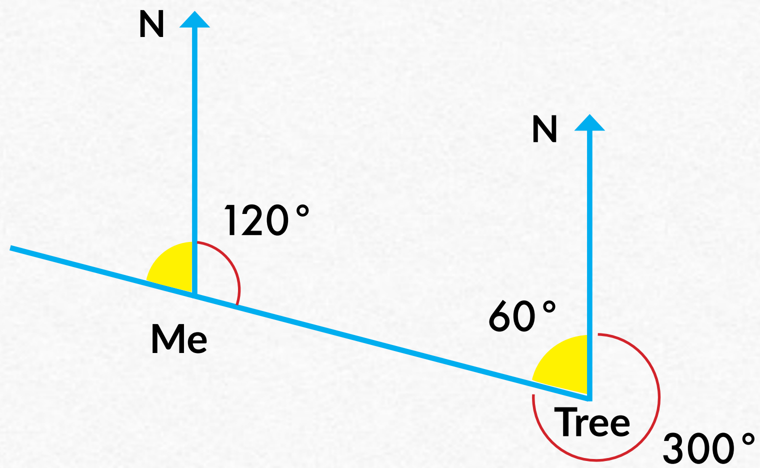
25. Now copy and complete this table, giving the degrees for each point of the compass:

Point	Degrees	Point	Degrees
N	0°	S	180°
NNE	022.5°	SSW	
NE	045°	SW	
ENE		WSW	
E	090°	W	270°
ESE		WNW	
SE		NW	
SSE		NNW	

Bearings

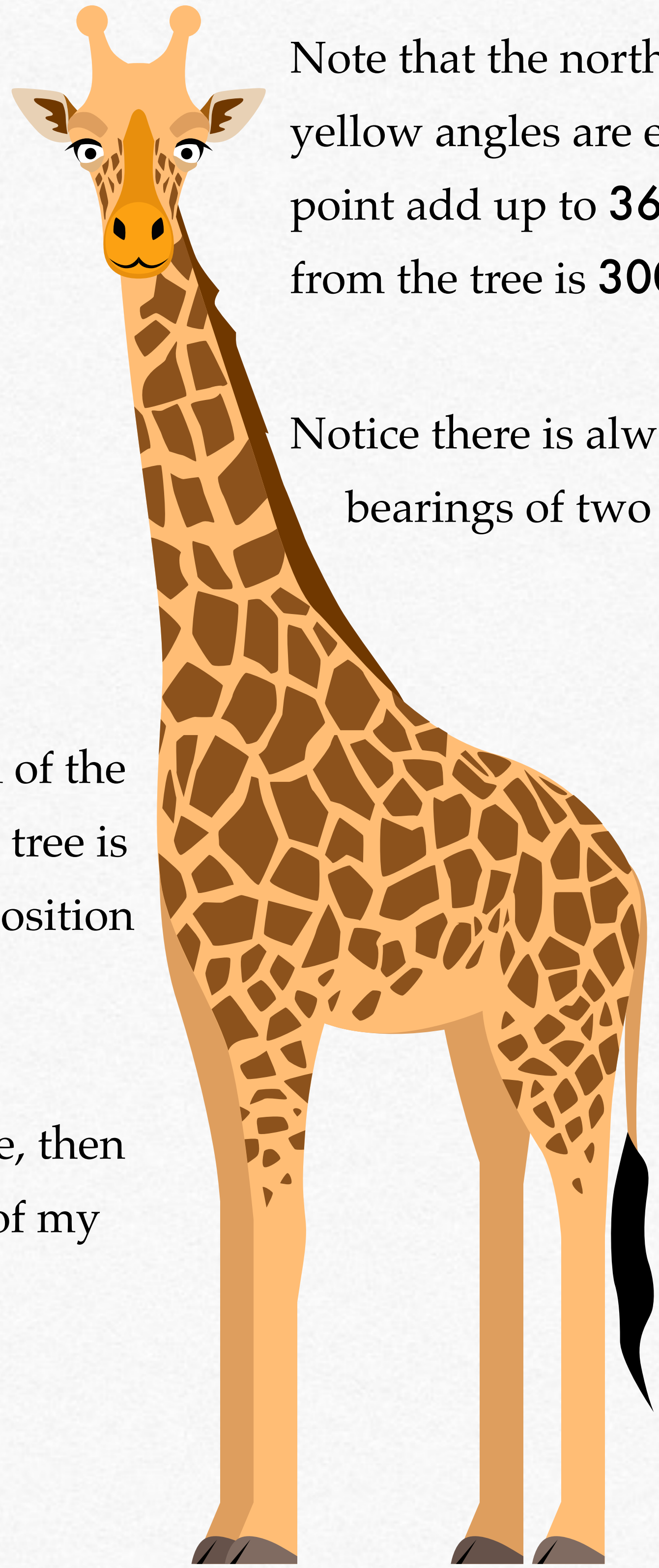
Degrees are used to take bearings, measuring clockwise from north. Taking bearings is one way to find out exactly where you are on a map.

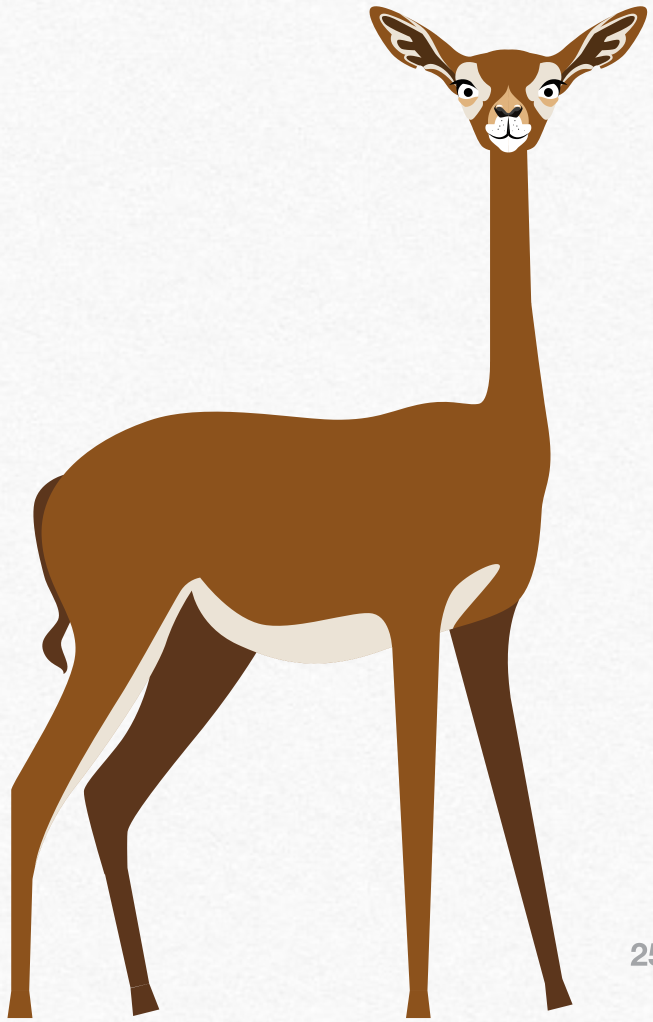
Look at this diagram:



I am standing at a point and looking at the direction of the tree with my compass. The compass direction of the tree is $120°$ from north. The bearing of the tree from my position is therefore $120°$.

If my friend is standing at the tree and looking at me, then she will see me at a direction of $300°$. The bearing of my position from the tree is $300°$.





Where I am standing, I use the angle fact that angles on a straight line add up to $180°$ to calculate that the yellow angle is $60°$.

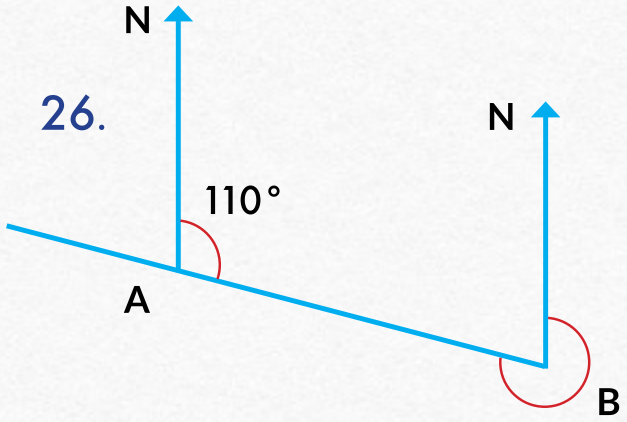
Note that the north lines are parallel and therefore the two yellow angles are equal. Use the angle fact that angles at a point add up to $360°$ to calculate that the bearing of me from the tree is $300°$.

Notice there is always a difference of $180°$ between the bearings of two points looking at each other.

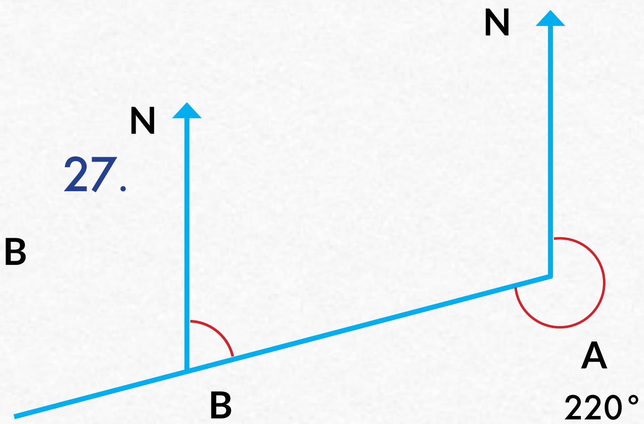
Exercises

In these drawings you are given the bearing of B from A. Calculate the bearing of A from B.

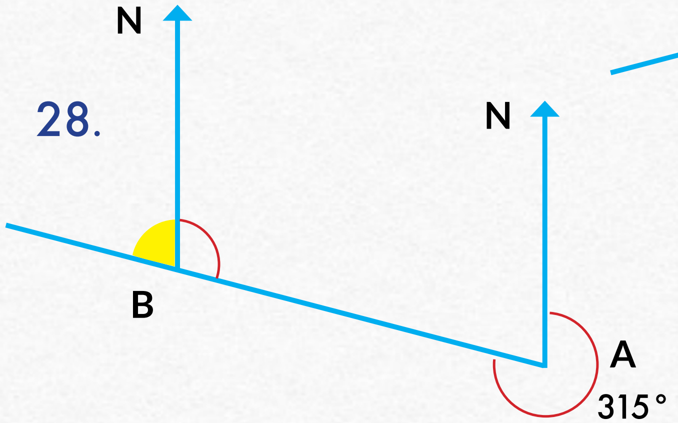
26.



27.



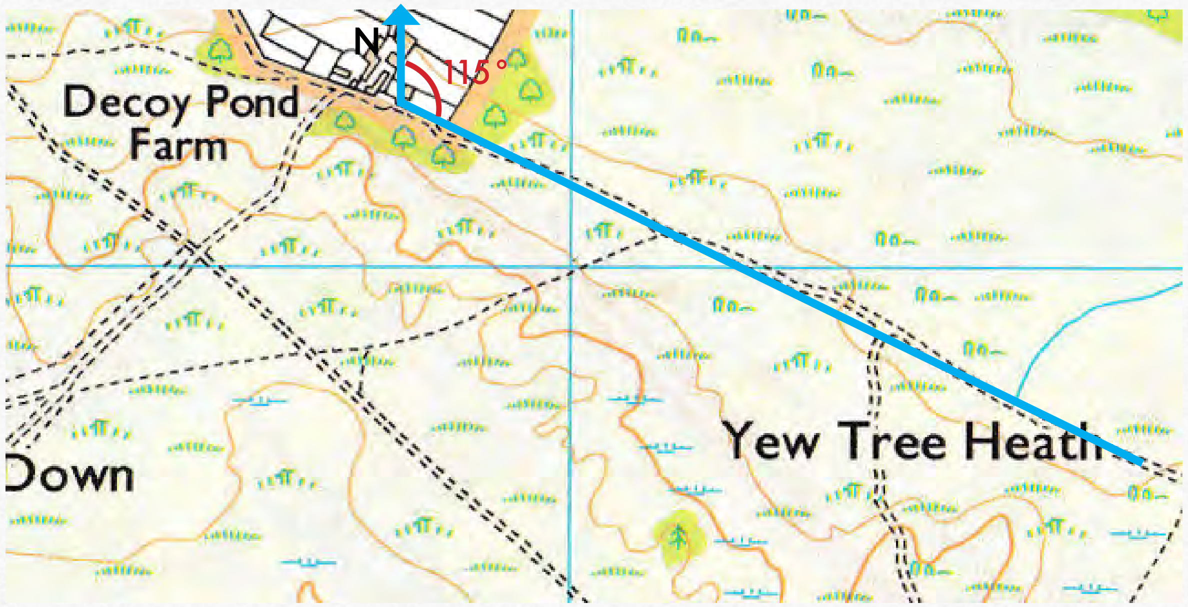
28.



From your answers, can you see that if the bearing at point A is less than $180°$, add on $180°$ to find the bearing at point B? If however the bearing at point A is more than $180°$, subtract $180°$ to find the bearing at point B. Now think about using this on a map.

Here you are at a meeting of two paths running over a heath, but you are not sure exactly where. You can see a farm not far away and you take a bearing of the nearest part of the farm. It is $295°$.

If a bearing of $295°$ points to a farm, then plot a bearing of $115°$ ($295° - 180°$) at the farm on the map with a long bearing line. Anywhere on that line will give a bearing $295°$ towards the farm.



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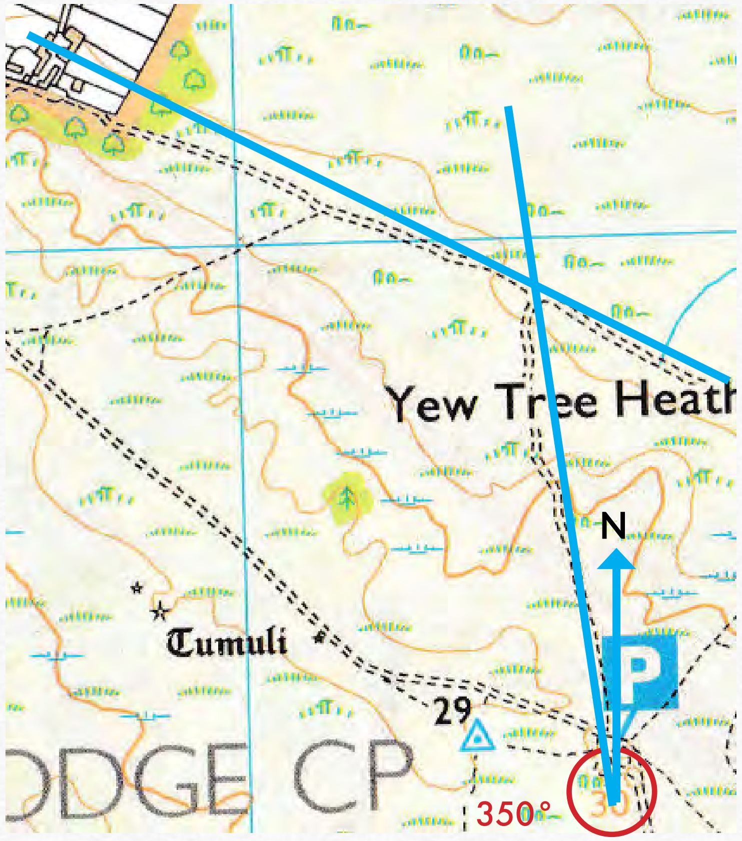
Now the line runs through two possible points where paths meet and so you will have to take another bearing to find where you are.

You can see a car park and so you take a bearing of the nearest part of that. The bearing is **170°**. **170°** is less than **180°**, so add on **180°**: **170°** + **180°= 350°**. Plot the bearing of **350°** at the car park on the map.



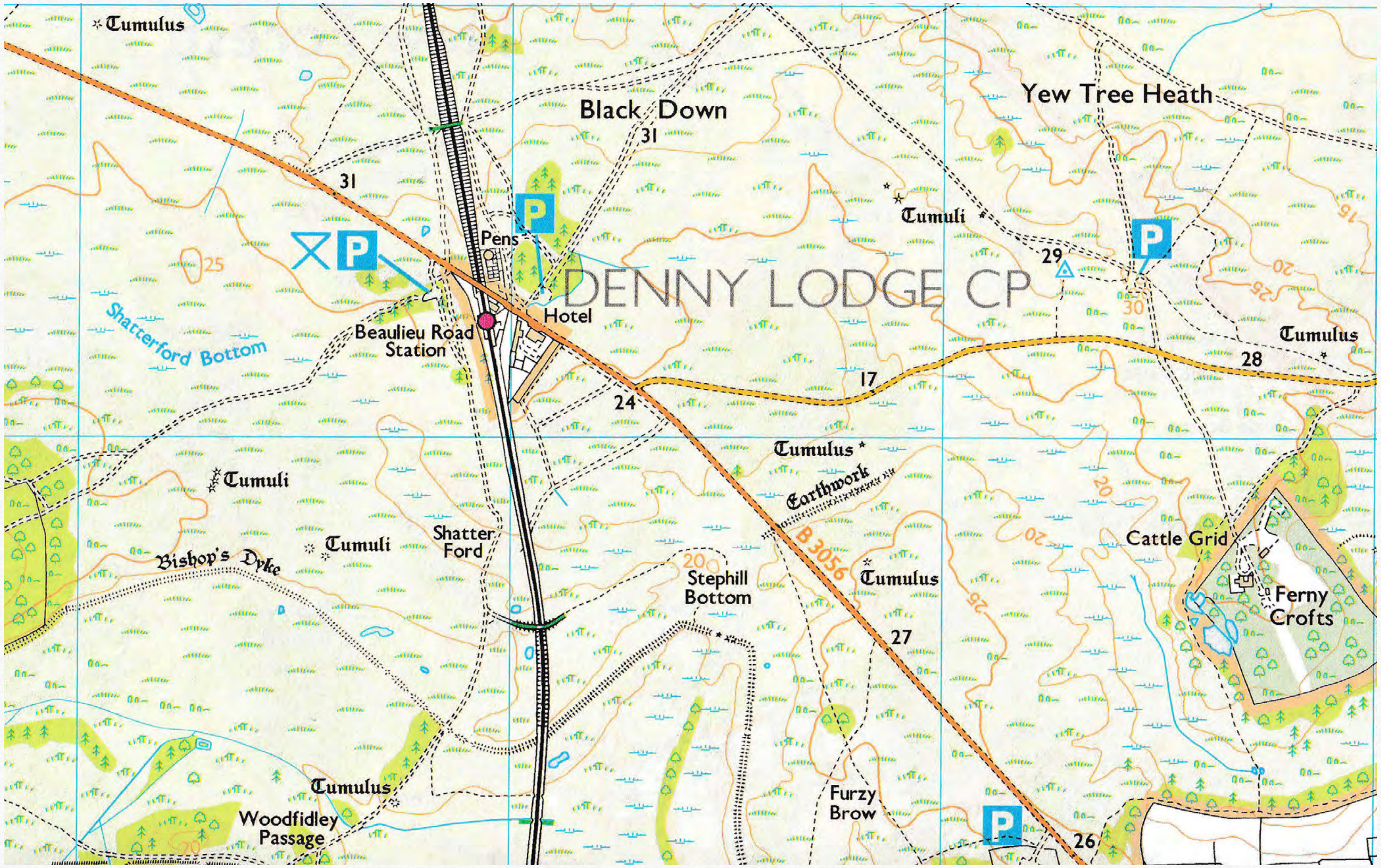
Now you can see exactly where you are on the map.

It is best to use three facts to find your exact location. Sometimes you will take three bearings but, in this case, I was at a point when two paths crossed so that was my third fact.

Exercises

29. Tilly is lost. She knows she is not far from Beaulieu. She takes three bearings of things around her.

The bearing of a railway bridge is **240°**, the bearing of the nearest pond is **115°** and the bearing of a car park is **060°**. Draw these bearings on this map and find Tilly's location.



30. Uta is on a yacht in the Solent. She needs to know exactly where she is and takes three bearings.

The bearing of the end of the jetty by Sconce Point is **160°**, the bearing of the Hurst Castle is **225°** and the bearing of the jetty by Keyhaven Marshes is **310°**. Draw these bearings on this map and find Uta's location.

In what direction should she sail in order to enter Keyhaven Lake without going aground?

Answers to Part 4

1. Betty has forgotten that the clocks should have changed overnight. Her traditional alarm clock tells her that the time is a quarter past eight on the last Sunday in October, what is the actual time?

Clocks go back in October therefore:

8:15 - 1 = 7:15

Answer: The actual time is 7:15 am

2. Cherie had a really relaxed weekend. She got up as usual on Monday morning but to her surprise when she arrived at work she was told that she was an hour late. Why do you think that was and what was the date?

Answer: She should have put her clocks forward on Saturday night. It is the Monday after the last Sunday in March.

3. If it is **5 pm** in London, what is the time in:

(a) Los Angeles (UTC **−8**) (b) South Africa (UTC **+2**)
(c) Thailand (UTC **+7**)?

(a) Los Angeles is in Zone **−8**, **17:00 − 8 = 09:00**

Answer: The time is 9 am

(b) South Africa is in Zone **+ 2**, **17:00 + 2 = 19:00**

Answer: The time is 7 pm

(c) Thailand is in Zone **+7**, **17:00 + 7 = 24:00**

Answer: The time is midnight

4. Della's boyfriend is working in Miami (UTC **−5**). She wants to talk to him at **6 pm** his time. What time will that be in the U.K.?

Miami is in Zone **−5**, so Della must add **5** to the Miami time.

Answer: 6 pm in Miami will be 11 pm in the UK

5. Eithne is travelling in New Zealand (UTC +12). She wants to call her mum in the UK on her birthday. If she decides to call her mum at 8 pm UK time, what will be the time in New Zealand?

New Zealand is in Zone +12, so Eithne must add 12 hours to the UK time.

Answer: 8 pm UK time will be 8 am in New Zealand but Eithne must call the morning after the date of Eithne's mum's birthday in New Zealand

6. In March, Freya travels to Dubai (UTC+4). Her parents live in Manchester, England (UTC). and her boyfriend is travelling in South America (UTC−3). Freya has set up a card that reminds of the times in each country so that she can call at a convenient time of day:

(a) Fill in the missing times on the card.

Dubai	UK	Brazil
12 noon	8 am	5 am
4 pm	12 noon	9 am
11 pm	7 pm	4 pm

(b) In the last Sunday in March, UK clocks go forward to British Summer Time, but they do not in Dubai or Brazil. If Freya wants to call her parents on Sunday evening at 7 pm UK time, what time will it be in Dubai?

Clocks have gone forward, so when it is 7 pm on Sunday it would have been 6 pm if the clocks hadn't changed. So the time difference between UK and Dubai will now be one hour less, +3.

Answer: The time in the Dubai will be 10 pm

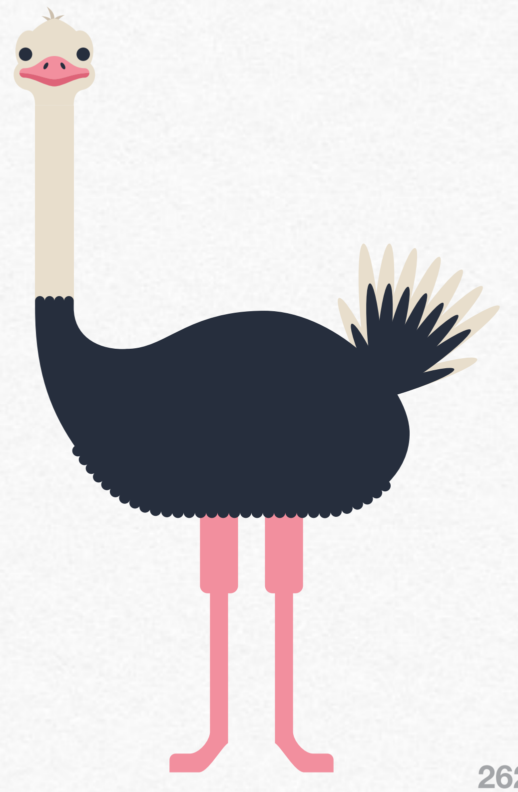
7. Georgie wants to find out how fast she walks. She timed herself as she walked 1 mile. It took her 20 minutes, how fast did she walk?

20 minutes = $^1/_3$ hour

$$\textit{Speed} = \frac{\textit{distance}}{\textit{time}}$$

$$= 1 \div {}^1/_3$$

$$= 3 \text{ miles per hour}$$



Answer: Georgie walks at 3 mph

8. Hattie knows that she can walk the 2 miles to the gym in 30 minutes. At what speed does she walk?

30 minutes = ½ hour

$$Speed = \frac{distance}{time}$$

$= 2 \div \frac{1}{2}$

$= 4$ miles per hour

Answer: Hattie walks at 4 mph

9. Kailie walks at 3 miles an hour. She has arrived at Exeter by bus and wants to visit the museum. Her guide book tells her that it is 2 miles from the central bus station to the museum. How long does it take her to walk to the museum?

$$Time = \frac{distance}{speed}$$

$= \frac{2}{3}$ hour

$= 40$ minutes

Answer: It takes Kailie 40 minutes to walk to the museum

10. Lisa is camping in the New Forest. She can see form her map that it is 2 ½ miles from the campsite to the village of Boldre where she can buy supplies. If Lisa walks at 4 miles an hour, how long will it take her to get to the village and back, allowing 15 minutes to do her shopping?

$$Time = \frac{distance}{speed}$$

$= {}^{2.5}/_{4}$

$= 0.625$ hours ($\times$ 60 to convert to minutes)

$= 37.5$ minutes

Total time $= 37.5 \times 2 + 15$

$= 75 + 15$

$= 90$ mins

$= 1$ hr 30 mins

Answer: It will take Lisa an hour and a half

11. From the signpost on the previous page work out at what time Mina will arrive at Sproxton if she left Oswalkirk at **9:15 am** and her walking speed is **3** ½ miles an hour.

Sproxton to Oswalkirk = 1.25 + 0.5 = 1.75 miles

$$Time = \frac{distance}{speed}$$

$$= {}^{1.75}/_{3.5}$$

$$= 0.5 \text{ hours} = 30 \text{ minutes}$$

9:15 + 30 mins = 9:45

Answer: Mina will arrive at 9:45 am

12. If Nina is not going to break the speed limit, how long will it take her to drive the **1** miles through Half Moon village?

$$Time = \frac{distance}{speed}$$

$$= {}^{1.75}/_{30}$$

$$= 0.05833\ldots \text{ hours}$$

$$= 3.5 \text{ minutes}$$

Answer: It will take Mina 3 and a half minutes

Food for Thought

The signpost with the distances is in Yorkshire. This county in Northern Britain is associated with many famous people and events in British History.

The Wars of the Roses in the fifteenth century were between the House of Lancashire and the House of York.

The Bronte sisters lived in Yorkshire.

The book, and the subsequent television series 'All Creatures Great and Small' is set in Yorkshire.

Have you ever explored the Yorkshire Dales?

13. Using the Bahts / **£** graph work out the cost of:

a) One night at a midrange hotel at **2,500฿**

b) Two market stall meals at **80฿** each and two bottles of beer at **100฿** each

c) A restaurant meal for **4** people costing **250฿** each

d) Seven days motorbike hire at **200฿** per day

(a) **Answer: 2,500฿ is approximately £62**

(b) 2 x 80 + 2 x 100 = 360฿

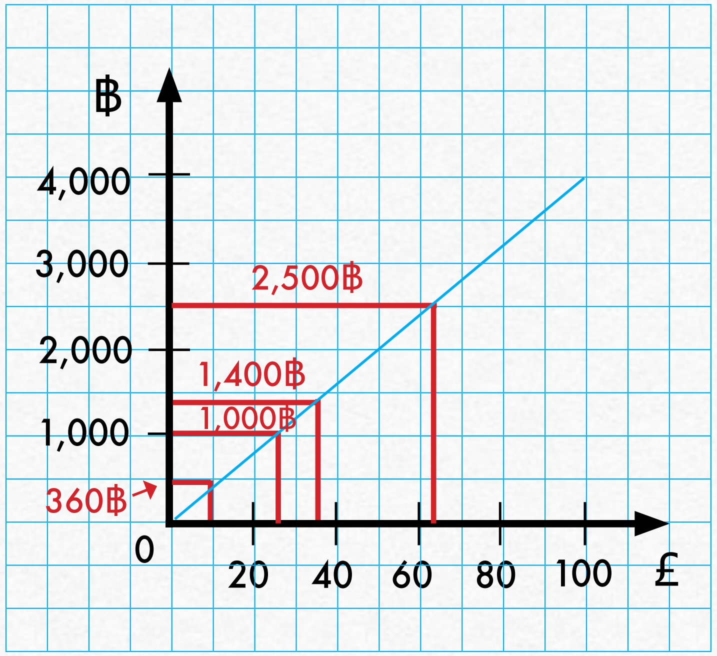
Answer: 360฿ is approximately £10

(c) 4 x 250 = 1,000฿

Answer: 1,000฿ is approximately £25

(d) 7 x 200 = 1,400฿

Answer: 1,400฿ is approximately £35



14. Draw a Turkish lire to pound sterling graph using the exchange rate:

£1 equals 11.5 Turkish Lire

Estimate down

£1 equals 10 Turkish Lire

Use your graph to work out the cost in pounds of:

a) a mid-range hotel room costing **220 TL**

b) a local meal for two costing **80 TL** each

c) four day trips to Gallipoli costing **90 TL** each

d) a four day Gulet boat trip costing **650 TL**

(a) **Answer: 220 TL is approximately £22**

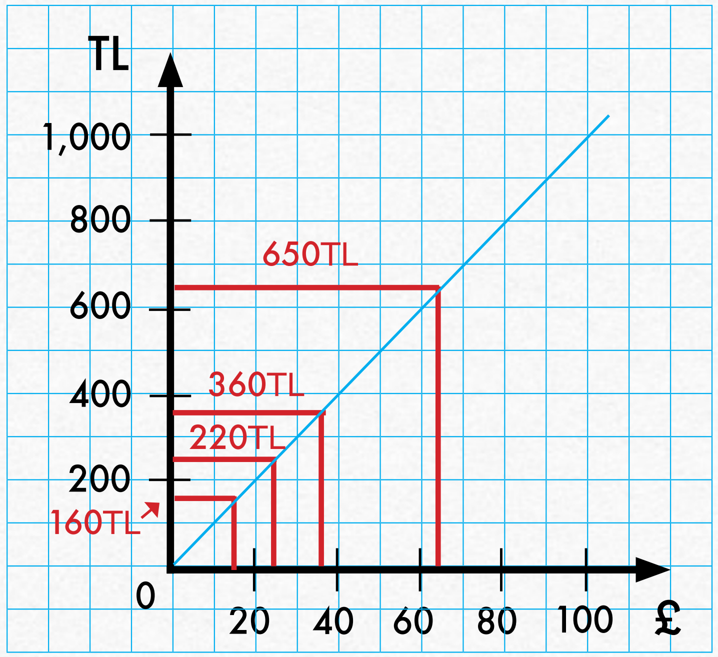
(b) 2 x 80 = 160 TL

Answer: 160 TL is approximately £16

(c) 4 x 90 TL = 360 TL

Answer: 360 TL is approximately £36

(d) **Answer: 650 TL is approximately £65**



15. Draw an Australian dollar to pound sterling graph using the exchange rate:

£1 equals 1.79 Australian dollars (AUD $)

Estimate down:

£1 equals AUD $1.75

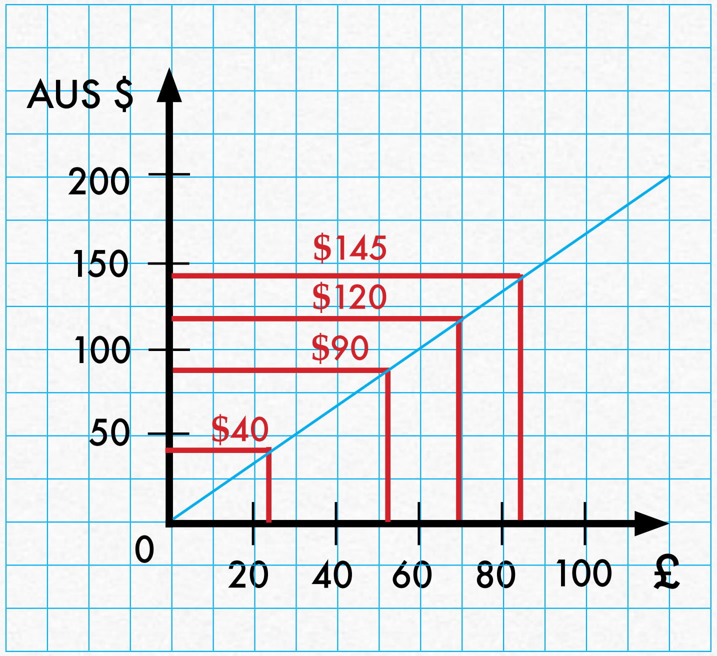
Use your graph to work out the cost in pounds of:

(a) a hostel dorm bed costing AUD **$40**

(b) a café meal for three costing AUD **$30** each

(c) four tickets to a gig costing AUD **$30** each

(d) a round Sydney bike tour costing AUD **$145**



(a) **Answer: AUD $40 is approximately £23**

(b) 3 x 30 = $90

Answer: AUD $90 is approximately £52

(c) 4 x 30 = $120

Answer: AUD $120 is approximately £69

(d) **Answer: AUD $145 is approximately £83**

16. Now answer the exercises **Q13 – Q15** by using a calculator and without rounding the exchange rate down.

[13.] Work out the cost of:

(a) One night at a midrange hotel at **2,500฿**

£1 equals 43.15฿

£? equals 2,500฿

2,500 ÷ 43.15 = £57.937. . .

Answer: 2,500฿ equals £57.94

(b) Two market stall meals at **80฿** each and two bottles of beer at **100**B each

360 ÷ 43.15 = £8.3429. . .

Answer: 360฿ equals £8.34

(c) A restaurant meal for **4** people costing **250฿** each

1,000 ÷ 43.15 = £23.1749. . .

Answer: 1,000฿ equals £23.17

(d) **7** days motorbike hire at **200฿** per day

1,400 ÷ 43.15 = £32.444. . .

Answer: £32.44

[14.] Work out the cost in pounds of:

(a) a midrange hotel room costing **220 TL**

£1 equals 11.5 TL

£? equals 220 TL

220 ÷ 11.5 = £19.130. . .

Answer: 220 TL is £19.13

(b) a local meal for two costing **80 TL** each

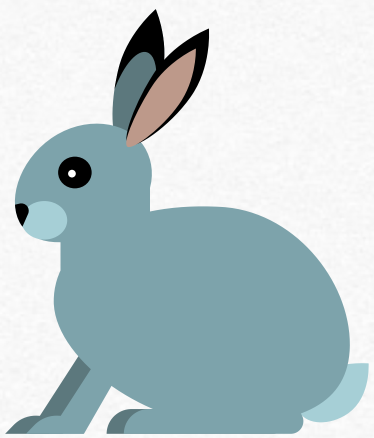
160 ÷ 11.5 = £13.913. . .

Answer: 80 TL is £13.91

(c) four day trips to Gallipoli costing **90 TL** each

360 ÷ 11.5 = £31.304. . .

Answer: 360 TL is £31.30



(d) a four day Gulet boat trip costing 650 TL

650 ÷ 11.5 = £56.5217. . .

Answer: £56.52

[15.] Work out the cost in pounds of:

(a) a hostel dorm bed costing AUD $40

£1 equals AUD $1.79
£? equals AUD $40
40 ÷ 1.79 = £22.346. . .

Answer: $40 is £22.35

(b) a café meal for three costing AUD $30 each

90 ÷ 1.79 = £50.279. . .

Answer: $90 is £50.28

(c) four tickets to a gig costing AUD $30 each

120 ÷ 1.79 = £67.039. . .

Answer: $120 is £67.04

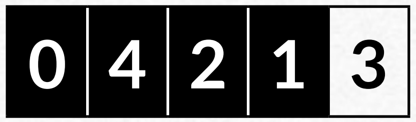
(d) a round Sydney guided bike tour costing AUD $145

145 ÷ 1.79 = £81.005. . .

Answer: $145 is £81.01

Give your answers to 1 d.p. unless the answer is money.

17. Peta had put her trip recorder to zero last time she filled up. It now reads:



Peta now puts 35.7 litres into her tank. How many miles per litre has she done?

$$\textit{Miles per litre} = \frac{\textit{miles}}{\textit{litres}}$$

$$= {}^{421.3}/_{35.7}$$

$$= 11.801\ldots \text{ MPL}$$

Answer: Peta has done 11.8 MPL

What is that in MPG?

Multiply by **4.55** to change MPL to MPG

$11.801\ldots \times 4.55 = 53.695\ldots$

Answer: Peta has done 53.70 MPL

18. If the cost of a litre of petrol is **125.8p**, what is the cost per mile for Peta's car?

Peta filled her tank with **35.7** litres of petrol, so the cost was:

Cost to fill up = No of litres of petrol x price per litre

$$= 35.7 \times 125.8$$

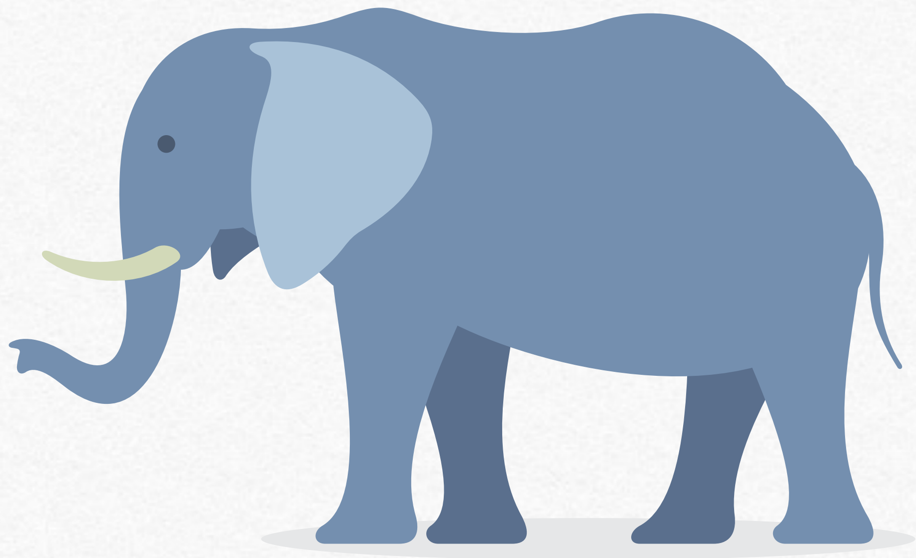
$$= 4{,}491.06\text{p} = £44.91$$

$$\textit{Cost per mile} = \frac{\textit{Cost}}{\textit{No of miles}}$$

$$= {}^{4{,}491.06}/_{421.3}$$

$$= 10.66\text{p} \approx 11\text{p}$$

Answer: 11p per mile



19. Peta is driving to London. She passes a sign saying that she has **145** miles to go and her petrol gauge shows the tank is a quarter full. If her tank holds **50** litres, has she got enough fuel to complete the trip?

Amount of fuel in tank $= \frac{1}{4} \times 50$

$= 12.5\ l$

No of miles = amount of fuel x MPL

$= 12.5 \times 11.8$

$= 147.5$

Answer: Peta has only just enough fuel to get to London but would only have enough for 2.5 miles more after arriving, so she should fill up when she can.

20. Rana has driven herself and two friends from London to Liverpool. The distance is **220** miles and her car does **50** MPG. If the cost of a litre of petrol is **125.8p**. What should she ask each friend to pay as their share of the cost?

Cost per litre = 125.8 p or £1.258

Cost per gallon $= 1.258 \times 4.55$

= £5.7239. . .

= £5.72

Rana's car does **50** MPG so for a journey of **220** miles:

Gallons used $= 220 \div 50$

$= 4.4$

Cost = 4.4 x £5.72

= £25.168

Cost per person = £25.168 ÷ 3

= £8.389. . .

Answer: Rana should ask each friend to pay her £10. Although the fuel cost is £8.39 it is reasonable to ask for a little more to cover her other expenses such as insurance and car maintenance.

21. On the OS Explorer map of the New Forest find the shortest route to walk from the car park at Clayhill Heath to Stag Park. Calculate the real distance from the car park at Clayhill Heath to Stag Park if the distance on the map measures **21 cm**. How long would it take to walk this distance at **3 mph**?

Distance on map = 21 cm

Map	Real
4 cm	$^5/_8$ mile
1 cm	$^5/_8 \div 4 = {^5/_{32}}$ miles
21 cm	$21 \times {^5/_{32}}$ mile = 3.281. . . miles

Answer: It is about 3.3 miles from Clayhill Heath car park to Stag Park.

How long would it take to walk this distance at **3 mph**?

$t = {^d/_s}$

$= {^{3.3}/_3}$

$= 1.1$ hours

(Calculate the minutes by multiplying 0.1 hours by 60 mins)

Answer: It would take 1 hour and 6 minutes to walk

22. On the OS Explorer map of the New Forest find Pondhead camp site and Bishops Dyke. If the various paths and tracks from the camp site at Pondhead to the start of Bishops Dyke measure **25 cm** on the map, calculate the distance in real life. How long would it take to cycle this distance at **12 mph**?

Distance on map = 25 cm

Map	Real
4 cm	$^5/_8$ mile
1 cm	$^5/_8 \div 4 = {^5/_{32}}$ miles
25 cm	$25 \times {^5/_{32}}$ mile = 3.906. . . miles

Answer: It is about 3.9 miles from the campsite at Pondhead to the start of Bishops Dyke.

How long would it take to cycle this distance at **12 mph**?

$t = {^d/_s}$

$= {^{3.9}/_{12}}$

$= 0.325$ hours (x 60 to convert to minutes)

$= 19.5$ minutes

Answer: It would take about 20 mins to cycle

23. Sienna is in charge of catering. At **5 pm** she realises that she needs some extra potatoes and heads off from the campsite next to Pondhead to the shops at Clayhill. If she walks at **3.5 mph**, takes **10** minutes over her shopping and then will need **30** minutes to prepare the potatoes, what is the earliest time that she can call the rest of her group to eat supper? The distance on the map of how Sienna would walk from the campsite to Clayhill measures **10 cm**.

Distance on map = 10 cm

Map	Real
4 cm	$^5/_8$ mile
1 cm	$^5/_{32}$ mile
10 cm	10 x $^5/_{32}$ mile = 1.56. . . miles

Distance to shops = **1.6 miles**

$t = {}^d/_s$

$= {}^{1.6}/_{3.5}$

= 0.457. . . hours (x 60 to convert to minutes)

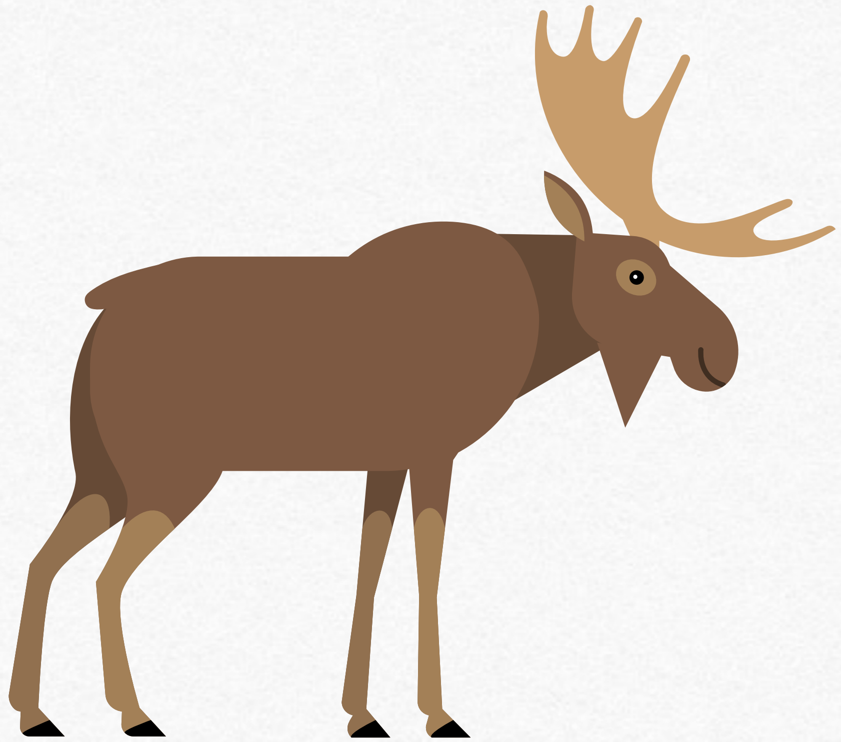
= 27 minutes, so say 30 mins

Total time = 30 mins + 10 mins + 30 mins + 30 mins

= 100 mins

= 1 hour 40 mins

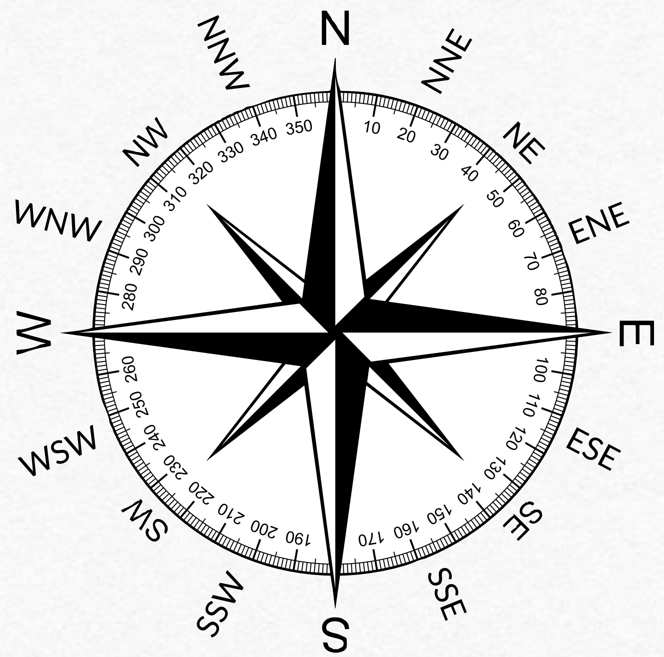
Answer: 6:40 pm is the earliest time that Sienna can call the group to supper.



24. Copy this compass, and mark on it all the missing bearings.

First mark NE, SE, SW and NW.

The NNE is between N and NE, ENE is between NE and E, ESE between E and SE and SSE between S and SE and so on.

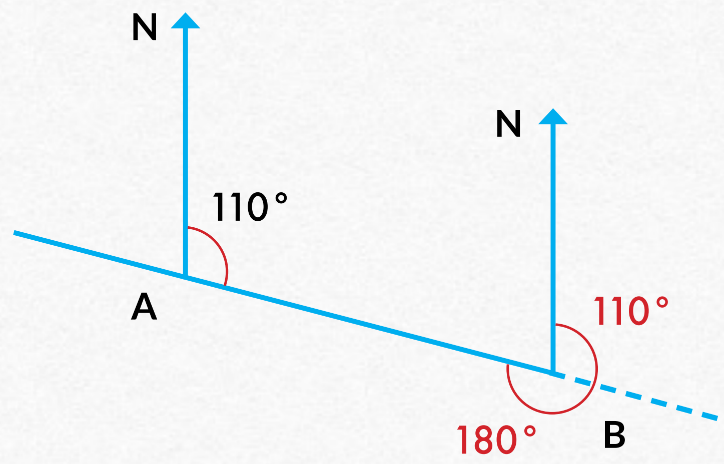


25. Now copy and complete this table, giving the degrees for each point of the compass:

Point	Degrees	Point	Degrees
N	0°	S	180°
NNE	022.5°	SSW	202.5°
NE	045°	SW	225°
ENE	067.5	WSW	247.5°
E	090°	W	270°
ESE	112.5°	WNW	292.5°
SE	135°	NW	315°
SSE	157.5°	NNW	337.5°

In these drawings you are given the bearing of B from A. Calculate the bearing of A from B.

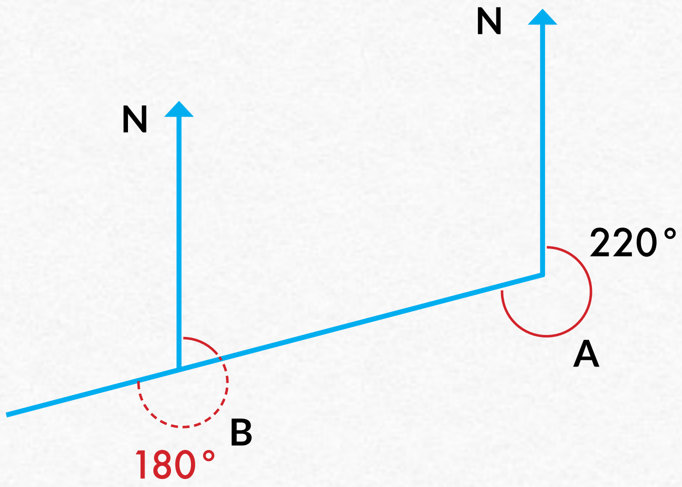
26.



110° + 180° = 290°

Answer: The bearing of A from B is 290°

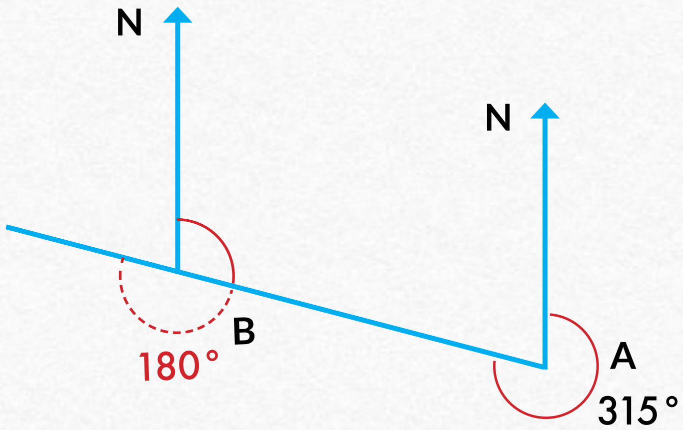
27.



$220° - 180° = 40°$

Answer: The bearing of A from B is = 040°

28.



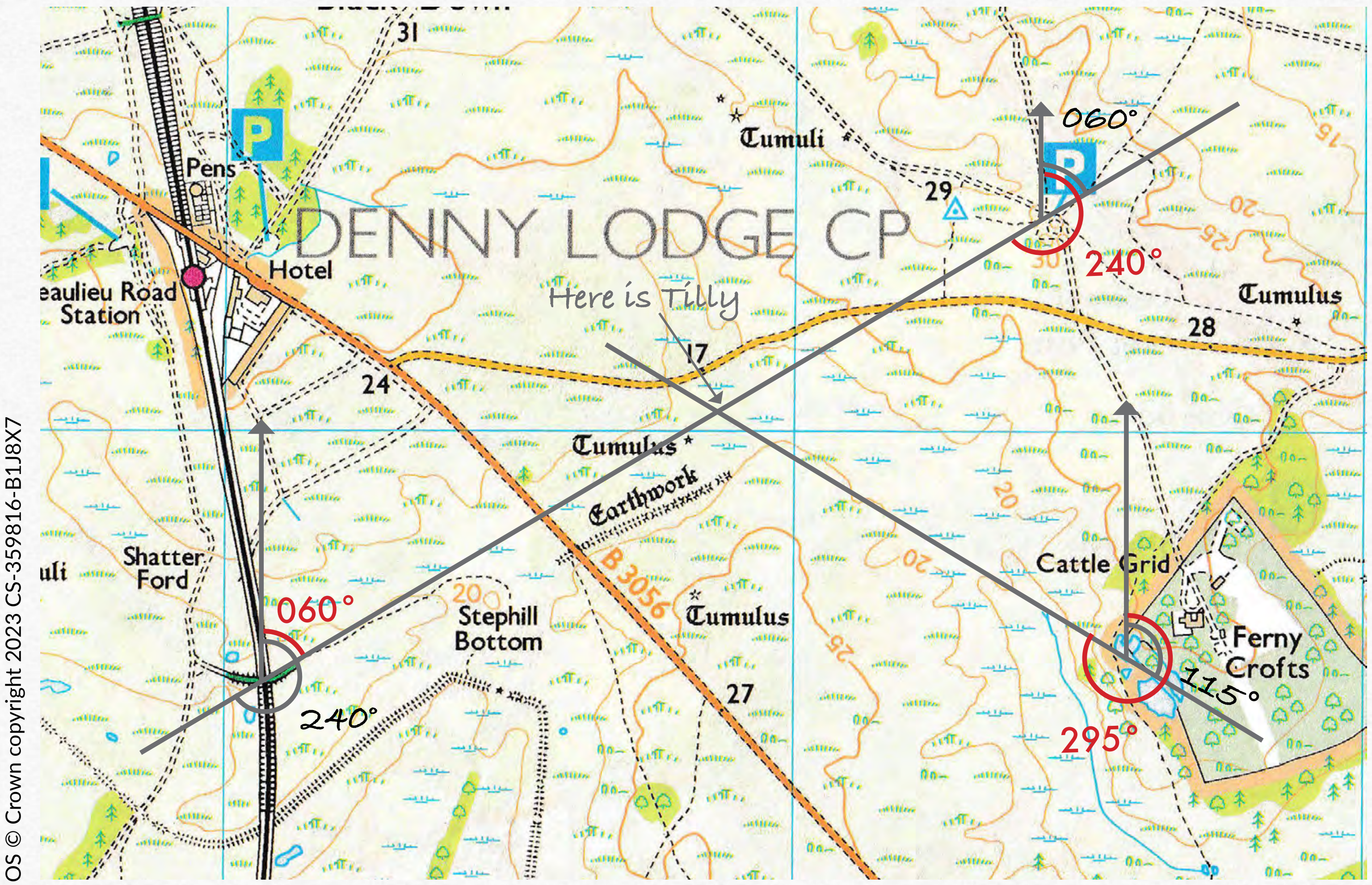
$315° - 180° = 135°$

Answer: The bearing of A from B is 135°

29. Tilly is lost. She knows she is not far from Beaulieu. She takes three bearings of things around her.

The bearing of a railway bridge is **240°**, the bearing of the nearest pond is **115°** and the bearing of a car park is **060°**. Draw these bearings on this map and find Tilly's location.

First draw North lines at the **3** points that Tilly has taken her bearings from. Then draw bearings as follows at each point: **240° − 180° = 060°** bearing at the railway bridge; **115° + 180° = 295°** bearing at the pond; and **60° + 180° = 240°** bearing at the car park. Where the three lines meet is Tilly's location. Do not worry if your lines do not meet at an exact point as Tilly will not have been able to take very precise bearings.



30. Uta is on a yacht in the Solent. She needs to know exactly where she is and takes three bearings.

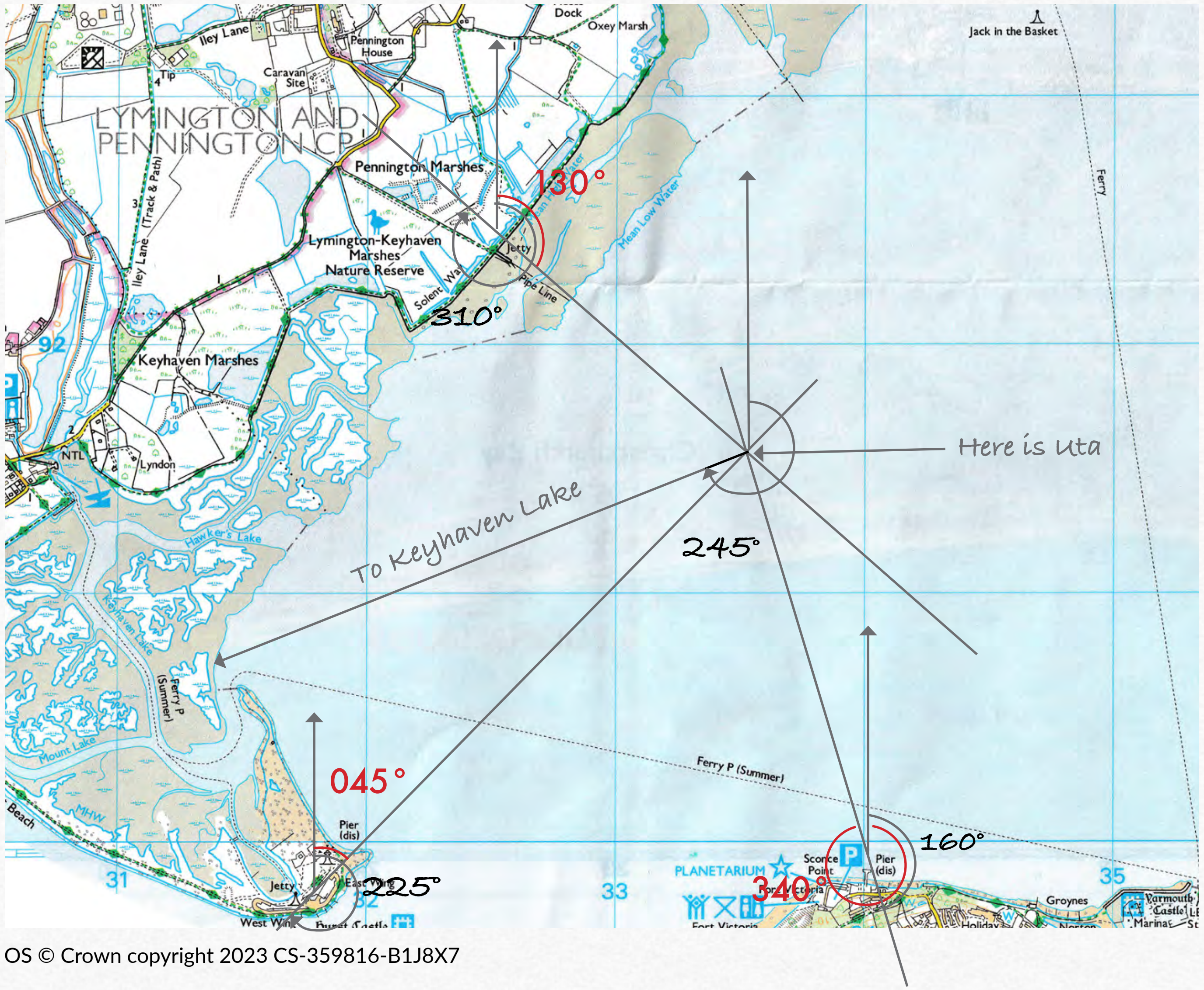
The bearing of the end of the jetty by Sconce Point is **160°**, the bearing of the Hurst Castle is **225°** and the bearing of the jetty by Keyhaven Marshes is **310°**. Draw these bearings on this map and find Uta's location.

In what direction should she sail in order to enter Keyhaven Lake without going aground?

First draw North lines at the **3** points that Uta has taken her bearings from. Then draw long lines at the correct bearing: **160° + 180° = 340°** bearing at Sconce Point; **225° − 180° = 045°** bearing at Hurst Castle; and **310° − 180° = 130°** bearing at Keyhaven Marshes jetty. Where the three lines meet is Uta's location. Now draw a line form Uta's location to the mouth of Keyhaven lake, avoiding all the marshy bits. Measure the bearing of the line.

Answer: Uta should sail on a bearing of 245°

You can now see how important bearings are to sailors and navigators.



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Food for Thought

Laura Dekker, born 1995, is the youngest female to successfully sail around the world single handed.

Laura's first boat was an Optimist dinghy that she received for her 6th birthday and named *Guppy*. In 2007, Laura bought her first yacht, a Hurley 700 also named *Guppy*, with a loan from her father.

In May 2008, aged 13, she sailed *Guppy* from the Netherlands across the English Channel without telling her father. Local authorities in Lowestoft placed her in a children's home and asked him to retrieve her. He returned her to her boat and then flew home without her. Laura then left England on her own. There was a strong wind blowing and she arrived back in Rotterdam the following morning.

Laura's round the world challenge began on 21 August 2010, when she was 15 years old. Laura successfully completed her solo circumnavigation in her 12.4-metre (40 ft) two-masted yacht named *Guppy* 518 days later, arriving back in Sint Maarten at the age of 16.

Laura has since founded the Laura Dekker World Sailing Foundation in order to show the coming generation what this world has to offer and how they can be a strong part of it.

YOUR BRAIN WORKOUT

Q1

If it is **1 pm** in London, what is the time in San Francisco that is (UTC-**8**)?

YOUR BRAIN WORKOUT

Q2

If I can jog for a mile in 12 minutes, what is my speed in mph?

YOUR BRAIN WORKOUT

Q3

How long does it take to drive 9 miles at 36 mph?

YOUR BRAIN WORKOUT

Q4

If **£1 = €1.2** (Euro), what is the value in pounds of a ticket to the Louvre museum costing **€24**?

YOUR BRAIN WORKOUT

Q5

I have driven for **200** miles and used **25** litres of petrol. What is my petrol consumption in miles per litre?

YOUR BRAIN WORKOUT

Q6

A map has a scale of **4 cm** representing **1 km**. What is the actual distance in **km** of a length on the map of **10 cm**?

YOUR BRAIN WORKOUT

Q7

What is the compass direction SW as a bearing in degrees?

YOUR BRAIN WORKOUT

Q8

If the bearing of a tree from an explorer is **312°**, what is the bearing of the explorer from the tree?

YOUR BRAIN WORKOUT

Answers

Q1 1 pm = 1300 hours; 13 − 8 = 5 am

Q2 12 mins = $^{12}/_{60}$ = $^{1}/_{5}$ hour;
speed = 1 ÷ $^{1}/_{5}$ = 5 mph

Q3 time = $^{9}/_{36}$ = $^{1}/_{4}$ hour = 15 mins

Q4 €24 ÷ 1.2 = £20

Q5 200 ÷ 25 = 8 miles per litre

Q6 10 ÷ 4 = 2.5 km

Q7 225°

Q8 312 − 180 = 132°

PART 5

MATHS TO SAVE THE PLANET AND YOURSELF

Your Very Own Planet Earth

Have you ever taken time to think about how fortunate you are to live on planet Earth? Wherever you are in the world, you can breathe fresh air of exactly the right mix of elements to support your life. You can see what you are doing by the natural light from the sun. In most locations, the climate is neither too hot nor too cold for you to live comfortably. You can find food by hunting, fishing or agriculture. You can drink fresh water from the many streams and rivers, or from wells.

But will this always be the case?

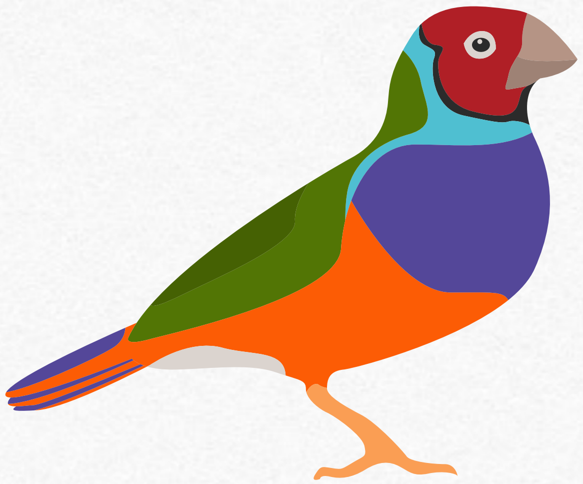
It clearly has not always been true. We know that climate has changed over time and that at times in the past, human life was not sustainable.

You have probably heard the term 'ice age'. An ice age is long period when the reduction in the temperature on the Earth results in the creation of glaciers, the expansion of the polar ice caps and the formation of continental ice sheets.

Food for Thought

A glacier is sometimes called a river of ice. This is the Skaftafell Glacier, Vatnajokull National Park in Iceland.

There are also occasional long warm periods within an ice age which are called 'interglacial' periods. And when we say 'long' periods we mean long, as in many thousands of years.



Weather and Climate

To understand changes to the planet you need to consider the facts and the science behind them.

You will have heard the terms ‘weather’ and ‘climate’ used equally but **weather** refers to short term conditions such as temperature, pressure, humidity, cloud cover, wind, rain etc. while **climate** is the weather of a specific region averaged over a long period of time. Although a change in the weather can happen very quickly, and sometimes unexpectedly, changes to climate take a long time and can be predicted by looking for patterns in the data collected.

In this Part, you are going to look at what a weather forecast tells you and then at some data collected over time before considering what you can do to help conserve natural resources.

You can find data on the weather from news programmes, weather reporting websites and from apps on a smart phone.

The image below is typical of an app. It tells you the forecast highest and lowest temperatures and if the weather is predicted to be sunny, cloudy or raining. The percentages next to some days tells you the probability that it will rain on that day.

Examples

Ani is planning a trip to London for a week. What does the data that she sees on her phone app tell her about what to pack?

London

Monday		29	15
Tuesday		27	15
Wednesday		26	14
Thursday	60%	23	14
Friday		20	13
Saturday	40%	19	12
Sunday	30%	20	13

Answer: Ani can see that the weather will be very changeable, being sunny on Monday but a mix of sun and cloud thereafter and even rain.

The app shows a 60% chance of rain on Thursday, which means that rain is likely (the probability being more than 50%) and so Ani should make sure she has a waterproof jacket.

The temperature will be very warm on Monday but then decreases to 10°C lower by Saturday, so Ani will need to pack both warm and light clothing.

It will be chillier in the evening and at night so Ani will need to have something warm as well as pretty for that special Friday night out.

Exercise

1. Bea is going to Paris for a week as part of her Art course. Her app shows the weather forecast:

Paris

Monday		30	14
Tuesday		30	17
Wednesday		33	19
Thursday	80%	27	20
Friday	80%	25	18
Saturday	60%	25	16
Sunday		27	17

What does this tell Bea about what she should pack? Which would be the best day for:

- a trip down the river Seine
- A walk round the City centre
- Visits to Art galleries?

Average Temperature

As you know, the actual temperature changes over the **24** hours of a day, and so forecasts give the predicted highest and lowest temperatures.

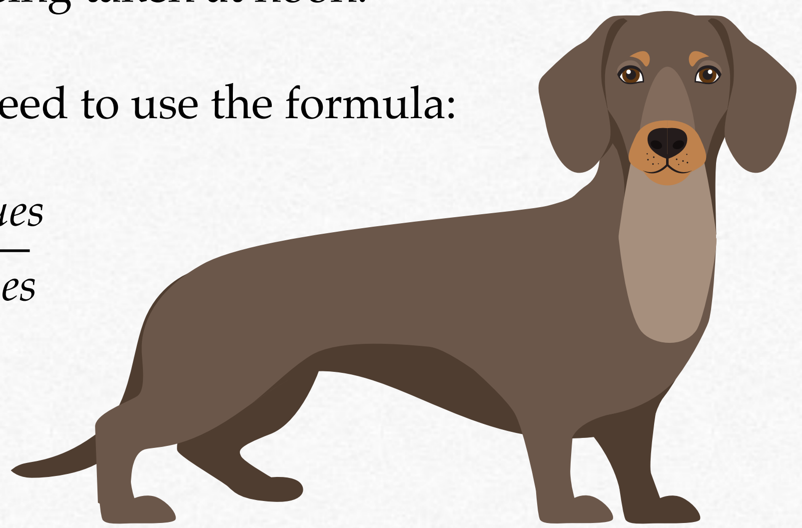
Food for Thought

Do you remember that when we use the word average, it is the result of adding up all the values and then dividing by the number of items or values. This is more technically called the mean or mean average. There are two other ways of expressing averages: mode and median. If you are using these, then you should make it clear.

When comparing weather at various times of year, the average weekly temperature can be a useful measure, with each daily temperature being taken at noon.

To find the average you need to use the formula:

$$\textit{Average} = \frac{\textit{Sum of the values}}{\textit{Number of values}}$$



Example:

Carrie has recorded the temperature at noon every day as part of her Geography field work. These are her results, by Saturday:

M	Tu	W	Th	F	Sa	Su
16	14	18	21	22	25	

What was the average temperature in °C?

$$\textit{Average} = \frac{\textit{Sum of the values}}{\textit{Number of values}}$$

$$= \frac{16 + 14 + 18 + 21 + 22 + 25}{6}$$

$$= {}^{116}/_{6}$$

$$= 19.33\ldots{}^\circ C$$

Answer: The average = 19.3 °C

Carrie can see that the temperature is increasing. What would be a sensible estimate of Sunday's temperature looking at the figures?

From Tuesday the temperature is rising between 1 and 4 degrees per day so Carrie estimates 2 °C rise

Answer: 27 °C

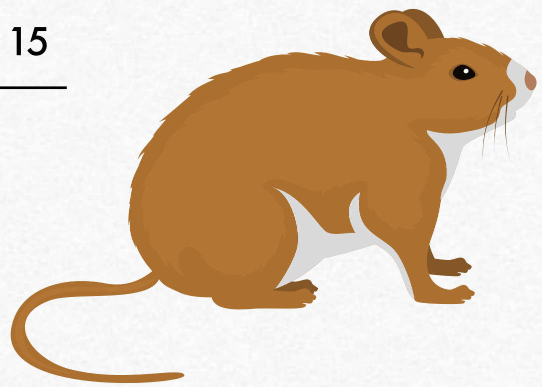
In fact, the temperature on the Sunday dropped to 15 °C. What was the average in °C for the 7 days?

$$\textit{Average} = \frac{\textit{Sum of the values}}{\textit{Number of the values}}$$

$$= \frac{16 + 14 + 18 + 21 + 22 + 15}{7}$$

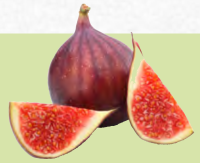
$$= {}^{131}/_{7}$$

$$= 18.71\ldots\,°\mathrm{C}$$



Answer: The average for 7 days was 18.7 °C

You can see that although there was a big drop in temperature on the Sunday, the average for 7 days is only a little less than the average to 6 days. Average is a useful measure as it does not change greatly by relatively small variations.



Food for Thought

The average can, however, be unduly affected by very much higher or lower values that are not typical. These are known as **outliers**. The most common cause of outliers is an error in either the recording or the taking of the data and so you should always check if you spot unexpected results or **outliers**.

Exercises

2 Using the figures from Ani's app forecast for London in the example, what was the average temperature forecast for the week Ani was to spend in London?

3 Using the figures in **Q1**, what was the average temperature forecast for the week Bea spent in Paris?

Note that those temperatures were for a **forecast**. To study climate, you need to consider **actual** records.

4 Daria measured the temperature at 12 noon for a week in September. These are her results in °C.

M	Tu	W	Th	F	Sa	Su
24	21	16	12	13	14	12

What was the average temperature in °C?

5 In 2020, Eliana had worked out the average noon temperature for each month. These are her results in °C:

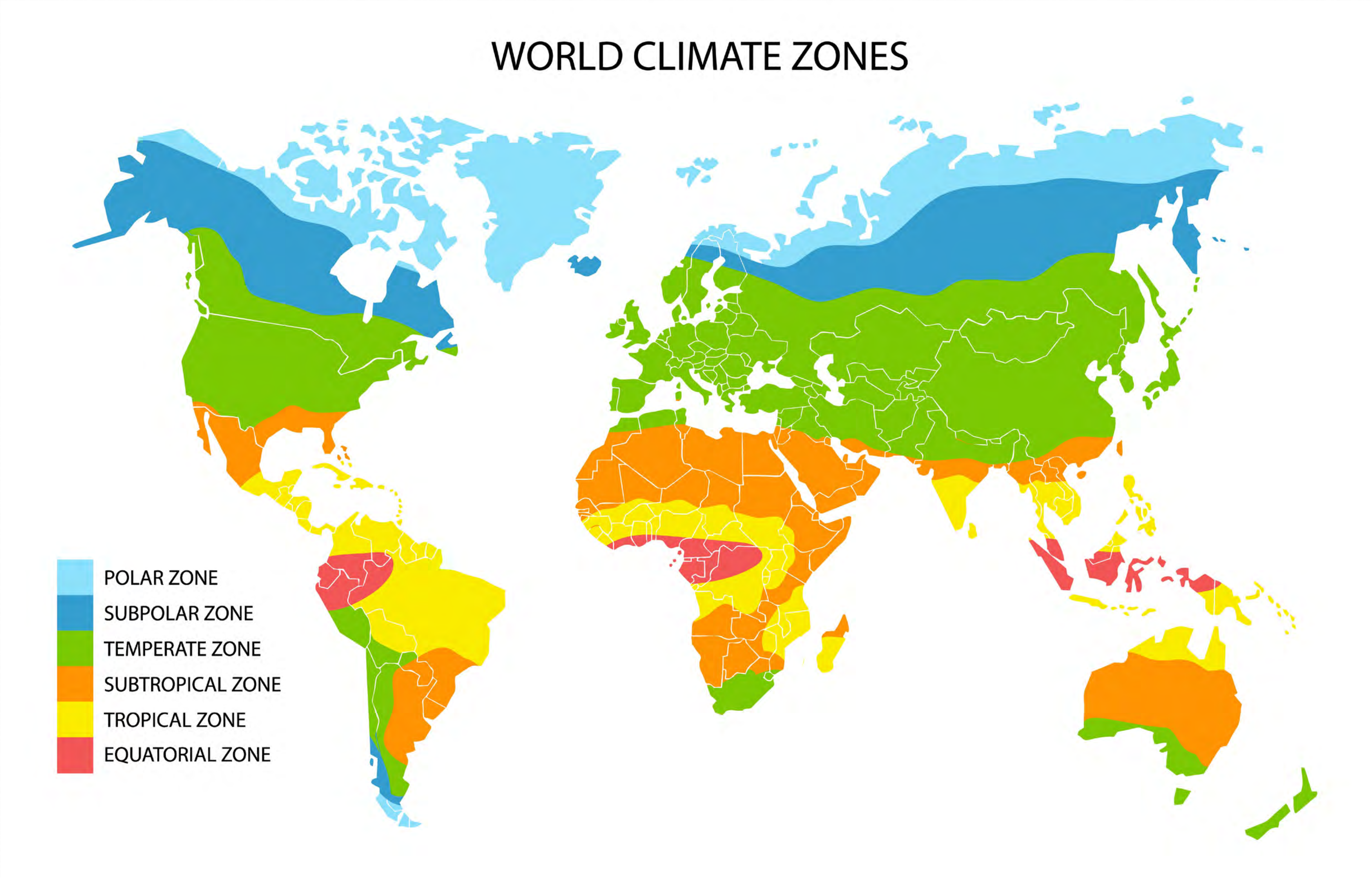
Jan	Feb	Mar	Apr	May	Jun
−8	−7	−3	0	6	8

Jul	Aug	Sep	Oct	Nov	Dec
10	12	8	5	−3	−6

What was the average temperature in °C?

6 Daria and Eliana clearly live in very different climates. Suggest where each might live?

You may have studied different climates around the world in Geography. This world map shows the various zones.

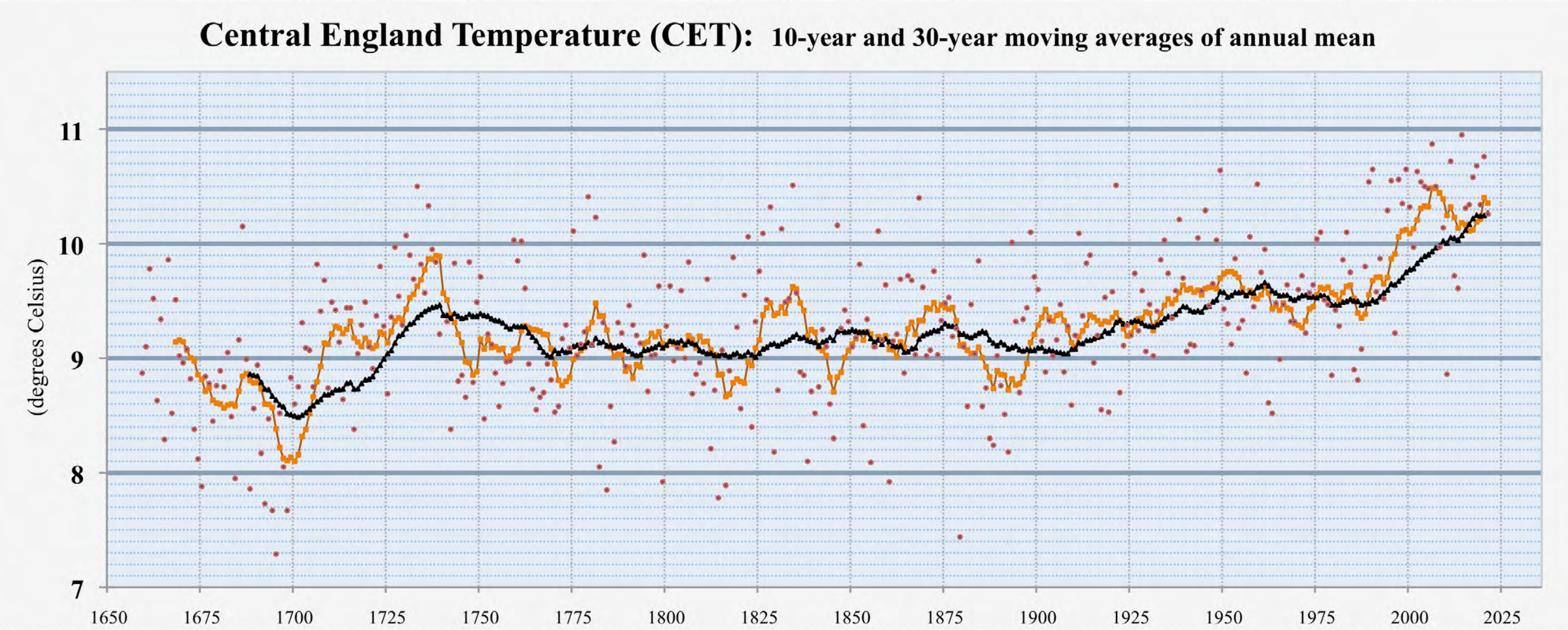


Food for Thought

Modern fabric technology makes it simple for you to find appropriate clothing to keep you warm in winter. Spare a thought for your great grandmothers who would have had very different winter wardrobes from those that you have now.

Accurate information about how climate changes over time is found by measurement. However, temperature measurements on a worldwide basis only started in the second half of the 19th century. The longest temperature series (from Central England) only goes back to the mid 17th century. Any other record of previous temperatures can only be deduced by applied science, such as looking at ice core data and tree rings.

Look at the record of temperature in England, below. The red dots show the mean average temperature in each year and the orange line shows the average annual, or yearly, temperature over 10 years. This is called the 10-year moving average. You can see that although there are big differences in the annual average temperature, there are trends when the average temperature rises and others when it falls. (*Data source: Met Office U.K.* (31 *July* 2019). "*mean CET ranked coldest to warmest from* 1659 *to* 2019".
Met Office, Hadley Centre for Climate Prediction and Research.)



Example

From the graph, find the lowest and highest average annual temperatures. Estimate how many years apart these are.

Answer: The lowest mean annual temperature was 7.3° in about 1690.

The highest mean annual temperature was 11° in about 2015.

These are about 325 years apart.

You could use your answer above to say that the mean annual temperature has risen 3.7°C over 325 years, but that would be incorrect as you can see that there are other times in between these years when the temperature has been very close to these two extreme values.

Exercise

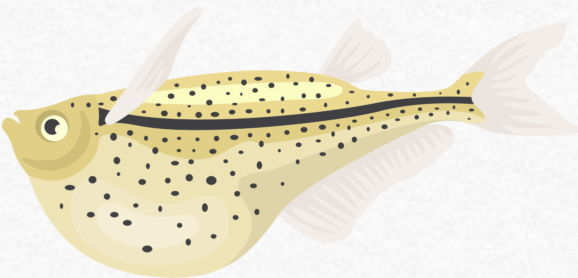
7 From the graph on the previous page, find the 25-year period with the most mean annual temperatures between 10 and 11°C.

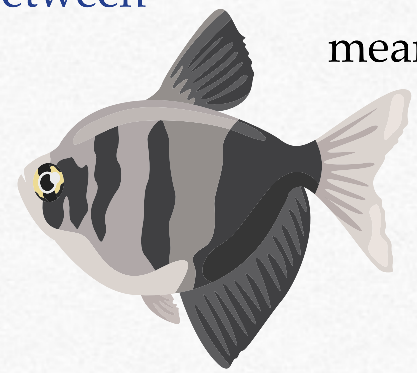
8 From the graph on the previous page, find the 25-year period with the most mean annual temperatures between 7 and 8°C.

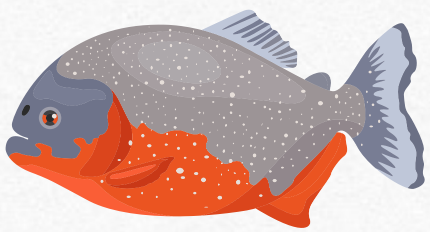
Food for Thought

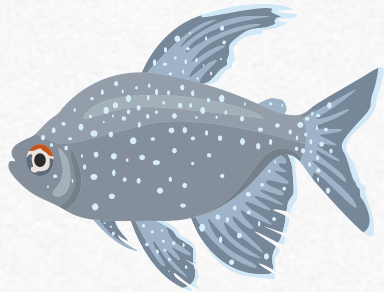
From about 1300 to 1850 there was a period known as the Little Ice Age when the average temperature in the Northern Hemisphere was slightly cooler that the previous and later periods. There were three particularly cold intervals one beginning about 1650, another about 1770, and the last in 1850, all separated by intervals of slight warming.

You can see that looking at actual temperatures tells you something about each year and that looking at a 25-year periods tells you something more, but a more useful measure to analyse trends is looking at average temperature over a number of years. Now consider the orange line on the graph that shows the 10-year moving average of the mean annual temperature.









When the orange line moves up it shows an increase of the average mean temperature, the steeper the sloping line, the more rapid the rise.

Conversely, as the line moves down it shows a drop in temperature. The steeper the downward slope, the faster the drop in temperature.

Exercises

9 From the graph, find the period when the 10-year moving average of the mean average temperature:

a) dropped most rapidly? b) rose most rapidly?

You will have noticed that the last questions were quite hard as there were several periods when the orange line rose and fell at about the same rate. Look now at the black line on the graph. This shows the 30-year moving average of the mean annual temperature and is even better for looking at trends, as over the longer period the variations in the data are ironed out.

Exercise

10 From the graph, find the period when the 30-year moving average of the mean average temperature:

a) dropped most rapidly? b) rose most rapidly?

Food for Thought

Between 1607 and 1814, Londoners made the most of several winters when the River Thames in London froze over for several weeks by holding Frost Fairs. At these fairs there were shops, bars, ice skating rinks and other attractions all held on the ice!

Climate Change

Although you will have noted a steep rise in mean annual temperature in the eighteenth century, you will also have noted that the rise in mean average temperature in the most recent 25 year period is also rapid. However the overall difference in temperature between the lowest and highest points of the black line is less than 2 °C.

Is this proof that the Earth is warming?

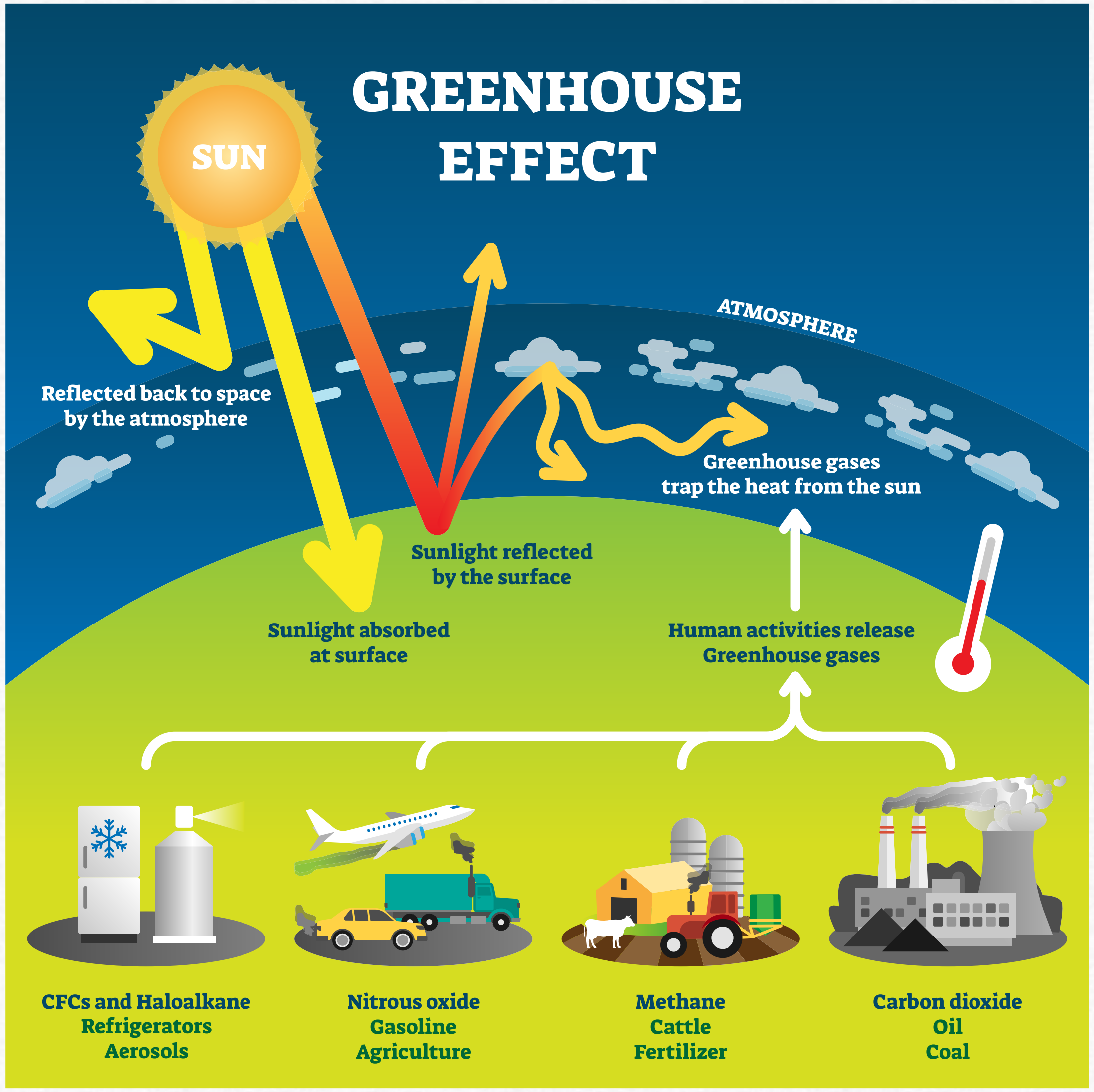
How serious is a 2°C rise in global temperature?

This book is not going to attempt to answer these questions. Climate does change over time and from the data collecting at measuring stations all around the world there is evidence that temperatures are currently rising. The clearest result of this is that the polar ice caps are melting.

The steady rise in 30-year moving average of the mean annual temperature in Britain started around 1900, as did the increase in industry, and the emission of greenhouse gases, most importantly carbon dioxide and methane.

It has been proven that greenhouse gases keep heat trapped close to the Earth rather than passing through the atmosphere back into space.

It is therefore important that all possible action is taken to ensure that human activity does not result in changes to the Earth's climate.



Food for Thought

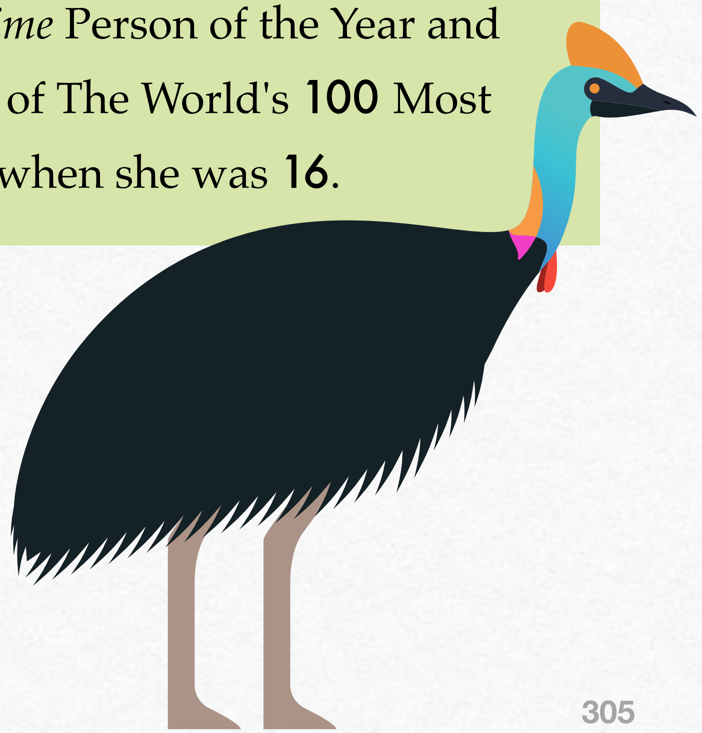
Greta Thunberg first started to challenge world leaders to take action to mitigate climate change when she was 15. She started spending school days outside the Swedish parliament holding a sign reading *Skolstrejk för klimatet* (School strike for climate).

Soon other students around the world took notice and together organised a climate movement, Fridays for Future, where school pupils protested against their governments rather than going to school on Fridays.

By 2019, there were coordinated multi-city protests around the world, each involving over a million students.

Greta spoke at the 2019 UN Climate Action Summit. She addressed the British, European and French parliaments and met world leaders. Some politicians and journalists have mocked her views and statements, often because of her age. Greta responded: "It's quite hilarious when the only thing people can do is mock you, or talk about your appearance or personality, as it means they have no argument or nothing else to say."

Greta has received numerous honours and awards becoming the youngest *Time* Person of the Year and inclusion in the *Forbes list* of The World's 100 Most Powerful Women (2019) when she was 16.



How to Help the Planet

Whilst governments can make big decisions about cutting down the emissions of greenhouse gases, there are some things that everyone can do.

Water

Health experts estimate that the average person needs between **2** – **3** litres of water per day. In the UK, tap water is perfectly safe to drink. If you want a different taste, adding a sprig of mint or a slice of fruit to your water can make a pleasant change.

Apart from drinking, do you know how much water you use in a day? How can you save your use of water?

Use a measuring jug or something similar to find out.

Exercises

11 Ffion brushes her teeth for **2** minutes twice a day but leaves the tap running whilst she brushes.

Using measuring jug, run a tap for **15** seconds. You may have to empty and refill the jug at least once.

Now you have measured how much water comes out of the tap in **15** seconds, work out how much water comes out in a minute.

a) How much water does Ffion waste by leaving the tap running when she brushes her teeth?

b) How much water does she waste in a week?

c) How much does she waste in a year?

12 If the cost of tap water in the UK is **0.15p** per litre, how much does the wasted tap water cost Ffion in a year?

13 Put a bucket in the shower and run it for **15** seconds. Calculate how many litres is used in a one minute shower.

a) Gigi showers twice a day for ten minutes each time. Calculate how much water she uses in her showers per week.

b) Calculate the amount of water that she uses in her showers each year.

14 Hebe prefers to bath. Her bath tub is **150 cm** long and **50 cm** wide. She likes to fill her bath to a depth of **15 cm** before she steps in.

a) Knowing that $1000 cm^3 = 1$ litres, how many litres of water does Hebe use in her bath?

b) Calculate the number of litres that she uses in a week if she bathes every day with an extra bath three times a week after her sports practice.

c) Calculate the number of litres that she uses in her baths each year.

15 Calculate the cost of your own showers or baths, or perhaps a combination of the two each year?

16 a) When you make yourself a hot drink, how much water do you put into the kettle?

b) After you have poured out the boiled water, how much is left in the kettle? How much could you have saved?

c) Think about the number of hot drinks that you have in a week and work out how much water you could save if you only boiled a cupful of water each time.

d) How much water and thus how much money could you save in a year?

17 Write yourself a water saving plan. Work out how you could alter your household's habits to save water.

a) How much water could your household save in a week?

b) How much in a year?

c) How much money would that save?

Food

You obviously need to eat, but have you ever thought about the energy needed to bring your food onto the shelf in the supermarket?

One measurement you can use is to look at 'food miles', that it the distance between where something is grown to where it's eaten. Clearly if a food item has travelled a very long way, then a considerable amount of energy will have been used in the transport.

Exercises

18 Isla is choosing tomatoes at her local supermarket in Birmingham.

Use an online atlas or phone app to find how many more miles it is to Birmingham from Morocco compared to Kent?

19 Julia is very conscientious about eating locally grown seasonal food. For her Christmas party she is deciding between serving:

- Fresh raspberries and cream
- Blackberries with frosted butter
- Baked apples with seasonal spices

Which would you advise Julia to serve?

Food for Thought

Food miles only tell part of the story. If tomatoes are grown in England but in an artificially heated greenhouse, they may use more energy than tomatoes grown in sunnier climates that are then imported.

Plastic

Plastic waste is a very real problem. Plastic does not rot like paper and cardboard and only certain types of plastic can be recycled. Many schools and colleges are looking at how to reduce their use of single use plastics

Exercises

20 Kola's college used to sell **500 ml** bottles of water in their canteen for **60p** a bottle. Now they sell college branded reusable water bottles for **£5** each which can be filled from a water fountain for free.

Kola used to buy a bottle of water with her lunch every day for the **30** weeks she was at college.

a) How much money did Kola save in a **30**-week academic year by buying a reusable bottle.

b) If the college has **1,500** students and all made the same change as Kola, how many single use plastic bottles were no longer used?

21 Lea goes to a music festival every year. The festival served drinks in single use plastic glasses. Last year, **1.2** million glasses were left behind scattered all over the festival site and it took **300** volunteers **5** days to clear them all away. How many plastic glasses is that per volunteer per day?

(This year the festival is to be plastic free.)

Before you answer the next question, remind yourself of what some big numbers mean:

1 million is one thousand thousands or 1,000,000

1 billion is one thousand million or 1,000,000,000

22 Mina's college carried out a plastics survey. They found that of the 178 households that took part on average each threw away 116 pieces of plastic each week.

a) How many pieces of plastic is that in total over each week?

b) If each household threw away the same amount of plastic every week, how many pieces is that per household in a year?

c) If there are approximately 25 million households in the UK, how many billion pieces of plastic will be thrown away in a year?

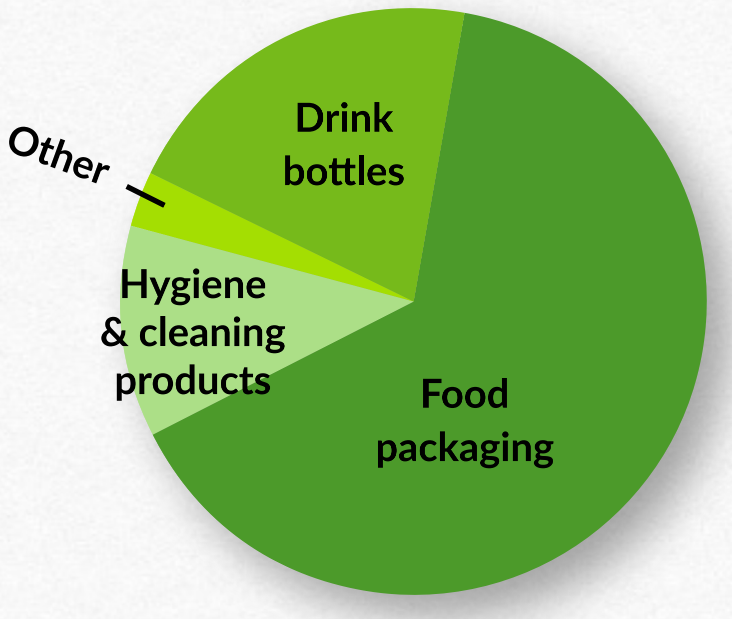
Food for Thought

One million plastic bottles are bought around the world every minute. It is estimated that less than one third of all plastic bottles will be recycled. This photograph is from Thilafushi, an artificial island in the Indian Ocean created from waste.

Do you remember pie charts? A pie chart is in the shape of a circle, divided into slices. Each slice shows what fraction or percentage of the total is represented.

Example

Mina analyses the data from the plastics survey and shows it on this pie chart:



(a) Roughly what percentage of the plastic items came from drink bottles?

You can see that the angle for drink bottles is just under $90°$.

$90°$ is a quarter of a circle

$\frac{1}{4}$ is equal to 25%

Answer: The percentage of the plastic items that came from drink bottles is roughly 25%

(b) If two thirds of the food packaging were flimsy plastic that could not be recycled, what percentage of the items is this?

Food packing is roughly two thirds of the whole circle.

$^2/_3$ of $^2/_3$ is equal to $^2/_3 \times {}^2/_3 = {}^4/_9$

$^4/_9$ of $100\% = {}^4/_9 \times 100\%$

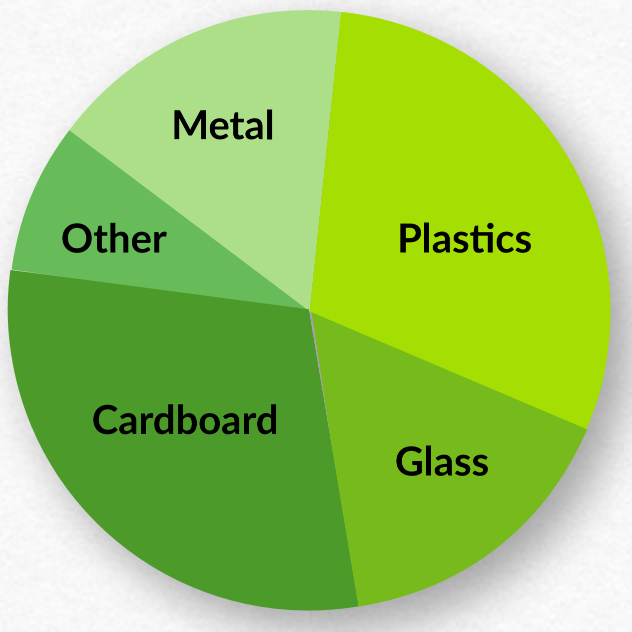
$= {}^{400}/_9\%$

$= 44\%$

Answer: The percentage of the plastic items that are flimsy plastic that cannot be recycled is roughly 44%

Exercises

23 Nona has also carried out a survey. She asked participants to count the number of items of each type of material that they put in their recycling every two weeks. She showed her results as a pie chart:

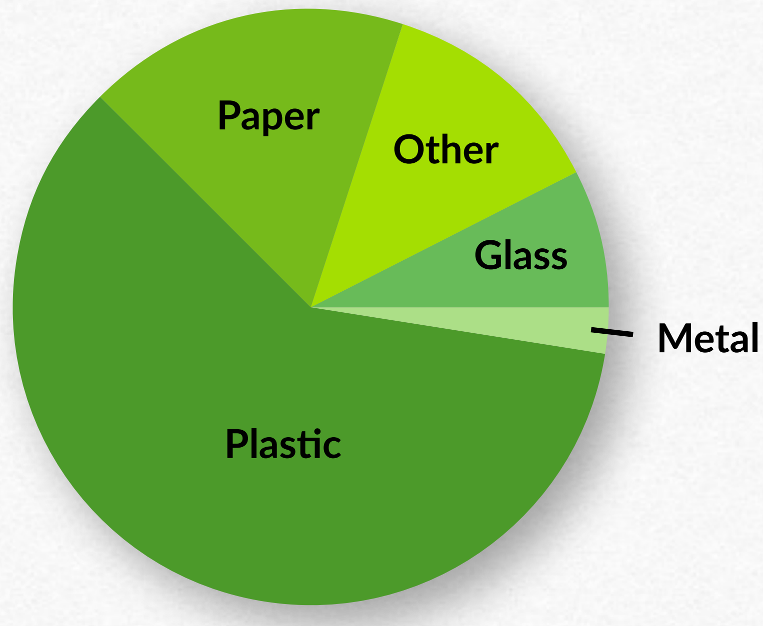


a) What type of material was the most common?

b) Roughly how many more cardboard items were recycled than glass items?

c) Roughly what percentage of all the recycled items were plastic?

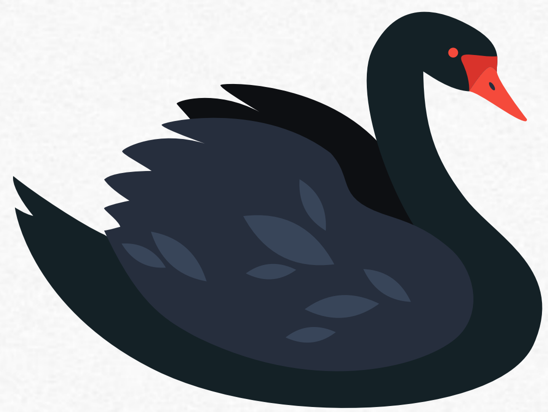
24 Oona volunteers for her local environmental group once a month. They have just completed a litter picking exercise on the local beach. Oona has drawn up this pie chart of the number of items to show the results:



a) What type of material was most litter made of?

b) Roughly what percentage of all the litter was plastic?

c) Roughly how many more plastic items were found than glass?




"We're Green"

Energy

As well as thinking about saving water, eating produce that is sourced locally, using non disposable materials and recycling responsibly, you can also think amount the amount of energy you use.

Electricity usage in the UK is measured in **kWh** or kilowatt-hour. Your electricity supplier measures the amount of energy you have used in **kWh** in order to work out your bill.

All electrical appliances in your home use energy, but some more than others. If you know how many **kWh** each one uses then you can adjust your usage and save money as well as energy.

According to the Department for Business, Energy & Industrial Strategy (BEIS), the average household uses just under **4,000 kWh** per year, but that will vary from a one-bedroom studio flat that will use about **2,500 kWh** and a four bedroom family house that might use **4,500 kWh**. These figures do not include heating. If you home is heated by electricity (rather than gas or any other fuel) then these amounts will be higher.

Food for Thought

This cat may be happy but her owner is likely to find that this is an expensive way to keep warm.

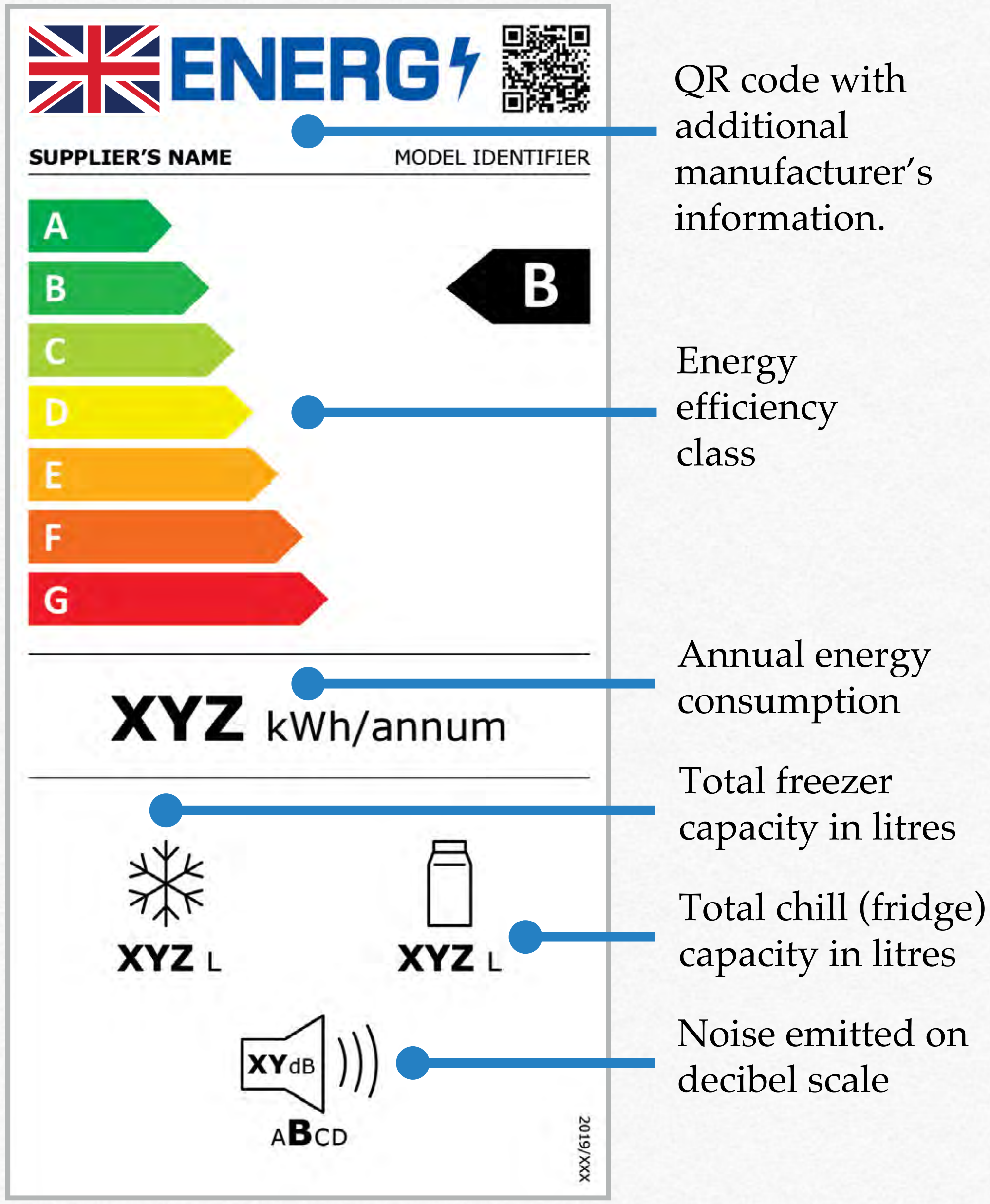
You might have expected a bigger variation between the smallest home and the largest. The reason that there is not that big a difference is because there are some things every home must have, and the most expensive is a refrigerator, which runs all day. However, there is a considerable amount of difference as to the amount of energy used depending on size and efficiency.

This table shows you how, one average, the energy usage of a household is distributed though the various appliances:

Type of appliance	Percentage of household energy use
Cold appliances (fridge/freezers)	63%
Wet appliances (washing machines and dishwashers)	10%
Cooking (ovens, hotplates and microwaves	7%
Lighting (lamps and ceiling lights)	6%
Consumer electronics (TV, laptop, phones, games consoles etc)	4%

Energy labels

To help you decide which appliance to buy, you should look at the energy label. These changed in 2021 and now look like this:



The above example is for a fridge freezer, different appliances have different information below the energy summary.

The Energy rating is an Energy Efficiency Index (EEI) and it is calculated as a percentage:

$$EEI = \frac{\textit{actual energy consumption of the item}}{\textit{standard energy consumption of the type of appliance}} \times 100\%$$

If you want to work out how much energy your appliance will cost you then you need to look at the detail in the manufacturer's information.

Example

Petal buys a fridge freezer with a E rating that uses 254 kWh/ annum. What will this cost her per year if her energy supplier charges her 17.5p per kWh?

Cost of running the fridge = 254×17.5

=4,445p or £44.45 per year

Answer: The fridge freezer will cost £44.45 per year

A fridge freezer is on all the time and therefore it is relatively simple to work out the annual cost. Other appliances are not in full use all day. To calculate their cost you need to estimate the hours they are in use.

Example

Quita is buying a new TV. She decides to buy a 43″smart TV with energy consumption of 100 W. Quita works out that she watches television for 30 hours a week. If her energy supplier charges her 18 p per kWh what will be the annual cost of watching the television?

First Quita has to work out the number of kWh. To do this she multiplies the Watts by the number of hours per week then by 52, as there are 52 weeks in a year. She then divides by 1000 to turn W into kW:

$$\textit{Number of } \mathrm{kWh} = \frac{\textit{Watts} \times \textit{hours per week} \times 52}{1000}$$

$$= \frac{100 \times 30 \times 52}{1000}$$

$$= 156 \text{ kWh}$$

$$\text{Cost per year} = 156 \times 0.18$$

$$= £28.08$$

Answer: The TV will cost Quita £28 per year in electricity

Exercises

25 Rosie's studio flat has an under the counter fridge with an energy rating of **115 kWh/a**. What will this cost her per month if her landlord charges her electricity at **25p** per **kWh**?

26 Samira lives at home with her mother and grandfather. Their television has just broken down and they are deciding which model to replace it with. Samira works out that between them the household watches roughly **40** hours of television a week. Their electricity supplier charges them **19 p** per **kWh**. What will be the annual cost of:

a) a **55″** TV with energy consumption of **125 W**?

b) a **27″** TV with energy consumption of **26 W**?

27 Thalia has a new washing machine with energy consumption of **0.545 kWh** per cycle, if the eco setting is used. What does this cost her per wash if she pays **18.5p** per **kWh** for her electricity?

28 Uma has an old washing machine that is rated at **1.2 kW**. If she uses her washing machine **3** times a week and each cycle takes **2** hours, what does this cost her per month if she pays **22p** per **kWh** for her electricity?

29 Vee likes to dry her clothes in the tumble dryer. Her dryer has an energy consumption of **5.2 kWh** for a full load. What does one hour of drying cost if her electricity supplier charges **20p** per **kWh**?

What does Vee pay per week if she does a wash four times a week and uses the dryer for an hour and a half for each wash?

30 Winona does not use a tumble dryer but either hangs up her clothes outside on a line or uses an inside clothes airer. Winona also does a clothes wash four times a week. What does she save per year compared to Vee?

Food for Thought

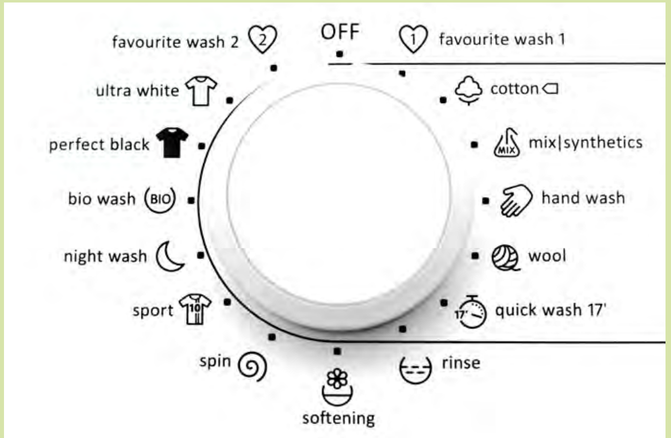
These amounts that you have worked out for laundry are a useful starting point. You should note that a tumble dryer is one of the most expensive appliances in the home and you may want to think how you can save money. Modern appliances offer you plenty of options. Consider:

Separating your cottons and synthetics, as synthetics can be washed at a lower temperature

Only putting on the wash when you have a full load

Washing clothes on a lower temperature

Using the speed wash cycle



Using a faster spin before you use the tumble dryer

Dry your clothes by hanging them on the washing line


DONATION

Healthy Planet, Healthy You!

It is all very well to want to help the whole planet, but sometimes you need to start by looking after yourself.

Do you know what are the important numbers to help you to maintain good health? There are five useful measurements: body temperature, blood pressure, heart rate, breathing rate and weight (or body mass).

Temperature

Normal body temperature for an adult may change from person to person and throughout the day. Normally your temperature should be approximately 37 °C. A temperature of 38 °C is considered high.

A high temperature almost always indicates that your body is fighting an infection or illness and should be taken seriously.

If you are unwell then either you or whoever is looking after you could monitor your temperature by taking it every regularly, every four or six hours.

Look at the temperature chart in this next example.

Food for Thought

There are very small differences in 'normal'body temperature between individuals. To find what is normal for you, take your temperature in the morning when you wake up. This is known as your 'basal body temperature' or BBT.

For women, ovulation triggers a slight rise in their BBT between 0.3 – 0.6 °C which lasts until their next period. Measuring your BBT over a couple of months can help you find out when you are due to ovulate (that is when an egg is released from one of your ovaries). It is not reliable enough to use for contraceptive purpose but understanding these rhythms that occur naturally is a useful part of monitoring your own health. A woman's menstrual cycle is 28 days on average but not all women are the same and your cycle may be shorter or longer than this but still be perfectly normal.

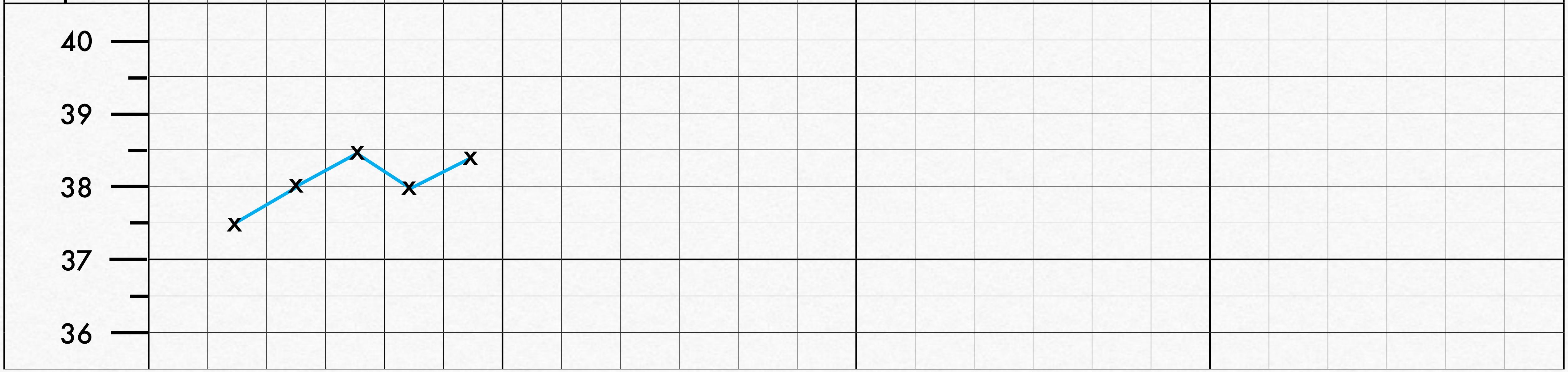
If you are ever worried that what is happening to your body is not normal then you should always seek help.

Example

Xandra is looking after her little sister who is unwell. She takes her temperature every four hours and has recorded it on a chart.

Answer:

Day	Monday						Tuesday						Wednesday						Thursday					
Time	4	8	12	16	20	0	4	8	12	16	20	0	4	8	12	16	20	0	4	8	12	16	20	0
Temp °C		37.5	38	38.5	38	38.4																		



When did Xandra's sister first say she felt unwell?

Answer: As the first temperature taken was at eight o'clock her little sister must have said she did not feel well when she woke up before she had to go to school

When did Xandra decide to call the doctor?

Answer: Xandra may have called the doctor at 4 pm when her sister's temperature rose above 38°.

Exercise

31 Xandra continued to take her sister's temperature. This is what she recorded:

Tuesday					
4	**8**	**12**	**16**	**20**	**0**
37.5	38.5	39	38.8	39	38.5
Wednesday					
4	**8**	**12**	**16**	**20**	**0**
38	37.8	38.1	37.6	37.6	37.6
Thursday					
4	**8**	**12**	**16**	**20**	**0**
37.3	37	37.3	37		

a) Copy the chart from the previous page and complete it with the temperatures Xandra recorded.

b) When was her sister's temperature highest?

c) When did her sister's temperature first come back down to normal?

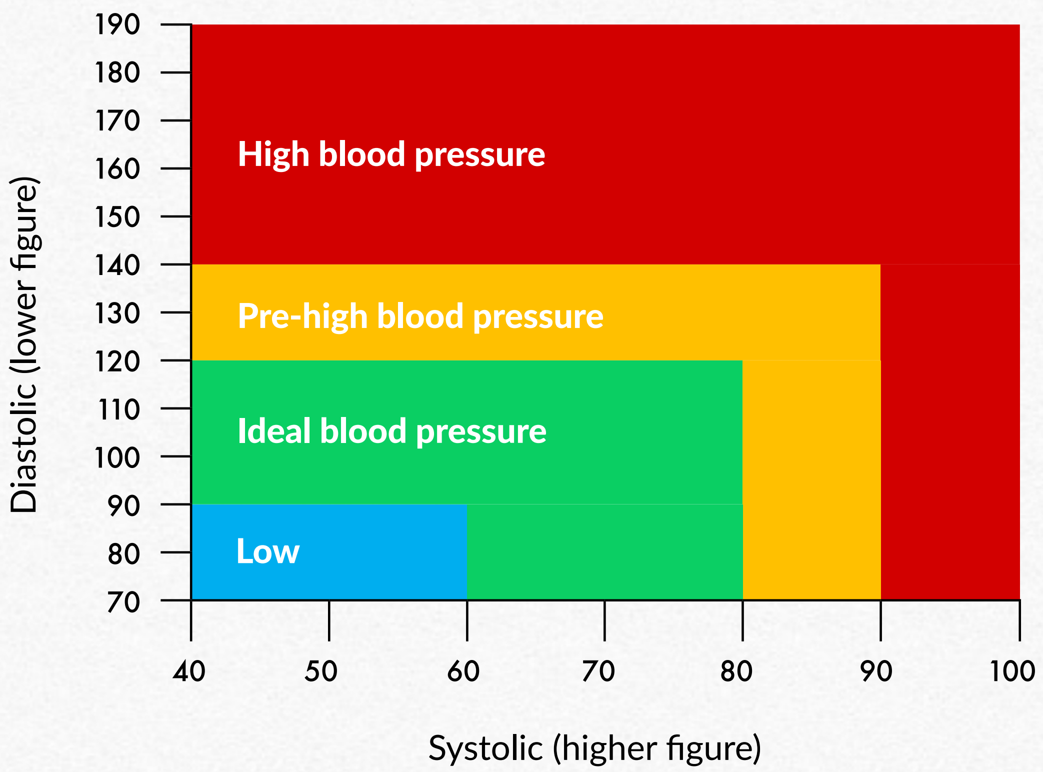
d) When did Xandra decide that her sister had recovered?

Blood Pressure

As the heart pumps blood around your body it presses against the walls your arteries. This pressure can be measured and gives an indication of the efficiency of the normal work of your body. Blood pressure was usually measured only by health professionals, but many households now have a blood pressure machine so that individuals can monitor their own health.

Blood pressure is usually given by two numbers, the first (systolic) is that maximum pressure over one heartbeat and the lower (diastolic) is the minimum pressure between two heartbeats. When either number is too high then hypertension, or high blood pressure, is diagnosed, if either number is too low then hypotension is diagnosed. Hypertension is more common than hypotension and can be an indicator of various medical conditions.

This table shows the ranges for each category of blood pressure:



Example

Yuka has blood pressure of **120** over **80**. Use the chart to find which range Yuka's blood pressure is in.

Answer: Yuka's blood pressure is in the normal range but right at the top end.

Exercise

32 Doctor Zulia measures the blood pressure of three of her patients, the results are:

Patient A

Patient B

Patient C

What does Doctor Zulia tell each patient?

Food for Thought

If you are concerned that your blood pressure is too high, then following a healthy lifestyle can help to bring it down:

- eating a low-fat, balanced diet – including plenty of fresh fruit and vegetables
- being more active

As well as

- consuming less salt
- giving up smoking
- cutting down on alcohol
- losing weight

Heart rate

Heart rate, or your pulse, is the number of beats of the heart per minute (**bpm**). Heart rate can vary according to activity, typically going up during exercise and coming down during rest and sleep. You can measure your heartrate by feeling for the pulse in your wrist or neck and counting the beats for a minute. However, if you have a smart watch, then this will be monitoring it for you.

Most adults have a resting heartbeat of between **60** and **100 bpm**. The fitter you are then the lower your resting heart rate.

Exercise is good for your heart as well as for your overall health.

Breathing Rate

Your breathing rate is measured by counting the number of breaths for one minute by counting how many times your chest rises. The normal breathing range for a healthy adult at rest are between 12 and 16 breaths per minute. Breathing rates can change when exercising and may increase with fever, illness or other medical conditions.

Weight or body mass

A person's weight increases as they grow. Being underweight or overweight can be an indication of an underlying health problem which is why children are regularly weighed as they grow up.

'Normal' weight for an adult depends on many factors including gender, race and build. Body mass is an index that considers your height as well as you weight. It is not an exact measurement but one that is a useful starting point to reassure yourself that you are a healthy size.

If you look around at your friends, you will see that you are all different heights and of different builds but perfectly healthy.

If you look at pictures of young women in magazines or on social media you will see some that are very skinny. This is not the normal build for all healthy young women and sometimes is achieved only by such rigorous dieting that it causes poor mental and physical health.

It is normal for a woman to gain and lose weight at various times in her life, but it is easier to gain weight than to lose it and therefore it is sensible to keep an eye on yourself and be sure to eat properly and take reasonable amounts of exercise in order to stay healthy.

For example, when you get your first job you might find yourself in an office and not able to exercise as much as before. You might also find that you are eating more convenience foods which are often less healthy.

Always make sure that despite your commitment to your work that you still find time for yourself.

Find ways of walking either as part of your journey to or from work or at your lunch break, or see if there is a gym near your work, your employer might even give you membership.

Packing your own lunch will not only save you money but will keep you eating sensibly.

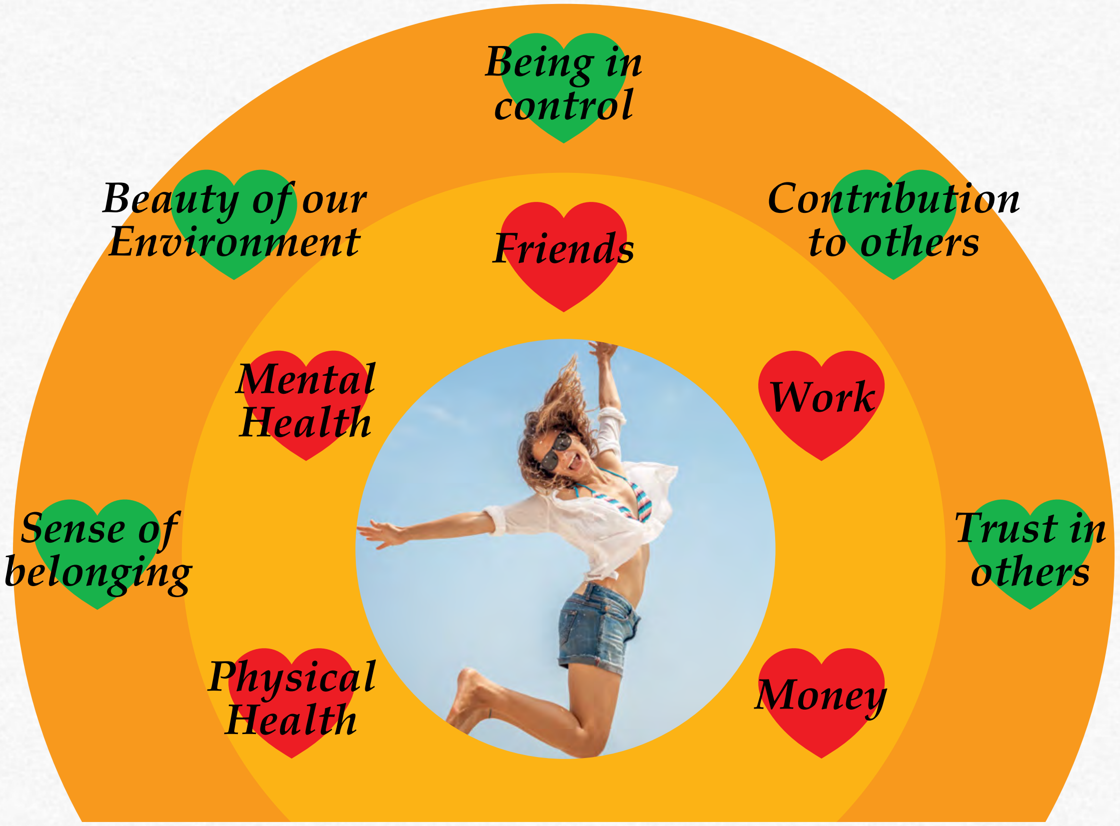
The missing number

These five indicators of health: temperature, blood pressure, heart rate, breathing rate and weight can all be measured.

Do you know what vital ingredient is so important to your health but cannot be measured?

You cannot measure happiness or how you feel. You do, however, know when you are unhappy or feeling low. How you are feeling can be called 'wellbeing'.

You may not be able to measure wellbeing, but we know the factors that contribute towards it:



If you score each of the ten wellbeing indicators out of ten and then add them together, then you can measure your wellbeing!

Food for Thought

We all need to find our own inspiration to help maintain a heathy level of wellbeing. This poem has hung in the author's bedroom since her teens:

Desiderata

GO PLACIDLY amid the noise and the haste, and remember what peace there may be in silence. As far as possible, without surrender, be on good terms with all persons.

Speak your truth quietly and clearly; and listen to others, even to the dull and the ignorant; they too have their story.

Avoid loud and aggressive persons; they are vexatious to the spirit. If you compare yourself with others, you may become vain or bitter, for always there will be greater and lesser persons than yourself.

Enjoy your achievements as well as your plans. Keep interested in your own career, however humble; it is a real possession in the changing fortunes of time.

Exercise caution in your business affairs, for the world is full of trickery. But let this not blind you to what virtue there is; many persons strive for high ideals, and everywhere life is full of heroism.

Be yourself. Especially do not feign affection. Neither be cynical about love; for in the face of all aridity and disenchantment, it is as perennial as the grass.

Take kindly the counsel of the years, gracefully surrendering the things of youth.

Nurture strength of spirit to shield you in sudden misfortune. But do not distress yourself with dark imaginings. Many fears are born of fatigue and loneliness.

Beyond a wholesome discipline, be gentle with yourself. You are a child of the universe no less than the trees and the stars; you have a right to be here.

And whether or not it is clear to you, no doubt the universe is unfolding as it should. Therefore be at peace with God, whatever you conceive Him to be. And whatever your labours and aspirations, in the noisy confusion of life, keep peace in your soul. With all its sham, drudgery and broken dreams, it is still a beautiful world. Be cheerful. Strive to be happy.

By Max Ehrmann © 1927

Answers to Part 5

1. Bea is going to Paris for a week as part of her Art course. Her app shows the weather forecast:

Paris			
Monday		30	14
Tuesday		30	17
Wednesday		33	19
Thursday	80%	27	20
Friday	80%	25	18
Saturday	60%	25	16
Sunday		27	17

What does this tell Bea about what she should pack?

Answer: Bea can see that the weather goes from full sunshine to thunderstorms but remains warm. She will therefore pack light clothing, a waterproof jacket and an umbrella.

Which would be the best day for:

- a trip down the river Seine

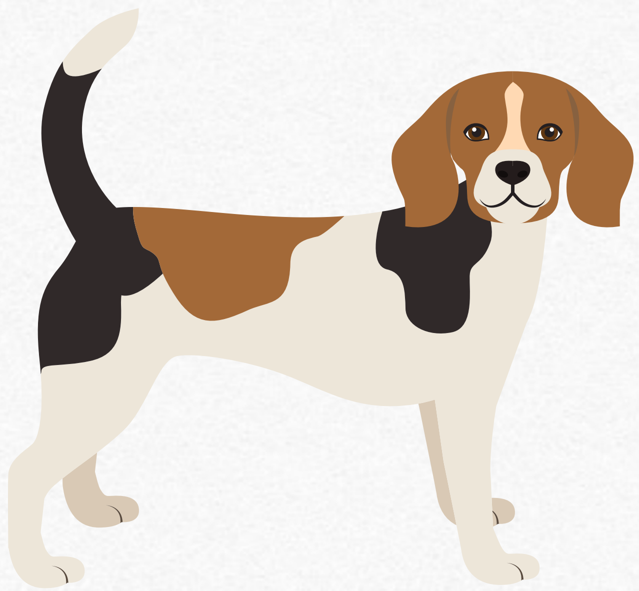
Answer: Monday or Wednesday as the forecast says dry and sunny

- A walk round the City centre

Answer: Tuesday as it is not as sunny and rain is unlikely

- Visits to Art galleries?

Answer: Thursday, Friday and Saturday as rain is likely



2 Using the figures from Ani's app forecast for London in the example, what was the average temperature forecast for the week Ani was to spend in London?

$$\textit{Average} = \frac{\textit{Sum of the values}}{\textit{Number of the values}}$$

$$= \frac{29 + 27 + 26 + 23 + 20 + 19 + 20}{7}$$

$$= {}^{164}/_{7}$$

$$= 23.428\ldots{}^\circ C$$

Answer: The average temperature was forecast to be 23.4 °C

3 Using the app figures in **Q1**, what was the average temperature forecast for the week Bea spent in Paris?

$$\textit{Average} = \frac{\textit{Sum of the values}}{\textit{Number of the values}}$$

$$= \frac{30 + 30 + 33 + 27 + 25 + 25 + 27}{7}$$

$$= {}^{197}/_{7}$$

$$= 28.142\ldots$$

Answer: The average temperature was forecast to be 28.1 °C

4 Daria measured the temperature at 12 noon for a week in September. These are her results in °C.

M	Tu	W	Th	F	Sa	Su
24	21	16	12	13	14	12

What was the average temperature in °C?

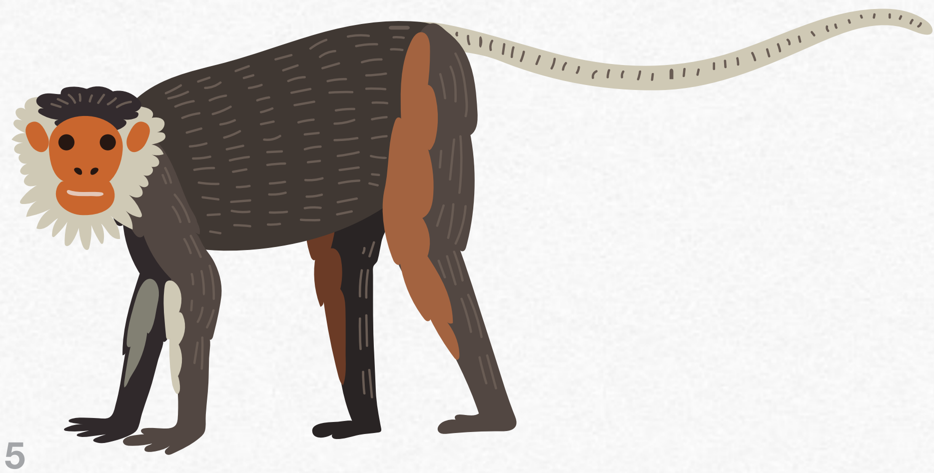
$$Average = \frac{\textit{Sum of the values}}{\textit{Number of the values}}$$

$$= \frac{24 + 21 + 16 + 12 + 13 + 14 + 12}{7}$$

$$= {}^{112}/_{7}$$

$$= 16$$

Answer: The average temperature was 16 °C



5 In 2020, Eliana had worked out the average noon temperature for each month. These are her results in °C:

Jan	Feb	Mar	Apr	May	Jun
−8	−7	−3	0	6	8

Jul	Aug	Sep	Oct	Nov	Dec
10	12	8	5	−3	−6

What was the average temperature in °C?

$$Average = \frac{\textit{Sum of the values}}{\textit{Number of the values}}$$

$$= \frac{-8 - 7 - 3 + 0 + 6 + 8 + 10 + 12 + 8 + 5 - 3 - 6}{12}$$

$$= {}^{22}/_{12}$$

$$= 1.833\ldots °C$$

Answer: The average temperature was 1.8 °C

6 Daria and Eliana clearly live in very different climates. Suggest where each might live?

Answer: Daria took her temperatures in Southern England and Eliana in Davos, Switzerland but any comparable places could be correct.

7 From the graph on the previous page, find the 25-year period with the most mean annual temperatures between 10 and 11 °C.

Answer: The 25-year period with the most mean annual temperatures between 10 and 11 °C is 2000–2025 with fourteen (and that is only to 2019).

Next was 1975–2000 with ten mean annual temperatures between 10 and 11 °C.

8 From the graph on the previous page, find the 25-year period with the most mean annual temperatures between 7 and 8 °C.

Answer: The 25-year period with the most mean annual temperatures between 7 and 8 °C is 1675–1700 with seven.

1775–1800 and 1800–1825 both had two mean annual temperatures between 7 and 8 °C.

9 From the graph, find the period when the 10-year moving average of the mean average temperature:

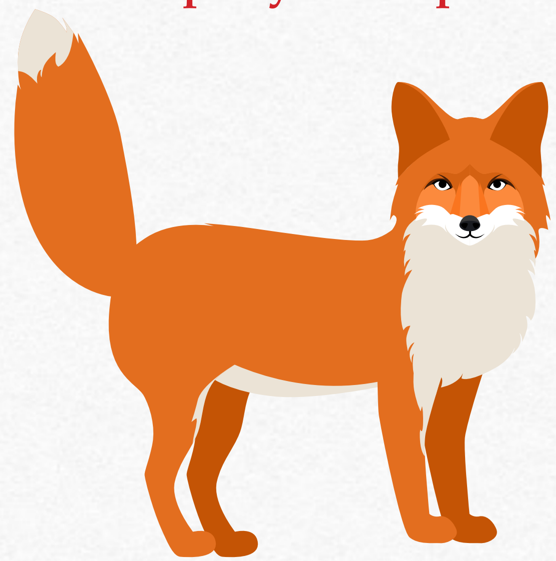
a) dropped most rapidly?

Answer: The 10 year moving average of the mean average temperature dropped most rapidly in the period around 1730–1748

(The temperature also dropped rapidly in the period 1690–1698 but the period around 1730–1748 was the longest and steepest drop)

b) rose most rapidly?

Answer: The 10 year moving average of the mean average temperature rose most rapidly in the period around 1700–1710



Contentment

10 From the graph, find the period when the **30**-year moving average of the mean average temperature:

a) dropped most rapidly?

Answer: The 30 year moving average of the mean average temperature dropped most rapidly in the period around 1688–1700 and again around 1760 to 1770

b) rose most rapidly?

Answer: The 30 year moving average of the mean average temperature rose most rapidly in the period around 1720–1730, which was part of a longer, overall rise in temperature from 1700–1740. 1990–2019 shows another long rise in temperature with a more rapid rise in 2014–2019.

11 Ffion brushes her teeth for **2** minutes twice a day but leaves the tap running whilst she brushes.

Using measuring jug, run a tap for **15** seconds. You may have to empty and refill the jug at least once.

Now you have measured how much water comes out of the tap in **15** seconds, work out how much water comes out in a minute.

The answer will depend on your experiment but is likely to be about 4 litres per minute.

a) How much water does Ffion waste by leaving the tap running when she brushes her teeth?

4 litres x 2 mins = 8 litres

Answer: Ffion wastes 8 litres of water each time she brushes her teeth

b) How much water does she waste in a week?

Ffion brushes her teeth twice a day and there are **7** days in a week.

8 litres x 2 x 7 = 112 litres

Answer: Ffion wastes around 112 litres of water in a week

c) How much does she waste in a year?

Ffion brushes her teeth twice a day and there are 365 days in a year.

8 litres x 2 x 365 = 5,840 litres

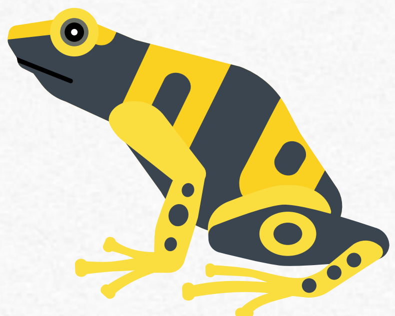
Answer: Ffion wastes 5,840 litres of water in a year

12 If the cost of tap water in the UK is 0.15p per litre, how much does the wasted tap water cost Ffion in a year?

Cost will be the number of litres multiplied by the cost per litre = 5,840 x 0.15p
= 876p
= £8.76

Answer: The wasted water will cost Ffion £8.76 per year.

That might not sound very much but if Ffion lives with a large family and everyone leaves the tap running, then how much money will be wasted?



13 Put a bucket in the shower and run it for 15 seconds. Calculate how many litres is used in a one minute shower.

The answer will depend on your experiment but is likely to be about 6 litres per minute

a) Gigi showers twice a day for ten minutes each time. Calculate how much water she uses in her showers per week.

Water used in each shower = 6 x 10
= 60 litres

Water used per week = 60 x 2 x 7
= 840 litres

Answer: Gigi's showers use 840 litres of water each week

b) Calculate the amount of water that she uses in her showers each year.

Water used = 60 x 2 x 365
= 43,800 litres

Answer: Gigi's showers use 43,800 litres of water each year

14 Hebe prefers to bath. Her bath tub is **150 cm** long and **50 cm** wide. She likes to fill her bath to a depth of **15 cm** before she steps in.

a) Knowing that $1{,}000\ cm^3 = 1$ litres, how many litres of water does Hebe use in her bath?

Amount of water = 150 x 50 x 15
= 112,500 cm^3
= 112.5 litres

Divide by 1,000 to turn cm^3 to litres

Answer: Hebe uses 112.5 litres in each bath

b) Calculate the number of litres that she uses in a week if she baths every day with an extra bath three times a week after her sports practice.

No of baths = 7 + 3 = 10
Amount of water = 112.5 x 10
= 1,125 litres

Answer: Hebe uses 1,125 litres in a week for her baths

c) Calculate the number of litres that she uses in her baths each year.

This time multiply the amount of litres in a week by 52 as there are 52 weeks in a year.

Amount of water = 1,125 x 52
= 58,500 litres

Answer: Hebe uses 58,500 litres in a year for her baths

15 Calculate the cost of your own showers or baths, or perhaps a combination of the two each year?

The answer will depend on the number of showers or baths and the volume of water you use.

You then need to find out what your water supplier charges you per litre. All houses are entitled to have a water meter.

If your house does not yet have a water metre installed, look up your suppliers website and find what the cost per litre would be.

16 a) When you make yourself a hot drink, how much water do you put into the kettle?

The answer will depend on your habits but most people fill the kettle with at least a litre of water.

b) After you have poured out the boiled water, how much is left in the kettle? How much could you have saved?

Answer: Most hot drinks have a volume of 250 ml, so therefore 750 ml of water will have been boiled unnecessarily.

c) Think about the number of hot drinks that you have in a week and work out how much water you could save if you only boiled a cupful of water each time.

If you have three hot drinks a day then you could save

3 x 750 ml per day or:
Per week, saving = 3 x 750 x 7
= 15,750 ml

Answer: You could save 15,750 ml or 15.75 litres a week

d) How much water and thus how much money could you save in a year?

Per year, saving = 3 x 750 x 365
= 821,250 ml
= 821.25 litres

Cost saved = 821.25 x 0.15p
= 123p or £1.23

Answer: In a year about 820 litres of water could be saved which costs £1.23

This again does not seem very much, but consider the amount of energy used unnecessarily in boiling all that unused water plus is everyone in your home or office wasting as much water and energy?

17 Write yourself a water saving plan. Work out how you could alter your household's habits to save water.

Your plan should start with a survey of how much water your household uses a week.

Your survey could look like this:

Question	Answer per day	Answer per week	Average litres per activity	Litres in total per week
1. How many baths do we take a week?			100 litres per bath	
2. How many showers do we take a week?			60 litres per shower	
3. How many times a day/week do we brush our teeth a) with the tap running? b) without the tap running?			a) 4 litres per minute b) 1 litre	
4. How much water do we put in the kettle but don't drink?			0.75 l per kettle	
5. How many times do we use the washing machine?			65 litres a load	
6. How many times do we use the dishwasher?			20 litres a load	
7. How many litres do we use to water the garden?			15 litres a minute	
8. Any other area where we could save water?				

You also use water to drink, to flush the W.C. and to wash up but you are not likely to be able to save water in any of these areas.

a) How much water could your household save in a week?

b) How much in a year?

c) How much money would that save?

This will depend on your household, but if you water the garden with a hose then that is likely to be the biggest water guzzler of all.

Remember that no one is suggesting that you do not shower or bath, only that you consider if you could use less water overall.

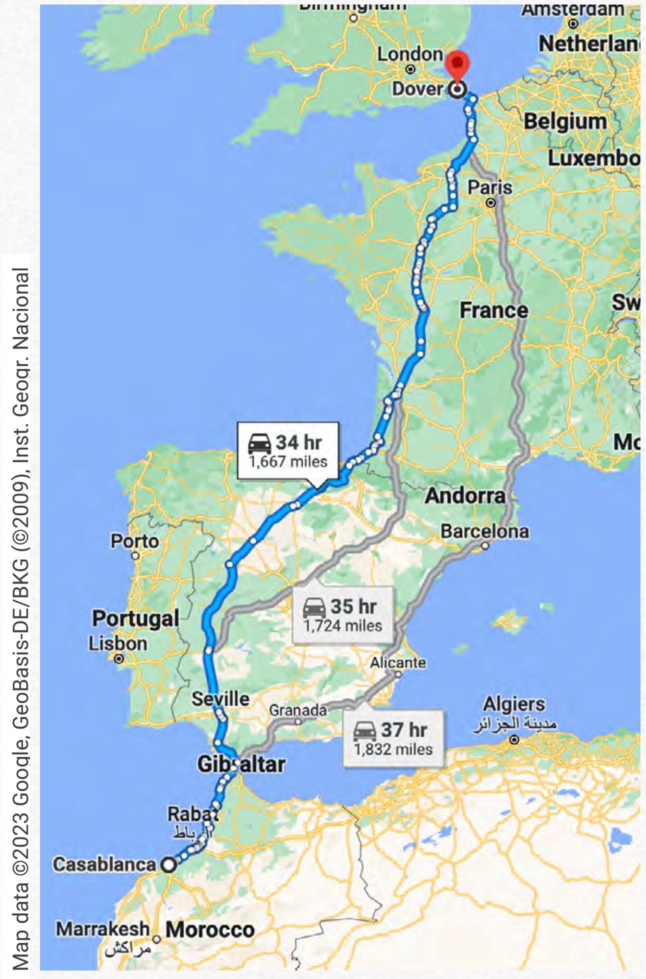
Food for Thought

As you let that tap run, spare a thought for women in other continents. Did you know that **40%** of the **783** million people in sub-Saharan Africa do not have access to clean drinking water?

18 Isla is choosing tomatoes at her local supermarket in Birmingham.

Use an online atlas or phone app to find how many more miles it is to Birmingham from Morocco compared to Kent?

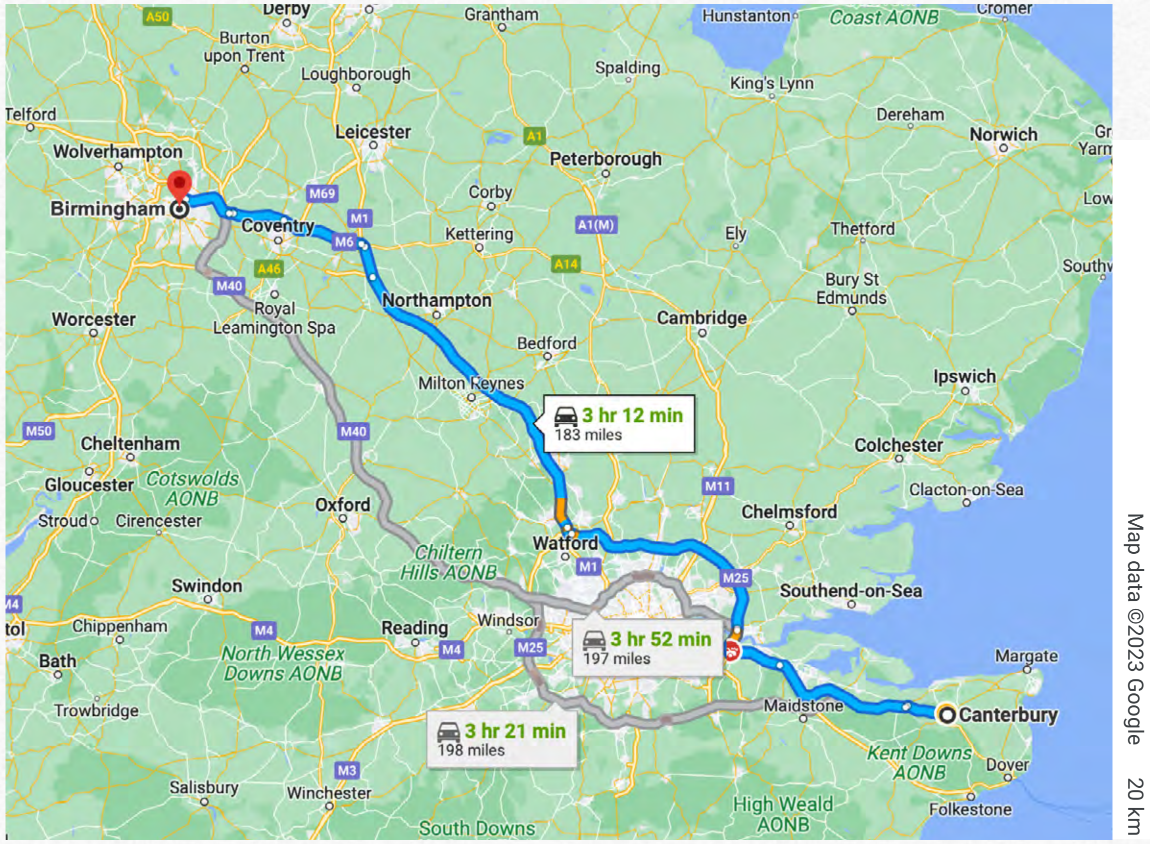
Your answer will be an approximation as you do not know exactly where in Kent or Morocco the tomatoes are grown.



If you assume to tomatoes will be flown then take the distance from the capital of Morocco, Casablanca, to London which is about **1,300** miles.

However, the tomatoes could be delivered by truck. The distance by road from Casablanca to Dover, where they will probably enter the country, is about **1,700** miles. You'll also need to add the distance from Dover to Birmingham, which is 200 miles. So the total distance is about **1,900** miles.

We do not know where in Kent so if we take the capital, Canterbury, then the distance by road from Canterbury to Birmingham is about **184** miles.



So by road the difference is **1,900–180 = 1,720** miles.

Answer: It is 1,720 more miles to Birmingham from Morocco compared to Kent

19 Julia is very conscientious about eating locally grown seasonal food. For her Christmas party she is deciding between serving:

- Fresh raspberries and cream
- Blackberries with frosted butter
- Baked apples with seasonal spices

Which would you advise Julia to serve?

You may see all these fruit in the supermarket all year round but only one is harvested in Britain in December.

Answer: Julia should serve Baked apples with seasonal spices.

Food for Thought

Until the 1950s, almost the only fruits that were in the shops were those grown locally. The cook in the household had to plan meals around what was available, not what everyone might have preferred.

Julia's great-great-grandmother kept this record of what fruits were available month by month in her kitchen garden in Suffolk:

January and February (Stored) Apples, Pears

March and April None

May Rhubarb

June Cherries, Gooseberries, Raspberries, Redcurrants, Rhubarb, Strawberries

July Blackcurrants, Cherries, Gooseberries, Loganberries, Raspberries, Redcurrants, Rhubarb, Strawberries

August Blackcurrants, Gooseberries, Greengages, Loganberries, Plums, Redcurrants, Rhubarb, Strawberries, Elderberries, Blueberries, Tayberries, Pears

September Blackberries, Greengages, Damsons, Pears, Plums, Blueberries, Tayberries, Loganberries

October Apples, Pears, Blackberries, Damson, Quince, Tayberries

November Cranberries and stored Apples, Pears, Quince

December Cranberries and stored Apples, Pears, Quince

20 Kola's college used to sell **500 ml** bottles of water in their canteen for **60p** a bottle. Now they sell college branded reusable water bottles for **£5** each which can be filled from a water fountain for free.

Kola used to buy a bottle of water with her lunch every day for the **30** weeks she was at college.

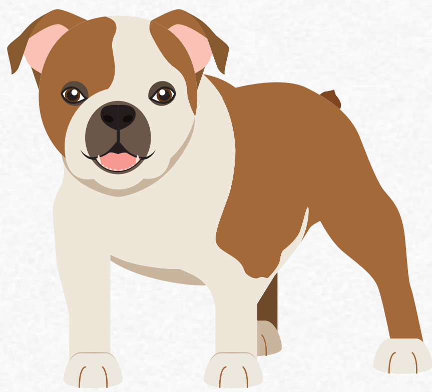
a) How much did Kola save in a **30** week academic year by buying a reusable bottle.

Kola should multiply the number of weekdays in a week by the number of weeks by the cost of a bottle.

Cost of buying water = 5 x 30 x 0.60
= £90

She will save the cost of buying water less the **£5** cost of a reusable bottle

Answer: Kola saved £85



b) If the college has **1,500** student and all made the same change as Kola, how many single use plastic bottles would be no longer used?

The **1,500** students would use **1,500 x 5 x 30** bottles

Answer: 225,000 single use bottles would be no longer used

21 Lea goes to a music festival every year. The festival served drinks in single use plastic glasses. Last year, **1.2** million glasses were left behind scattered all over the festival site and it took **300** volunteers **5** days to clear them all away. How many plastic glasses is that per volunteer per day?

The number of volunteer days = 300 x 5 = 1,500

The number of glasses per volunteer per day
= 1,200,000 ÷ 1,500
= 800

Answer: Each volunteer collected on average 800 plastic glasses per day.

22 Mina's college carried out a plastic survey. They found that of the **178** households that took part on average each threw away **116** pieces of plastic each week.

a) How many pieces of plastic is that in total each week?

$178 \times 116 = 20,648$

Answer: 20,648 pieces of plastic were thrown away each week

b) If each household threw away the same amount of plastic every week, how many pieces is that per household in a year?

$116 \times 52 = 6,032$

Answer: Each household would throw away 6,032 pieces of plastic in a year

c) If there are approximately **25** million households in the UK, how many billion pieces of plastic will be thrown away in a year?

Multiply your answer to (b) by the number of households in the UK

$6,032 \times 25$ million $= 150,800$ million

$= 150.8$ billion

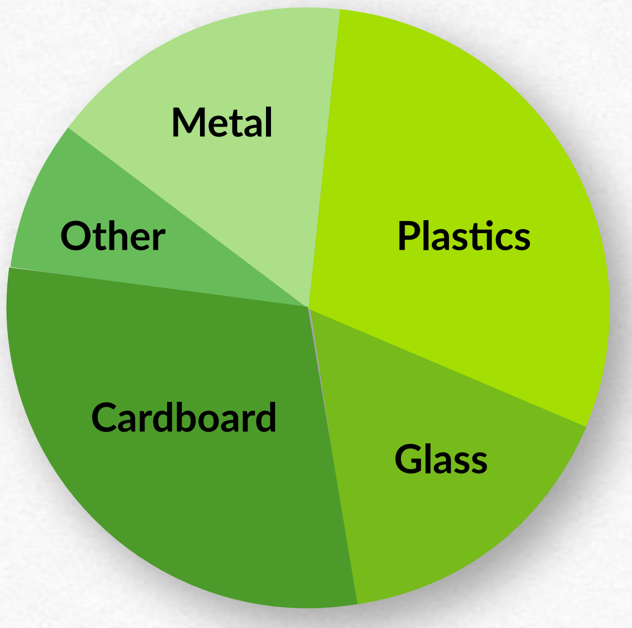
Answer: In the UK approximately 150 billion pieces of plastic are thrown away each year

Food for Thought

You might think that when you put your plastic in the recycling bin, that it all gets disposed off in a responsible manner. Sadly, this is not the case. Much recycling gets sent abroad and there can get dumped rather than recycled.

Plastic waste often ends up in the sea and can be a threat to sea life.

23 Nona has also carried out a survey. She asked participants to count the number of items of each type of material that they put in their recycling every two weeks. She showed her results as a pie chart:



a) What type of material was the most common?

Answer: Plastic and cardbboard had almost the same number of items and were the most common

b) Roughly how many more cardboard items were recycled than glass items?

If you are confident then you can estimate but you may prefer to use a protractor or angle measurer.

The angle for cardboard is about $110°$ and the angle for glass is about $55°$.

Answer: Roughly double the amount of cardboard items were recycled than glass items

c) Roughly what percentage of all the recycled items were plastic?

Using a protractor the angle for plastics is $110°$

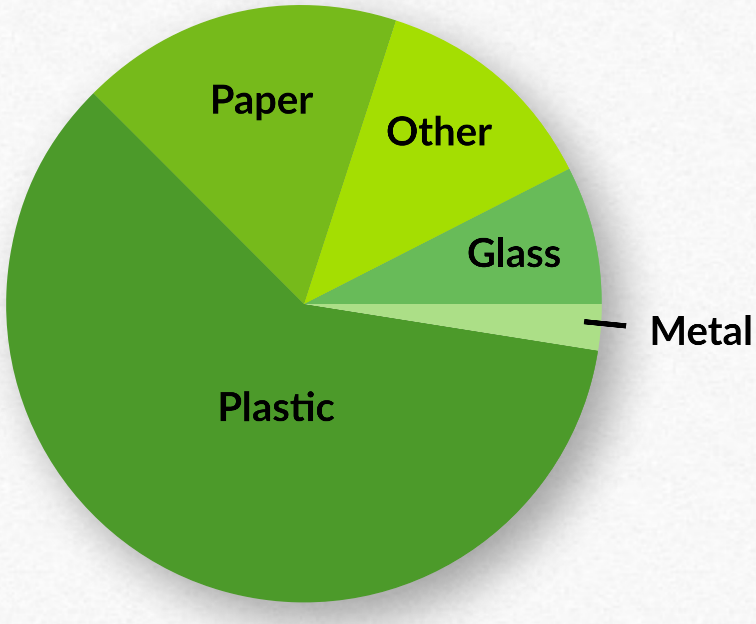
$^{110}/_{360} \times 100 = 30.55\%$

Answer: Roughly 30% of all the recycled items were plastic

Food for Thought

Have you considered joining a local volunteer group to help improve your environment?

24 Oona volunteers for her local environmental group once a month. They have just completed a litter picking exercise on the local beach. Oona has drawn up this pie chart of the number of items to show the results:



a) What type of material was most litter made of?

Answer: Most litter was made of plastic

b) Roughly what percentage of all the litter was plastic?

The angle is about 220°, to find the percentage, make a fraction and multiply by 100.

Percentage plastic = $^{220}/_{360} \times 100\ \%$

$= 61.11..\%$

$\approx 60\%$

Answer: Roughly 60% of all the litter was plastic

c) Roughly how many more plastic items were found than glass?

The angle for glass is just about 20°

$220° \div 20° \approx 10$

Answer: Roughly 10 times as many plastic items than glass items were found

That will be a big change from your grandparents'time, when glass was the most common type of litter. Why do you think that is?

If you do some research, you will see how much plastics production has increased over the last fifty years, and how little is recycled.


“Just heard - I passed my maths exam”

25 Rosie's studio flat has an under the counter fridge with an energy rating of **115 kWh**/a. What will this cost her per month if her landlord charges her electricity at **25p** per **kWh**?

Rosie should first divide the **kWh**/annum by **12** to get the **kWh** per month

Number of **kWh** = $^{115}/_{12}$

= 9.5833. . . kWh per month

And then without changing the display, multiply by **0.25** to give the cost in pounds.

Cost per month = 9.5833. . . x 0.25

= £2.395. . .

Answer: The fridge will cost her £2.40 per month

26 Samira lives at home with her mother and grandfather. Their television has just broken down and they are deciding which model to replace it with. Samira works out that between them the household watches roughly **40** hours of television a week. Their electricity supplier charges them **19p** per **kWh**. What will be the annual cost of:

a) a **55″** TV with energy consumption of **125 W**?

$$\text{Number of kWh} = \frac{\text{Watts x hours per week x } 52}{1000}$$

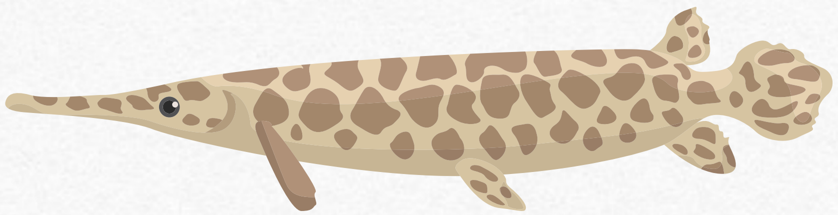
$$= 125 \times 40 \times 52$$

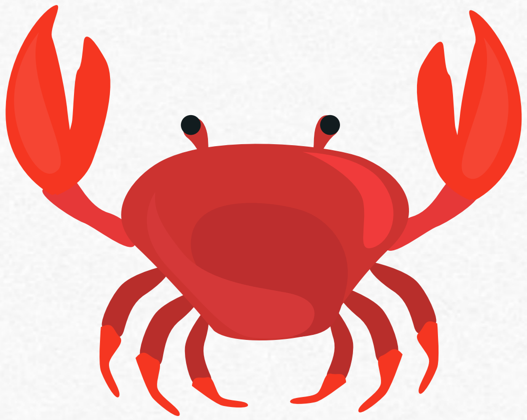
$$= 260 \text{ kWh}$$

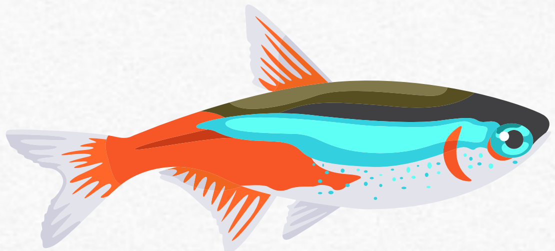
Cost per year = 260 x 0.19

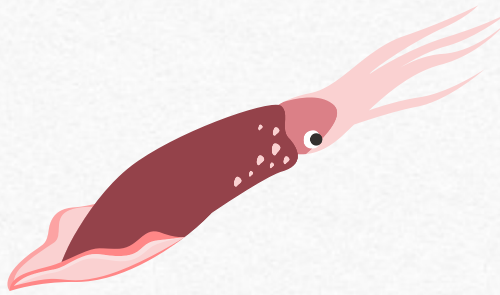
= £49.40

Answer: The TV will cost Samira's family £49.40 per year









b) a **27″** TV with energy consumption of **26 W**?

$$\text{Number of kWh} = \frac{\text{Watts x hours per week} \times 52}{1000}$$

$$= \frac{26 \times 40 \times 52}{1000}$$

$$= 54.08 \text{ kWh}$$

Cost per year = 54.08 x 0.19
= £10.275. . .

Answer: The TV will cost Samira's family £10.28 per year.

27 Thalia has a new washing machine with energy consumption of **0.545 kWh** per cycle, if the eco setting is used. What does this cost her per wash if she pays **18.5p** per **kWh** for her electricity?

Cost per wash = 0.545 x 18.5p
= 10.08. . .p

Answer: Each wash will cost Thalia 10p

28 Uma has an old washing machine that is rated at **1.2 kW**. If she uses her washing machine **3** times a week and each cycle takes **2** hours, what does this cost her per month if she pays **22 p** per **kWh** for her electricity?

No of kWh per week = 1.2 x 3 x 2
= 7.2 kWh

Weekly cost = 7.2 x 22p
= 158.4p = £1.58

Answer: Her washing costs Uma £1.58 per week.

29 Vee likes to dry her clothes in the tumble dryer. Her dryer has an energy consumption of **5.2 kWh** for a full load. What does one hour of drying cost if her electricity supplier charges **20p** per **kWh**?

Cost = 5.2 x 0.20
= £1.04

Answer: One hour of dring costs Vee £1.04

What does Vee pay per week if she does a wash four times a week and uses the dryer for an hour and a half for each wash?

Cost = £1.04 x 1.5 x 4
= £6.24

Answer: Vee pays £6.24 for drying her clothes each week

30 Winona does not use a tumble dryer but either hangs up her clothes outside on a line or uses an inside clothes airer. Winona also does a clothes wash four times a week. What does she save per year compared to Vee?

Over a year, Winona will save **52** times what it costs Vee each week

Saving = 52 x £6.24
= £324.48

Answer: Winona will save £325 a year

Food for Thought

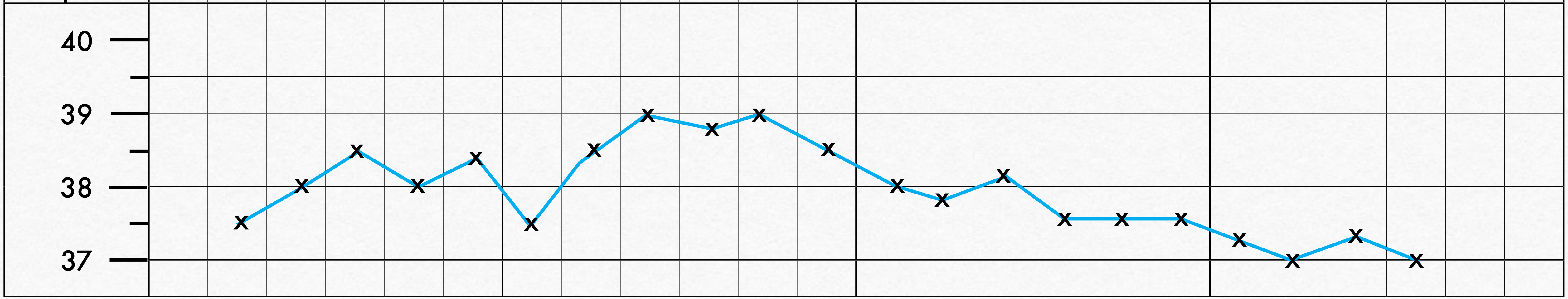
Have you asked your energy supplier for a smart meter to help you track how much electricity and gas you are using? Being tech savvy with how you use your energy keeps you in control:

31 Xandra continued to take her sister's temperature.

(a) Copy the chart from the previous page and complete it with the temperatures Xandra recorded.

Answer:

Day	Monday						Tuesday						Wednesday						Thursday					
Time	4	8	12	16	20	0	4	8	12	16	20	0	4	8	12	16	20	0	4	8	12	16	20	0
Temp °C		37.5	38	38.5	38	38.4	37.5	38.5	39	38.8	39	38.5	38	37.8	38.1	37.6	37.6	37.6	37.3	37	37.3	37		



(b) When was her sister's temperature highest?

Answer: Her temperature was highest at 12 noon and 8pm on Tuesday

(c) When did her sister's temperature first come back down to normal?

Answer: Her temperature was back to normal from 4 pm on Wednesday

(d) When did Xandra decide that her sister had recovered?

Answer: Sometime on Thursday when she complained of being hungry and ran around the house as normal

32 Doctor Zulia measures the blood pressure of three of her patients, the results are:

Patient A

Patient B

Patient C

What does Doctor Zulia tell each patient?

Answer:

Patient A has pre-high blood pressure

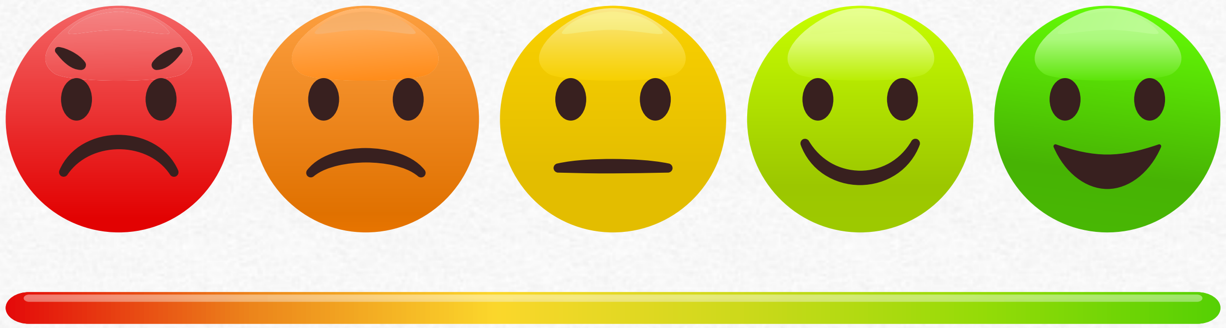
Patient B has ideal blood pressure

Patient C has high blood pressure

Wellbeing

The concept of wellbeing and mental health is increasingly being seen as being as important to our lives as physical health.

Once you have added up your scores, you can use a simple scale to measure your wellbeing:



0	20	40	60	80	100

In Crisis	Struggling	Surviving	Thriving	Excelling

Remember to keep an eye on your friends and colleagues. If you feel they might not score as much as surviving then talk to them. Remember, together we can do great things!

"I can do things you cannot, you can do things I cannot. Together we can do great things!" MOTHER TERESA

Mother Teresa was the founder of the Order of the Missionaries of Charity, a Roman Catholic congregation of women dedicated to helping the poor. She is considered one of the 20th Century's greatest humanitarians.

YOUR BRAIN WORKOUT

Q1

What is the average temperature for this week in Glasgow?

Mon	Tues	Wed	Thurs
9°C	10°C	12°C	11°C

Fri	Sat	Sun
8°C	11°C	9°C

YOUR BRAIN WORKOUT

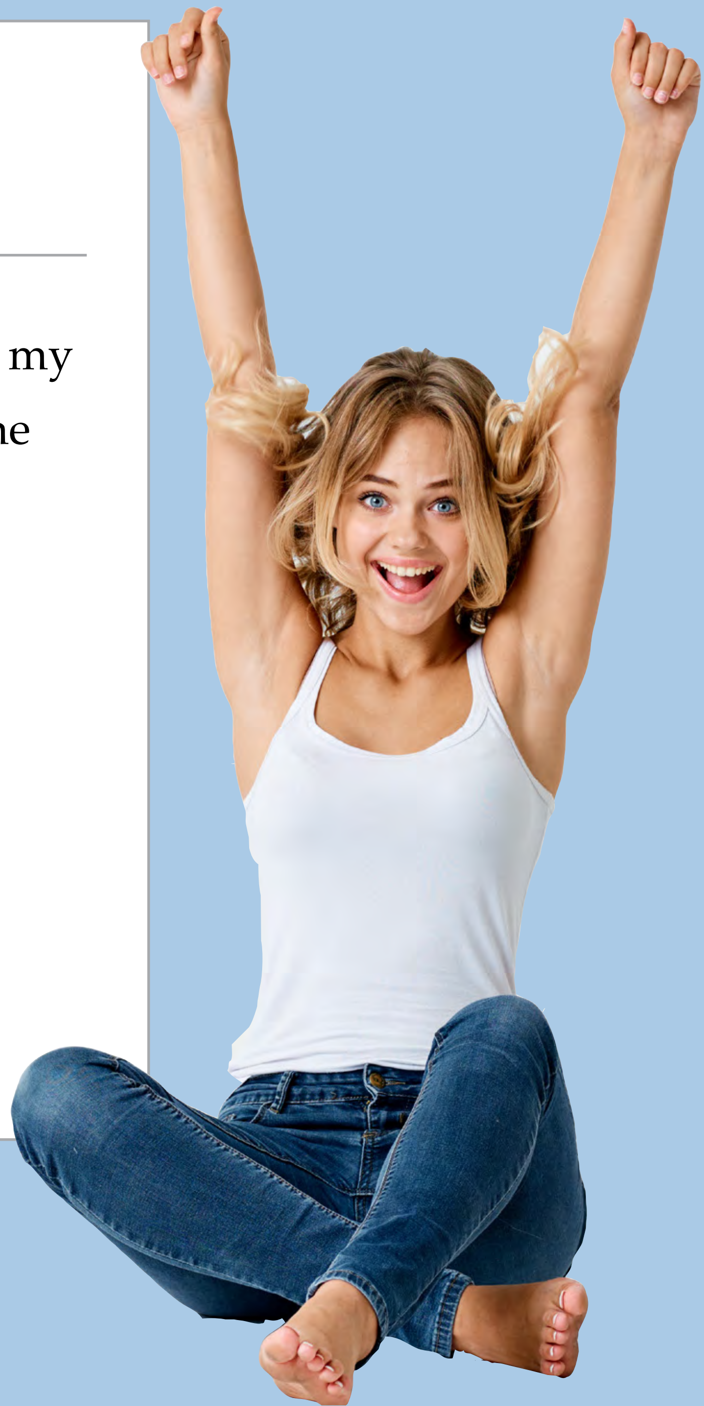
Q2

I water my plants every evening using **2** litres of water each time. How much water is that in a week?

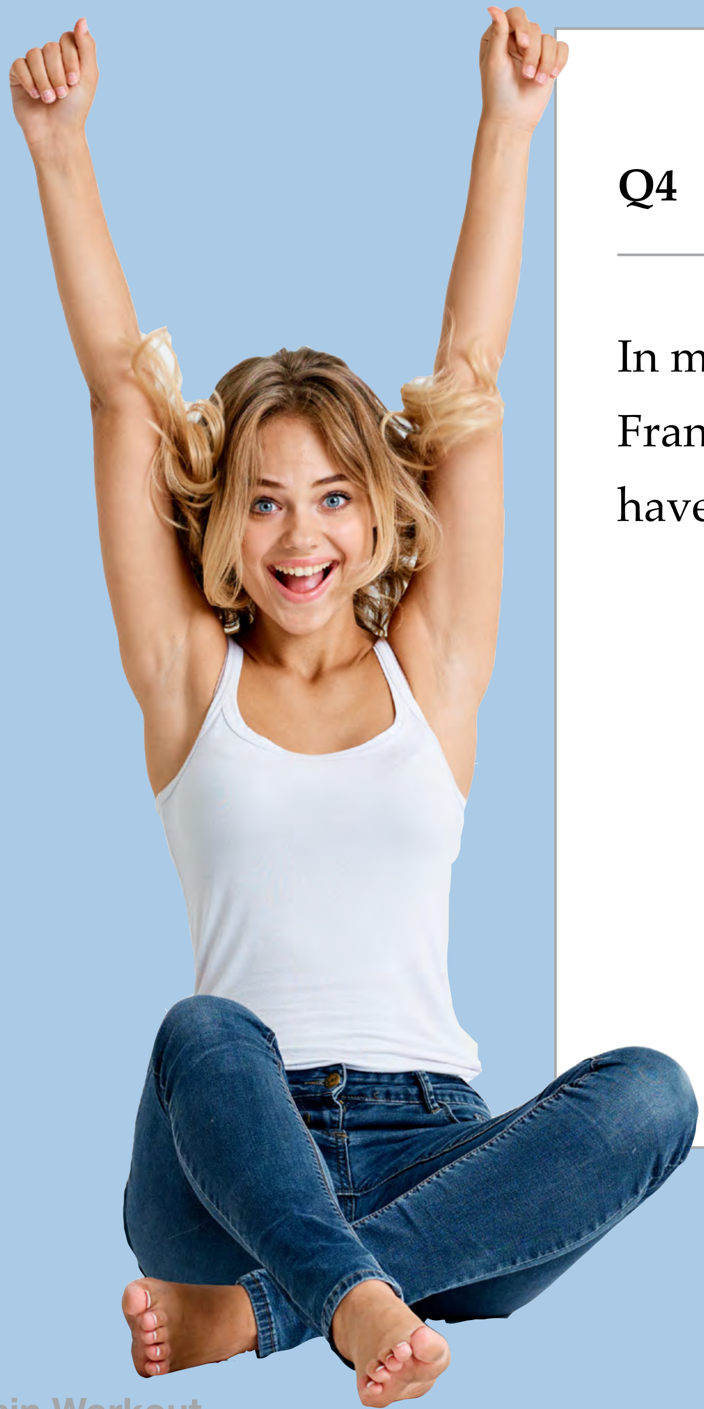
YOUR BRAIN WORKOUT

Q3

I estimate that I use 700 litres a year watering my plants. If water costs 0.15p per litre, what is the total coat per year?



YOUR BRAIN WORKOUT



Q4

In my supermarket I can buy apples grown in France or grown in South Africa. Which apples have more 'food miles'?

YOUR BRAIN WORKOUT

Q5

In a school with **500** pupils, each pupil buys a bottle of water every week day for the **36** weeks when they go to school. How many bottles of water is that?

YOUR BRAIN WORKOUT

Q6

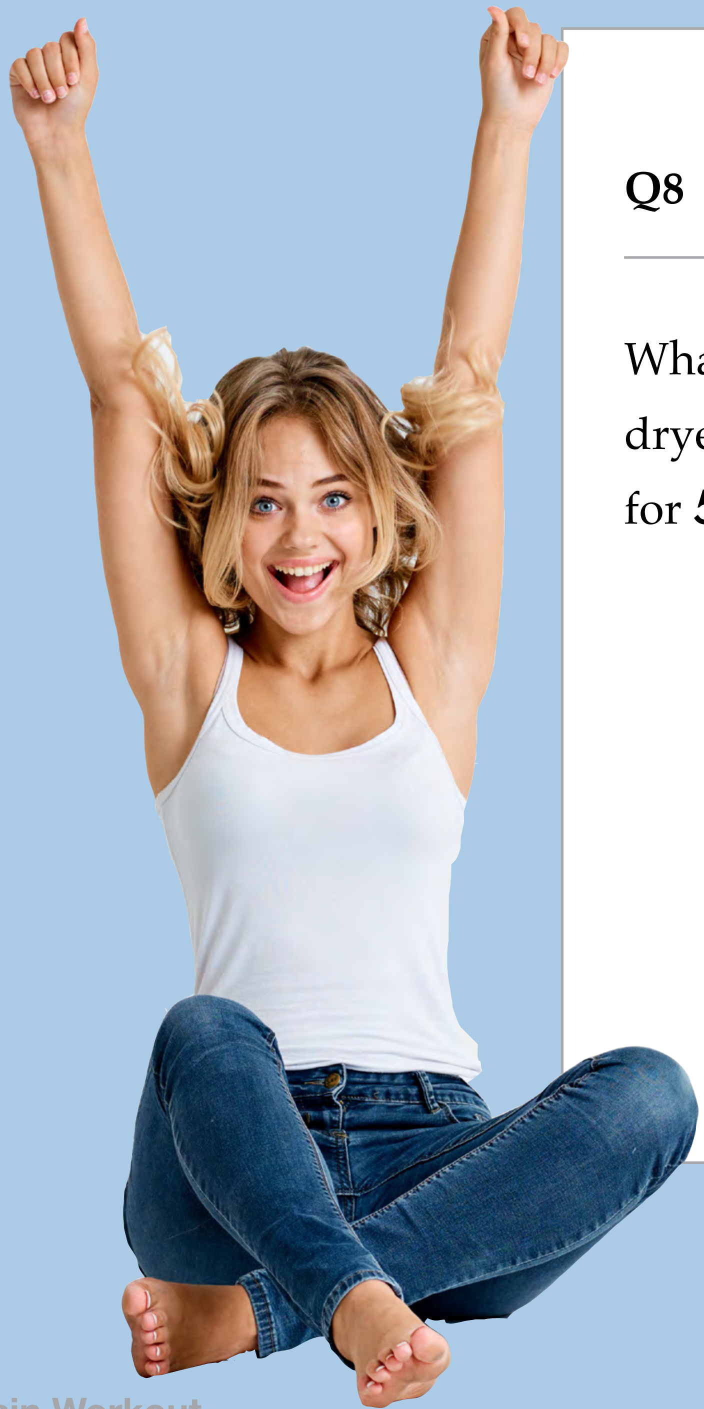
Students in a college fill the recycling boxes with 100,000 empty 500 ml bottles of water a year. How many tonnes is that per year if 30,000 bottles make a tonne?

YOUR BRAIN WORKOUT

Q7

What is the annual cost of running a fridge freezer that uses **200 kWh** per annum if energy is charged at **20p** per **kWh**?

YOUR BRAIN WORKOUT



Q8

What is the annual cost of running a tumble dryer with an energy consumption of 5.2 kWh for 5 hours a week if energy costs 20p per kWh?

YOUR BRAIN WORKOUT

Answers

Q1 $\frac{9 + 10 + 12 + 11 + 8 + 11 + 9}{7} = {}^{70}/_{7} = 10\,°C$

Q2 2 x 7 = 14 litres

Q3 700 x 0.15 p = £1.05

Q4 South Africa

Q5 500 x 5 x 36 = 90,000

Q6 $\frac{100{,}000}{30{,}000}$ = About 3.3 tonnes

Q7 200 x 20 p = £40

Q8 5.2 x 5 x 52 x 0.2 = £270.40

Underwater patterns.

From the birds and the bees to the tides of the seas,
From the winds howling high to the stars in the sky…
We can be romantic, but we must be practical:
Our Universe is based on formulae mathematical.

The authors hope that this Step has given you the tools to go out into the world with the confidence to make your own money: make the most of your earnings and manage your spending. They also hope that you step into the world with other qualities.

Money Stuff

MATHS PLAN STEP 6

I judge myself competent in the following:

Reflection

Rotation

Symmetry

Opening a Bank Account

Using a Spreadsheet to Track Your Finances

Loan or Credit Card

Pythagorus Theorum

Foreign Exchange

How to Read a Map

How to Help the Planet

Signed ....

Date ....

Money Stuff

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26 (c,r)	Shutterstock	**Klabold**
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27	Sasha Marie Spyrou	
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28 (r)	Shutterstock	**skyNext**
29	Adobe Stock	**banjongseal168**
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33 (t,r)	Shutterstock	**Luzzmar**
33 (b,l)	Shutterstock	**Alexey Kljatov**
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33 (b,r)	Shutterstock	**Nigel Jarvis**
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P36 (r,l)	Shutterstock	**19 STUDIO**
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38	Shutterstock	**essevu**
42	Adobe Stock	**yaophotograph**
43	Adobe Stock	**knelson20**
44	Shutterstock	**Glass and Nature**
45 (l)	Shutterstock	**Ambidextr**
45 (r)	Shutterstock	**flowersmile**
47 (l)	Shutterstock	**Sergio Bertino**

48	Shutterstock	**Halfpoint**
49 (r)	Shutterstock	**George Rudy**
50 (t,l)	Adobe Stock	**neirfy**
50 (b,l)	Adobe Stock	**banphote**
50 (r), 74	Shutterstock	**Africa Studio**
51, 75	Shutterstock	**pics five**
52 (t,l and r)	Adobe Stock	**Cool Hand Creative**
52 (b,l)	Sasha Marie Spyrou	
52 (b,r)	Shutterstock	**New Africa**
53	Shutterstock	**Featureflash Photo Agency**
58	Shutterstock	**Benedikt Juerges**
59 (r)	Shutterstock	**Andrew Krasovitckii**
63	Shutterstock	**roberaten**
66 (l)	Adobe Stock	**ksenyasavva**
66 (r)	Shutterstock	**Jaros**
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81–9	Shutterstock	**Alliance Images**
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92 (l)	Shutterstock	**Zhenya Voevodina**
92 (t,r)	Shutterstock	**Andrey_Popov**
92 (b,r)	Sasha Marie Spyrou	
93 (t)	Shutterstock	**Katiek**
93 (b)	Shutterstock	**SunshineVector**
94	Shutterstock	**Andrey Arkusha**
95	Shutterstock	**Nadya_Art**
96	Sasha Marie Spyrou	
97	Adobe Stock	**Dmytro**
98	Adobe Stock	**HBS**
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102	Shutterstock	**Daria Riabets**
103	Sasha Marie Spyrou	
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111	Sasha Marie Spyrou	
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115	Shutterstock	**Sunflower Light Pro**
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148-55	Shutterstock	**Boris Ryaposov**
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166	Shutterstock	**XanderSt**
167 (l)	Shutterstock	**Walter Kopplinger**
167 (r)	Shutterstock	**Daria Riabets**
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169	Shutterstock	**attaphong**
170	Shutterstock	**stockcreations**
171	Shutterstock	**Mahmoud Rafik Adel**
172	Shutterstock	**MVolodymyr**
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176	Shutterstock	**T.Den_Team**
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180	Shutterstock	**Dmitry Skutin**
181 (r)	Shutterstock	**Viktory Viktor**
182 (l)	Shutterstock	**Nasared**
184	Shutterstock	**Nadya_Art**
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193	Sasha Marie Spyrou	
196	Shutterstock	**Africa Studio**
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325 (r), 356 (t,r)	Shutterstock	**Me dia**
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