BY SERENA ALEXANDER

BA (Hons) Architecture, Dept of Engineering, University of Bristol, PGCE (Secondary Mathematics) University of Southampton

MATHEMATICS CONSULTANT:

MARGARET BROWN, MA, PhD,

DSc(hon) EdD(hon), FAcSS, FKC, OBE,

Emeritus Professor of Mathematics Education, King's College London

EDITOR: LINDSAY NICHOLSON, MBE,

BSc Hons Astrophysics, University College London

CREATOR: DAME SHIRLEY CONRAN, DBE,

Hon Fellow UCL

THE DO-IT-YOURSELF 6 STEP MATHS PLAN FOR REAL LIFE

THIS IS STEP 5





A BRIEF DESCRIPTION OF MONEY STUFF ONLINE MATHS COURSE

Maths is increasingly important in the modern world.

The MONEY STUFF Teach-Yourself Maths Course is online, FREE, and doesn't need a teacher.

This tested and proven Maths Course links maths to money to help people in real life. It was designed for girls who don't like maths, but actually it is suitable for anyone of any age, especially anyone who lacks confidence and is anxious about maths. It can be used on any tablet or computer and can also be individually printed.

Up to 15% of people in the UK are dyslexic. MONEY STUFF has been specifically designed so that dyslexics can read it easily. The entire computer production team was dyslexic.



How to get where you want

"The ocean's always trying to kill you. It doesn't take a break."

Those are the words of Tracy Edwards who skippered the first ever all-female crew on the yacht Maiden in the Whitbread Round the World Race of 1989-90.

The **2019** documentary film 'Maiden' tells the story of the race. Tracy, then **24**, had never skippered a yacht before having formerly sailed as a crew member and cook. She had to attract other young women to join her, find a financial backer and learn to navigate.

The documentary tells the story of their **32,000** mile journey, at times in terrifying **50** foot waves. But it was not just the ocean that Tracy and her crew had to fight, they also had their own social upheavals and were always under attack from the media for their daring to participate in the 'man's world' of ocean racing.

Tracy had to rely on her huge determination to succeed, requiring real grit to ensure the survival of the yacht and its crew whilst also coping with the other pressures upon her.

The race started in September 1989 from Southampton with 23 yachts from 13 countries. The race is sailed around the world in six legs, and Maiden won two of them. She finished the final leg second in her class and was second overall, the best result for any British boat in 17 years.

Tracy Edwards, who had as a child been suspended from school **26** times and was finally expelled, was awarded the Yachtsman of the Year Trophy, the first woman to receive it, and was made an MBE.

Tracy's story shows that if you want something badly enough then you can achieve it, but you have to work for it. Decide what you need to do, what you need to learn, who you need to talk to, make a plan and stick to it. It may still need reviewing and adapting but that is part of your journey to success.

Where in the world will you get to?

Maiden was refurbished in **2018** and set off on a two-year tour around the world to promote the need for girls' education.



!!! Watch out for prices !!!

The cost of living has been zig-zagging upwards for hundreds of years. In the sixteenth century, Queen Elizabeth I worried about the increasing costs of feeding and equipping her army and navy. Today, you can still expect prices to rise unsteadily in the unforeseeable future.

What causes prices to rise? Many reasons, including bad weather, which increases farmers' food prices. So workers need higher wages, which means that the cost of the goods they make will increase. If the prices of bricks, cement and steel increase then so will the cost of housing and rents.

Sometimes the price rises are so small you don't notice them – but you will certainly notice if your home energy bill shoots up in a few months and mum starts switching off the lights and heating.

These rising prices are called **inflation**.

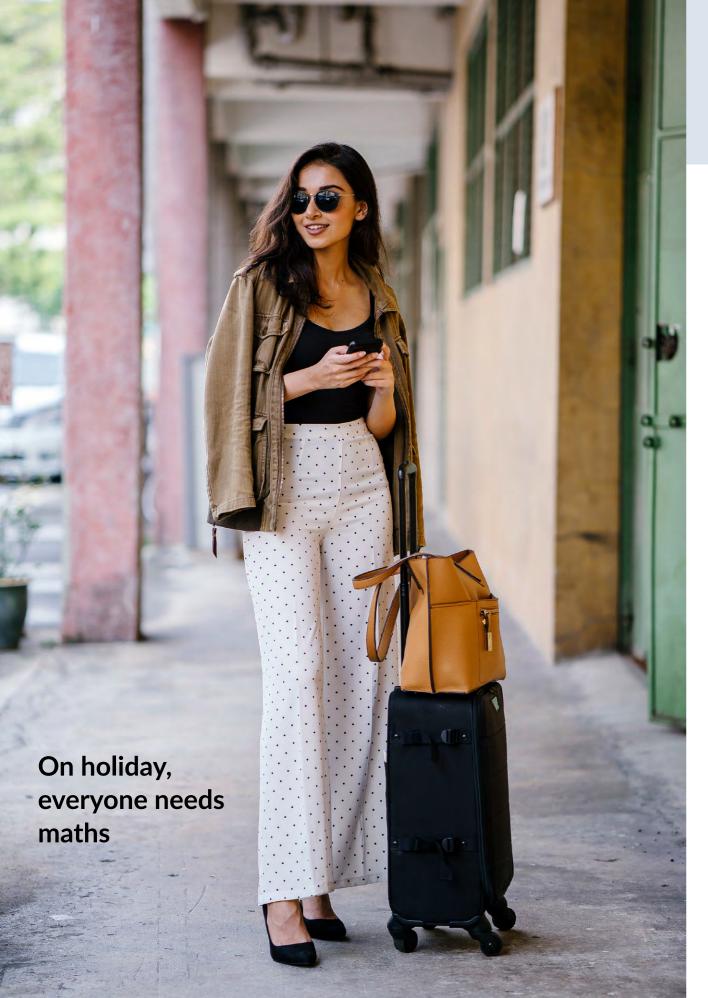
When I started to write this maths course, the prices I used in the exercises were the same as the prices in the shops – but by the time I had finished **Step 1**, the shop prices had risen - so the exercise prices were out-of-date. That is why the prices in **MONEY STUFF** are not current prices; they are historically correct prices, paid by your grandmother and mother in the early **21st** century.

In maths, as in life, people have different ways to write numbers. For example, you can write a fraction as either $\frac{1}{2}$ with a diagonal line, as we do, or as $\frac{1}{2}$ which you may also see. Whichever you use, the meaning is the same. Likewise, some people write 1,000 or 1,000,000 as we do, with commas to break up the digits, others prefer just to leave a space, like this 1 000 or 1 000 000. The choice is yours – that's the joy of maths!

Shop prices will alter throughout your life.

But the maths you need to shop will never alter.

Dame Shirley Conran



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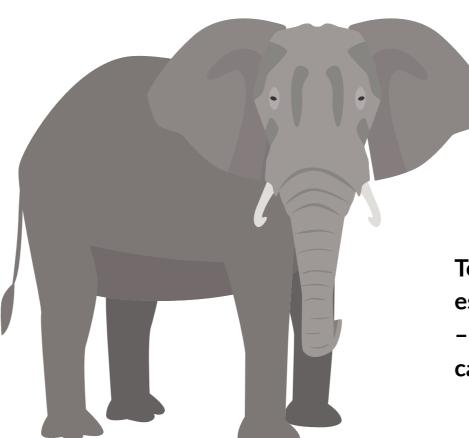
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To make calculations – especially when hunting – animals have an innate calculation system.

Introduction and Thanks

When I was asked by a despairing mother to find a good maths textbook for her mutinous 14-year-old daughter, I went to my local Waterstones and discovered in one afternoon in 2004 how inefficient the UK maths learning system is: I was astonished by the badness of the maths textbooks - with one exception.



In my opinion far the best textbook writer – the only one – was maths school teacher and qualified architect **Serena Alexander** BA (Hons) Architecture, Dept of Engineering, University of Bristol, PGCE (Secondary Mathematics)

University of Southampton. In her acknowledgements, Serena thanked all her students who had tested her work BEFORE it was printed. I am an instructions writer – so I know that testing is exasperating, time consuming and very expensive; endless patience, tenacity and money is needed.

So I started a voluntary group, Maths Action, to improve maths learning in the UK and then I started the Maths Anxiety Trust, which produced the online, FREE, teachyourself MONEY STUFF Maths Course with 4 Steps, all carefully tested.

Astonishingly, two 17th Century dishonest beliefs still persist in Britain today: that only boys are born with maths ability and that girls don't need maths in their adult lives. Believe me, many Year 9 girls still believe that rubbish.

So we decided to produce two further Steps for the MONEY STUFF COURSE. This was to teach girls – future women – the importance and the excitement of further mathematics, if they are to get the same chances as men. In 2022, the bosses of the FTSE – the hundred biggest businesses in Britain – numbered 7 women bosses and 93 men bosses. So which group has the most power?

The NHS then discovered that I had a tumour the size of a tennis ball in my brain. I was unable to write these important Steps and without them this series would not be a complete course at international level.

So Serena Alexander was asked to write MONEY STUFF STEP 5 and STEP 6.

We all held our breath.

Serena said yes.

Nervously, I asked Serena what her fee would be.

Serena said, "Nothing. It needs doing."

Members of the Maths Anxiety Trust know how long textbooks take to write and test, so we are very grateful to Serena.

Also working for no pay on **MONEY STUFF** were two distinguished women, who want girls to use maths to get a better life. The mathematics consultant is Margaret Brown and the editor-in-chief is Lindsay Nicholson.



The Emeritus Professor of
Mathematics at Kings College,
London, Margaret Brown MA,
PhD, DSc(hon) EdD(hon),
FAcSS, FKC, OBE, has a
mathematical background in
teaching and writing good

maths books. The modest and discreet Margaret seems to have her finger in every important maths pie in Britain, but she never talks about those pies. When recently I asked Margaret to come with me to a small business lunch with the Royal Society to discuss their important, Government-funded project to improve maths ability in Britain, I only discovered over the risotto that The Royal Society had already consulted Margaret.



Lindsay Nicholson MBE, BSc
Hons Astrophysics, University
College London is a qualified
astrophysicist and still works
closely with her famous college.
As well as being a top journalist,
top editor and writer of a

bestselling autobiography, Lindsay was editor-in-chief for the Hearst Publishing Empire in Europe and so is familiar with international finance at Board level.

Serena is a maths textbook STAR and she is **OUR** star and we hope she realises how much we appreciate her abilities and her generosity. We hope that will soon be appreciated by many, many people who Serena will never meet – whose lives will be improved by the gifted and generous SERENA ALEXANDER.

Dame Shirley Conran





Serena could not have had better champions at her side.



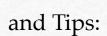
Stepping into Step 5

As you embark on STEP 5 you should be aware of the importance of numbers, mathematics and money.

Just as you have learnt some general knowledge on topics such as literature, geography, famous people, history, etc., you need to have some general knowledge on mathematics.

To help you with this you will see a new feature in addition to Notes and Quick Tips, called Food for Thought:













Food for Thought

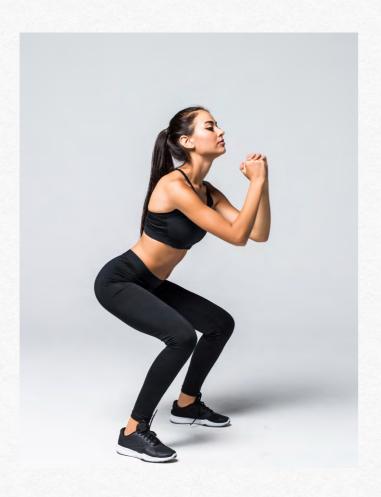
Food for Thought boxes give you additional general knowledge such as information about the history of mathematics, how mathematics and money may be used in different cultures and civilisations, information about facts and symbols that may be new to you and sometimes just a moment to reflect on the beauty of mathematics.

Stepping into Step 5



Mental Arithmetic Strategies

You know the importance of your daily physical workout to keep your body in shape:



a daily maths workout helps keep your brain in shape.

This first part reminds you of some basic calculations that you can use for your brain workout.



In Book 1 you focused on the meaning of numbers and how to calculate. You might like to have a read through of what you learned before you start this Part.

Do you know the meaning of strategy?

A strategy is a plan of action designed to achieve a long-term or overall aim.

If your aim is to calculate quickly and accurately, then you need some strategies to help you achieve that.

Daily Workout

Addition and Subtraction

Mental Arithmetic Strategy 1:

Finding 10s

When adding a row of digits, it is always worth looking for any pairs of numbers that add up to 10 or a multiple of 10.

A multiple is a number that is the result of multiplying one whole number by another whole number. Therefore 10 is a multiple of 2 and 5 (and also a multiple of 1 and of 10). From that it follows that 20, 30, 40, etc. are multiples of 10.

Example

$$7 + 5 + 3 + 5$$

You will probably spot that 7 + 3 = 10 and 5 + 5 = 10

So in your head you will say to yourself:

$$7 + 5 + 3 + 5 = 7 + 3 + 5 + 5$$

= 10 + 10
= 20

Answer: 7 + 5 + 3 + 5 = 20

Another example

Can you see how to make 10s here?

$$46 - 6 = 40$$

$$46 - 6 = 40$$
 $91 + 109 = 200$

In your head you might say to yourself:

$$46 + 91 - 6 + 109 = 46 - 6 + 91 + 109$$

= $40 + 200$
= 240

Exercises

Now try these:

- (a) 2+6+8+4
 - (b) 1+8+2+9
 - (c) 4+7+5+6+3+5
- 2 (a) 113 + 2 + 7 + 18
 - (b) 2+3+16+7+4+18
 - (c) 35 + 6 + 164 + 15

Mental Arithmetic Strategy 2:

Near multiples of 10

For example

Consider **65 + 49**

If you were writing this in columns, then you would add the units: 65 + 9 = 74

And then the tens: 74 + 40 = 114

But it can be easier to recognise that a number that ends in 9 is almost a multiple of 10, and split the other number up, so 65 = 64 + 1:

Another example



As **19** is one less than **20** you can subtract **20** and then add back the **1**:

$$64 - 19 = 64 - 20 + 1$$

$$= 44 + 1$$

$$= 45$$



Food for Thought

- 1. Why is ten such an important number? It is because our number system is based on 10. You might find it interesting that almost every civilization all over the world has a number system based on 10. Can you think why?
- **2**. The ancient Babylonians expanded this to a number system based on **60**. What legacy from the Babylonians in still in use all around the world today?

After any subtraction, you should quickly check your answer by doing the **inverse** addition.

Check:
$$19 + 45 = 19 + 1 + 44 = 64$$

(Answers on next page)



Answers:

- 1. We all have 10 fingers
- 2. Time: 60 seconds in a minute

60 minutes in an hour





Exercises

(c)
$$64 - 19$$

(b)
$$83 - 39$$

(c)
$$98 + 49$$

Put strategy 1 together with strategy 2 for these:

(b)
$$33 + 18 - 9 + 2$$

(c)
$$79 + 35 + 21 - 15$$

Mental Arithmetic Strategy 3:

Subtraction by number bonds

Knowing your number bonds for numbers up to 10, means knowing which two numbers you add together to make the number. For example knowing 5 is made from 1 + 4 or 2 + 3.

Knowing these bonds will also make tricky subtractions easier.

Example

72 - 5

What two helpful numbers can 5 be split into?

Because there is a 2 in 72, choose 2 + 3 rather than 1 + 4.

Therefore
$$72 - 5 = 72 - 2 - 3$$

= $70 - 3$
= 67

Exercises

6 (a) 28 – 6

(b) 45 - 8

(c) **57 - 15**

7 (a) 64 – 25

(b) **95 - 37**

(c) 83 - 56



Mental Arithmetic Strategy 4:

Use of place value

If none of the above methods seems appropriate, then you can always add and subtract digit by digit, but it is often easiest to do this row by row.

Examples:

Consider the addition 50 + 301 + 6

First add the units, then the tens and then the hundreds

Add units: **6 + 1 7**

Then add tens on: + 50 57

Then add hundred on: + 300 357

Animals seem to have their own mental maths ability

Consider the subtraction 450 - 135

Start with 450

First subtract hundreds: 450 – 100 350

Then subtract tens: – 30 320

Then subract units: - 5 315

Answer: 450 - 135 = 315

Remember to check by adding: 135 + 315 = 450



Food for Thought

If you are writing rough notes to help your calculation it can be tempting to write:

$$450 - 135 = 350 = 320 = 315$$

But you can see this is WRONG

The numbers each side of the equal signs are not equal.

It is just as important to use maths grammar correctly as it is to use English grammar correctly and the = sign is an important maths punctuation mark in a maths sentence (or equation).

For calculations like these you could use a → instead:

$$450 - 135 \rightarrow 350 \rightarrow 320 \rightarrow 315$$

Exercises

- 8 (a) 32 + 57
 - (b) 134 + 45
 - (c) 60 + 123 + 4
- 9 (a) 350 45
 - (b) 366 144
 - (c) 842 267

Mental Arithmetic Strategy 5:

Relationship to multiplication

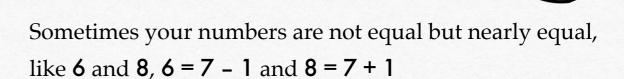
As you know, multiplication is the result of repeated addition.

Consider
$$5 + 5 + 5$$

This is 3 fives or 3×5

$$5 + 5 + 5 = 3 \times 5$$

= 15



$$6 + 7 + 8 = 7 \times 3$$



Food for Thought

Can you see that if you add three consecutive numbers the sum is always three times the middle number?

WHY?

If the middle number is? then the three numbers are:

When you add these up:

$$? + ? + ? - 1 + 1 = ? + ? + ?$$

$$= 3 x ?$$

Exercises

Now try these:

(b)
$$7 + 8 + 9$$

(c)
$$8+8+8+8+8$$

(b)
$$45 + 30 + 15$$

(c)
$$9 + 11 + 11 + 13$$



Food for Thought

There is no 'right' way to do mental arithmetic. Everyone's mind works differently. For these last questions, use whatever strategy you find works for you.

(c)
$$199 + 401$$

(e)
$$19+7+7+7$$

Multiplication and Division

That last strategy used multiplication. In Step 1 you looked at using your times tables. You might want to look at that again before this next bit.

Mental Arithmetic Strategy 6:

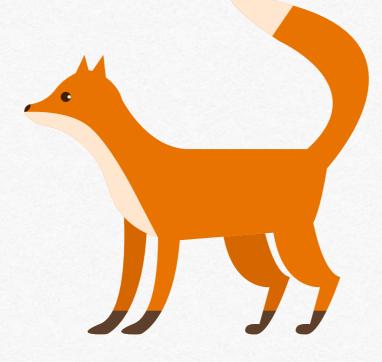
Collecting tens

In Mental Arithmetic Strategy 1 and 2, you looked at multiples of 10. We know it is easy to multiply by ten and multiples of 10. In a multiplication, you may be able to combine numbers to make a multiple of ten:

For example

$$2 \times 12 \times 5 = 12 \times 10$$

= 120





Multiplication, like addition, is commutative. That is a posh word which means that the order of numbers in the calculation does not matter. $2 \times 12 \times 5$ is the same as $12 \times 2 \times 5$.

You need to be careful with subtraction and division as they are not commutative:

12 - 5 is not equal to **5 - 12** and

 $12 \div 3$ is not the same as $3 \div 12$

Another example

$$15 \times 8 = 3 \times 5 \times 8$$

= 3 × 40
= 120

Exercises

- 13 (a) 5 x 2 x 6
 - (b) $7 \times 4 \times 5$
 - (c) $4 \times 5 \times 5 \times 3$

14 (a) $50 \times 5 \times 2 \times 40$

- (b) $40 \times 500 \times 80 \times 5$
- (c) $6 \times 50 \times 500 \times 4$

Many of the useful strategies for multiplication and division use **factors**. Do you remember what these are?

A **factor** is any whole number that can **EXACTLY divide** into a larger number.

Therefore 3 and 4 are factors of 12 because $12 \div 3 = 4$ and $12 \div 4 = 3$

Mental Arithmetic Strategy 7:

Using factors

In that last example (15×8), there was not a 5 in the original calculation, but there was a multiple of 5. It therefore made sense to split that into its factors. It can also be useful to split larger numbers into smaller factors to make the multiplication or division easier.

Example

$$50 \times 1,800 = 50 \times 2 \times 900$$

= 100×900
= $90,000$



Food for Thought

Remember for numbers more than one thousand (1,000) you should separate the thousands by a comma between every three digits, counting from the right or counting from the decimal point.

In most English-speaking countries, including the UK, USA and Australia, commas are used as thousands separators. Other countries may use spaces (as did the UK before the 1980s) or full stops as thousands separators - so be careful when abroad.

Example

$$4,500 \div 15 =$$

 $15 = 5 \times 3$ therefore think of the sum as

4,500 divided by **5** and then divided by **3**.

$$4,500 \div 5 = 900$$

$$900 \div 3 = 300$$

Exercises

15 (a) 8 x 15

(b) 120 x 25

(c) 450 x 160

(c)
$$1,050 \div 15$$



Mental Arithmetic Strategy 8:

Extending from known times tables

If you know your times tables up to twelve then you can use these to extend to larger multiplications.

For example

Consider 6 x 13

As you know that $6 \times 12 = 72$

Then
$$6 \times 13 = 6 \times 12 + 6$$

= $72 + 6$
= 78

Another example

You know that
$$24 = 2 \times 12$$

Then $4 \times 24 = 4 \times 12 \times 2 = 48 \times 2$
= 96



Quick tip

You can check your answer by looking at the last digit.

For 6×13 , note $6 \times 3 = 18$ so your answer must have an 8 as its last digit.

For 4×24 , $4 \times 4 = 16$ so your answer must have 6 as its last digit.

Exercises

(c)
$$7 \times 36$$



Mental Arithmetic Strategy 9:

The inverse

Many people find it easier to multiply rather than divide. Just as subtraction is the inverse, or opposite, of addition, division is the **inverse** of multiplication.

Think again about when you first learned your times tables. As you learned each multiplication, you also learned three more number bonds.

As
$$7 \times 8 = 56$$

Then
$$8 \times 7 = 56$$

And
$$56 \div 7 = 8$$

And
$$56 \div 8 = 7$$

And as
$$8 = 2 \times 4$$

Then
$$56 \div 4 = 7 \times 2$$

The same is true for larger numbers:

$$15 \times 7 = 105$$

Then
$$7 \times 15 = 105$$

And
$$105 \div 7 = 15$$

Also
$$105 \div 15 = 7$$

And as
$$15 = 5 \times 3$$
 then $5 \times 3 \times 7 = 105$

Then

$$3 \times 35 = 105$$
 and $5 \times 21 = 105$

$$105 \div 35 = 3$$
 and $105 \div 21 = 5$

Therefore, if you are unsure about a division, you can use the inverse – that's a maths terms meaning opposite.



For example

Say to yourself: $2 \times 70 = 630$

And then you can see that as $9 \times 7 = 63$

Then
$$9 \times 70 = 630$$

And
$$630 \div 70 = 9$$



Do you remember how to divide by numbers ending in noughts? You can knock off the same number of noughts from both the numbers in the sum.

Why does this work?

If you remember that a fraction is another way of thinking of division, consider $630 \div 70$ which can be written as $^{630}/_{70}$

10 is a common factor of 630 and 70 so you can see that $^{630}/_{70} = ^{63}/_{7} = 63 \div 7$

Therefore you could think in your head:

$$630 \div 70 = 63 \div 7$$

Exercises

(c)
$$14,400 \div 120$$

(c)
$$84,000 \div 400$$

Mental Arithmetic Strategy 10:

Mixed operations

You can add, subtract, multiply and divide, but there are times when you need to do at least two of these in one calculation to solve a problem. The rule is that if the calculation contains a mixture, then you divide and multiply before you add or subtract. If the calculation includes brackets or indices, then you do the calculation in this order:

Brackets that exist in the calculation you are given.

Indices

Divide

Multiply

Add

Subtract This is known as **BIDMAS**.

(But we are not going to worry about brackets and indices now.)

Exercises

- 21 (a) 12 x 4 3
 - (b) $12 4 \times 3$
 - (c) $12 \div 4 + 3$
- 22 (a) 120 40 x 2
 - (b) 12,000 800 ÷ 4
 - (c) $2,400 \div 80 + 40 \times 20$

Now use any strategy that you like to answer these:

- 23 (a) 19 x 50
 - (b) $720 \div 36$
 - (c) 6 x 35
- 24 (a) 540 ÷ 18
 - (b) $150 540 \div 18$
 - (c) $4,800 \div 60$

Now we are going to see why it is so important to be good at mental arithmetic.

This picture gives a clue





Working with Money

Probably the most important reason for being good at mental arithmetic is that is allows you to be in control of your own money.

If you are not in control, then it is all too easy to spend more than you earn and to get into debt. It is not nearly so easy to get out of debt so this is best avoided.

Debt is different from borrowing – borrowing is something you may do with planning when you want to buy an expensive item, like a car or a house, or even a yacht. We will look more at borrowing in Step **6**.

And that amounts larger than £1 are written using a decimal point:

£5.65 means £5 and 65p

In most countries in Europe the currency is the Euro:

€24.15 means **€24** and 15 cents, although French-speaking countries use the term centime, (usually shortened to cent – pronounced "sent.")

In the USA the unit of currency is the US dollar. Many other countries have their own dollars, such as the Australian dollar.

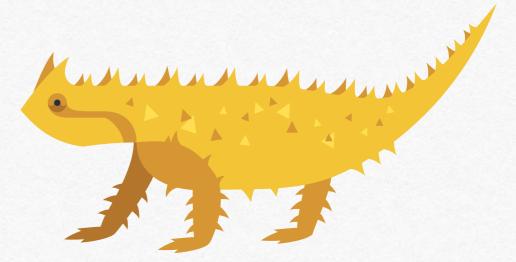
They are all written the same way:

\$9,875.20 means \$9,875 and 20 cents.

When working with money you know that there are:

100p (pence) in £1







The decimal point or decimal separator is not the same in all countries. In the UK and all English-speaking countries, the decimal point is a full stop, but in other countries, particularly continental Europe it is written as a comma: €4,90 means 4 euro and 90 cents.

You will know that the cost of anything is rarely a neat number of pounds. Most prices are a mix of pounds and pence and so includes a decimal.

When working out your finances, going shopping or budgeting for an event you will need to work with decimals.

This does NOT mean that you should automatically reach for your calculator. You can use all the strategies that you have tried in this chapter to calculate with money using the simple strategy:

Ignore the decimal point when you calculate but put it back in the answer before the last two digits.

For example

One net of oranges cost £1.20, what is the cost of 3 nets?

$$120 \times 3 = 360$$

Answer: Cost of 3 nets = £3.60

Another example

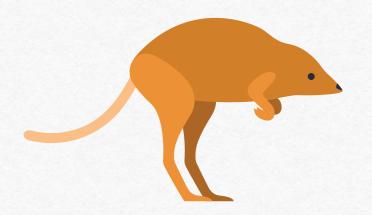
I need to buy a pack of 6 pens costing £2.25 and a pack of drawing pins costing 65p. How much will I need to spend in total?

$$225 + 65 = 290$$

Answer: Total amount = £2.90

One more example

My total bill for petrol is £17.42. How much change will I get from a £20 note?





Remember to change the £20 to 2,000p

$$2,000 - 1,742 = 258$$

Exercises

25 I buy a pack of 6 pens for £2.40. What is the value of one pen?



If you are not sure whether to add, subtract, multiply or divide, look for a clue in the wording of the question. You know that $6 \times 10^{\circ}$ x 'cost of one' = £2.40 so you will need to divide or use the inverse.

26 I fill up my car with 10 litres of petrol costing £1.19 a litre. How much do I spend?

27 I use a ten pound note to buy stationery costing £7.64. How much change do I receive?



Calculating change will always be a subtraction.

28 The price of a packet of biscuits is £1.40. What is the total cost of four packets of biscuits?



The word 'total' suggest that you should use addition but as all items are the same price you could also simply multiply.

29 My usual brand of washing powder is £7.80 for 5 litres. The supermarkets own brand is £5.25 for 5 litres. How much will I save by buying the supermarket's own brand?



A saving suggests that one price is less than another which means a subtraction.

Working with Money

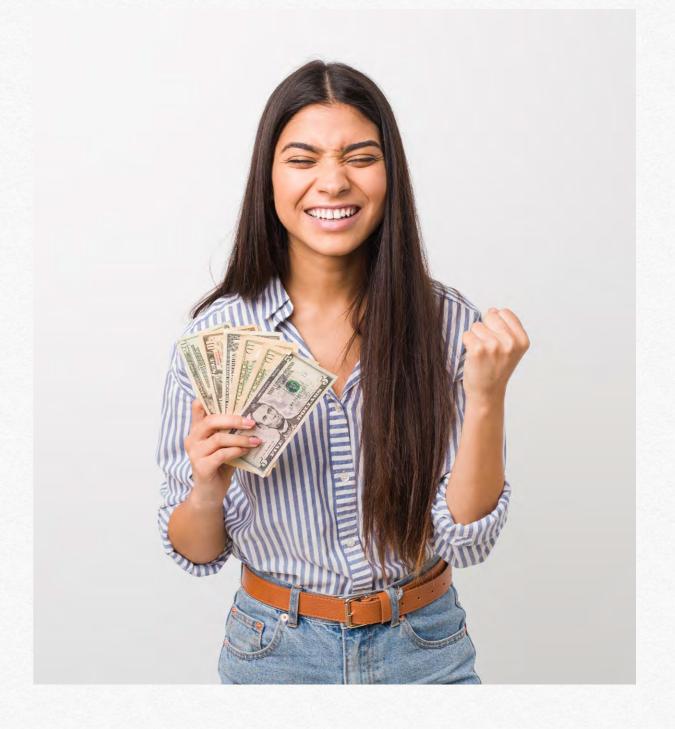
30 What is the total cost of **5 kg** of potatoes at **40p** per kilo and **2 kg** of carrots at **12p** per kilo?



Another 'total' to calculate so use addition but note the little word 'at' followed by a cost per **kg**. So, you will need to multiply and then add.



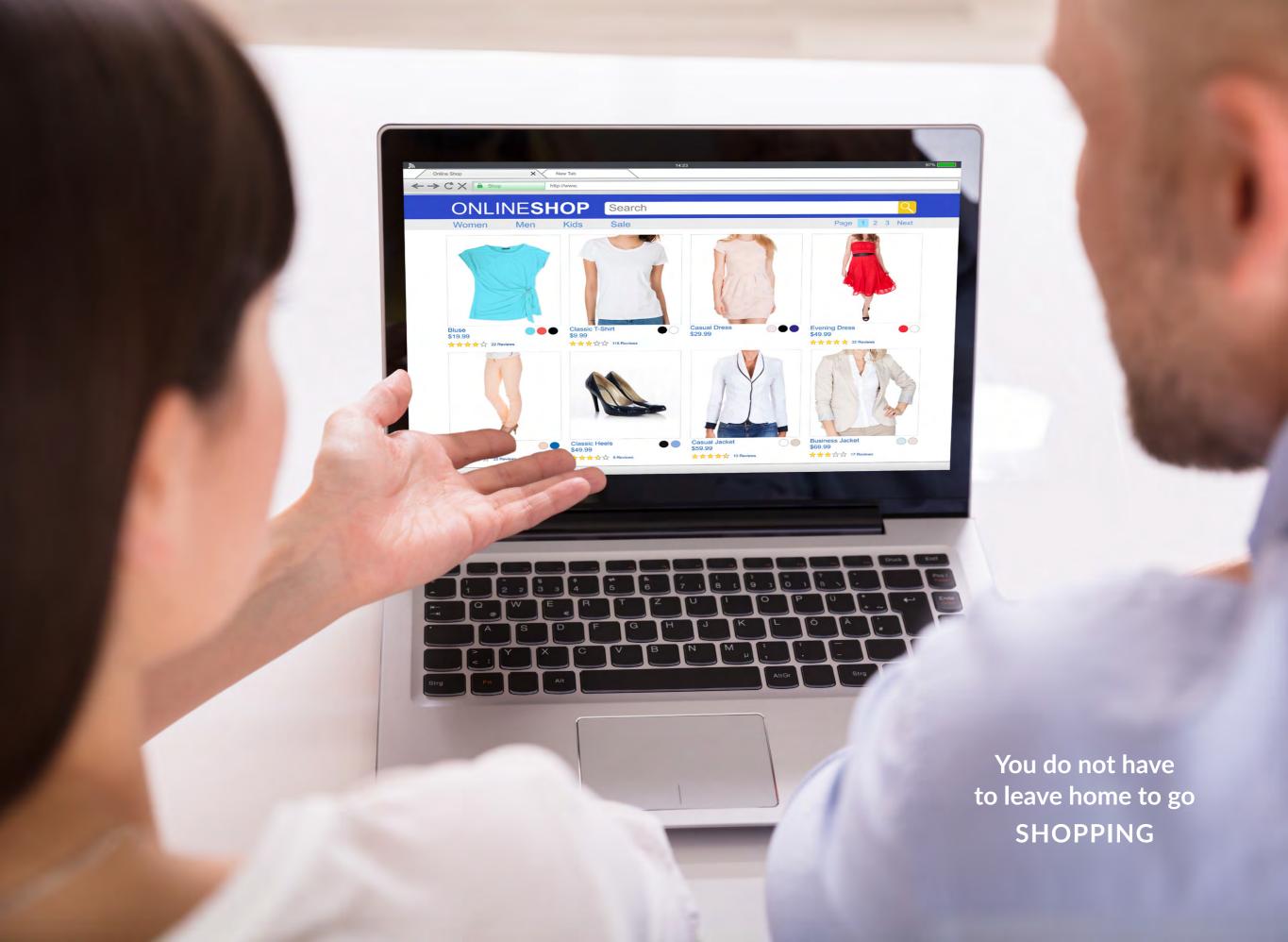
The last questions are typical of using money in Real Life. Now think about when you have used money recently. Set yourself some similar questions and then answer them **IN YOUR HEAD**.



Careful money management helps you to be a WINNER!

Remember the saying: *Take care of the pennies and the pounds* will take care of themselves.

Working with Money 31



Rounding (and Estimating)

When you look at prices you will doubtless have seen they are often not whole numbers but can include either 99p, 90p, 95p or 49p.

You rarely need to work out how much you can spend by calculating to the nearest penny. You should round off to the nearest pound:

£22.99 ≈ £23

£19.99 ≈ £20

£30.90 ≈ £31

What about £21.49?

You could either say £21.49 ≈ £21

Or round off to the nearest **50p** depending on how precise you need to be:

 $£21.49 \approx £21.50$



Have you seen the symbol ≈ before? This wiggly equal sign means 'is approximately equal to'. It is a useful symbol to use when you are **estimating** and then there is no need to write 'to the nearest pound'.

Of course, as this is all in your head, you do not need to write anything down.

Look at the prices on the following page and answer some these next questions. Here is an example to start you off:

Example

I have **£50**. Is that enough to buy shirt A and trainers X? If not, roughly how much more do I need?

Shirt A **£22.99** ≈ **£23**

Trainers X £29.99 \approx £30

£23 + £30 = £53

Answer: No, I will need £3 more

Exercises

In questions 31–35 you need to use the following list of prices for shirts and trainers.

Shirts		Trainers	
A	£22.99	X	£29.99
В	£17.99	Y	£39.95
C	£19.99	Z	£14.49
D	£21.49		

31 So far I have saved £25. Approximately how much more will I need to save to buy trainers X?



'How much more' indicates a difference or subtraction.

32 I have £40. Which two different styles of shirt could I buy?



There may be more than one answer. In Real Life, money problems do not always have one solution.

33 Estimate the total cost of Shirt A and Trainers Y to the nearest £?



Use the rounded costs to add together not the exact pence.

- **34** Pick your favourite shirt and pair of trainers. Estimate the total cost of your two choices?
- **35** Using estimations, what is the cheapest combination of shirt and trainers? What is the most expensive? What is the difference?



Have a look online at some items that you might wish to purchase. Set yourself some Real Life problems like the ones above.



Real Life Problems

You can use mental arithmetic to solve everyday problems.

Example

How many seconds are there in an hour?

60 seconds in a minute

60 minutes in an hour

Number of seconds =
$$60 \times 60$$

= 3,600



Mental arithmetic is when you do the calculation in your head, without writing anything down. However, you may just want to write down the calculation you are going to do in your head, as looking at it can help you to decide the best strategy.

Before we look at another example, let us think about remainders. In Real Life, when you carry out a division, you do not always get an exact answer but an answer with something left over, in other words, a remainder.

Example

A pack of **12** pens costs **£2.50**. What is the value of one pen?

As ten is almost 12, you could round up the answer and say $250 \div 12 \approx 21$

Answer: Value of one pen is about 21p.

Another example

A farmer is collecting eggs. She packs them in boxes of 12. If she collects 103 eggs, how many boxes will she need?

$$103 \div 12 = 8 \text{ remainder } 7$$

Answer: She will need 8 boxes (and have 7 eggs left over)

Sometimes you can put in a decimal point and continue dividing until you have an answer.

You could also write the answer as a fraction

Example

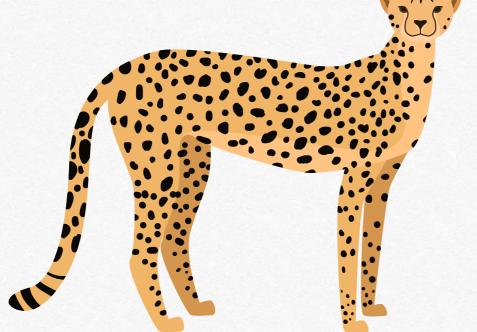
A pack of **8** batteries costs £10. What is the value of one battery?

Answer a)
$$10 \div 8 = 1$$
 remainder 2 $Cost \approx £1$

Answer b)
$$10 \div 8 = 1^{2}/_{8} = 1^{1}/_{4}$$
 Cost = £1 $^{1}/_{4}$

Answer c)
$$10.00 \div 8 = 1.25$$
 Cost = £1.25

Answer c) is the best for this problem as it is about money.





If you end up with a remainder you can do one of several things depending on the problem:

- a) Ignore the remainder (also called rounding down)
- b) Round the answer to the nearest whole number
- c) Put in a decimal point and keep dividing
- d) Give the remainder as a fraction
- e) Give the answer with the remainder (like the farmer and her eggs)

Exercises

36 Aria has driven to Scotland. She first drove 25 miles to the nearest service station and filled up with petrol. She then drove for 119 miles and stopped for lunch. After another 55 miles she had to stop again as her little brother needed a pee, then she drove another 121 miles to reach her holiday cottage. How far has she driven in total?



In school, you probably did most of your calculations for distance using millimetres, centimetres, metres and kilometres. However, as you know, in the UK long distances are measured in miles. If you are travelling in Europe then you should use kilometres.

Have you ever thought why the UK did not change to kilometres when the metric system was adopted?

The answer is that it would be too expensive to change all the road signs!

37 On her trip Aria put 25 litres of petrol in the tank. That cost £30. What was the cost per litre?



Make sure you have the division the right way round. It is cost **per** litre so that is **cost divided by litres**.

- 38 Benedicta has bought a 3 m length of shelving. She is going to cut it up to make 4 shelves of equal length.

 Roughly how long will each shelf be?
- 39 Chantal buys a bunch of 6 bananas for £1.10. What is the cost of one banana to the nearest penny?
- **40** Daria has a budget of £15 for a top up of her weekly shop. On her list she has:

4 pints of milk	£1.09	
1 dozen eggs	£2.95	
2.5 kg baking potatoes	£1.75	•
1 kg of tomatoes	£2.89	
1 cucumber	£0.79	
350 g cheese	£2.59	
Pack of 100 tea bags	£2.39	**
6 bottles spring water	£2.99	

Has Daria budgeted enough to buy everything on her list?

41 If not, where could she economise or what could she leave out?

- **42** Emma is organising a party. She has invited **49** guests (which makes a total of **50** with Emma herself). If they are to sit on tables of **8**, how many tables will there be?
- **43** The caterer has offered Emma a choice of menus.

Starter A at £6.99 for 5 people
Starter B at £7.49 for 5 people
Main Dish C at £12.90 per person
Main Dish D at £13.99 per person

- a) For the party of **50** people, estimate the cheapest meal, consisting of a starter and main dish, that Emma could order.
- b) Estimate the cost of the most expensive meal on offer.
- c) Emma chooses Starter B and Main Dish C. What should she budget for the meal?
- 44 Fran is working out her daily calorie intake. She sees that a 1 kg carton of muesli has a total of 2,500 calories and 1 litre a semi skimmed milk has 450 calories. What does she estimate the calorie content of a 50 g bowl of muesli





Food For Thought

Note the word **approximately** in the next problem. People are different. They do not all need the same amount of calories to keep healthy. You will work out what is best for you.

45 An average healthy woman should have an intake of approximately **2,000** calories a day.

Georgie has totted up her intake today as:

Breakfast Muesli pot 308 calories

Chicken sandwich 482 calories

Latte 99 calories

Banana 115 calories

Estimate how many calories should she now allow for her

evening meal?



Food For Thought

The last questions explored situations in Real Life when it helps to be able to calculate mentally and the usefulness of estimation.

There are many other areas of Real Life where you need to calculate such as in sport, art and travel.

What else can you think of?



Real Life Problems

Answers to Part 1

These worked answers show the thinking that your head might have been doing to work out the answer. But do not worry if you got the correct answer but your head did something different!

1 (a)
$$2+6+8+4$$

 $8+2+6+4=10+10$

Answer: 20

(b)
$$1 + 8 + 2 + 9$$

 $1 + 9 + 8 + 2 = 10 + 10$

Answer: 20

(c)
$$4+7+5+6+3+5$$

 $4+6+7+3+5+5=10+10+10$

Answer: 30

Answer: 140

Answer: 50

Answer: 220

(a)
$$7+9$$

 $6+1+9=6+10$

Answer: 16

(c)
$$64 - 19$$

 $64 - 20 + 1 = 44 + 1$

4 (a)
$$45 + 29$$

 $45 + 30 - 1 = 75 - 1$

Answer: 74



(b)
$$83 - 39$$

 $83 - 40 + 1 = 43 + 1$

Answer: 44

Answer: 147

Answer: 46

(b)
$$33 + 18 - 9 + 2$$

 $18 + 2 + 33 - 10 + 1 = 20 + 23 + 1$

Answer: 44

(c)
$$79 + 35 + 21 - 15$$

 $79 + 21 + 35 - 15 = 100 + 20$

Answer: 120

Answer: 22

(b)
$$45 - 8$$

 $45 - 5 - 3 = 37$
 $40 - 3 = 37$

Answer: 37

(c)
$$57 - 15$$

 $57 - 5 - 10 = 42$



$$64 - 4 - 20 - 1 = 39$$

$$95 - 5 - 30 - 2 = 58$$

Answer: 58

(c)
$$83 - 56$$

$$83 - 3 - 50 - 3 = 27$$

Answer: 27

$$32 + 50 + 7 = 89$$

Answer: 89

(b)
$$134 + 45$$

$$134 + 40 + 5 = 179$$

Answer: 179



(c)
$$60 + 123 + 4$$

$$60 + 100 + 20 + 3 + 4 = 187$$

Answer: 187

$$350 - 40 - 5 = 305$$

Answer: 305

$$366 - 100 - 40 - 4 = 222$$

Answer: 222

$$842 - 200 - 60 - 7 = 575$$

Answer: 575

$$3 \times 6 = 18$$

(b)
$$7 + 8 + 9$$

$$7 + 1 = 8$$
 $9 - 1 = 8$

$$3 \times 8 = 24$$

(c)
$$8+8+8+8+8$$

 $5 \times 8 = 40$

Answer: 40

Answer: 36

(b)
$$45 + 30 + 15$$

 $30 + 15 = 45$ $30 - 15 = 15$
 $3 \times 30 = 90$

Answer: 90

(c)
$$9 + 11 + 11 + 13$$

 $9 + 2 = 11$ $13 - 2 = 11$
 $4 \times 11 = 44$

Answer: 44

Answer: 74

Answer: 111

(c)
$$199 + 401$$

 $199 + 1 + 400 = 200 + 400$

Answer: 600



(e)
$$19+7+7+7$$

 $19+21=40$

Answer: 4,002

13 (a)
$$5 \times 2 \times 6$$

 $10 \times 6 = 60$

Answer: 60

(b)
$$7 \times 4 \times 5$$

 $7 \times 2 \times 2 \times 5 = 14 \times 10$

Answer: 140

(c)
$$4 \times 5 \times 5 \times 3$$

 $2 \times 5 \times 2 \times 5 \times 3 = 10 \times 10 \times 3$

Answer: 300

14 (a)
$$50 \times 5 \times 2 \times 40$$

 $50 \times 2 \times 10 \times 5 \times 4 = 100 \times 10 \times 20$

Answer: 20,000

(b)
$$40 \times 500 \times 80 \times 5$$

 $40 \times 5 \times 100 \times 10 \times 8 \times 5 = 200 \times 1,000 \times 40$

Answer: 8,000,000

(c)
$$6 \times 50 \times 500 \times 4$$

 $6 \times 5 \times 10 \times 100 \times 5 \times 4 = 30 \times 1,000 \times 20$

Answer: 600,000

15 (a)
$$8 \times 15$$

 $8 \times 5 \times 3 = 40 \times 3$

Answer: 120

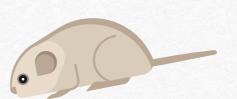
(b)
$$120 \times 25$$

 $120 \times 5 \times 5 = 600 \times 5$

Answer: 3,000

(c)
$$450 \times 160$$

 $450 \times 2 \times 80 = 900 \times 80$



Answer: 72,000

16 (a)
$$108 \div 18$$

 $108 \div 2 \text{ then } \div 9 = 54 \div 9$

Answer: 6

(b)
$$400 \div 50$$

 $400 \div 10 \text{ then } \div 5 = 40 \div 5$

Answer: 8

(c)
$$1,050 \div 15$$

 $1,050 \div 5 \text{ then } \div 3 = 210 \div 3$

Answer: 70

17 (a)
$$7 \times 13$$

 $7 \times 12 + 7 = 84 + 7$



Answer: 91

(b)
$$24 \times 5$$

 $12 \times 2 \times 5 = 12 \times 10$

Answer: 120



(c)
$$7 \times 36$$

 $7 \times 12 \times 3 = 84 \times 3$

Answer: 252

18 (a)
$$50 \times 66$$

 $50 \times 6 \times 11 = 300 \times 11$

Answer: 3,300



(b)
$$16 \times 9$$

 $2 \times 8 \times 9 = 2 \times 72$

Answer: 144

(c)
$$22 \times 12$$

 $2 \times 11 \times 12 = 2 \times 132$



19 (a)
$$560 \div 80 = 560 \div 80 = 56 \div 8$$

8 x ? = 56

(b)
$$7,200 \div 900 = 7,200 \div 900 = 72 \div 9$$

 $9 \times ? = 72$

Answer: 8

(c)
$$14,400 \div 120 = 14,400 \div 120 = 1,440 \div 12$$

 $12 \times ? = 1,440$

Answer: 120

20 (a)
$$10,000 \div 400 = 10,000 \div 400 = 100 \div 4$$

 $4 \times ? = 100$

Answer: 25

(b)
$$12,000 \div 80 = 12,000 \div 80 = 1,200 \div 8$$

8 x ? = 1,200

Answer: 150

(c)
$$84,000 \div 400 = 84,000 \div 400 = 840 \div 4$$

 $4 \times ? = 840$

Answer: 210

21 (a)
$$12 \times 4 - 3$$

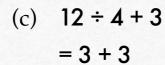
= $48 - 3$

Answer: 45

(b)
$$12 - 4 \times 3$$

= $12 - 12$

Answer: 0



Answer: 6

22 (a)
$$120 - 40 \times 2$$

= $120 - 80$



(b)
$$12,000 - 800 \div 4$$

= $12,000 - 200$

Answer: 11,800

(c)
$$2,400 \div 80 + 40 \times 20$$

= $2,400 \div 80 + 40 \times 20 = 240 \div 8 + 800$
= $30 + 800$

Answer: 830

23 (a)
$$19 \times 50$$

 $20 \times 50 - 50 = 1,000 - 50$

Answer: 950

(b)
$$720 \div 36$$

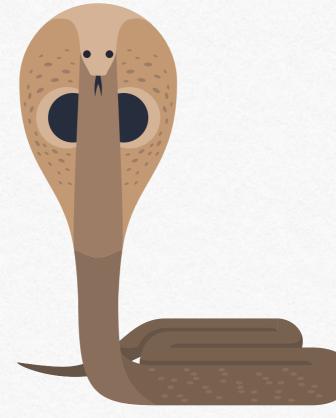
 $720 \div 12 \div 3 = 60 \div 3$

Answer: 20

(c)
$$6 \times 35$$

 $6 \times 5 \times 7 = 30 \times 7$

Answer: 210



24 (a)
$$540 \div 18$$

 $540 \div 9 \div 2 = 60 \div 2$

Answer: 30

(b)
$$150 - 540 \div 18$$

Division first: we know $540 \div 18 = 30$ from (a) $150 - 30 = 120$

Answer: 120

(c)
$$4,800 \div 60 = 4,800 \div 60 = 480 \div 6$$

Answer: 80

25 I buy a pack of 6 pens for £2.40.

What is the value of one pen?

As in the earlier questions, as you are going to do these in your head, your head might do something different with the numbers to the example answer here. That is fine if you get the same answer.

$$? = 40$$

Answer: 40p or £0.40

26 I fill up my car with 10 litres of petrol costing £1.19 a litre. How much do I spend?

This example answer uses the strategy of ignoring the decimal point in the calculation but putting it back in the answer in front of the last two digits.

$$10 \times 119 = 1,190$$

Answer: £11.90

27 I use a ten pound note to buy stationery costing £7.64. How much change do I receive?

As well as ignoring the decimal point I must work in the same units and so I have changed the £10 to 1,000p

$$1,000 - 764 = 1,000 - 700 - 60 - 4$$
$$= 300 - 60 - 4$$
$$= 240 - 4$$
$$= 236$$

Answer: 236p or £2.36

28 The price of a packet of biscuits is £1.40. What is the total cost of four packets of biscuits?

$$140 \times 4 = 100 \times 4 + 40 \times 4$$

= $400 + 160$
= 560

Answer: 560p or £5.60

29 My usual brand of washing powder is £7.80 for 5 litres. The supermarkets own brand is £5.25 for 5 litres. How much will I save by buying the supermarket's own brand?

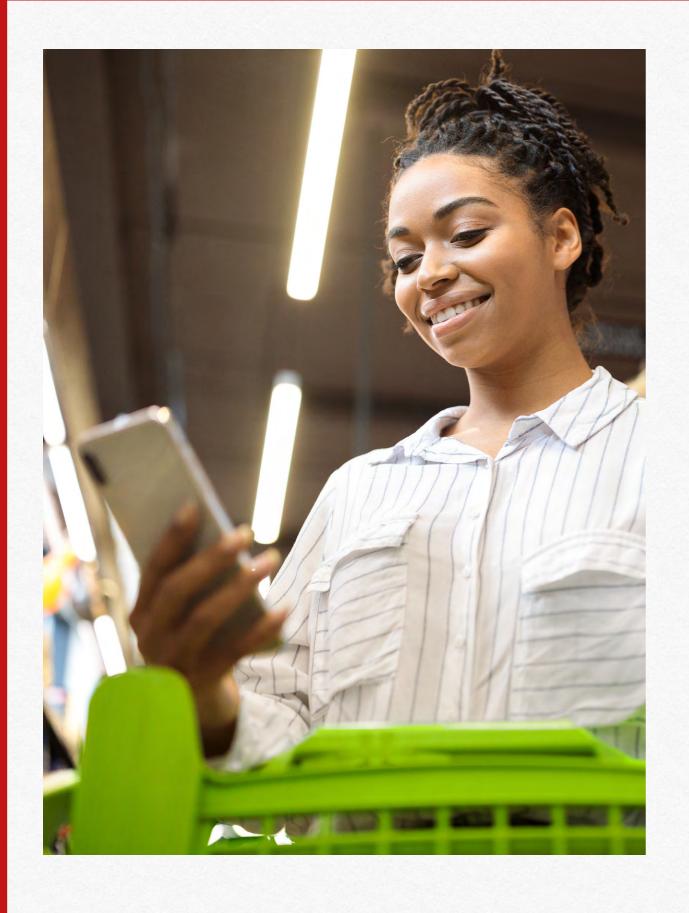
The calculation will be a subtraction with no decimal points. Your head will probably do something like this:

Answer: 255p or £2.55

Remember to check by adding:

$$255 + 525 = 780$$





30 What is the total cost of **5 kg** of potatoes at **40p** per kilo and **2 kg** of carrots at **12p** per kilo?

Potatoes cost $5 \times 40 = 200$

Carrots cost $2 \times 12 = 24$

Total cost 200 + 24 = 224

Answer: 224p or £2.24

BUT – your food shopping is not just about price but also about quality.

In questions 31–35 you need to use the following list of prices for shirts and trainers.

Start by inserting rounded prices to make it easier.

Sh	irts	Trainers			
A	£22.99 ≈ £23	X	£29.99 ≈ £30		
В	£17.99 ≈ £18	Y	£39.95 ≈ £40		
C	£19.99 ≈ £20	Z	£14.49 ≈ £14.50		
D	£21.49 ≈ £21.50				

31 So far I have saved £25. Approximately how much more will I need to save to buy trainers X?

$$30 - 25 = 5$$

Answer: £5

32 I have £40, which two different styles of shirts could I buy?

As in Real Life there is more than one answer:

In these next questions you can see how simple it is once you have rounded your prices

33 Estimate the total cost of shirt A and trainers Y to the nearest £?

$$23 + 40 = 63$$

Answer: £63

34 Pick your favourites out of the four shirts and three trainers. Estimate the total cost cost of your two choices.

My choice is shirt A and trainers Z

My answer is £37.50 but yours might be different

35 Using estimations, what is the cheapest combination of shirt and trainers? What is the most expensive? What is the difference?

Cheapest are shirt B and trainers Z

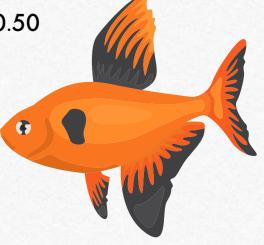
$$£18 + £14.50 = £32.50$$

Most expensive are shirt A and trainers Y

$$£23 + £40 = £63$$

Difference is £63 - £32.50 = 30.50

Answers: £32.50; £63; £30.50





36 Aria has driven to Scotland. She first drove 25 miles to the nearest service station and filled up with petrol. She then drove for 119 miles and stopped for lunch. After another 55 miles she had to stop again as her little brother needed a pee, then she drove another 121 miles to reach her holiday cottage. How far has she driven in total?

First identify all the numbers that you need:

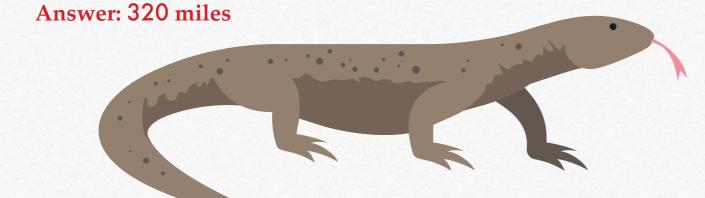
Consider your strategy. Looking at last digits is a good start

Answers to Part 1

You may notice that there are pairs of numbers that make multiples of ten:

$$25 + 55 + 119 + 121 = 80 + 240$$

= 320



37 On her trip Aria put 25 litres of petrol in the tank. That cost £30. What was the cost per litre?

Even though the cost is a whole number of pounds, you can see the answer is not, so you will need to work in pence.

Think of
$$3,000 \div 25$$
, as $3,000 \div 5$ then $\div 5$

3,000 divide by
$$5 \rightarrow 600$$

Divide by 5 again
$$\rightarrow$$
 120

Replace the decimal infront of the last two digits → 1.20

Answer: £1.20 per litre

38 Benedicta has bought a 3 m length of shelving. She is going to cut it up to make 4 shelves of equal length. Roughly how long will each shelf be?

This time work in cm. 3 m = 300 cm

Think of
$$300 \div 4$$
 as, $300 \div 2$ then $\div 2$

300 divide by
$$2 \rightarrow 150$$

Divide by 2 again
$$\rightarrow$$
 75

Answer: 75 cm

39 Chantal buys a bunch of 6 bananas for £1.10. What is the cost of one banana to the nearest penny?

You are going to work in pence

$$110 \div 6 = 110 \div 2 \div 3$$

Divide by 2 55

Divide by 3 18 r 1



Answer: To the nearest p the cost is 18p

You will find in Real Life answers are often not whole numbers

40 Daria has a budget of **£15** for a top up of her weekly shop. On her list she has:

4 pints of milk	£1.09
1 dozen eggs	£2.95
2.5 kg baking potatoes	£1.75
1 kg of tomatoes	£2.89
1 cucumber	£0.79
350 g cheese	£2.59
Pack of 100 tea bags	£2.39
6 bottles spring water	£2.99

Has Daria budgeted enough to buy everything on her list?

Estimating to the nearest pound will give you:

$$1 + 3 + 2 + 3 + 1 + 3 + 2 + 3 = 18$$

Answer: Daria does not have enough. She has £15 which is £3 less than the £18 needed for her shopping list.

41 If not, where could Daria economise or what could she leave out?

Answer: She could economise by, for example:

Buying 6 eggs and only 50 tea bags will save about £2.70 so close enough to be in budget. She could also look for cheaper options.

Or she could leave out the spring water. In the UK, the water that comes out of the tap is very good to drink. There have been many taste tests that find that people can rarely tell the difference between tap water and bottled water – but bottled water is more expensive and then you have a plastic bottle to recycle. You may have a different answer which is just as good.

42 Emma is organising a party. She has invited **49** guests (which makes a total of **50** with Emma herself). If they are to sit on tables of **8**, how many tables will there be?

Think: guests per table

$$50 \div 8 = 6$$
 remainder 2

Emma must round up even though the remainder is **2** because she cannot have **2** guests without a place to sit!

Answer: 7 tables

(She will probably make that **6** tables of **7** guests and **1** table of **8**)

43 The caterer has offered Emma a choice of menus.

Start by estimating all the options and calculating how much it will cost for the party of 50 people. Then answer the questions

Starter A at £6.99 for 5 people: $7 \times 10 = £70$

Starter B at £7.49 for 5 people: $7.5 \times 10 = £75$

Main Dish C at £12.90 per person: $13 \times 50 = £650$

Main Dish D at £13.99 per person: $14 \times 50 = £700$

a) For the party of **50** people, estimate the cheapest meal, consisting of a starter and main dish, that Emma could order.

Answer: Starter A and Main Dish C

$$£70 + £650 = £720$$

b) Estimate the cost of the most expensive meal on offer.

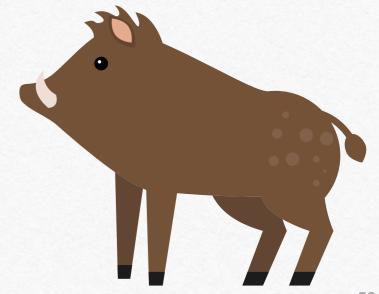
Answer: Starter B and Main Dish D

$$£75 + £700 = £775$$

c) Emma chooses Starter B and Main Dish C. What should she budget for the meal?

$$£75 + £650 = £25 + £50 + £650$$

Answer: £725



44 Fran is working out her daily calorie intake. She sees that a 1 kg carton of muesli has a total of 2,500 calories and 1 litre a semi skimmed milk has 450 calories. What does she estimate the calorie content of a 50 g bowl of muesli with 200 ml milk to be?

This sounds simple but there is quite a lot of mental arithmetic to do. Keep it simple by saying:

Muesli: How many 50 g in a 1 kg? (1 kg = 1,000 g) $1,000 \div 50 = 20$

Calories = $2,500 \div 20 = 125$

Milk: How many 200 ml in 1 litre? (1 l = 1,000 ml)

$$1,000 \div 200 = 5$$

Calories = $450 \div 5 = 90$

Total calories = 125 + 90 = 215

Answer: 215 calories

45 An average healthy woman should have an intake of approximately **2,000** calories a day.

Georgie has totted up her intake today as:

Breakfast Muesli pot 308 calories

Chicken sandwich 482 calories

Latte 99 calories

Banana 115 calories

Estimate how many calories should she now allow for her evening meal?

First estimate to the nearest 100:

300 + 500 + 100 + 100 = 1,000

2,000 - 1,000 = 1,000

Answer: 1,000 calories

Calories are not a very exact measure so there is no point making exact calculations. There are much better ways of spending your time!



Q1

Write down what you need to for each question and then check the answer to see if you were correct.

$$124 + 37 = ?$$

Q2

Do each sum in your head and then check the answer to see if you were correct.

$$3 + 34 + 17 + 26 = ?$$







$$180 \div 15 = ?$$





$$146 + 19 = ?$$

$$5 + 7 + 9 = ?$$







$$315,000 \div 900 = ?$$





Answers

Q1
$$124 + 37 = 161$$

$$Q2 3 + 34 + 17 + 26 = 80$$

Q3
$$25 \times 7 = 175$$

$$Q4 180 \div 15 = 12$$

$$Q6 5 + 7 + 9 = 21$$

Q7
$$35 \times 8 = 280$$

Q8
$$315,000 \div 900 = 350$$



Written Strategies

You will have found out in Part 1 just how good you are at mental calculations. You know that practice will make you get better, but there will also be times when you want to use pen and paper to calculate.

REMEMBER – any of the mental strategies that you have learnt can just as well be used as written strategies, for example:

Jot down the calculation

Write it out differently to make it simpler:

- Collect tens
- Use factors
- Calculate hundreds then tens, then units
- Ignore the decimal point and put it back in the answer

Check that your answer makes sense.

For Real Life problems, it is a good idea to start by saying to yourself: What do I think the answer might be?

For example

I have been offered a job that pays £21,000 a year. What will my monthly pay be?

As there are 12 months in a year I need to calculate:

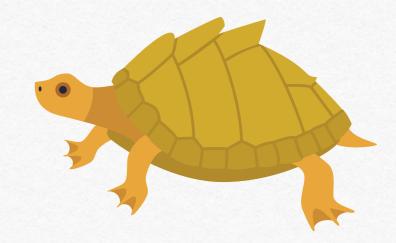
As 12 is a bit more than 10 and $21,000 \div 10 = 2,100$ then I expect my answer to be just less than £2,000.

As 12 is a multiple of 3 and 4 I am going to use factors:

Think 21,000
$$\div$$
 3 then \div 4 = 7,000 \div 4

$$= 7 \times 1,000 \div 4$$

$$= 7 \times 250$$



You do not have to write down all those stages. Some you will DO IN YOUR HEAD but it can help to write down some stages just so you do not lose track.



Food for Thought

You know that the monthly amount that you have calculated will not be what you take home as you will have to pay tax and National Insurance. We will look at how to work those out in Part **7**.

Exercises

1 Hannah is a newly qualified teacher and her starting salary is £26,400 a year. What will her monthly salary be?



Food for Thought

Have you ever wondered what teachers get paid?

A newly qualified teacher starts on a salary of at least £25,714, or £32,157 in Inner London. With experience their pay increases, and they can also get paid more if they are promoted. A headteacher, in Inner London could earn as much as £125,000.

- 2 (a) Iris is saving £300 a month. How much will she save in a year?
- (b) Iris wants to buy a flat and needs to have a extra £10,000 to have a big enough deposit. How many months will it take her to save at least £10,000?



Quick Tip

For this next question, remember that when prices are not exact, such as £499, it is a perfectly sensible strategy to round them to a sensible number, such as £500.

- 3 Janis is shopping for her new home office. She has chosen a desk costing £157, a chair with good back support for £79.99, a mid range laptop at £799 and a printer/scanner costing £129.
- (a) Estimate, to the nearest £10, the total cost of Janis' purchases?
- (b) Janis had set herself a budget of £1,500. Roughly how much will she have left?

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- (c) What other essentials do you think Janis will need to buy?
- 4 Kia is redecorating her bedroom. She has worked out that the area of her walls is 22.5m². The paint she has chosen says that 1 litre will cover 10m² and that she will need to give her walls 2 coats of paint. A 5 litre pot of paint costs £26 and a 2.5 litre pot of paint costs £16.
- (a) How much paint does Kia need?
- (b) What is the least amount of paint she should buy and what will it cost?







Kia should also budget for paint brushes, rollers and a paint tray as well as paint cleaner or white spirit, not only for her brushes but for her hands too. Most of us make a bit of a mess when painting but don't let that put you off!

In those problems you probably used a bit of mental strategy and a bit of jotting down the numbers. There will be other times when you will need to do more careful calculation on paper.



Addition and Subtraction

In a previous Step you looked at how to set out written addition and subtraction calculations. In this part you are going to be particularly looking at problems with money and therefore we will just brush up on calculations with decimals here.

Firstly, remember **place value** and how the columns are in groups of three, each with hundreds, tens and units. Also remember that there are invisible columns, that contain a comma:

H = Hundreds, T = Tens, U = Units

Н	T	U		Н	T	U		Н	T	U		Н	T	U
Bi	illio	ns		M	illio	าร		Thousands						
		4	,	8	0	9	,	2	7	3	,	5	6	1

4,809,273,561

Written Strategies 71

You read this number as four billion, eight hundred and nine million, two hundred and seventy-three thousand, five hundred and sixty-one.

When working with money, you are likely to have 2 digits after the decimal point. With other measurements, such as weight or length, you may have three.

Н	T	J		t	h	th
6	4	5	•	1	9	5

These columns after the decimal point are tenths, hundredths and thousandths:

This number is:

six hundred and forty-five point one, nine five.

Note there is NO HIDDEN COLUMN between the units and the tenths but there is a decimal point. The decimal point sits on the line between the two columns.

You read the digits after the decimal point as numbers, they are not grouped in hundreds, tens and units.

Addition

Examples

What is the sum of 3.527, 342 and 34.7

(I expect my answer to be about 5 + 350 + 35 = 390)

	Н	T	U	t	h	th	
			3	5	2	7	
	3	4	2	0	0	0	
+		3	4	7	0	0	



When you write your calculation out on paper, it is useful to write in the missing noughts after the decimal point. This is essential in subtraction but if you get used to doing this for addition then you will always remember.

Adding the thousandths and hundredths is straightforward but in the tenths column 5 + 7 is 12 so you write 2 in the tenths column and carry 1 into the units column by writing it below the line. Write a decimal point in the answer space, in line with all the decimal points above.

	Н	T	U	t	h	th	
			3	5	2	7	
	3	4	2	0	0	0	
+		3	4	7	0	0	
	3	8	0	2	2	7	
		1	1				

Answer: 380.227

Subtraction

Setting out a subtraction is just the same as for addition. What can be tricky in subtraction is when the numbers on the top line are less than the numbers on the bottom, and you have to shift and regroup:

Example

What is the difference between 5,194 and 248?



The difference is the answer to a subtraction.

(I expect my answer to be just less than 5,000)

When you look at the units column you can see that you cannot take **8** from **4** so you shift one and regroup to make **14** in the units and **8** in the tens column:

When you get to the hundreds column you have to shift again:



Remember to do a quick check: 248 + 4,946 = 5,194

So far so good, but what about when you have **0** in the top line? You cannot take **1** away from **0**.

In Real Life money problems this is something you have to do a lot, as you work out change so you need a quick trick.

Consider this example

Take **5.45** from **10**.

(I expect my answer to be just under 5)

First write out your calculation adding the extra **0**s.

You can see that you need to shift so that the **0** in the hundredths column becomes **10**, but where do you take the **1** from?

Ignore the decimal point and run your finger along the digits to the left until you get to a whole number: 1.

Now you shift 1 from 100 which regroups as 99. So cross out 100 and write 99 as two little 9s in place of the zeros that you crossed out.

Your subtraction grid looks like this:

And now the subtraction is simple:

Finally, just check mentally by addition.

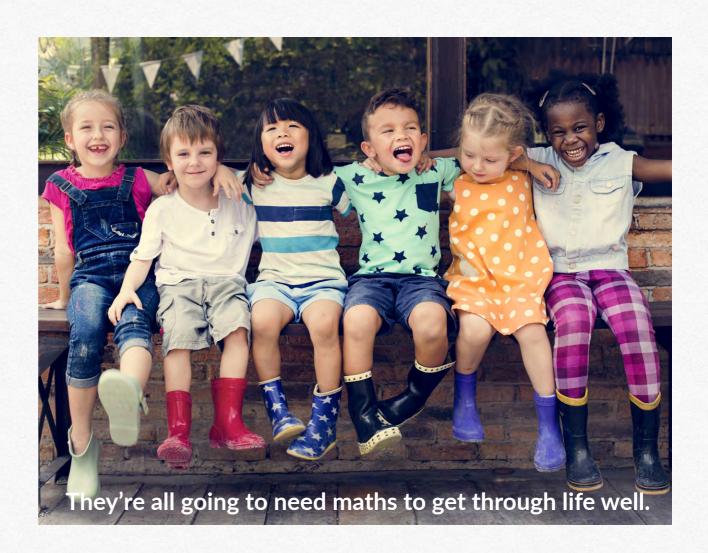
Answer: 5.45 + 4.55 = 10.00

Exercises

To help answer the questions here in Part 2, write the column headings above the digits. You do not have to

do this for ever, but you might find that it helps to make sure that every digit is in the correct place at first.

- 5 What is the sum of 45, 256.2 and 15.95?
- 6 What is the difference between 6,351 and 1,435?
- 7 What is 10 take away 3.84?
- 8 What is 4 less 1.738?
- 9 What is the total value of **4.9**, **495** and **44.49**?
- 10 Take 5.178 from 20.



Subtraction

Multiplication

Just like addition, it is important to put the numbers in the correct columns before you start. Then work with the units first, then the tens, and then the hundreds. Remember to put the carried numbers under the line.

You should always start with an estimate as that will help you to make sure that you have enough columns.

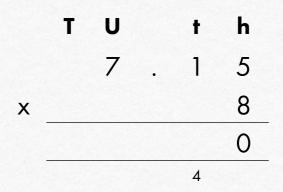
When multiplying with decimals, you may get a zero or nought in the columns to the right of the decimal point. The zeros help keep the decimal in the correct position, so do not ignore them until you write your final answer.

The following method can only be used when multiplying a decimal with a whole number.

Example

Multiply 7.15 by 8

(Estimate $7 \times 8 = 56$, you will need an extra column)



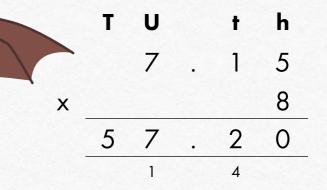
Step 1:

 $8 \times 5 = 40$, write 0 in the hundredths column and carry the 4.

	T	U	t	h
		7	1	5
X				8
			2	0
		1	4	

Step 2:

 $8 \times 1 = 8$, then add 4 to make 12. Write 2 in the tenths column and carry the 1.



Step 3:

 $8 \times 7 = 56$, then add 1 to make 57. Write 7 in the units column 5 in the tens column. Place the decimal point in the answer in line with the decimal point in the top line. Remember this only works when only one of the numbers in the sum has a decimal point

Answer: $7.15 \times 8 = 57.20$

BUT as there is no value to the **0** at the end you can write your answer as:

$$7.15 \times 8 = 57.2$$



If that question had been about money, for example: What is the cost of **8** items each costing **£7.15**, then you MUST write your answer as **£57.20** not **£57.2**

You may need to multiply by a multiple of 10. It is easiest to do this in two parts. First multiply by the 10, 100 or 1,000 in your head and then set the multiplication in a grid to work out the final answer. Remember, when multiplying

a decimal by 10, 100, 1,000..., move the decimal point a place to the right for each zero in the 10, 100, 1,000 etc.

Example

Calculate 14.75 x 60

(estimate
$$15 \times 60 = 15 \times 6 \times 10 = 90 \times 10 = 900$$
)

$$14.75 \times 60 = 14.75 \times 10 \times 6$$

= 147.5×6

	н	I	U	t	
	1	4	7	5	
x				6	
	8	8	5	0	
	2	4	3		

Answer: $14.75 \times 60 = 885$

Exercises

11 8.35 x 4

12 12.07 x 50

13 78.18 x 5

14 126.19 x 60

15 15.48 x 500



Division

When solving a division in writing, the number columns are still important. Make sure that your answer is in the correct place and, just as with any other calculation, you need to be careful with any zeroes.

Example

Divide $3,054 \div 6$ (Estimate $3,000 \div 6 = 500$)

Step 1:

 $3 \div 6$ does not go, so take the 3 to the hundreds column.

$$30 \div 6 = 5$$



Step 2:

 $5 \div 6$ does not go, so write 0 in the tens column and take the 5 to the units column.

Step 3:

 $54 \div 6 = 9$. Write 9 in the units column.

Answer: $3,054 \div 6 = 509$

Look back at Part 1 Use your head to remind yourself about dealing with remainders.

With written calculations it can be best to add a decimal point and **0**s and keep dividing:

Example

$$1,245 \div 4$$

(Estimate: $1,200 \div 4 = 300$)

You can see that with the extra **0**s you have reached an exact answer to your division.

Answer: $1,245 \div 4 = 311.25$

If you have to divide by a power of 10, then just like with multiplication, divide by 10, 100 or 1,000 first.

Example

$$1,245 \div 600$$

(Estimate: $1,200 \div 600 = 2$)

$$1,245 \div 600 = 1,245 \div 100 \div 6$$

= $12.45 \div 6$

Answer: $1,245 \div 600 = 2.075$

Exercises

Working with Money

Whether you are working in pounds (\mathfrak{L}) , Dollars (\mathfrak{L}) or Euros (\mathfrak{L}) there are some basic rules that you should remember when working with money:

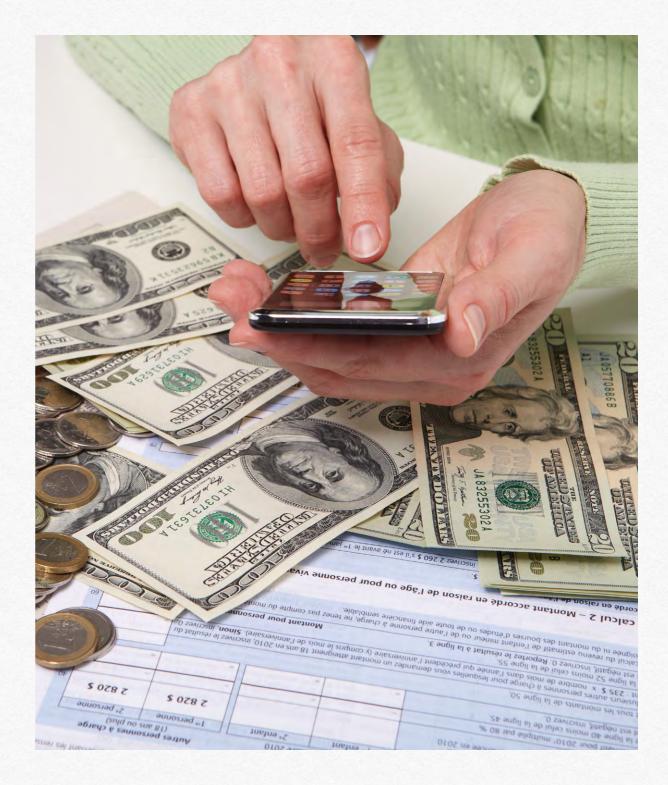
Always include the units BUT do not mix them up

If you are not writing an exact amount of pounds then your answer should have two decimal places after the decimal point. If your answer gives you more, then you should round to the nearest penny:

Example

£11 ÷ 8 = £1.375
= £1.38 to the nearest
$$p$$

Always estimate first and then double check your answer. In Real Life you do not want to muddle the tens and hundreds, or even worse the hundreds and thousands.



Working with Money 80

Real Life Problems

Now you are ready for some Real Life problems that you will probably want to do by writing out your calculations. If you can do some of the calculation in your head then that is fine, but it is a good idea to write some of that down, so that you can carry out those final checks.

Some problems are about **Profit** and **Loss**.

To calculate this, you need to add up **all** the income that you have made from your sales and then to deduct **all** the costs or expenses.

If your income is **more** than your expenses, then you have made a **profit**.

If your income is **less** than your expenses, then you will have made a **loss**.

Exercises

When answering these questions, you need to work out whether you have to add, subtract, multiply or divide. There are some clues in the wording.

- 21 Lena is going skiing and needs to buy some winter underwear. What is the total cost of three thermal vests at £7.95 each, two pairs of ski socks at 9.45 a pair and a hat and glove set at £24?
- **22** (a) Mina spends £3.64 on a drink and a snack from the corner shop. What change should she receive from a £5 note?
- (b) Naomi has bought some office stationery that comes to a total of £13.68. What change should she receive from a £20 note?
- (c) Olivia has spent £32.43 at the supermarket. What change should she receive if she pays with two £20 notes?





23 Phil has a small balcony at her new flat and wants to grow some plants in pots. She visits the garden centre and decides to buy:

1 terracotta trough	£27.49
Set of assorted pots	£25
Bag of compost	£15
Lavender	£12.95
Mixed heathers	£12.50
Strawberry plants	£9
Mixed herbs	£12

What is the total cost of her purchases?



Food for Thought

You can grow your own herbs almost anywhere including your kitchen windowsill. Strawberries are also easy to grow and look pretty as well as tasting yummy!

24 Queenie has cooked some cakes for a charity bake sale. She has worked out that the costs of her ingredients came to a total of £8.56. She has made 3 cakes; each is divided into 8 slices and each slice sells for £2.25. How much profit can Queenie contribute to the charity?



Food for Thought

What other costs might Queenie have that she should add to her expenses?



- 25 Rosa is the accountant for her local football club and has approved the purchase of 6 new training nets at £24.99 each and a set of 12 professional quality footballs at £19.95 each.
- (a) Estimate the cost of the new equipment.
- (b) Calculate the exact cost of the new equipment.



Food for Thought

Although an estimate is fine to get the rough amount, accounts need to have exact figures.



Quick Tip

Take care with some of these next questions as there are some numbers here that you do not need in your calculation in order to answer the question.

26 Shirley has opened a new fitness studio. She has bought 30 fitness mats at £24.95 each, 8 sets of 6 hand dumbbells at £39.50 a set and 6 sets of 12 resistance bands for £19.99 a set.

What is the total cost of this new equipment?



27 Tracey is captain of the village cricket team. She has bought 20 boxes of 6 cricket balls costing £690 in total.

What is the cost of one ball?

28 Tracey's club members each need a set of protective pads, gloves, a helmet and a cricket bat. Tracey can buy 20 sets for a total of £2,708. She sells the sets back to her members but makes a profit of £25.60 on each one. How much does she sell the sets to members for?

- 29 Ursula has purchased 30 million vaccines at a total cost of €94.5 million.
- (a) What is the cost of one vaccine?
- (b) How much will it cost to vaccinate a town of 100,000 people?



The term vaccination derives from the Latin word vacca meaning cow.

The reason for this is that the first vaccinations were against a disease called smallpox. Smallpox was a contagious disease that even when not fatal caused terrible scars.

A scientist, Edward Jenner discovered that a vaccination using a different and far less dangerous disease called cowpox provided immunisation against smallpox.

Vaccination against smallpox became compulsory in England and Wales in **1841** and by the twentieth century was almost unknown here. Vaccinations were also given around the world and the World Health Organisation declared the disease totally eradicated in **1980**.

- 30 Vicky runs a stationery business. She buys cards in packs of 500 for £360 and then sells them in packs of 6 for £4.99.
- (a) How much profit does she make on each pack of **6** cards?
- (b) Vicky forgot to include the cost of repackaging the cards. She received a bill for £125 for all 500 cards. Has she made a total profit or loss and how much?





Have you heard the word Polymath? It is nothing to do with maths but is a Greek word meaning 'having learnt much' and describes an individual who has expertise in a variety of different fields.

Women frequently have to be polymaths as they juggle their careers with family commitments and raising children. Consider Ursula von Leyen, a German doctor of medicine and a politician.



After leaving school, she started studying economics in Germany but moved to London where she studied at the London School of Economics.. Later she switched to studying medicine.

She is married and is the mother of **7** children, staying at home when they were small. She later returned to work in medicine and then became involved in politics.

Ursula von der Leyen was appointed as president of the European Commission in **2019** becoming the first woman in the role.

Real Life Problems 85

Using a Spreadsheet

You can of course always use your hands to use a calculator, there is no need to explain how to do that.

You can also use a spreadsheet on a computer for calculations, particularly for calculating business profits.

The most common of these is Excel. You will probably find it installed on your computer. Simply find the programme or app:



And open it. Select 'blank workbook' and you will get a blank spreadsheet, which is basically a page divided into rows and columns. If you want to know more there are lots guides to help you available in books or online. Consider the question below about Tracey and the cricket sets:

Tracey can buy **20** sets for a total of **£2,708**. She sells the sets back to her members but makes a profit of **£25.60** on each one. How much does she sell the sets to members for?



A spreadsheet is at its most useful when you use a simple formula to carry out repeated calculations.

On a spreadsheet the symbol * is used for multiply and / is used for divide.

Set up the information on a spreadsheet

	Α	В	С	D	E
1		C	ricket set	ts	
2		Total Cost	No	Cost per se	et
3		£2,708.00	20		
4					
5					
6					

Using a Spreadsheet 86

To find the cost per item, you need the spreadsheet to do a calculation. In cell D3 type: = B3/C3

Now add the profit and prepare the final calculation. In cell D7 type: =D3+D5

	Α	В	С	D	E
1		C	ricket set	ts	
2		Total Cost	No	Cost per se	t
3		£2,708.00	20	£135.40	
4					
5		Profit		£25.60	
6					
7		Selling Pric	е		

Press enter and you should get the same answer as to question 28.

Another useful thing that spreadsheets can do is to copy formulae (formulae is the plural of formula), so you do not have write them out each time.

Look at the shopping list for Shirley's fitness centre:

	Α	В	С	D	E	F	G
1		Fitnes	s Stuc	lio			
2		Item		Cost		Total	
3		Fitness mats	30	£24.95			
4		Dumbbells	8	£39.50			
5		Fitness mats	6	£19.99			
6							
7							

First see that the prices are all written as money. However, they were entered as numbers e.g. **24.95**, then all the cells that are to be money were highlighted and then under Format Cells they are changed to currency.

Now in cell **F3** type **=C3** * **D3**

F3		X ✓	fx	=C3 * E)3)		
	Α	В	С	D	E	F	G
1		Fitnes	s Stud	io			
2		Item		Cost		Total	
3		Fitness mats	30	£24.95		£748.50	
4		Dumbbells	8	£39.50		£316.00	
5		Fitness mats	6	£19.99		£119.94	

This will multiply the number of items by the cost of one item.

You can then copy this formula into the next two cells.

Notice how you can see the formula in the row above.

Now you need to total your shopping.

In cell **F7** type the formula **=SUM** (**F3:F5**) which means all the amounts in cells **F3** to **F5** are added together.

If you prefer you could use the formula:

$$= F3 + F4 + F5$$

You should have the same answer as you had for **Q26**.

Now use a spreadsheet to check your answers to some of the earlier questions.

Exercises

31 Wendy is going shopping for her boys.

Use a spreadsheet to work out the total amount of her shopping:

5 catapults at £6.99 each

7 footballs at £12.99 each

8 sets of marbles at £8.59 each

12 pirate head scarves at £6.99 each



Do you know the story of Peter Pan, Wendy and the lost boys? It was written a long time ago when the toys parents bought for their boys were very different from those you would buy now. Have you ever used a catapult? It is useful for hand-eye coordination.

Using a Spreadsheet 88

32 Xin is working out her monthly budget by looking at her weekly expenditure. She has entered her total weekly spending for 4 weeks on a spreadsheet. Complete the spreadsheet to calculate what she has spent in a month.

	Α	В	С	D	E	F
1			Month	ly expen	diture	
2		Week	1	2	3	4
3		Rent		£850.00		
4		Electrics			£32.50	
5		Council tax	£75.00			
6		Groceries	£45.64	£72.81	£53.76	£48.53
7		Tights/Clothes	£9.99	£32.45	£7.45	£45.75
8		Travel			£45.00	
9		Other	£9.99	£15.54	£6.50	£25.50

33 Yetta and Zoe have gone on holiday together. This what they have each spent on the holiday.

Yetta: £150 Campsite

£35.65 Supermarket

Zoe: £65 Travel

£42.50 Pizzeria

£63.24 Supermarket

Use a spreadsheet to work out the total amount spent and who owes who what.



Using a Spreadsheet 89

Answers to Part 2

These worked answers for Q1 – 4 show the thinking that your head and hands might have been doing to work out the answer. But do not worry if you head and hands did something different, as long as you reach the correct answer!

1 Hannah is a newly qualified teacher and her starting salary is £26,400 a year. What will her monthly salary be?

$$26,400 \div 12 = 26,400 \div 4 \div 3$$

= 6,600 \div 3
= 2,200

Answer: Hannah's monthly salary is £2,200

2 (a) Iris is saving £300 a month. How much will she save in a year?

$$300 \times 12 = 3,600$$

Answer: £3,600

(b) Iris wants to buy a flat and needs to have a £10,000 to have a big enough deposit. How many months will it take her to save at least £10,000?

$$10,000 \div 300 = 100 \div 3$$

= 33 r 1

Answer: 34 months (note you need to round up your answer to a whole number of months as after 33 months she will not have saved £10,000)

- Janis is shopping for her new home office. She has chosen a desk costing £157, a chair with good back support for £79.99, a mid range laptop at £799 and a printer/scanner costing £129.
- (a) Estimate, to the nearest £10, the total cost of Janis' purchases?

$$160 + 80 + 800 + 130 = 240 + 930$$

= 1,130 + 40
= 1,170

Answer: £1,170

(b) Janis had set herself a budget of £1,500. Roughly how much will she have left?

$$1,500 - 1,170 = 1,500 - 1,000 - 100 - 70$$

= 330

Answer: About £330

(c) What other essentials do you think Janis will need to buy?

She will need paper and ink for her printer, plus pens and notepads. She may need a good desk light, also a filing cabinet and files. She may need a headset with speakers and a microphone depending on her laptop.

However, Janis will find lots of other things that she needs to buy and must not get carried away with expensive accessories!



4 Kia is redecorating her bedroom. She has worked out that the area of her walls is 22.5 m². The paint she has chosen says that 1 litre will cover 10 m² and that she will need to give her walls 2 coats of paint. A 5 litre pot of paint costs £26 and a 2.5 litre pot of paint costs £16.

(a) How much paint does Kia need?

Double the wall area: $22.5 \times 2 = 45 \text{ m}^2$ 1 | paint covers 10 m²: $45 \div 10 = 4.5 \text{ |}$

Answer: She will need 4.5 | of paint

(b) What is the least amount of paint Kia should buy and what will it cost?

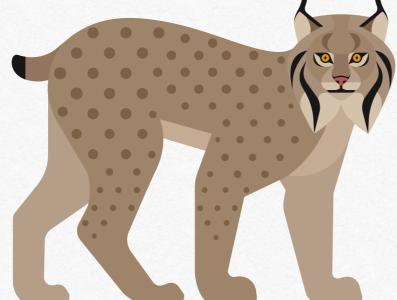
Answer: She should buy $5 \mid$ of paint and it is cheapest to buy a $5 \mid$ pot at £26 rather than $2 \times 2.5 \mid$.

5 What is the sum of 45, 256.2 and 15.95?

Write the digits in the correct columns, fill in the missing zeros and add, writing any carried numbers under the line.

H T U t h
4 5 . 0 0
2 5 6 . 2 0
+ 1 5 . 9 5
3 1 7 . 1 5

Answer: 317.15



6 What is the difference between 6,351 and 1,435?

Just as before, write the digits in the correct columns. Make sure that you have the larger number above the smaller. Now deduct the small number from the big one.

You will need to shift in order to subtract 5 from 11 rather than 1 and then again to subtract 4 from 13 rather than 3.

Answer: 4,916

(Check: 4,916 + 1,435 = 6,351)

7 What is 10 take away 3.84?

You will need to shift in order to subtract **4** from **10** rather than **0**. The whole number from which you shift is **100**. Regroup **100** as **99**:

$$100 - 1 = 99$$

Answer: 6.16

(Check 6.16 + 3.84 = 10.00)

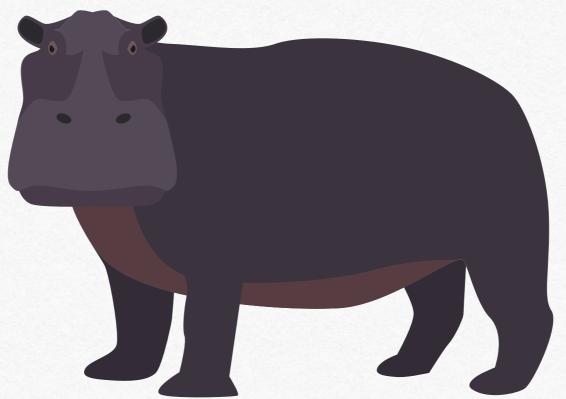
8 What is 4 less 1.738?

This time the whole number from which you shift 1 is 400. Regroup 400 as 399:

$$400 - 1 = 399$$

Answer: 2.262

(Check 2.262 + 1.738 = 4.000)



9 What is the total value of **4.9**, **495** and **44.49**?

No surprises here! Put the digits in the correct columns and add.

Answer: 544.39

10 Take 5.178 from 20.

This time the whole number from which you shift 1 is 2,000. Regroup 2,000 as 1,999:

$$2,000 - 1 = 1,999$$



Answer: 14.822

(Check: 14.822 + 5.178 = 20.000)

11 8.35 x 4

This one should be straightforward

As the last digit is **0**, and as this is not money, it does not need to be written down.

Answer: 33.4

12 12.07 x 50

$$12.07 \times 50 = 12.07 \times 10 \times 5$$

= 120.7×5

Answer: 603.5

13 78.18 x 5

When you estimate $80 \times 5 = 400$, you will need a hundreds column

Again, you do not need to write the last 0.

Answer: 390.9

$$126.19 \times 60 = 126.19 \times 10 \times 6$$

= 1,261.9 x 6

Be careful with the carrying here, there is lots of it.

Answer: 7,571.4



15 15.48 x 500

$$15.48 \times 500 = 15.48 \times 100 \times 5$$

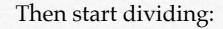
= 1,548 x 5

This time you do need to write the final zero because it is not after a decimal point.

Answer: 7,740

16 51.28 ÷ 4

First set up your grid:



Step 1: 4 goes into 5 once, remainder 1.

Step 2: 4 goes into 11 twice, remainder 3.

Step 3: 4 goes into 32 eight times

Step 4: 4 goes into 8 twice:

Answer: 12.82

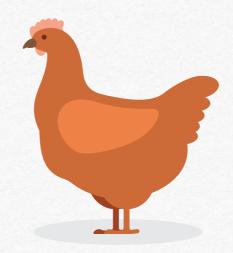
17 106.02 ÷ 9

You have an extra column this time, but just keep dividing as before:

Answer: 11.78

$$70 \div 50 = 70 \div 10 \div 5$$

= $7 \div 5$



You are going to add one **0** after the decimal point. It will only be one as all numbers ending in **0** can be divided by **5**.

Answer: 1.4

$$412 \div 80 = 412 \div 10 \div 8$$

= $41.2 \div 8$

You may have one or more **0**s so leave enough space:

Answer: 5.15

$$1,265 \div 400 = 1,265 \div 100 \div 4$$

= $12.65 \div 4$

Again, you will need to add **0**s but a maximum of two as any number ending in **00** can be divided by **4**.

Answer: 3.1625

21 Lena is going skiing and needs to buy some winter underwear. What is the total cost of three thermal vests at £7.95 each, two pairs of ski socks at 9.45 a pair and a hat and glove set at £24?

Total means that you need to add, but first you need to find the cost of all those thermal vests and socks by multiplying.

(Estimate:
$$3 \times 8 + 2 \times 9.5 + 24 = 24 + 19 + 24$$

= $24 + 43$
= 67)



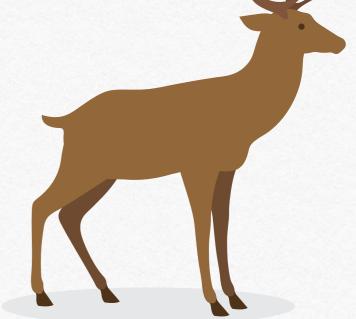
Vests

Socks

Even though £24 has no pence you should put in the 0s:

Total:

Answer: £66.75



- 22 For these questions, just as in the earlier examples, you need to add the extra 0s for the whole number of pounds. Start by borrowing 1 and then find the whole number and subtract 1:
- (a) Mina spends £3.64 on a drink and a snack from the corner shop. What change should she receive from a £5 note?

The whole number from which you shift 1 is 50.

$$50 - 1 = 49$$

Answer: £1.36

(b) Naomi has bought some office stationery that comes to a total of £13.68. What change should she receive from a £20 note?

The whole number from which you shift 1 is 200.

$$200 - 1 = 199$$

Answer: £6.32

(c) Olivia has spent £32.43 at the supermarket. What change should she receive if she pays with two £20 notes?

The whole number from which you shift 1 is 400.

$$400 - 1 = 399$$

Answer: £7.57

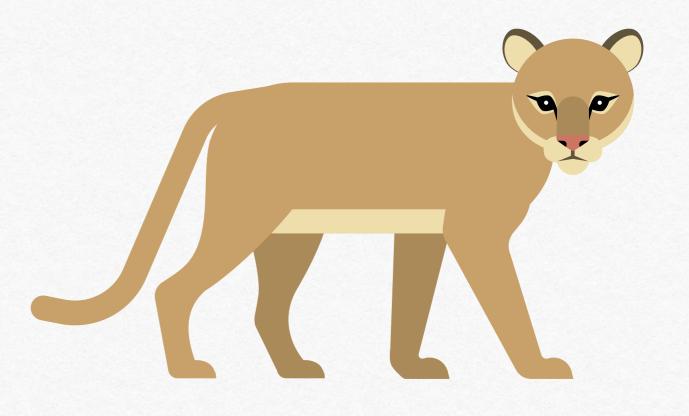
23 Phil has a small balcony at her new flat and wants to grow some plants in pots. She visits the garden centre and decides to buy:

1 terracotta trough	£27.49
Set of assorted pots	£25
Bag of compost	£15
Lavender	£12.95
Mixed heathers	£12.50
Strawberry plants	£9
Mixed herbs	£12

What is the total cost of her purchases?

Carefully put your numbers in the right column and add:

Answer: £113.94



24 Queenie has cooked some cakes for a charity bake sale. She has worked out that the costs of her ingredients came to a total of £8.56. She has made 3 cakes; each is divided into 8 slices and each slice sells for £2.25. How much profit can Queenie contribute to the charity?

You can do part of this in your head:

Income from the cakes =
$$8 \times 3 \times 2.25$$

= 8×6.75

(Estimate: $8 \times 7 = 56$)

The income from selling the slices of cake was £54.00

To find the profit you then need a subtraction.

Answer: The profit for Queenie's charity is £45.44



Food for Thought

What other costs might Queenie have that she should add to her expenses?

She will have used her oven and thus energy to cook the cakes, although probably not very much. However, if Queenie were running a cake making business she would need to consider her energy bills.

- 25 Rosa is the accountant for her local football club and has approved the purchase of 6 new training nets at £24.99 each and a set of 12 professional quality footballs at £19.95 each.
- (a) Estimate the cost of the new equipment.

Estimate:
$$6 \times 25 + 12 \times 20 = 150 + 240$$

= 390

(b) Calculate the exact cost of the new equipment.

You need two multiplications and an addition:

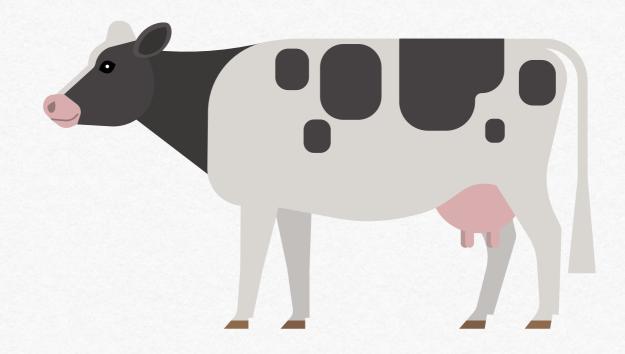
For the footballs as you are multiplying by **12** you may have to carry two digits rather than one:

Nets

Footballs

Total:

Answer: £389.34



26 Shirley has opened a new fitness studio. She has bought 30 fitness mats at £24.95 each, 8 sets of 6 hand dumbbells at £39.50 a set and 6 sets of 12 resistance bands for £19.99 a set.

What is the total cost of this new equipment?

The number of items in a set is not relevant to answering the question

(Estimate:
$$30 \times 25 + 8 \times 40 + 6 \times 20 = 750 + 320 + 120$$

= 1,070 + 120
= 1,190)

Mats:
$$24.95 \times 30 = 24.95 \times 10 \times 3$$

= 249.5×3

Dumbbells:

Resistance bands:

Total:

Answer: £1,184.44

27 Tracey is captain of the village cricket team. She has bought 20 boxes of 6 cricket balls costing £690 in total.

What is the cost of one ball?

(Estimate:
$$700 \div 20 \text{ then } \div 6 = 35 \div 6 \approx 6$$
)

$$690 \div 20 \text{ then } \div 6 = 69 \div 2 \text{ then } \div 6$$

= $34.5 \div 6$



Answer: £5.75

28 Tracey's club members each need a set of protective pads, gloves, a helmet and a cricket bat. Tracey can buy 20 sets for a total of £2,708. She sells the sets back to her members but makes a profit of £25.60 on each one. How much does she sell the sets to members for?

Cost per set:

(Estimate =
$$2,700 \div 20 = 270 \div 2$$

= 135)

$$2,708 \div 20 = 2,708 \div 10 \text{ then } \div 2$$

= $270.8 \div 2$

Price for members:

Answer: £161.00

29 Ursula has purchased 30 million vaccines at a total cost of €94.5 million.

(a) What is the cost of one vaccine?

As both figures are in millions, you simply need to look at the whole number of millions:

Cost of 1 vaccine =
$$94.5 \div 30 = 94.5 \div 10$$
 then $\div 3 = 9.45 \div 3$

Answer: €3.15

(b) How much will it cost to vaccinate a town of 100,000 people?

To be sure you do not get confused with all the **0**s multiply by **100** first and then by **1,000**.

Cost =
$$3.15 \times 100,000$$

= $3.15 \times 100 \times 1,000$
= $315 \times 1,000$
= $315,000$

Answer: €315,000

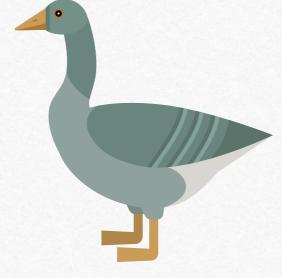


- 30 Vicky runs a stationery business. She buys cards in packs of 500 for £360 and then sells them in packs of 6 for £4.99.
- (a) How much profit does she make on each pack of 6 cards?

Cost of 6 cards =
$$360 \div 500 \times 6$$

= $360 \div 100$ then $\div 5 \times 6$
= $3.6 \div 5 \times 6$

Profit =
$$4.99 - 4.32$$



You may be able to do this in your head or you could write it out:

Answer: £0.67

(b) Vicky forgot to include the cost of repackaging the cards. She received a bill for £125 for all 500 cards. Has she made a total profit or loss and how much?

When calculating total profit, it is best to work with total costs and total sales. In this case Vicky cannot make all her **500** cards into packs of **6**, so first work out how many packs Vicky can sell:

No of packs =
$$500 \div 6$$

= $83 \text{ r } 2$

So Vicky can only sell 83 packs

Total costs for 500 cards =
$$£360 + £125$$

= $£485$

Total sales income for 83 packs = 4.99×83

You could revise your long multiplication. But as £4.99 is almost £5 it is quicker to multiply 83 by 5 and then subtract 83p

Do not get the £ and p muddled. You need to subtract 83p from £415:

As costs are £485 and sales are £414.17 Vicky will make a loss.

Answer: Vicky makes a loss of £70.83



Many small businesses do not survive due to the total costs not being taken into account. It is important to make sure that you are aware of all costs. In addition there are 'hidden costs' such as rent, light and heat.

Also, Vicky might not sell all her packs of cards, so she needs to have a high enough profit on each pack to cover any lack of sales. When she works out her selling prices she needs to consider her 'profit margin' carefully.

Hopefully, she will have done some marketing to ensure that her cards are going to be popular and therefore she should sell more. **31** Wendy is going shopping for her boys.

Use a spreadsheet to work out the total amount of her shopping:

5 catapults at £6.99 each

7 footballs at £12.99 each

8 sets of marbles at £8.59 each

12 pirate head scarves at £6.99 each

Here is the completed spreadsheet. You may have the columns in a different order, but the total should be the same:

	Α	В	С	D	E	F
1			Wer	ndy's shopp	ing	
2		5	catapaults	£6.99		£34.95
3		7	footballs	£12.99		£90.93
4		8	marbles	£8.59		£68.72
5		12	scarves	£6.99		£83.88
6						
7						£278.48
8						

32 Xin is working out her monthly budget by looking at her weekly expenditure. She has entered her total weekly spending for 4 weeks on a spreadsheet. Complete the spreadsheet to calculate what she has spent in a month.

Here is the completed spreadsheet. It is always a good idea to make sure that both your horizontal rows (i.e. total for each item) and your vertical columns (i.e. amount spent each week) add up to the same amount – and a spread sheet makes that simple!

	Α	В	С	D	E	F	Н
1		M	lonthly	expend	liture		
2		Week	1	2	3	4	Total
3		Rent		£850.00			£850.00
4		Electrics			£32.50		£32.50
5		Council tax	£75.00				£75.00
6		Groceries	£45.64	£72.81	£53.76	£48.53	£220.74
7		Tights/Clothes	£9.99	£32.45	£7.45	£45.75	£95.64
8		Travel			£45.00		£45.00
9		Other	£9.99	£15.54	£6.50	£25.50	£57.53
10							
11			£140.62	£970.80	£145.21	£119.78	£1,376.41

Answers to Part 2

33 Yetta and Zoe have gone on holiday together. This what they have each spent on the holiday.

Yetta: £150 Campsite

£35.65 Supermarket

Zoe: £65 Travel

£42.50 Pizzeria

£63.24 Supermarket

Use a spreadsheet to work out the total amount spent and who owes who what.

Below is the spread sheet. You can see that each girl works out what she has spent for the pair of them. They then add up their totals and divide by two to split them equally. As Yetta has spent more, Zoe owes her £7.45

	Α	В	С	D	E	F
1	Yetta		Zoe			
2	Campsite	£150.00	Travel	£65.00		
3	Supermarket	£35.65	Pizzeria	£42.50		
4			Supermarket	£63.24		
5						
6	Total Yetta	£185.65	Total Zoe	£170.74	Total both	£356.39
7		£178.20		£178.20	½ each	£178.20
8	Yetta's difference	£7.45	Zoe's difference	- £7.45		



Answers to Part 2

100



Q1

Write down what you need to in order to answer each question and then check to see if you were correct.

£20 - £8.67 = ?

Q2

$$8.45 \times 60 = ?$$





Q3

$$15 \div 8 = ?$$

Q4

What is the total cost of **5** basil plants each costing **£0.85** and **6** mint plants each costing **£1.19**?





Q5

A politician earns £86,799 per year. What is her monthly salary?

Q6

Vicky buys 120 big brown padded envelopes wholesale for a total of £65.99 and sells them in packs of 10 for £7.95. What is her total profit?





Q7

Tracey is mending her garden shed. She needs to buy 6 planks costing £5.95 each, wood screws costing £4.95, wood preserver at £22.45 a pot, brushes at £5.95 a set and a litre of white spirit for £1.99. Estimate her total costs and then work out her exact costs.

Q8

Tracey puts her costs in a spreadsheet:

	Α	В	С	D	E	F
1		Item	No	Cost		Total
2		Planks	6	£5.95		
3		Box of screws	1	£4.95		
4		Preserver	1	£22.45		
5		Brush set	1	£5.95		
6		White spirit	1	£1.99		
7						

What formula should she put in (a) cell **F2** (b) cell **F7**?





Answers

- Q1 20 8.67 = 11.33
- Q2 $8.45 \times 60 = 507$
- Q3 $15 \div 8 = 1.875$

= £71.04

- $Q4 5 \times £0.85 + 6 \times £1.19 = £11.39$
- Q5 $£86,799 \div 12 = £7,233.25$
- Q6 $12 \times £7.95 = £95.40$ £95.40 - £65.99 = £29.41
- Q7 Estimate: $6 \times 6 + 5 + 22 + 6 + 2 = 36 + 35$ = 71 $6 \times £5.95 = £35.70$ £35.70 + £4.95 + £22.45 + £5.95 + £1.99
- Q8 (a) = C2*D2 (b) = SUM (F2:F6)



Do you enjoy word puzzles such as word ladders or crosswords? Do you also enjoy number puzzles? Solving puzzles is a good way to keep your brain exercised but also can impress others.

However, many people are a bit nervous about tackling number puzzles. Once you become familiar with them, you will find that you do not have to have a mathematics degree or be a maths genius to solve them. Many problems are more about logic than maths, and you already have enough knowledge of number to be able to solve many of these puzzles.

In this part we are going to look at some mathematical general knowledge and then some tricks that you can use in order to impress.



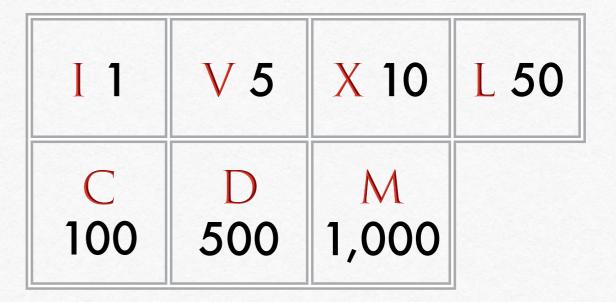
Roman Numerals

When you are travelling either in the UK or overseas it is not unusual to see old buildings with the dates of their construction shown in Roman numerals:



It is very impressive if you can say to your companions 'that was built in ...' whatever the date may be.

The Romans used seven letters that they combined together to make numbers.



Romans would write these letters one after the other to build numbers:

Example

The numbers are usually written in order of size, largest first:

However, the three symbols I, X and C may be written ONCE before a symbol of higher value to subtract its value. There are **6** possibilities:

4 is IV (i.e. 5 less 1)
9 is IX (i.e. 10 – 1)
40 is XL (i.e. 50 less 10)
90 is XC (i.e. 100 – 10).
400 is CD (i.e. 500 less 100)
900 is CM (i.e. 1,000 - 100).

When reading a number, you know that the letters go in descending value, so if you get a letter in the wrong order, it must be being used as a subtraction.

As most Roman numbers you will see are dates, let us look at the possible years. Note that for the date in years, no thousand separator comma is used.

MM would be 2000

MCM would be 1000 + 900 = 1900

MDCCC would be 1000 + 500 + 300 = 1800

MDCC would be 1000 + 500 + 200 = 1700

To read number written in Roman Numerals, you should break it into thousands, hundreds, tens and units.

Look at the date on this building:



Example MCMXCIV

This breaks down into:

M CM XC IV

M = 1000

CM = 900

XC = 90

IV = 4

The year is 1994



You probably know that the ancient Romans were a highly successful civilisation. Their engineering skills were very well developed enabling them to build roads, cities and amazing aqueducts and water systems across Europe and what is now North Africa. However, their number system would mean that they would never be sophisticated mathematicians, because they could not multiply simply. For a calculation such as 8×12 they would have to add successive eights 12 times or:

VIII + VIII + VIII + etc.

Now you can do 8×12 in a nanosecond = 96.

So you are better at mental arithmetic than the sophisticated Romans.



Roman Numerals 123

Exercises

Write down the value of these numbers written in Roman Numerals. Remember to first break each long numeral into thousands, hundreds, tens and units, then translate to western numbers and add.

1 What is the number of this archway in the Colosseum, Rome?



2 Sandor Palace, Budapest



3 Is this an ancient archway?



4 How old is this Gaelic chapel?



Roman Numerals 124

Factors and Multiples

From earlier parts you have seen how important factors and multiples are. You can break down a tricky multiplication or division by using factors. It is therefore important to recognise when a number has factors and what these are.

There are some rules about this which are worth learning.





Rules of Divisibility

If a number has, for example, **5** as a factor then the number can be divided by **5**. Therefore, these rules are called 'rules of divisibility'. We will first look at times tables to understand the rules.

Is 2 a factor?

The two times table gives the sequence:

These numbers all result from multiplying by two and they are all even. Therefore, you can say:

All numbers that are even can be divided by 2.

Is 3 a factor?

The three times table gives the sequence:

If you add the digits of the two-digit numbers together:

$$12 \rightarrow 1+2 = 3$$

 $15 \rightarrow 1+5 = 6$
 $18 \rightarrow 1+8 = 9$ and so on you get a new sequence:
 $3, 6, 9, 3, 6, 9, 3, 6, 9, ...$

You can see that all these numbers are multiples of **3**. The rule therefore is:

If the sum of the digits of a number is a multiple of **3**, the number can be divided by **3**.

For large numbers, when the sum of the digits is itself more than one digit, you will need to add the digits together again.

Example

Is 285 a multiple of 3?

285:
$$2 + 8 + 5 = 15 \rightarrow 1 + 5 = 6$$

Yes, **285** is a multiple of **3**.

Is 4 a factor?

The four times table gives the sequence:

and if you extend it beyond 100, you find it continues to:

As **100** can be divided by four, you can ignore the hundreds (or thousands, ten thousands, millions etc.) and just look at the last two digits.

The rule is:

If the last two digits of a number can be divided by **4** then the whole number can be divided by **4**.

Example

Is **34**,**556** a multiple of **4**?

Just look at the last two digits, **56**.

Can **56** be divided by **4**?

$$56 \div 4 = 14$$

Yes (also it is in the 8 times table and $8 = 2 \times 4$)

Yes, **34**,**556** is a multiple of **4**.



Is **5** a factor?

The five times table gives the sequence:

You can easily see that the rule is:

Any number that ends in 5 or 0 can be divided by five.

Is 6 a factor?

The six times table gives the sequence:

You know that $6 = 3 \times 2$, and so the numbers in the six times table must be multiples of both 2 and 3.

Therefore, the rule is:

If an even number has a digit sum that is a multiple of **3** then it can be divided by **6**.

Example

Can 3,456 by divided exactly by 6?

It is even so continue to see if it is also divisible by 3.

$$3 + 4 + 5 + 6 = 18$$
 which is a multiple of 3.

Yes **3,456** is a multiple of **6**.

Is 7 a factor?

There is rule for this but it is so complicated that it is best to know your **7** times table well and just divide.





Is 8 a factor?

The eight times table gives the sequence:

This is similar to the four times table (as $8 = 2 \times 4$) but 8 is not a factor of 100 but it is of 1,000

$$1,000 \div 8 = 125$$

So you cannot ignore the hundreds (but you can thousands etc.). Rather than checking by dividing by **8**, you could divide by **4**, or if you prefer divide by **2** and then by **2** again.

The rule is:

A number can be divided by **8** if you take the last three digits and divide by **4** and the answer is even.

Example

Is **8** a factor of **3,456**?

Just look at the last three digits, 456.

$$456 \div 4 = 228 \div 2 = 114$$

Which is even.

Yes, 8 is a factor of 3,456



Is 9 a factor?

The nine times table gives the sequence:

You know that $9 = 3 \times 3$ so your would expect the rule to be similar.

If you add the digits of the numbers in the **9** times table sequence you will find that they are all **9**.

The rule is:

If the digit sum of a number is **9**, or a multiple of **9**, then it can be divided by **9**.

Example

Can 1,234 be divided exactly by 9?

$$1 + 2 + 3 + 4 = 10$$

No, 1,234 cannot be divided by 9

Is 10 a factor?

The ten times table gives the sequence:

10, 20, 30, 40, 50, 60 etc...

This is very simple:

Any number that ends in 0 can be divided by 10.

Is 11 a factor?

Rather like **7**, there is a rule but it complicated, Luckily, you do not have to divide by **11** very often.



The twelve times table gives the sequence:

12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144

As twelve = 3×4 you can first check the digit sum and see if the number can be divided by 3, and then look at the last two digits and see if it can be divided by 4.

The rule is that if a number can be divided by both three and four then it can be divided by **12**.

Example

Is **678** a multiple of **12**?

6 + 7 + 8 = 21 which is a multiple of 3

 $78 \div 4 = 19 \text{ r } 2 \text{ so not a multiple of } 4.$

No, 678 is not a multiple of 12.



You will see from the examples that the questions were different in their wording but they all mean the same.

If a number has a **factor** \mathbf{x} , then it can be **divided by** x and is a **multiple of** x.

Exercise

- **6** Which of these numbers can be divided exactly by **4**?
- (a) **72** (b) **166** (c) **1,236** (d) **14,562**
- **7** Which of these numbers has **9** as a factor?
- (a) **74** (b) **135** (c) **1,536** (d) **24,012**
- **8** Which of these numbers is a multiple of **6**?
- (a) 114 (b) 501 (c) 1,204 (d) 24,012

- **9** Which of these numbers can be divided exactly by **12**?
- (a) 114 (b) 408 (c) 1,204 (d) 24,012

For this last question you can put together what you know about numbers that can be divided by **3** and numbers that can be divided by **5**:

- 10 Which of these numbers can be divided exactly by 15?
- (a) 135 (b) 500 (c) 1,245 (d) 24,010



Squares and Cubes

It is worth revising what square and cube numbers are and being able to recognise them.

A square number is the answer when a number is multiplied by itself:

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

A square can be written with an index number like this:

$$3 \times 3 = 3^2 = 9$$

The little **2** is called the index number and indicates that the two **3**s are multiplied together.

A cube number is the answer when a number is multiplied by itself three times:

$$1 \times 1 \times 1 = 1$$

$$2 \times 2 \times 2 = 8$$

A cube number can be written with an index number like this:

$$3 \times 3 \times 3 = 3^3 = 27$$

The little **3** is the index number and indicates that the three **3**s are multiplied together.

Exercise

- 11 Write down the first ten square numbers.
- 12 Write down the first ten cube numbers.

















Prime Numbers

Do you remember what a prime numbers is?

A prime number can only be divided by 1 or itself.

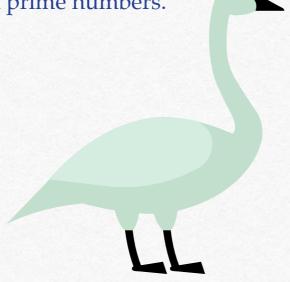
Or written another way:

A prime number has only **2** factors, itself and **1**.

You can see that 1 is not a prime number as it has only 1 factor, 1 itself.

Exercise

13 Write down the first ten prime numbers.





'Prime' means of the first importance, so it should not be a surprise that mathematicians have spent a lot of time looking at prime numbers. It is thought that the concept of prime numbers must be universal, so it could be a way of communicating to aliens that the inhabitants of Earth are intelligent..



Prime Numbers 134

Number Puzzles

Now that you have reminded yourself about numbers, have a go at some puzzles. You can use the information above to help you find the clues to answer them.

Exercise

- 14 Find a number that is both a square number and a cube number.
- 15 Write down all the prime numbers that are a multiple of 3.
- What is the smallest cube number that is a multiple of both 3 and 4?
- 17 Find the smallest number that is both a multiple of 2, 3 and 4 and a factor of 600?

- 18 A number is a factor of 630 and of 945. What is the largest number that could be?
- 19 Lighthouse A flashes every 15 seconds and Lighthouse B flashes every 25 seconds. If they flash together at midnight, how long is it until they flash at the same time again?
- 20 A florist has 24 red roses and 18 white roses. She wants to make the most possible identical bunches of both white and red roses with none left over. How many bunches can she make and how many roses will be in each?



Number Puzzles 135

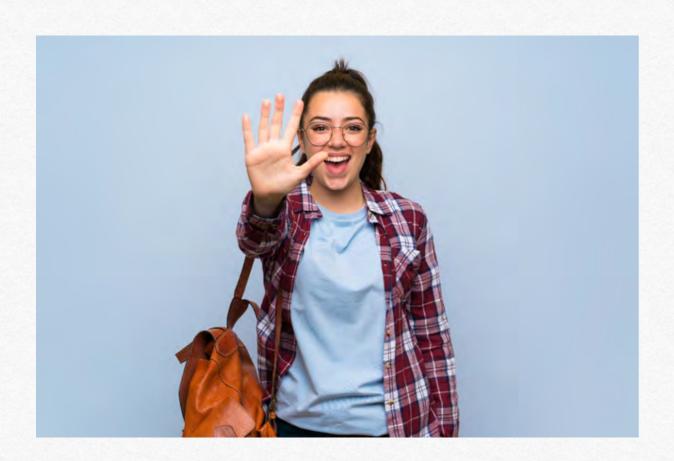
Number Tricks

These are a fun way to impress your friends.

Exercises

Try the following number tricks and then work out why they work.

21 Tell your friend to think of a number between 1 and 10.



Tell them to double it and then add 6.

Now tell them to divide the answer by **2** and subtract the number they first thought of.

You can now tell them that the answer is **3**. It always will be!

Can you work out why?

22 Ask your friend to think of a two-digit number between 10 and 100.

Now ask them to add the two digits together, so if they thought of 35, 3 + 5 = 8

Now ask them to subtract this answer from their original number (e.g. 35 - 8 = 27)

Ask them to add the digits in the resulting number together and write down that answer.

Show them a playing card, nine of clubs, is that their number? It is!

Can you work out why?

23 Ask your friend to think of a number between 1 and 10.

Tell them to multiply their number by **9**.

Now tell them to add the digits of that answer together and subtract **5**.

If 1 is A, 2 is B, 3 is C etc. ask them to think of the letter represented by their last answer.

Now write down a country that starts with that letter.

Write down an animal that starts with the last letter of the country.

Write down a colour that starts with the last letter of the animal.



You show them what they have written down before they show you – an orange kangaroo from Demark.

Can you work out why this is almost always the answer. Can you think of any other possible answer?

Number Tricks



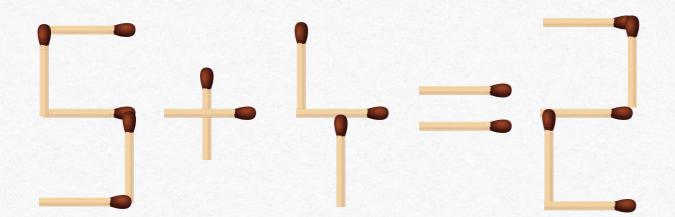
Food for Thought

You can find lots of tricks like these if you research online. They are all quite simple, but they always impress your audience.

There are also number tricks with matchsticks or toothpicks.

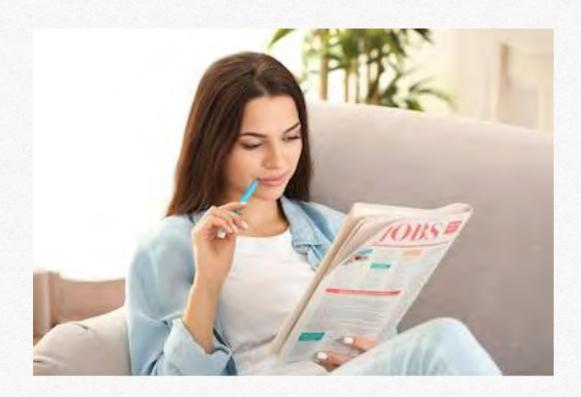
Exercise

24 Can you move one matchstick to make this equation correct?





Number Puzzles in Newspapers



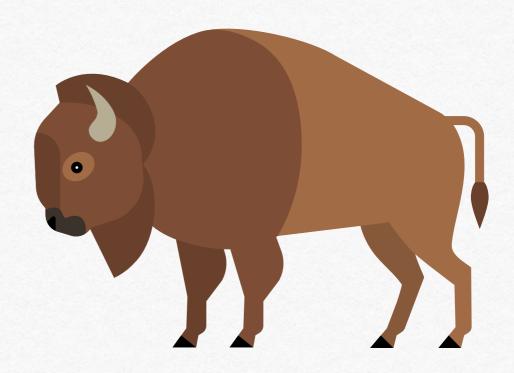
Newspapers are a useful source of information. They also have special sections that may be of interest to you, such as sport, finance or media. In the picture above the young woman is looking through the jobs section.

Most newspapers have puzzle pages. They may have crosswords, word searches, anagrams and other word games. They will also have number puzzles.

Most people only tackle the word puzzles.

Imagine arriving at an interview for college or for a new job and casually putting down a newspaper with the number puzzles solved. That will impress your potential colleagues.

Different newspapers have slightly different puzzles but they often have similar strategies to solve them. Let us have a look at some typical number puzzles and see how they can be solved.



30 Second Brain Teaser

There are various versions of these in different papers. To solve them you need to apply some of the mental strategies that you looked at earlier.

Here is an example:

$$88 \Rightarrow 11 \Rightarrow 12 \Rightarrow 4 \times 5 \Rightarrow 7 \Rightarrow 4$$



To solve this, simply take it step by step and write your answer to each stage in each box. As you get better, then you will probably be able to go a few steps before jotting down an answer.

At first, don't worry about the **30** second bit. Just concentrate on getting the right answers.

The divisions are an important checkpoint because if you have gone wrong you will not be able to divide exactly.

Let's have a go at that one:

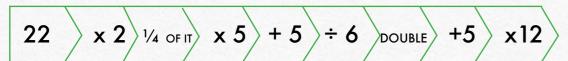
$$\begin{array}{c|c} 88 & \div 11 & \div 12 & \div 4 & \times 5 & \div 7 & \div 4 \\ \hline 8 & 20 & 5 & 25 & 32 & 8 \end{array}$$

ANSWER

8



Here is one with more stages and some different instructions:



ANSWER

Just take it step by step as before:

300

Now you are ready to try some on your own:

Exercises

25

$$36 \rightarrow 4$$
 DOUBLE $-6 \rightarrow x5 \rightarrow 3 \div 9$

ANSWER



26

$$236$$
 $\rightarrow 124$ $\div 4$ $\rightarrow 4$ $\rightarrow 4$ $\rightarrow 5$ $\rightarrow 1/2$ OF IT CUBE

ANSWER



27

$$18 \rightarrow x2 \rightarrow 4 \text{ OF IT } x5 \rightarrow +10 \rightarrow +11 \rightarrow 8 \text{ QUARE ROOT} x12 \rightarrow -13$$

ANSWER



28

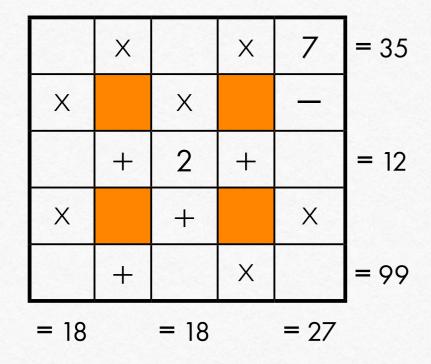
$$21$$
 $\frac{1}{3}$ OF IT $\frac{1}{3}$ OF IT $\frac{1}{3}$ SQUARE $\frac{1}{3}$ + 23 $\frac{1}{3}$ ÷ 6 $\frac{1}{3}$ CUBE ROOT $\frac{1}{3}$ + 19 $\frac{1}{3}$ ÷ 7 $\frac{1}{3}$ x 15

ANSWER

Number Square

Look at this one, sometimes called a number square.

You have to put all the digits from 1 to 9 in the missing boxes in order to make the numerical statements true.



You need to use what you know about numbers to solve the puzzle.

In the top row, $7 \times 5 = 35$

And only $1 \times 5 = 5$ and therefore 1 and 5 are the missing numbers. You will have to think which way round they should go.

$$1 \times 5 \times 7 = 35$$
 or

$$5 \times 1 \times 7 = 35$$

If 1 is in the middle column, then it will start 1×2 but there is no number you can add to make 18, so it must be 5 in the middle column.

Reading down the middle column will be:

$$5 \times 2 + 8 = 18$$

Now look at the bottom row. It has very high answer 99. You will need to use your highest number, 9, to make 99.

 $99 = 9 \times 11$. You will have to add 3 to the 8 to make 11 and the bottom row will read:

$$3 + 8 \times 9 = 99$$

Note: The puzzle setters do not follow BIDMAS rules, they simply do the sums in order, left to right or top to bottom.

You now need to fit 6 and 4 into the missing 2 spaces.

Here is the solution:

1	X	5	X	7	= 35
X		X		_	
6	+	2	+	4	= 12
X		+		Χ	
3	+	8	X	9	= 99
=18		=18		=27	

Exercise

29

=10

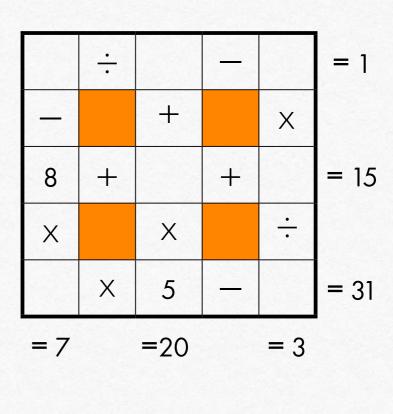
=21

Now here are two for you to solve:



30

=15



Doku Puzzles

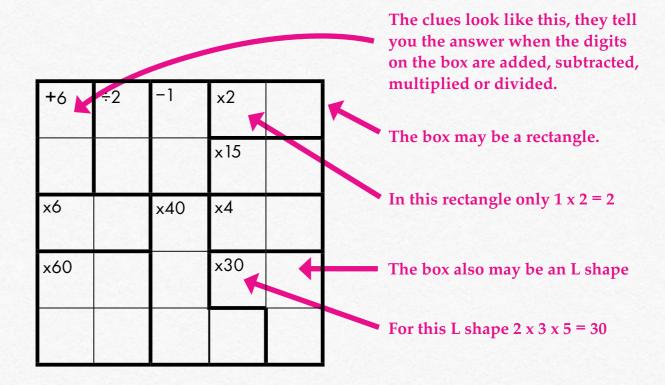
You will find several puzzles that have the suffix – doku. The origin of this is from an abbreviation to a Japanese phrase meaning – the digits must remain single. What all these puzzles have in common is that they are in a square, and in each line and row of the square, a digit must appear once only. How many digits depends on the size of the square. Here are a couple for you to try.

Ken KeMaths Doku

Similar puzzles, with names like Mathdoku and Calcudoku now appear in many newspapers and have been described as "Sudoku on steroids". But they are very do-able when you know how. In this example, we have a five by five grid.

The digits 1 to 5 appear in each row and column, but only once. There are clues to help you work out which digit goes where.

Look at this example:



The trick to solving these puzzles is not to try and solve them all in one go, like the brain teaser, but to jot down the possibilities, and from them deduce the solution.

Look at the first two rows of the puzzle. Start with the **x2 box**, since that is entirely in the top row. Only **2 x 1** give the answer **2** when multiplied. Jot the digits **1** and **2** in both the cells in the **x2 box**.'

For the **+6 box** two pairs of digits, only can add up to **6**: **1 + 5** and **2 + 4**. Remember **1** and **2** cannot be in the top row, so jot the digits **5** and **4** in the top cell, and the digits **1** and **2** in the bottom cell of the **+6 box**.

 $\div 2$ box: There are two pairs of digits that give the answer 2 when divided: $4 \div 2$, $2 \div 1$. Remember 1 and 2 cannot be in

the top row, so only **4** can be in the top cell and **2** must be in the lower cell of the ÷**2** box. Insert those numbers and circle them, since they are decided.

-1 box: There are four pairs of digits that give the answer 1 when subtracted: 5 - 4, 4 - 3, 3 - 2, 2 - 1. But 1, 2 and 4 are already used in the top row, so jot down 5 and 3 in the top cell, and 4 and 2 in the bottom cell of the -1 box.

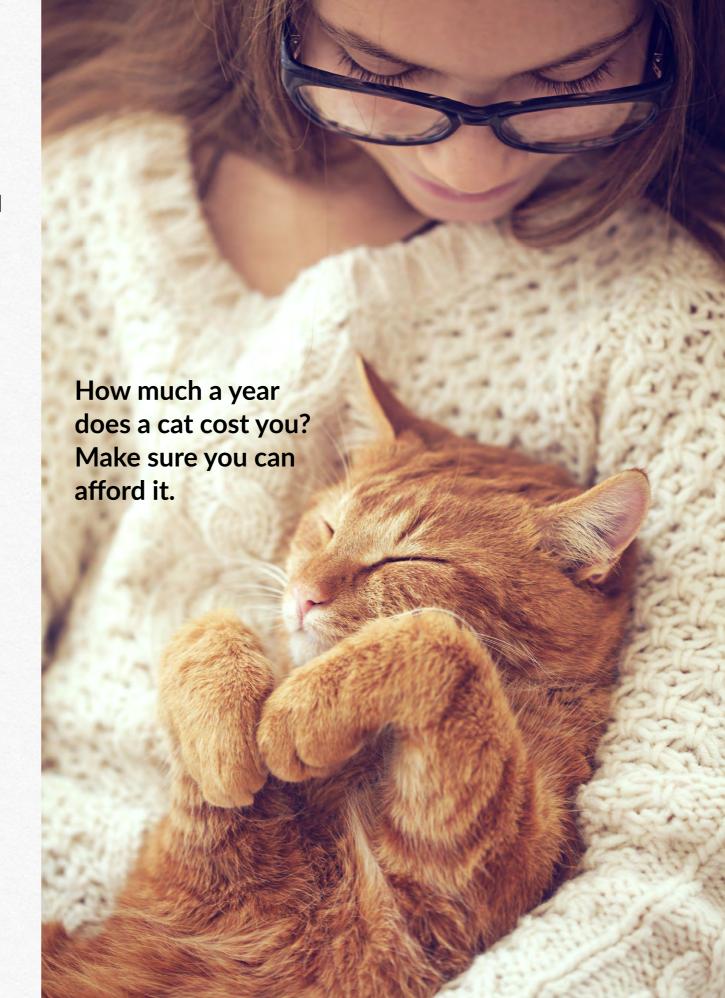
x15 box. Only 3 x 5 give the answer 15 when multiplied.

Jotting those options in the first two rows gives us:

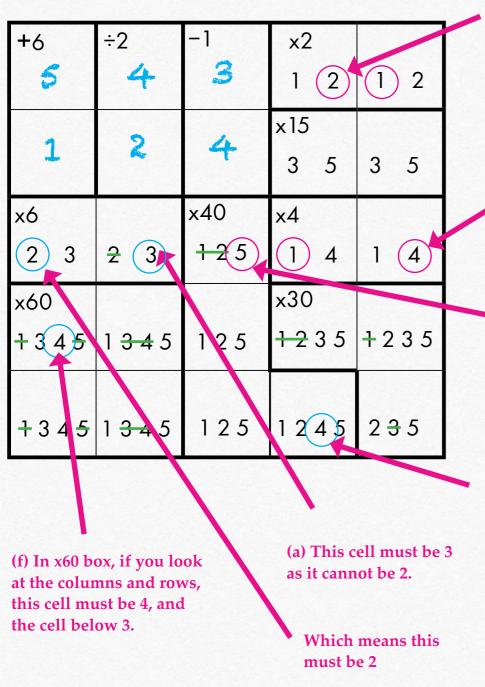
+6 5 4	÷2	-1 5 3	x2 2 1	2 1
1 2	2	4 2	x15 5 3	3 5

This is the only cell on the top row that can be 3. Therefore this cell must be 3, which means the top left cell must be 5. This is the only cell on the second row containing a 4. Therefore this cell must be 4

Now we have to look at the rest of the puzzle to solve the whole thing.



Now we add the various options for the next three rows and delete any that have been ruled out by the first **6** digits we have placed. Start with comment (a):



(e) Now we can identify the 2 and 1 in x2 box.

(d) Now the four on bottom row is known, we can identify the 1 and 4 in x4 box'

> (c) Look at this row and you'll see this cell must be 5

(b) x40 box must be made from 1 x 2 x 4 x 5. 4 is already used in the middle column, so this cell must be 4.

+6	÷2		-1		x2			
5		4	2	3		2		L
4		0		7	хl	5		
1		2	4	Ť	3	5	3	5
х6			x40		x4			
2	•	3	6	5		1	2	+
x60					x3	0		
4	1	5	1	2	3	5	23	3 5
0								
3	1	5	1	2	4	4	2	5

Now you can solve the last stage by guessing at a number and seeing if all the others then fit in:

If we make this cell 3, see what happens

+6 5	÷2	-1 3	x2 2	1
1	2	4	x15 3 5 3	3 5 5
x6	3	x40	x4 1	4
x60			x30	
4	1 5 1	122	3555	2353
3	155	121	4	252

The puzzle is solved!

Let's make it clear where we are now:



You may be familiar with the word 'deduce' from detective stories. When solving puzzles, whether they be in crime novels, word puzzles or number puzzles, you should remember the words of the great fictional detective, Sherlock Holmes:

'In solving a problem of this sort, the grand thing is to be able to reason backward. That is a very useful accomplishment, and a very easy one, but people do not practise it much. In the everyday affairs of life it is more useful to reason forward, and so the other comes to be neglected.'

Now here are two for you to solve:

Exercise

31

x8		x15		
	-1	+5	+7	
x6			+10	
	+9			x10
		x12		

32

+6	+9	x10		-1
	-2		×8	
×20		×15		
-3			-1	

Sudoku

All newspapers with puzzle pages include a Sudoku and lots of people enjoy solving them. They have been around for at least 100 years but really became popular about 20 years ago. They are usually graded as Easy, Hard or Fiendish/Killer and can be a very enjoyable way to passing small blocks of time, such as on a train or bus journey. There is even research that shows doing Sudoku can improve working memory in older people. Here's how to tackle them:

A Sudoku puzzle is solved when there is one of each digit in each row or column. However, the classic Sudoku is a 9 by 9 square, so you use the digits 1 to 9, and is in 9 blocks of 9. Each block must also have each of the digits 1 to 9 in it. There are no other clues, so you have to solve the puzzle just using this information.

There are, however, some tricks that can help.

Look at this puzzle that was in the photograph at the start of this Part.

8				3		1		
1					7		5	
		3	9	2		7		
	5		8	7		2	4	
2		9	4					
		8			5	3		
				6	2			
								1
3	1	6		5	8		2	

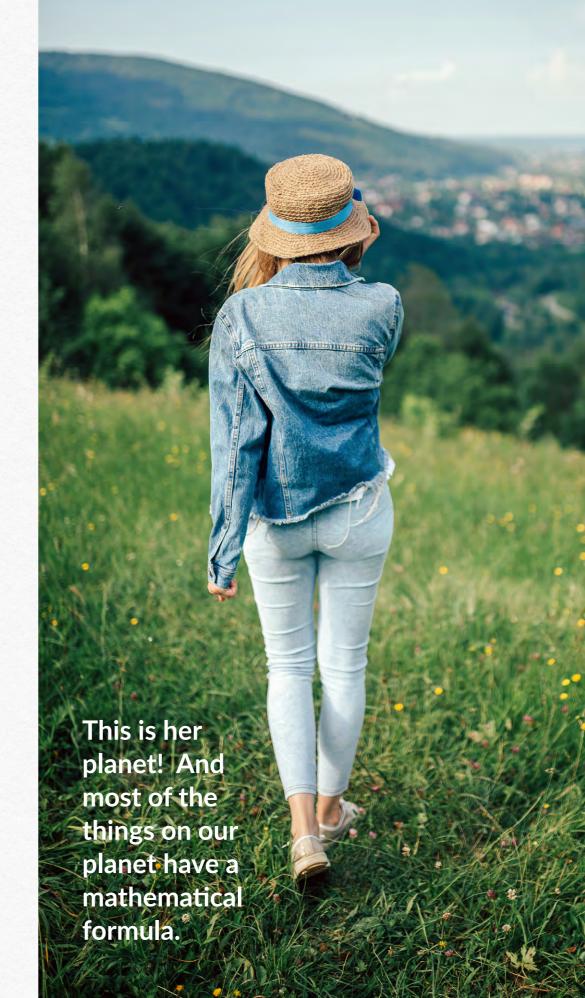
You need a method. This is one that you can try.

First, in your head look at which numbers occur in the grid several times. In this grid 1 is written 4 times. Let's mentally block out the rows, columns and blocks that have 1 in and see if there are any gaps.

8				3		1			
1					7		5		This must be 1
		3	9	2	14	7			
	5	1	8	7		2	4		
2		9	4			/			
		8			5	3			These must also be 1
			1	ó	2				Write in these two 1s and
								1	then mentally block out the row, column and blocks, as
3	1	6		5	8		2		done here in orange.

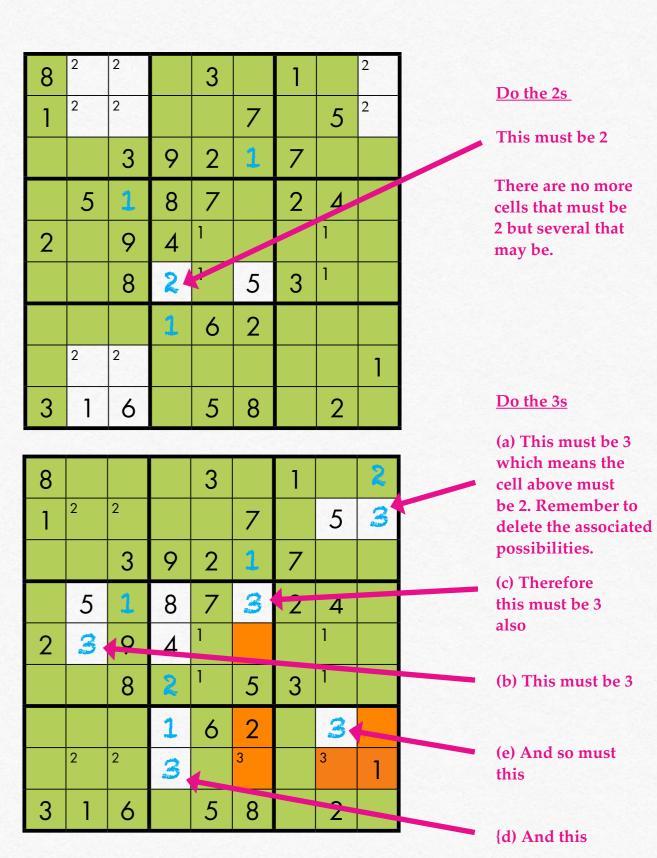
There are now very few places where the remaining 1s can go.

You only have a few squares left that could have 1 in them, so write a little note in them like you did for the Mathsduko.

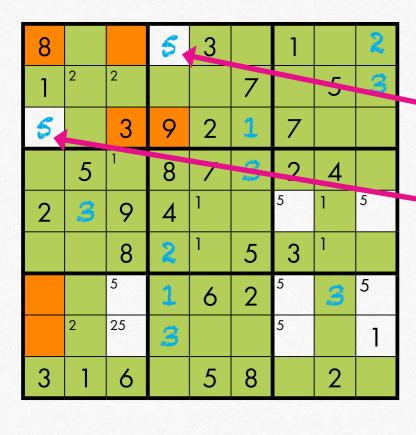


8				3		1		
1					7		5	
		3	9	2	1	7		
	5	1	8	7		2	4	
2		9	4	1			1	
		8		1	5	3	1	
			1	6	2			
								1
3	1	6		5	8		2	

The other numbers that occur frequently are 2, 3, 5 and 8, so do the same for them:



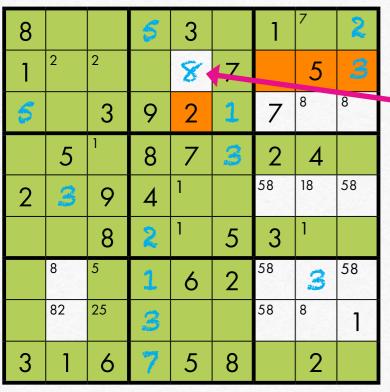
So we have all the 3s!



Do the 5s

This must be 5

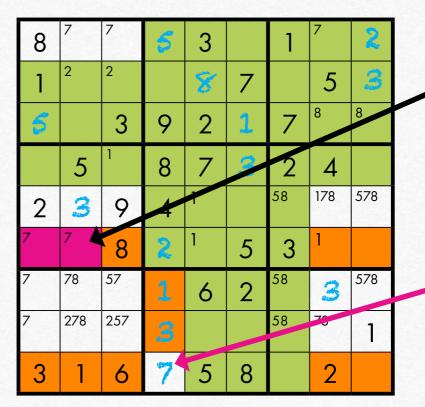
And therefore so must be this 5



Do the 8s

This must be 8

There are no more that must be 8 but some that may be.



And finally, the 4s, 6s and 9s

8	7	7	5	3	4	1	7	2
1	24	24		8	7	4	5	3
5	4	3	9	2	1	7	8	48
	5	1	8	7	3	2	4	
2	3	9	4	1		58	178	<i>57</i> 8
47	47	8	N	1	5	3	1	
47	478	457	1	6	2	458	2	4578
7	278	257	3	4		58	<i>7</i> 8	1
3	1	6	7	5	8	4	2	4

Do the 7s

Look at these two pink cells. One of these two cells must contain a 7, so no other 7s can be in this row.

This must be 7

There are no more that must be 7 but several that may be.

Do the 4s

This must be 4

And so must this

There are no more cells that must be 4 but many that may be.

Now do the 7s as the next most frequent

8	67	7	5	3	4	1	67	2
1	24	24	6	(C)	7	4	5	2
5	46	3	9	2	1	7	68	468
6	5	1	8	7	w	2	4	6
2	3	9	4	1	6	50	178	578
467	467	8	2	1	5	3	16	6
47	478	457	1	6	2	458	3	4578
7	278	257	(3)	4		568	768	1
3	1	6	7	5	8	4	2	4

Do the 6s

This must be 6

And so must this

As there are so few cells that have a 6 in, it follows that many of the remainder could be a 6.

Two cells that must be a **9**, and lots that could be. And that's it!

That is the end of your meticulous searching the cells.

It may look confusing but do NOT be daunted, as it will suddenly all come together very quickly.

Firstly, you can see that you have found **6** cells that have only one possible value, so write those in and delete the associated possibilities that you had written earlier:

8	679	7	V3	3	4	1	679	2
1	249	24	0	₩	7	49	5	3
5	46	3	9	2	1	7	68	468
6	5	1	8	7	3	2	4	69
2	3	9	4	1	6	58	178	579
467	467	8	2	9	5	3	16	6
479	4789	457	1	6	2	4589	2	45780
7	278	257	(1)	4	9	4568	768	1
3	1	6	7	5	8	49	2	49

Do the 9s

This must be 9

And so must this

	8	69	7	5	3	4	1	69	2
	1	249	24	6	Ì	7	49	5	3
	5	46	3	9	2	1	7	68	468
	6	5	1	8	7	3	2	4	9
	2	3	9	4	1	9	58	78	578
4		47	8	2	9	5	3	1	6
49	9	489	45	1	6	2	4589	3	4578
	7	28	25	3	4	9	568	68	1
	3	1	6	7	5	8	49	2	49

Doku Puzzles

Having done that, there are now **3** more cells with only one possible value, so do the same again:

8	69	7	5	3	4	1	69	2
1	249	24	6	W	7	49	5	3
5	46	3	9	2	1	7	68	468
6	5	1	8	7	3	2	4	9
2	3	9	4	1	6	58	<i>7</i> 8	578
4	7	8	2	9	5	3	1	6
9	489	45	1	6	2	4589	3	4578
7	28	25	3	4	9	568	68	1
3	1	6	7	5	8	49	2	4

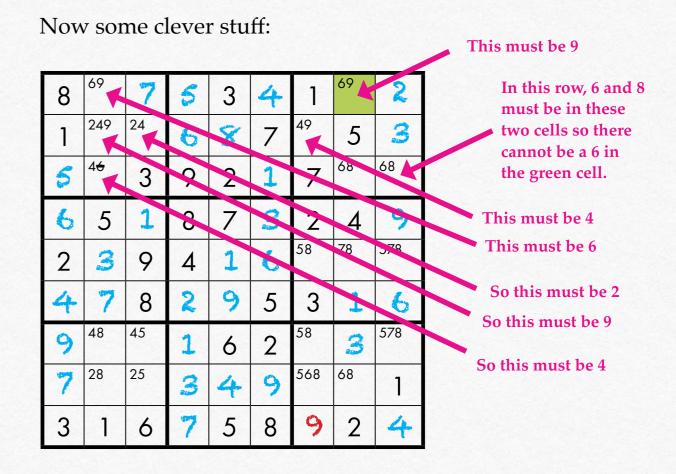
And do the same again:

8	69	7	5	3	4	1	69	2
1	249	24	6	8	7	49	5	3
5	46	3	9	2	1	7	68	68
6	5	1	8	7	3	2	4	9
2	3	9	4	1	6	58	78	578
4	7	8	2	9	5	3	1	6
9	48	45	1	6	2	58	3	578
7	28	25	3	4	9	568	68	1
3	1	6	7	5	8	49	2	4

At this point it is worth checking your blocks and rows to make sure that you have the right number of possible options in each. In the first row, there are two unfilled cells and two options: 6 or 9

In the second row there are 3 unfilled cells and 3 options: 2, 4 and 9.

Keep checking in this way to make sure that you have made no mistakes.



Which becomes this:

0	6	7	_	2	1	1	9	9
8	0		5	3	4			2
1	9	2	6	8	7	4	5	3
5	4	3	9	2	1	7	68	68
6	5	1	8	7	3	2	4	9
2	3	9	4	1	6	58	78	578
4	7	8	2	9	5	3	1	6
9	8	45	1	6	2	58	3	578
7	28	25	3	4	9	568	68	1
3	1	6	7	5	8	9	2	4

And now you should be able to complete the puzzle:

8	6	7	5	3	4	1	9	2
1	9	2	6	8	7	4	5	3
3	4	3	9	2	1	7	6	8
6	5	1	8	7	3	2	4	9
2	W	9	4	1	9	(X)	7	5
4	7	8	2	9	5	3	1	6
9	8	4	1	6	2	5	3	7
7	2	5	3	4	9	6	8	1
3	1	6	7	5	8	9	2	4

You may also find these puzzles as an app on your mobile phone and get lots more practice there. The advantage of that is that your app will tell your straight away if each digit you place is right or wrong.

Now here are three for you to solve.

Exercise

33 Easy

			5		1			2
6	9				8		3	1
5	1	2				4	7	
		1		3				9
3		4	8			1	5	7
	7		1	6	5	3		4
9	2		3		4		1	6
1		6	7		2	9		3
7	8	3	9	1	6			5

34 Hard

	6	1			2		9	
9					8			2
	8		6	7				4
5			9	1			4	
	4	9			3	5		
	1			6	4		8	3
3					6		1	
	9	6	2					7
	5		4		1	6	2	

			5			6	2	
			2		1		7	9
8	3							
		8	3	5		9		2
				7		4		
3	1	6		4				
6			7			2		8
	2	1						4
7		5			9			

35 Fiendish

Doku Puzzles

Top Tips for Solving Puzzles

- Start with easy puzzles. When you have become confident with these then try harder ones.
- Make sure that you have time, rushing those moments of careful reasoning is not a good idea.
- If today's puzzle does not work for you, just stop. You have better things to do.
- If you cannot see how a puzzle works, look at the answer and follow the reasoning backwards – this is good for cryptic crosswords too.

Finally, remember that those puzzles in the free newspaper that you pick up on your way to college or work are excellent Brain Training! You will start the day with your brain all nicely warmed up. "They sure needed maths to build that!" "And your bike, your jeans and your sweater!"



Answers to Part 3

1 What is the number of this archway in the Colosseum, Rome?



$$LII = 50 + 2 = 52$$

2 Sandor Palace



3 Is this an ancient archway?



4 How old is this Gaelic chapel?



Which of these numbers can be divided exactly by 4?

(a) 72 is in the 8 times table so must also be divisible by 4

(b)
$$66 \div 4 = 16 \text{ r2}$$

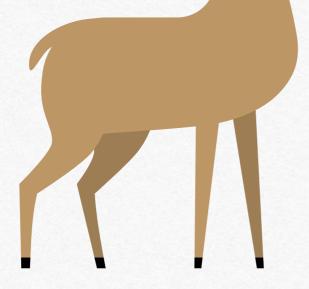
(c)
$$36 \div 4 = 9$$

(d)
$$62 \div 4 = 15 \text{ r}2$$



(a) No,
$$7 + 4 = 11$$

(b) Yes,
$$1 + 3 + 5 = 9$$



(c) No,
$$1 + 5 + 3 + 6 = 15$$

(d)
$$2 + 4 + 0 + 1 + 2 = 9$$

Which of these numbers is a multiple of **6**?

$$1 + 1 + 4 = 6$$
 which is a multiple of 3

$$1 + 2 + 0 + 4 = 7$$
 which is not a multiple of 3

$$2 + 4 + 0 + 1 + 2 = 9$$
 which is a multiple of 3

9 Which of these numbers can be divided exactly by **12**?

(a)
$$14 \div 4 = 3 \text{ r2 so } 114 \text{ is not a multiple of } 4$$

Answer: 114: No

(b)
$$8 \div 4 = 2$$
 so 408 is a multiple of 4

$$4 + 0 + 8 = 12$$
 which is a multiple of 3

Answer: 408: Yes

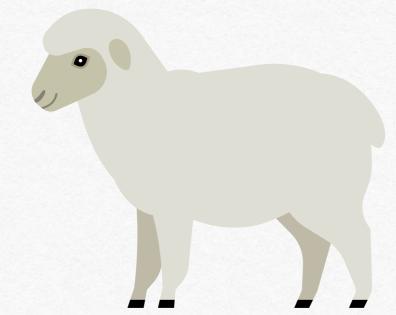
(c)
$$1 + 2 + 0 + 4 = 7$$
, not a multiple of 3

Answer: 1,204: No

(d)
$$12 \div 4 = 3$$
 so $24,012$ is a multiple of 4

$$2 + 4 + 0 + 1 + 2 = 9$$
 which is a multiple of 3

Answer: 24,012: Yes



10 Which of these numbers can be divided exactly by 15?

Numbers that can be divided by $\mathbf{5}$ must end on $\mathbf{0}$ or $\mathbf{5}$ and numbers that can be divided by $\mathbf{3}$ have a digit sum that is a multiple of $\mathbf{3}$. Both these properties must be met for a number to be a multiple of $\mathbf{15}$ (i.e. $\mathbf{3} \times \mathbf{5}$)

$$1 + 3 + 5 = 9$$
 which is a multiple of 3

Answer: 135: Yes

(b) 500 ends in 0

$$5 + 0 + 0 = 5$$
 so not a multiple of 3

Answer: 500: No

(c) 1,245 ends in 5

$$1 + 2 + 4 + 5 = 12$$
 which is a multiple of 3

Answer: 1,245: Yes

(d) 24,010 ends in 0

$$2 + 4 + 0 + 1 + 0 = 7$$
 which is not a multiple of 3

Answer: 24,010: No



You will have realised that you do not need to include the **0**s in the digit sum

11 Write down the first ten square numbers.

Answer: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

12 Write down the first ten cube numbers.

Answer: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000

Write down the first ten prime numbers.

Answer: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

14 Find a number that is both a square number and a cube number.

From your lists in questions 11 and 12 you can see:

Answer: 1 and 64 are both square numbers and cube numbers

15 Write down all the prime numbers that are a multiple of 3.

Other than 3, a multiple of 3 cannot be prime

Answer: 3

16 What is the smallest cube number that is a multiple of both 3 and 4?

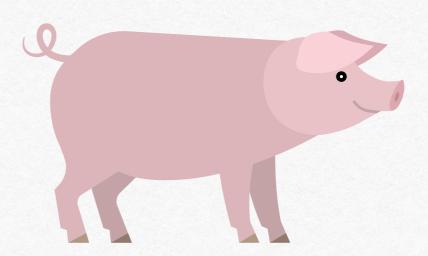
To be a multiple of $\bf 3$ and $\bf 4$, a number must have factors $\bf 3$ and $\bf 2 \times 2$.

To be a cube number, it must have $3 \times 3 \times 3$ as a factor and $2 \times 2 \times 2$

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 = 8 \times 27$$

= 216

Answer: 216



17 Find the smallest number that is both a multiple of 2, 3 and 4 and a factor of 600?

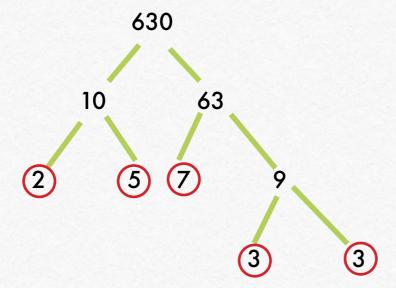
The smallest multiple of 2, 3 and $4 = 3 \times 4 = 12$ 12 is a factor of 600 (600 ÷ 12 = 50).

Answer: 12

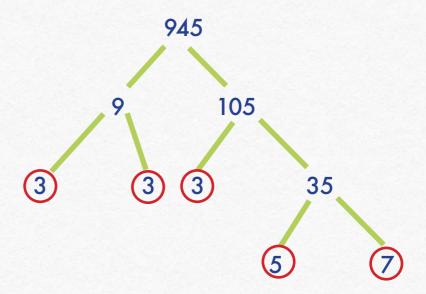
18 A number is a factor of 630 and of 945. What is the largest number that could be?

You could do this by lots of trail and error but you want to be smart, so look at factors.

First find the prime factors. Prime factors are factors which are prime numbers and therefore have no further factors themselves:



Now look at 945



$$630 = 2 \times 3 \times 3 \times 5 \times 7$$

 $945 = 3 \times 3 \times 3 \times 5 \times 7$

Common factors are
$$3 \times 3 \times 5 \times 7$$

= 315

Answer: 315

19 Lighthouse A flashes every 15 seconds and Lighthouse B flashes every 25 seconds. If they flash together at midnight, how long is it until they flash at the same time again?

You could make lists like this:

Lighthouse A: 0, 15, 30, 45, 60, 75 90, 105

Lighthouse B: 0, 25, 50, 75, 100

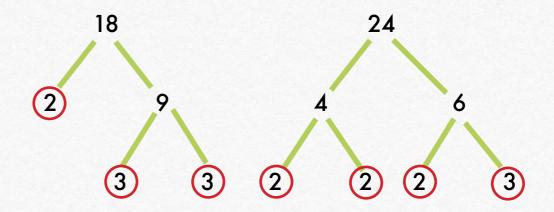
Or you could say, what is the smallest multiple of **25** that is also a multiple of **15**?

It must be 3×25 as $15 = 3 \times 5$

Answer: 75 seconds until the two lighthouses flash together again.

20 A florist has 24 red roses and 18 white roses. She wants to make the most possible identical bunches of both white and red roses. How many bunches can she make and how many roses will be in each?

It is not immediately obvious that you can use factors to answer this puzzle. But if you think that you are going to have to divide **24** into the same number of little bunches as **18** then you can see that you need to find a common factor.



 $18 = 2 \times 3 \times 3$

 $24 = 2 \times 2 \times 3 \times 3$

 $2 \times 3 = 6$ is common to both

Answer: The Florist can make 6 identical bunches with 4 red roses and 3 white roses in each.

21 Tell your friend to think of a number between 1 and 10.

Tell them to double it and then add 6.

Now tell them to divide the answer by **2** and subtract the number they first thought of.

You can now tell them that the answer is 3. It always will be!

Can you work out why?

To explain how this trick works you will use a bit of algebra. Let your friend's number be x

Double it and add 6 then gives you:

$$2x + 6$$

Dividing the answer by 2 gives you:

$$x + 3$$

Subtract the number they first thought of (i.e. x) then gives you **3**, so the answer is always **3**.

22. Ask your friend to think of a two-digit number between **10** and **100**.

Now ask them to add the two digits together, so if they thought of 35, 3 + 5 = 8

Now ask them to subtract this answer from their original number (e.g. 35 - 8 = 27)

Ask them to add the digits in the resulting number together and write down that answer.

Show them a playing card, nine of clubs, is that their number? It is!

Can you work out why?

Using algebra again any two-digit number written xy is actually equal to 10x + y.

If you add the digits together you have x + y.

Doing the subtraction:

$$10x + y - x - y$$
 will equal $9x$.

As you saw from the rules of divisibility, adding the digits together of any multiple of **9** will give the answer **9**.

23 Ask your friend to think of a number between 1 and 10.

Tell them to multiply their number by 9.

Now tell them to add the digits of that answer together and subtract **5**.

If 1 is A, 2 is B, 3 is C etc. ask them to think of the letter represented by their last answer.

Now write down a country that starts with that letter.

Write down an animal that starts with the last letter of the country.

Write down a colour that starts with the last letter of the animal.

You show them what they have written down before they show you – an orange kangaroo from Demark.

Can you work out why this is almost always the answer. Can you think of any other possible answer?

Rather like the last trick, this is all to do with nine. Having multiplied a number, x, by 9 to get 9x, adding the digits together will always give the total 9. Subtract 5 from 9 and you have 4.

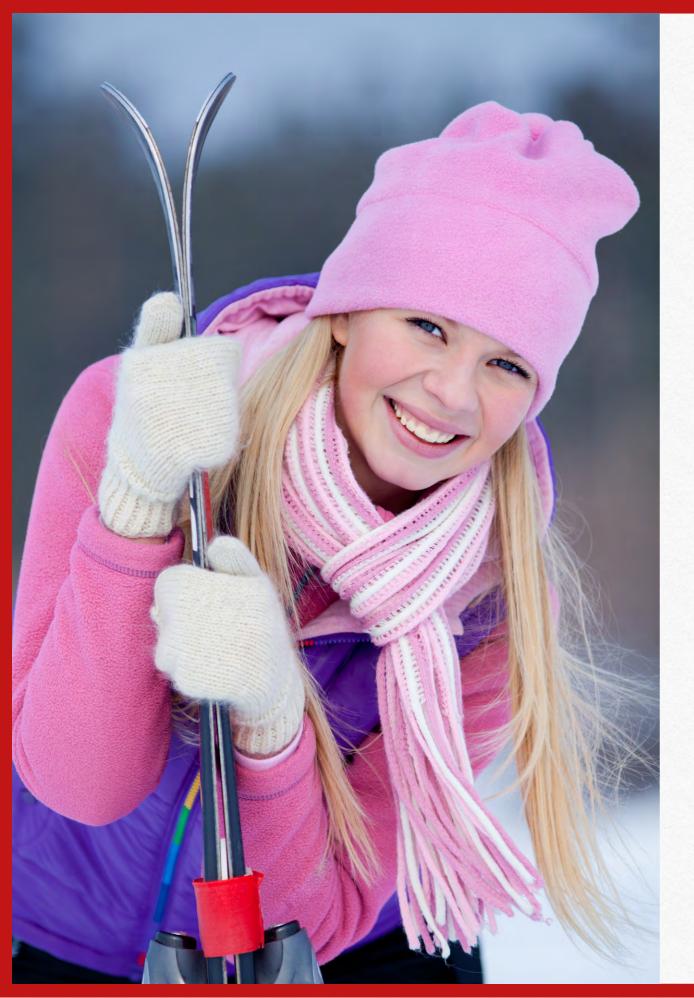
Using the method given, 4 will represent D.

Denmark is the only country that almost everyone thinks of that begins with D

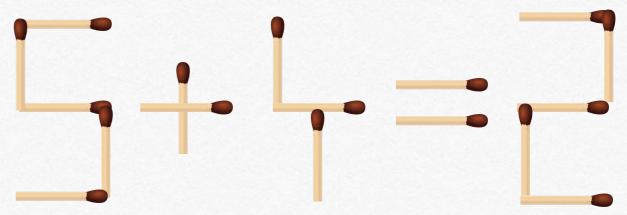
Similarly, most people then come up with Kangaroo for the animal followed by Orange for colour.

Very rarely someone might come up with Koala and thus a colour such as Amber or Aquamarine.

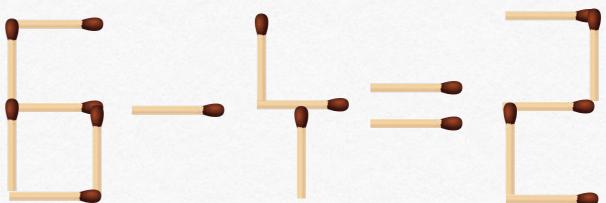
It is possible that someone might say Domenica or Djibouti for a country, but we have never known this to happen.



24 Can you move one matchstick to make this equation correct?



Answer:



$$\begin{array}{c|c} 36 & \div 4 & \text{DOUBLE} \\ 9 & 18 & 12 & 60 & 63 & 7 \end{array}$$

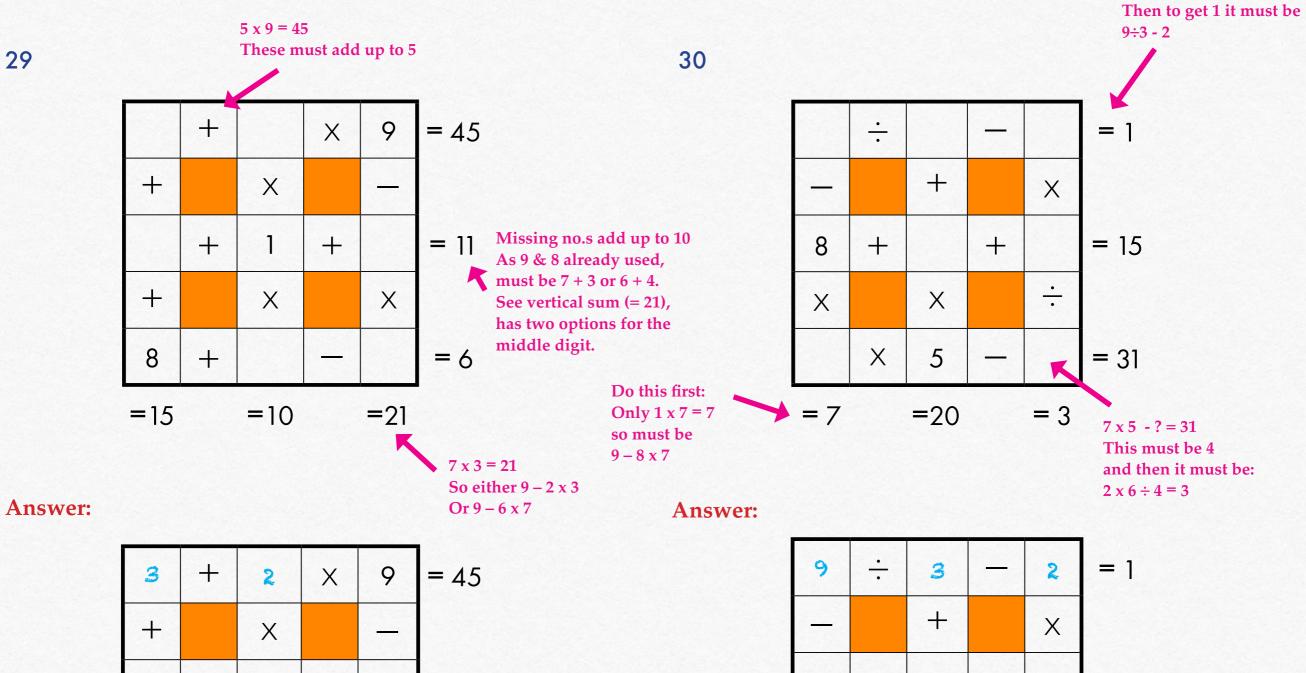
ANSWER

ANSWER

ANSWER

ANSWER





9	<u>.</u>	3	_	2	= 1
		+		X	
8	+	1	+	6	= 15
×		X		<u>.</u>	
7	Χ	5	_	4	= 31
= 7		=20		= 3	

29

31 First put in some options:

0		15		
x8		x 15		
2 4	2 4	135	135	135
	-1	+5	+7	
1		1234	2 4	2 4
x6			+10	
3 2		1234		1
	+9			x10
2 3				2 5
		x 12		
		3 4	3 4	25

So now we have:

x8		x15		
4	2	15	15	3
	-1	x5	+7	
1	5	3	2	4
x6	4	0	+10	1
3 2	+	2		•
	+9			x10
2 3				2 5
		x 12		
		3 4	3 4	2 5

b) x8 must be 1 x 2 x 4 1 is already in top row These must be 2 and 4

(a) Start here as x15 box in top row. Only $1 \times 3 \times 5 = 15$

(c) +7 must be 1 + 2 + 4Top cells must be 2 & 4because of x8, see (b)

So this can only be 1

And this can only be 1

d) Only 2 x 5 = 10
This means no other
2 or 5 can be in column.
This helps in +7 & x15 boxes.

e) Only 2 x 3 = 6 This means no other 2 or 3 can be in column. This helps in x8 box. See how the top green cell had to be 3 because because 1, 2 and 4 are already in that row. Therefore the bottom green cell must 2.

That left the top orange cell to be **5**, since all the other numbers are used up in that row. Therefore the bottom orange cell needs to be **4**.

So now we have more numbers to add and options to delete:

x8	2	x15 1 5	1 5	3
•	-1	+5	+7	
1	5	3	2	4
x6 3	4	2	+10	1
	+9			x10
2				5
		x12		6
		4	3	2

So now you can complete the puzzle:

x8 4	2	x15	1	3
1	-1 5	+5 3	+7	4
x6 3	4	2	+10 5	1
2	+9 3	1	4	x10
5	1	x12 4	3	2

32 First a few options

+6	+9	x10		-1	
123		125	125		
123			125		
	-2		x8		
123			124	1 2 4	
x20		x 15			
4 5	4 5	1 3 5	1 3 5	124	
-3			-1		
4 5	1 2	1 3 5			

Only $4 \times 5 = 20$, so 1 and 3 can only be in the green cells so the yellow cell must be 2

And this must be 5.

So now we have:

+6	+9	x10		-1
123		125	125	
123			125	
	-2		x8	
123			124	124
x20		x15		
5	4	1 3	1 3	2
-3			-1	
4	1	5	2 3	2 3

By deleting some of the options we can fill in some more numbers:

+6	+9	x10	4		
123		12	1.5		
123			1 5		
	-2		x8		
1 2 3	3 5	3 5	1 4	1 4	
x20		x15			
5	4	1 3	1 3	2	
-3			-1		
4	1	5	2	3	

1 and 5 must be in one of these pink cells, since there is a 2 in this column

So this must be 2

So this must be 4

So this must be 3

So now we have:

+6	+9	x10		-1
123		2	15	
123			1 5	
	-2		x8	
2	5	3	4	1
x20		x15		
x20 5	4	1	3	2
-3 4			-1	
4	1	5	2	3

And now you can compete the puzzle:

+6	+9	x10		-1
1	3	3 2		4
3	2	4	1	5
2	-2 5	3	×8 4	1
x20	4	×15	3	2
-3 4	1	45	-1 2	3

Sudoku

33 Easy

When you go through the digits 1 to 9, you will find all the ones are there and that you can then place all the 3s and 5s. From there you can go back and do the 4s and then the 2s. This is how your square should then look:

4	3		5		1			2
6	9		2	4	8	5	3	1
5	1	2			3	4	7	
2	5	1	4	3			2	9
3		4	8	2		1	5	7
2	7		1	6	5	3	2	4
9	2	5	3		4		1	6
1	4	6	7	5	2	9		3
7	8	3	9	1	6	2	4	5

As you can see that the bottom middle square just needs an **8**, do **8**s next and then you can complete the **2**s:

4	3	8	5		1			2
6	9		2	4	8	55	3	1
5	1	2			3	4	7	8
2	5	1	4	3		8		9
3		4	8	2		1	5	7
8	7		1	6	5	3	2	4
9	2	5	3	8	4		1	6
1	4	6	7	5	2	9	8	3
7	8	3	9	1	6	2	4	5

Now add the ninth digit in squares, rows and columns that already have eight:

4	3	8	5		1	6		2
6	9	7	2	4	8	5	3	1
5	1	2	6		3	4	7	8
2	5	1	4	3		8		9
3	6	4	8	2		1	5	7
8	7	9	1	6	5	3	2	4
9	2	5	3	8	4	7	1	6
1	4	6	7	5	2	9	8	3
7	8	3	9	1	6	2	4	5

And then doing that again and again until the puzzle is complete:

4	3	8	5	7	1	6	9	2
6	9	7	2	4	8	45	3	1
5	1	2	6	9	3	4	7	8
2	5	1	4	3	7	8	6	9
3	6	4	8	2	9	1	5	7
8	7	9	1	6	5	3	2	4
9	2	5	3	8	4	7	1	6
1	4	6	7	5	2	9	8	3
7	8	3	9	1	6	2	4	5

34 Hard Suduko

In this puzzle you can fill in all the 1s and 4s of the digits straight away but need to have noted the possible options for the remaining unfilled squares:

This is how your square should look once you have been through the digits 1 to 9:

4	6	1	3	35	2	378	9	58
9	37	357	1	4	8	37	6	2
2	8	235	6	7	9	1	35	4
5	23	8	9	1	7	2	4	6
267	4	9	8	2	3	5	7	1
27	1	2	5	6	4	9	8	3
3	2	4	7	589	6	8	1	59
1	9	6	2	358 (5	4	35	7
8	5	7	4	39	1	6	2	89

In red circles are the cells with only one remaining option, so fill them in and delete the associated possibilities:

4	6	1	3	5	2	78	9	58
9	37	357	1	4	8	37	6	2
2	8	35	6	7	9	1	35	4
5	3	8	9	1	7	2	4	6
67	4	9	8	2	3	5	7	1
7	1	2	55	6	4	9	8	3
3	2	4	7	59	6	8	1	59
1	9	6	2	38	5	4	35	7
8	5	7	4	39	1	6	2	9

Do the same again:

4	6	1	3	5	2	78	9	8
9	7	35	1	4	8	37	6	2
2	8	35	6	7	9	1	35	4
5	3	8	9	1	7	2	4	6
6	4	9	Ů0	2	3	5	7	1
7	1	2	5	6	4	9	8	3
3	2	4	7	9)	6	8	1	5
1	9	6	2	38	5	4	3	7
8	5	7	4	3	1	6	2	9

Do the same again and then complete the puzzle:

4	6	1	3	5	2	7	9	8
9	7	S	1	4	8	3	6	2
2	8	3	6	7	9	1	55	4
5	3	8	9	1	7	2	4	6
6	4	9	8	2	3	5	7	1
7	1	2	45	6	4	9	8	3
3	2	4	7	9	6	8	1	5
1	9	6	2	8	5	4	3	7
8	5	7	4	3	1	6	2	9

35 Fiendish Sudoku

This is how your square should look once you have been through the digits 1 to 9:

1	9	7	5	38	348	6	2	3
45	6	4	2	38	1	358	7	9
8	3	2	6	9	67	15	4	15
4	7	8	3	5	6	9	16	2
2	5	9	168	7	68	4	3568	356
3	1	6	9	4	2	578	58	57
6	4	3	7	13	345	2	9	8
9	2	1	68	368	3568	7	356	4
7	8	5	146	2	9	13	136	136

1	9	7	5	8	48	6	2	3
5	6	4	2	38	1	5 8	7	9
8	3	2	6	9	67	15	4	15
4	7	8	3	5	6	9	1	2
2	5	9	18	7	68	4	3568	356
3	1	6	9	4	2	578	58	57
6	4	3	7	1	5	2	9	8
9	2	1	8	368	358	7	3 5 6	4
7	8	5	146	2	9	13	136	16

In this block, 1 and 5 must lie in the bottom row, so you can delete 5 from the blue shaded cell.

In this block, 1, 3 and 6 must lie in the bottom row, so you can delete 3 and 6 from the blue shaded cell.

In red circles are the cells with only one option, so fill them in and delete the associated possibilities:

Next fill in the cell with only one option and delete the associated possibilities:

Now you can complete the puzzle:

1	9	7	5	8	4	6	2	3
5	6	4	2	3	1	8	7	9
8	3	2	6	9	7	15	4	15
4	7	8	3	5	6	9	1	2
2	5	9	18	7	68	4	3 5 68	3 5 6
3	1	6	9	4	2	57	5 8	57
6	4	S	7	1	5	2	9	8
9	2	1	8	36	3	7	5	4
7	8	5	4	2	9	13	36	16

By looking at the rows and columns, see that in this block, 5 and 7 must lie in the bottom pink cells, also see you can delete 5 from the blue shaded cells.

1	9	7	5	8	4	6	2	3
5	6	4	2	3	1	8	7	9
8	3	2	6	9	7	15	4	15
4	7	8	3	5	6	9	1	2
2	5	9	18	7	68	4	368	36
3	1	6	9	4	2	57	8	57
6	4	S	7	1	5	2	9	8
9	2	1	8	36	3	7	5	4
7	8	5	4	2	9	13	36	16

Happy puzzling!



Q1

What is this number carved into an old wall in Rome?



Q2

Is **2,022** divisible by:

- (a) **2**
- (b) **3**
- (c) **4**
- (d) **12**?



Your Brain Workout



Q3

What are the first two cube numbers that are multiples of **2**?

Q4

What is the smallest number that is a multiple of both 15 and 25?



Your Brain Workout

YOUR BRAIN WORKOUT



Q5

Find the answer to this brain teaser:

$$100 \rightarrow \div 4 > SQUARE \times 9 \rightarrow \div 8 > DOUBLE$$

YOUR BRAIN WORKOUT

Q6

Fill in the missing four numbers in this Number Square:

	x	1	+	5	=12
+		х		_	
2	х		÷	4	=4
х		_		х	
6	+		÷	9	=1
=54		=5		=9	-



YOUR BRAIN WORKOUT



Answers

- Q1 LXXXIX = 89
- Q2 2,022 is divisible by (a) 2 (b) 3 but not (c) 4 or (d) 12
- Q3 First two cube numbers that are multiples of 2 are 8 and 64
- Q4 Smallest multiple of both 15 and 25 is 75
- Q5 100 ÷ 4 $_{ROOT}^{SQUARE}$ x 9 + 35 ÷ 8 DOUBLE = 20
- Q6 $7 \times 1 + 5 = 12$
 - $2 \times 8 \div 4 = 4$
 - $6 + 3 \div 9 = 1$



PART 4
A QUICK GUIDE TO THE SALES

Everyone Loves a Bargain

There are times when people are so keen to get a bargain that they will camp outside the big stores waiting for them to open for a special product launch or for a seasonal sale.

Below, customers queuing in New York for a newly launched mobile phone.



Most of us have better things to do with our time than queuing. That does not mean that we should not pay attention to sales, when products can be sold at a substantial reduction, but we need to understand how to make the most of them.

Before you go bargain hunting:

- Make a list of what you NEED
- Make another list of what you might WANT
- Work out exactly how much you can afford to spend

Do some research on where you might find your wants and needs, when the discounts start and end and if you can shop online or do you have to go to the store.

Also have a look at pre-sale prices at a variety of different stores. You may find prices are already substantially cheaper from one supplier than another.

Everyone Loves a Bargain 185



Fractions, Decimals and Percentages

Before we go any further, let us remind ourselves of what those magically enticing "25% OFF" signs mean in terms of money.

A percentage is another way of writing a fraction.

While fractions can be any number of pieces of any sized part, for example: $\frac{1}{4}$, $\frac{2}{9}$, $\frac{3}{20}$, $\frac{8}{17}$ etc., a percentage is always a fraction out of one hundred.

25% is simply another way of saying $^{25}/100$

In a previous Step you looked at fractions, decimals and percentages and how to convert between them.

Calculating a percentage of an amount is often done most quickly by using a fraction.

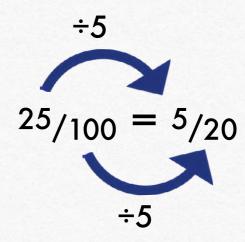
You know that you can write a fraction in its lowest times by diving the top number and bottom number of the fraction by the same number – or common factor.

For Example

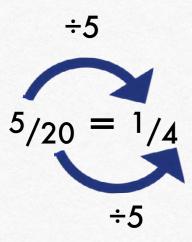
What is **25%** as a fraction in its lowest terms?

$$25\% = \frac{25}{100}$$

From your rules of divisibility, you can see that both **25** and **100** can be divided by **5**:



And both **5** and **20** can also be divided by **5**:



Therefore, we can say:

$$25\% = \frac{25}{100} = \frac{1}{4}$$

Answer: $25\% = \frac{1}{4}$



Food for Thought

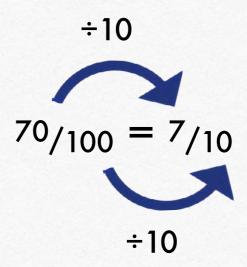
You might have noticed that both and **25** and **100** can be divided by **25** and then you can cancel your fraction down in one go. That is great but it does not matter if you use two or even more stages. If doing several divisions by small numbers works best for you then that is absolutely fine.

Another Example

What is **70%** as a fraction in its lowest terms?

$$70\% = \frac{70}{100}$$

Both 70 and 100 can be divided by 10



Answer: $70\% = \frac{70}{100} = \frac{7}{10}$

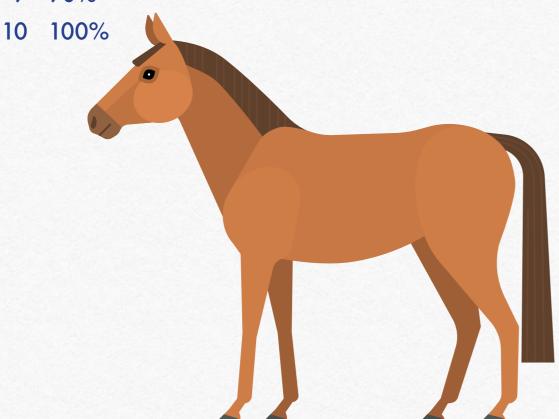
Now try some for yourself.



Exercises

Write each percentage as a fraction in its lowest terms.

- 1 10%
- 2 50%
- 3 20%
- 4 30%
- 5 60%
- 6 75%
- 7 80%
- 8 5%
- 9 90%



Calculating the Discount



When a price is marked with a percentage of the price taken off, this can translate into a sentence:

Normal price less the discount (% OFF) equals the sale price, that is the price to be paid.

Previously you learned how to calculate all percentages but in the sales the discounts tend to be those that you have just worked out in the last exercises. For these simple fractions it can be quickest to calculate the fraction by division.

Example

A dress normally costing £35 is reduced by 20% in the sale. Calculate the reduction.

$$20\% = \frac{1}{5}$$



To find 20% of an amount you divide by 5

20% of £35 =
$$\frac{1}{5}$$
 of 35 = 35 ÷ 5 = £7

That example was written out in full, but you will be doing this in your head when you are shopping. Your head will probably just say:

Remember these calculations work out the discount, not the final amount you will pay.

Some of the fractions that you worked out in the exercises are not unit fractions, for example:

$$40\% = \frac{2}{5}$$

There are two ways that you could consider calculating the discount, in your head:

First calculate 10% by dividing by 10, then multiply by 4

Calculate ²/5 by dividing by **5**, then multiply by **2**

Example

A blazer that usually costs £45 is marked down by 40% in the sale. What amount of money is saved?

Method 1:

$$45 \div 10 = 4.5$$

$$4.5 \times 4 = 18$$

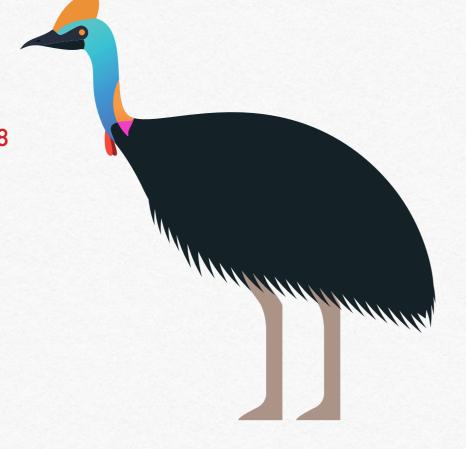
Answer: £18

Method 2

$$45 \div 5 = 9$$

$$9 \times 2 = 18$$

Answer: £18



Exercises

- 11 In a store, all prices on Black Friday are reduced by 20%, what is the discount on these goods that are normally priced as follows:
- (a) A shirt at £15
- (b) A pair of trainers at £30
- (c) A television at £300
- 12 In a store all prices on Cyber Monday are reduced by 25%, what is the discount on these goods that are normally priced as follows:
- (a) A mobile phone at £240
- (b) A tablet at £360
- (c) A pair of headphones at £32
- 13 A fashion shop is closing down and everything is to be sold, calculate these discounts:
- (a) 50% off a pair of shoes originally costing £35
- (b) 40% off a shirt originally costing £25
- (c) 60% off a coat originally costing £125
- (d) 75% off a dress originally costing £48

Calculating the Sale Price

Once you have calculated the discount or the saving you are going to make, that is not the end of the story. You need to know how much you are going to have to spend.

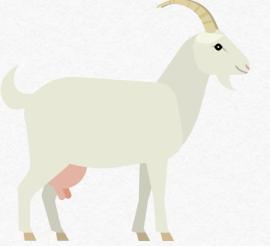
That needs another calculation, a subtraction. Let us have another look at that dress in the earlier example:

Example

A dress normally costing £35 is reduced by 20% in the sale. Calculate the reduced price.

Reduction will be $35 \div 5 = £7$

New price will be 35 - 7 = £22



Exercises

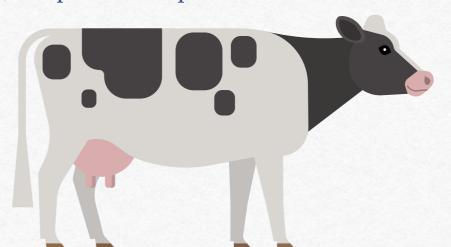
14 In a store, all prices on Black Friday are reduced by 20%, what is the reduced price on these goods that are normally priced as follows:

(You worked out these discounts in Q11)

- (a) A shirt at £15
- (b) A pair of trainers at £30
- (c) A television at £300
- 15 In a store, all prices on Cyber Monday are reduced by 25%, what is the reduced price on these goods that are normally priced as follows:

(You worked out these discounts in Q12)

- (a) A mobile phone at £240
- (b) A tablet at **£360**
- (c) A pair of headphones at £32



16 Find the sale price of these goods:

(These are new questions.)

- (a) A television normally costing £800 reduced by 10% in the sale.
- (b) A pair of socks normally costing £2.50 reduced by 10% in the sale.
- (c) A jumper normally costing £40 reduced by £25% in the sale.
- (d) A dress normally costing £40 marked down by 20% in the sale.



Food for Thought

Did you notice something in that last question? Two items had the same reduction of **10%** but the actual saving was very different.

The saving on the television was a whopping £80 whilst the saving on the socks was a measly 25 p.

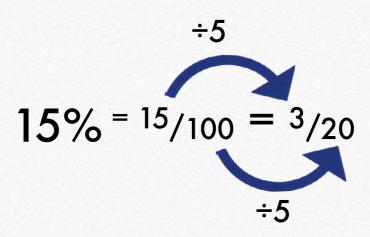
Just remember:

10% of nothing is nothing but 10% of a lot is worth having!

More Percentages

The percentages that you have calculated so far were either multiples of 10 or multiples of 25, so simple to divide.

That is not always the case, for example what about 15%?



Your head is unlikely to want to work that out whilst you are out shopping, so you need another way.

Can you see that 15% is 10% plus 5%?

So the best way to do this one is to find 10% and then halve that to find 5%.

Example

Today, there is **15**% off a packet of Thai fish cakes normally costing **£7**. By how much is the price reduced?

10% of £7 = 70p

$$5\%$$
 of £7 = $70 \div 2 = 35p$
Total discount = $70 + 35 = 105p$ or £1.05

You can use the same method to find other percentages such as 3%. This time you would find 1% and then multiply by 3 to get 3%.

In this next example, note that a percentage can be for a rise as well as for a deduction.

Example

I am earning £25,000 a year but have just been awarded a 3% pay rise. What will my new salary be?

This method of finding 1% of something and then scaling up is called the **unitary method**, as **unit** means 1. It is a useful fallback method for lots of calculations and we will use it again in the next part.

For this next set of questions do not worry about using a 'correct method'. Just use the method that works best for you – that is the correct one.

Do make sure that you read the questions carefully, sometimes you are asked for the discount, or additional amount, and sometimes you are asked for the actual sum of money after the reduction or the additional amount.

Exercises

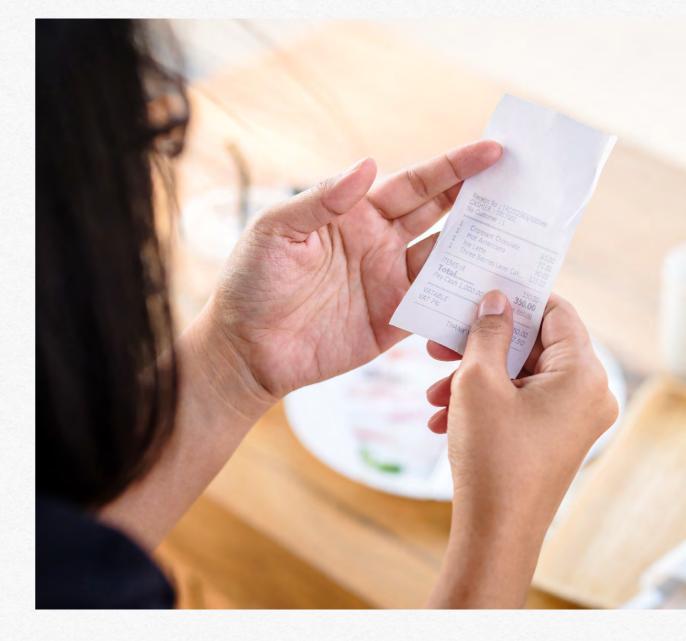
- 17 On the 15th February, all boxes of chocolates were reduced by 15%.
- (a) What is the reduction for a box of chocolates normally costing £5?
- (b) What is the reduced price for a box of chocolates normally costing £7.20?



The day after a special event such as Valentines Day or Mothers Day can be a good time to pick up bargains.

- 18 My annual salary was £30,000 but I have been awarded a pay rise of 4%. What will my new salary be?
- 19 The value of my car has decreased by 15%. If I bought it for £7,500, what is it worth now?
- 20 A restaurant adds 15% service charge to the bill. If my bill comes to £36, what service charge will be added?

You should always check about service charge when you are eating out or sitting down for drinks as it can be a bit of a shock when you get your final bill.



Always check your bill. Good practice for life.

More Percentages 195

Finding the Best Value

When we are shopping, we want to find the best value. If two products are identical, then the best value will be the one at the lowest price.





A: £25

B: £35 NOW 20% OFF Here we have two identical jeans, so which one is better value?

Jeans B:

New price =
$$35 - 7 = £28$$

Even with the discount, Jeans B is still more expensive so Jeans A is the best value.

Shops can often try to mislead you into thinking a big discount means better value, so make sure that you are able to work it out for yourself.

Exercises

21 Shop A is selling a blue jacket for £40. Shop B is selling the same jacket for £48 but with a 10% discount. Which is better value?

22 Deluxe Modes is selling a dress for £48 marked down by 25% in the sale whilst Cheap Tat is selling the same dress for £25. Which is better value?

23 I can buy a laptop online for £350 but with a 15% delivery charge added. I can buy the same laptop in store for £399. Which is better value?

24 Classy Clothes has a 30% off everything sale whilst Swanky Style has a 15% off everything sale. Before the sales, a jumper was £30 at Classy Clothes but the same jumper was priced at £24 at Swanky Style. Which is the better value in the sales?



Sometimes you want to work out which is better value when the items are not identical.

That is harder, as the items cannot be exactly compared.

You might feel that it is worth paying more for a good quality woollen jacket than cheap cotton one, as the woollen one will last longer.

Also think about buying food. When you compare prices, the cheaper product is not always the best. You need to look at the labels and make a decision based on quality as well as price.



Finding the Best Value 197

Real Life Problems

In this part we have looked at how a percentage can be added or subtracted to a value to give a new value.

Percentage deductions:

Sale prices

Price reductions

Loss of value

Percentage increase:

Pay rises

Service charge

Delivery charge



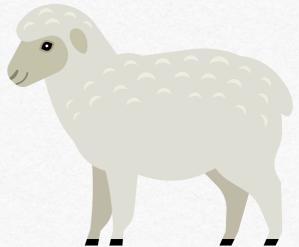
There are other percentages that you may come across, such as tax and commissions. For example, if you sell a house though an estate agent, the agent will charge you a commission.

Similarly, if you have a job selling houses, or other services, you may be paid a commission on the sales that you achieve.

Percentages are therefore an important part of real life that you need to be able to calculate.

In the exercises that you have done so far in this part, the prices were all whole numbers. You know that in the real world, prices are frequently a mix of pounds and pence. However, just as you did in the earlier parts, it is perfectly sensible to round these prices off to the nearest whole number.

Therefore, a refrigerator priced at £249.99 you would consider as £250 or a shirt selling at £19.99 you would think of as £20.



Real Life Problems



Remember, you do these kinds of calculations to give yourself an idea of what you can afford to spend. Therefore, you do not need to calculate everything to the nearest penny or pound.

Try this last set of exercises by rounding any non-exact numbers to a sensible whole number.

Exercises

- 25 A shirt normally costing £19.99 is marked down 10% in the sale. What is the reduced price?
- 26 April and Beattie have a meal in the pizzeria that comes to £25.87. Service charge of 15% is then added to the bill. What is the new total?
- 27 In the end of season sale all prices are marked down by 20%. What will Charlie pay for a refrigerator that originally sold for £249.99?

28 Donna is a doctor in training and her starting salary is £28,243. After two years, the salary should increase by 15%. Estimate what Donna's new salary will be?



- 29 Edwina wants to buy a new dress for her best friend's wedding. She has found one online for £49.99 but then sees the same dress in a shop for £55.49 but marked down 15% in a sale. Which purchase is the best value?
- 30 Fifi is a personal assistant with a salary of £25,150. Her boss earns £118,700 and has just awarded herself a 1% pay rise but tells Fifi that she is so pleased with her work that she is giving her a 4% pay rise. Who has the greater increase in pay?

Real Life Problems 199

Using a Calculator App

So far this part has encouraged you to carry out quick calculations with percentages without having to dig out a calculator.

However, there are times when you need exact answers and then a calculator is a really useful tool. (If you are at your desk, then you may wish to use a spreadsheet but we will look more at this later)

There are three types of calculator that you may have around.

1. Basic calculator

This one has the four functions: +, -, \times and \div and very little else. It is therefore simple to use. It is also small and fits easily into a handbag.



2. Scientific Calculator

This is probably the type that you used at school or college.



It has lots of functions, which can be useful, but is bigger and not as handy for you bag or pocket.

(Pink might not be your favourite colour but a pink calculator is easy to find in your bag or on your desk.)

Using a Calulator App 200

3. Calculator app

If you have a smartphone, there will almost certainly be a calculator app, it may be grouped as a utility:



If this key says C not AC it may mean that there is a number stored in the memory. Press the key and it should change to AC.

This is a good basic calculator and you generally have it to hand. It is particularly good for calculating percentages so after this introduction, this section is going to show you how this works.



Make sure that you clear your last calculation before starting on the next with the C/AC key.

The three calculators work differently as two of them follow the BIDMAS rule (i.e. multiply or divide before you add or subtract). If you enter:



into a basic calculator then you will get the answer 110 because the calculator has done:

$$8 + 3 = 11$$
 then $11 \times 10 = 110$

If you enter:



Into a scientific calculator or your app you will get the answer **38** because the calculator has done:

$$3 \times 10 = 30$$
 then $8 + 30 = 38$

You could think of it as putting brackets round the multiplication and doing that calculation first:

$$8 + (3 \times 10) =$$

You need to remember that if you are using a calculator app it may look like a basic calculator but calculates like a scientific calculator, using BIDMAS.

Before we do any examples or exercises, make sure that you understand what you are telling the calculator to do.

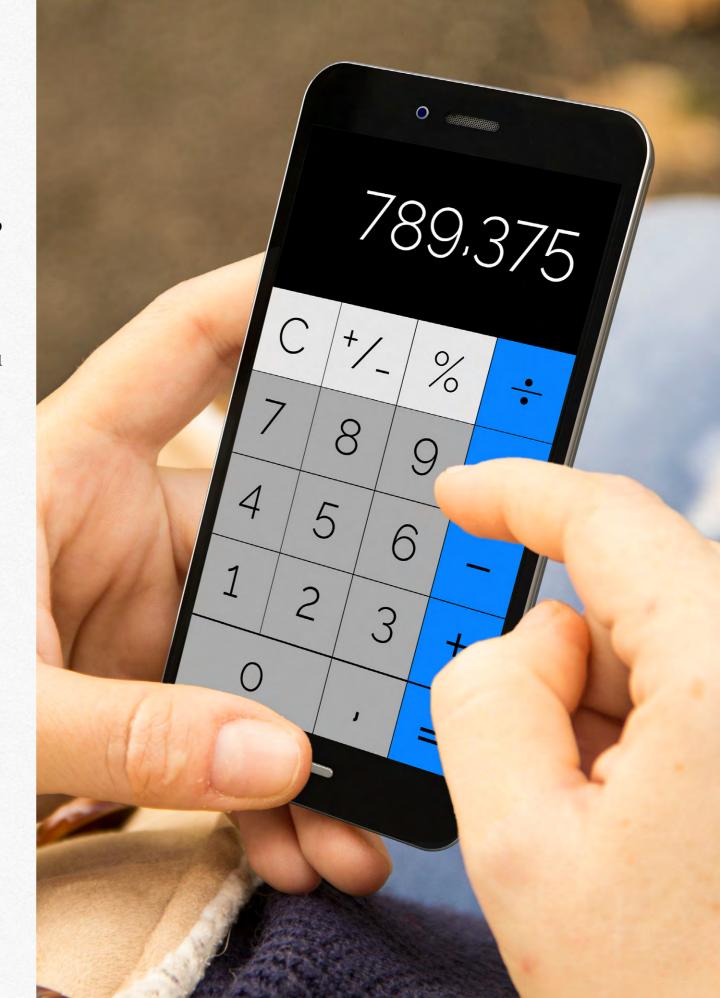


Food for Thought

The calculator will not make a mistake. If the answer looks wrong, then it is because the calculator was given the wrong instructions.

That is why it is always sensible to have an idea in your head of what you expect the answer to be (a rough estimate) before you press any buttons.

With percentages there are two ways to calculate a percentage of an amount.



Method 1

Just remember that you can write any percentage as a fraction over **100**:

$$74\% = \frac{74}{100} = 74 \div 100$$

Therefore, to find **74**% of **£120** you translate the calculation into a number sentence:

$$74 \div 100 \times 120 =$$

And press the keys as follows:

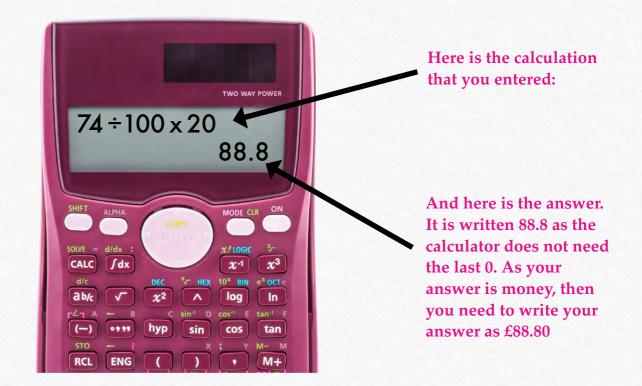


And you should get the answer **88.8**. As the question was about money, you must give your answer to **2** decimal places, or to the nearest penny.

Therefore:

$$74\%$$
 of £120 = £88.80

If you are using a **scientific calculator** then you should be able to see the calculation that you have entered as well as the answer:



Some scientific calculators will automatically give the answer as a fraction, if it is not a whole number. This is because a fraction is very often more exact than a decimal. If this is the case the answer to the above calcultion would be 74/5. In order to change the answer to a decimal, simply press the 'S \leftrightarrow D' button.

If you are using the calculator app on your phone or a basic calculator then you will not see the calculation.

Using a Calulator App 203

Method 2

You can also use the percentage buttons on your app or calculator.

To find **27**% of **£49** on a **basic calculator** (with a % button) or an app, you are going to enter the number sentence:

 $27\% \times 49 =$ by pressing the sequence:



And you will get the answer 13.23

$$27\%$$
 of £49 = £13.23

If you used the app, you may have noticed that the display changes from **27** to **0.27** when you pressed the % key.

That is because
$$27\% = \frac{27}{100} = 27 \div 100 = 0.27$$

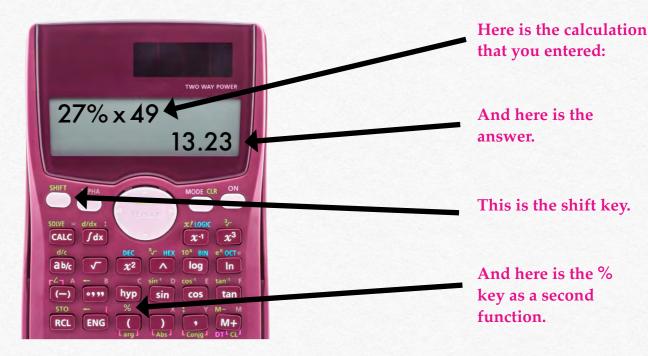
With a **scientific calculator**, you may find the percentage key is a **second function** and you have to use the SHIFT button to use it. If you are not sure what this means, then

you should look at the manual that came with the calculator (you can download one from the internet if you cannot find the original.)

A typical key sequence for the above calculation would be:



Here is an example of a scientific calculator, yours may be different.



As you enter the keys you will see the % sign appear.

From now on the examples will show how to use the app on a phone. You can do just the same thing using a calculator but note that they may work a little differently.

Example

Gail buys a shirt that normally costs £35.49 but is marked down 15% in the sale. How much does she save?

Saving is **15%** of **£35.49**

Gail will enter:



The display reads:



Gail rounds this to the nearest penny.

Answer: Gail saves £5.32

As we saw earlier, it is not just the saving that is important, what Gail really wants to know is how much the shirt will cost her.

When we worked this out earlier, we deducted the discount from the original price.

Using the calculator app makes this much easier.

The original price of the shirt is **100%**, then **15%** is deducted.

$$100 - 15 = 85$$

The price Gail will pay is 85% of the original price.

To find 85% of £35.49 she will enter:



The display reads:

30.1665

Gail rounds this to the nearest penny.

Answer: The shirt now costs Gail £30.17

Another example

Hetty and Ian are eating out. They each have a vegetarian pad that at £9.95 and soft drink at £2.95. 15% service is added to their bill. What is the total with service?

First Hetty will add up everything they have ordered. She has already estimated that it is about:

$$2 \times 10 + 2 \times 3 = 20 + 6 = 26$$

She calculates the exact amount by entering:

$$2 \times 9.95 + 2 \times 2.95$$

That makes a total of 25.80

That is 100%. Now 15% is to be added.

$$100 + 15 = 115$$
.

Without changing the display Hetty enters:



and gets the answer:

Answer: Hetty and Ian's final bill is £29.67



There are two reasons why the calculator app on a phone is so good for these calculations.

The first reason is that it calculates the multiplication first, so the calculation $2 \times 9.95 + 2 \times 2.95$ is worked out correctly in one simple set of key entering.

The second reason is that it then simply works out the percentage straight from the previous answer. You do not need to write down any stages or use the memory.

Quick and simple – we like it!

In these exercise be sure to read the question carefully. Make sure that know if you are to calculate a percentage, or a final amount once the percentage has been added on or taken away.

If the percentage is a discount then it will be taken away and, like Gail, you will subtract the percentage from 100. If it is added on, like a service charge, then like Hetty you will add the percentage to 100.

Exercises

31 Jyoti wants to buy a laptop. She has chosen one that normally costs £399.49 but is now marked down by 20%. How much has she saved?

32 Kay is a trainee beautician and is paid the minimum wage of £8.91 per hour. In April, the minimum wage is to rise by 2.2%. How much will Kay be paid per hour from April?

33 Louisa has saved £40 and has set out for the sales. Everything in Modish Madame is marked down by 35% today. Has Louisa saved enough to buy 2 shirts that were each £14.95 and a skirt that was £24.49?

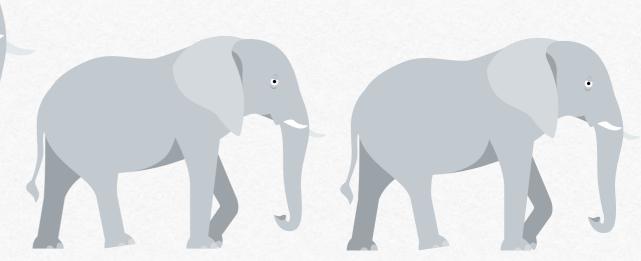


Sometimes a percentage is written with a fraction. Common examples are 12½% and 17½%. Just think of the fraction as a decimal and calculate just as before:

 $12\frac{1}{2}\% = 12.5\%$ and $17\frac{1}{2}\% = 17.5\%$

34 Mona has had a meal out with her friends. They order 5 set meals at £19.49 a head. Service at 12.5% is added to their bill. How much was the final bill including service?

35 Naomi is buying a new mobile phone. It would usually cost £179 but this CyberMonday there is a 17.5% discount. What does she pay for her new phone?



Finding the Original Amount

Previously you learnt how to calculate the original value before a percentage change by using the chart method. Using this method works well but it does mean writing things down which, when you want to calculate quickly you may not be able to do.

You can quickly calculate an original value with your calculator app by following the same steps as before but this time you will be dividing by a percentage.

Example

Olga bought a new coat which was marked down 30% in a sale. She paid £56. Olga's mother wants to know exactly how much money she actually saved.

As the coat was marked down **30%**, Olga has paid **70%** of the original price.

70% x original price = £56

To find the original price Olga therefore needs to divide by **70**%:

original value = $56 \div 70\%$



The display reads:



The original price of the coat was £80 and therefore Olga deducts the price she paid:



and she can tell her mother that she has saved £24.

Answer: Olga has saved £24

Another Example

15% service charge has been added to a restaurant bill to give a total of £72.34.

Peta does not think she has had good service and is not prepared to pay for service. What was the value of the bill before the service charge was added?

As 15% was added the mathematical sentence is:

115% of original bill = £72.34

Therefore, original bill = 72.34÷ 115%



The display reads:

62.9043478

Peta rounds the answer to the nearest penny.

Answer: The original bill was £62.90

When finding the original amount, remember:

If the final amount is from a percentage increase, then divide by 100% plus the percentage increase

If the final amount is from a percentage decrease then divide by 100% minus the percentage decrease.



Exercises

- 36 Rai has spent a total of £65.42 in a shop where all the prices had been reduced by 25% in a sale. What would she have spent if she had bought the same items before the sale?
- 37 Stella has bought stationery for her business and paid a total of £154.68. The amount includes Value Added Tax (VAT) of 20%. Stella needs to know the value of her stationery without VAT. What is the value before VAT was added?
- 38 Tania is selling home-made cakes on a market stall. She has priced her cakes so that she makes a 55% profit over her costs. If she sold all her cakes and took home £325.50, what were her costs? What was her profit for the day?
- 39 Una works as a freelance journalist. She has just written an article on the changing face of the cosmetic industry and sold it for £1,292.50. If the £1,292.50 includes an agency commission of 17.5%, how much has Una earned before adding on the commission?

40 A furniture supplier is offering a discount of **15%** on all orders over **£300**. Verity buys **4** bookcases and pays a total of **£316.20** once the discount has been deducted. What was the original cost of each bookcase?



Percentages and the pandemic.

In the 2020–21 pandemic, cases of the virus were regularly reported with the change from the previous week.

For example, in one week in February 2021: 'A further 12,000 new cases were recorded, with the seven-day total down 20%.'

You can work out the number of new cases in the previous week by dividing 12,000 by the difference between 20% and 100%:

No of new cases last week= 20,000 ÷ 80% = 25,000

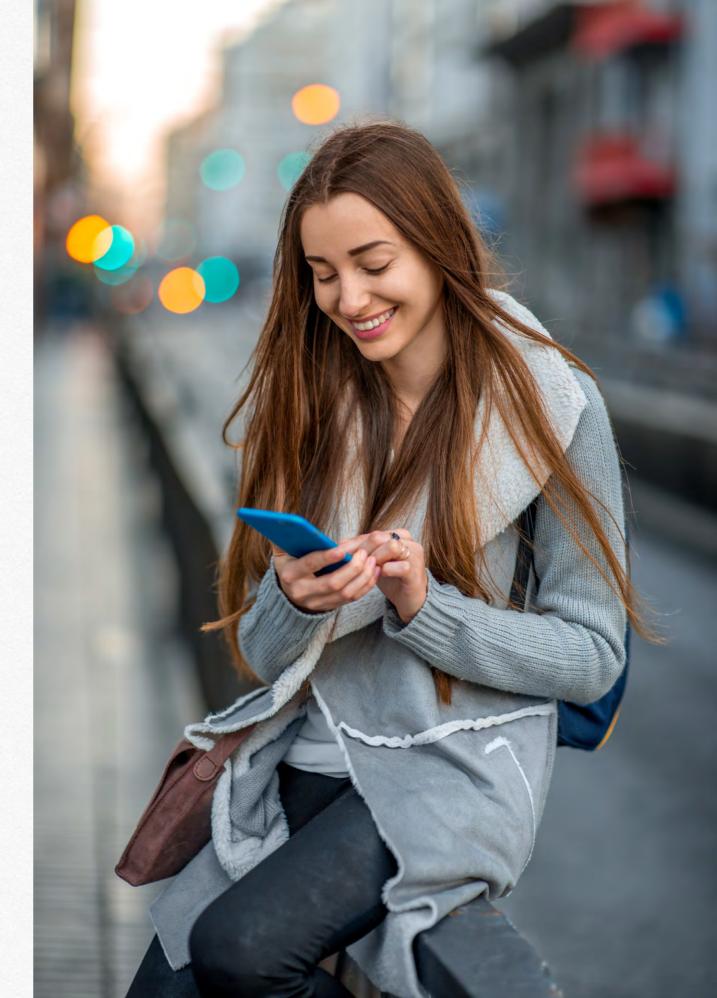


Food for Thought

Did you know that if you rotate your phone round **90%** then a whole lot of new functions are on your calculator app:



You will probably not need most of these, but it is good to know they are there if you ever do!



Answers to Part 4

These worked answers show the thinking that your head might have been doing to work out the answer. But do not worry if you head did something different, as long as you got the correct answer!

Write each percentage as a fraction in its lowest terms.

1 10%

Just as in the example, write 10% as a fraction out of 100. Then you should see that both 10 and 100 can be divided by 10.

$$\div 10$$

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$\div 10$$

Answer: ¹/₁₀

Therefore, to find 10% of something you will divide by 10.

2 50%

Write 50% as a fraction out of 100. Then you should see that both 50 and 100 can be divided by 50 (or you might prefer to divide by 10 and then by 5).

$$\div 50$$

$$50\% = 50/100 = 1/2$$

$$\div 50$$

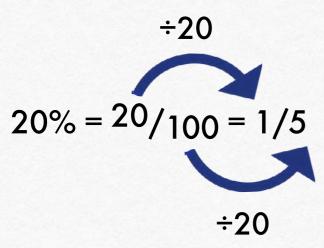
Answer: 1/2

Therefore, to find 50% of something you will divide by 2.



3 20%

Write 20% as a fraction out of 100. Then you should see that both 20 and 100 can be divided by 20 (or you might prefer to divide by 10 and then by 2).

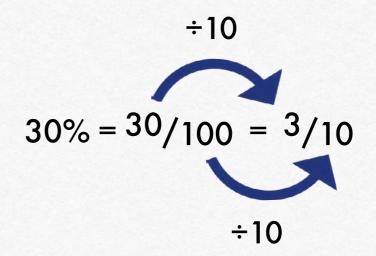


Answer: 1/5

Therefore, to find 20% of something you will divide by 5.

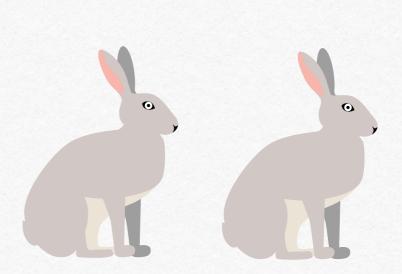
4 30%

Write 30% as a fraction out of 100. Then you should see that both 30 and 100 can be divided by 10.



Answer: $\frac{3}{10}$

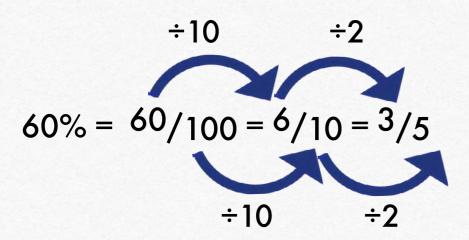
Therefore, to find 30% of something you will divide by 10 and multiply by 3.



5 60%

Write 60% as a fraction out of 100. Then you should see that both 60 and 100 can be divided by 10.

You could leave your answer as $^6/10$ or you could divide 6 and 10 by 2 to make $^3/_5$



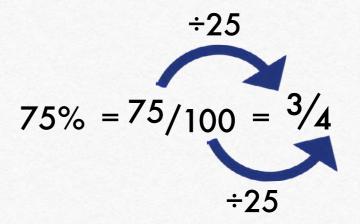
Answer: $\frac{6}{10}$ or $\frac{3}{5}$

Therefore, to find 60% of something you will divide by 10 and multiply by 6 (or divide by 5 and multiply by 3).



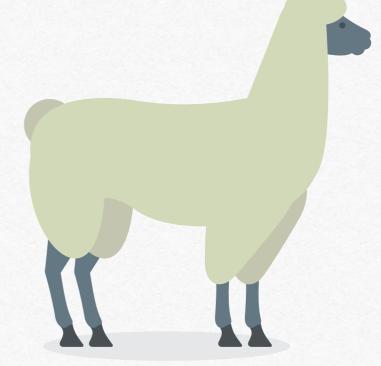
6 75%

Write **75%** as a fraction out of **100**. Then you should see that both **75** and **100** can be divided by **25**.



Answer: 3/4

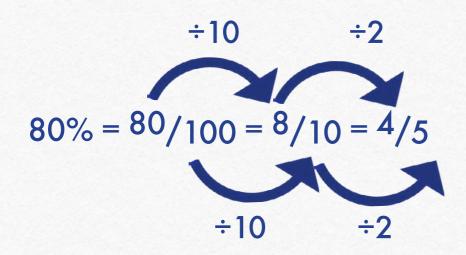
Therefore, to find 75% of something you will divide by 4 and multiply by 3.



7 80%

Write 80% as a fraction out of 100. Then you should see that both 80 and 100 can be divided by 10.

You could leave your answer as $^8/10$ or you could divide 8 and 10 by 2 to make $^4/5$

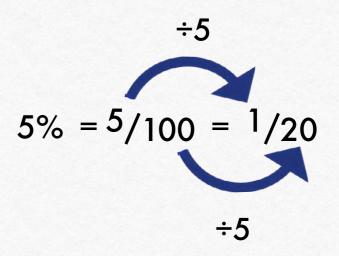


Answer: 8/10 or 4/5

Therefore, to find 80% of something you will divide by 10 and multiply by 8 (or divide by 5 and multiply by 4).

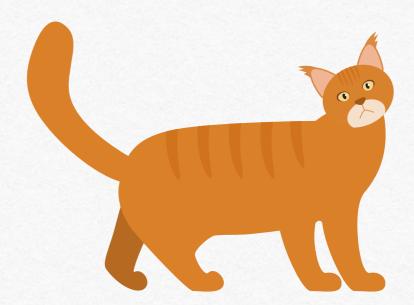
8 5%

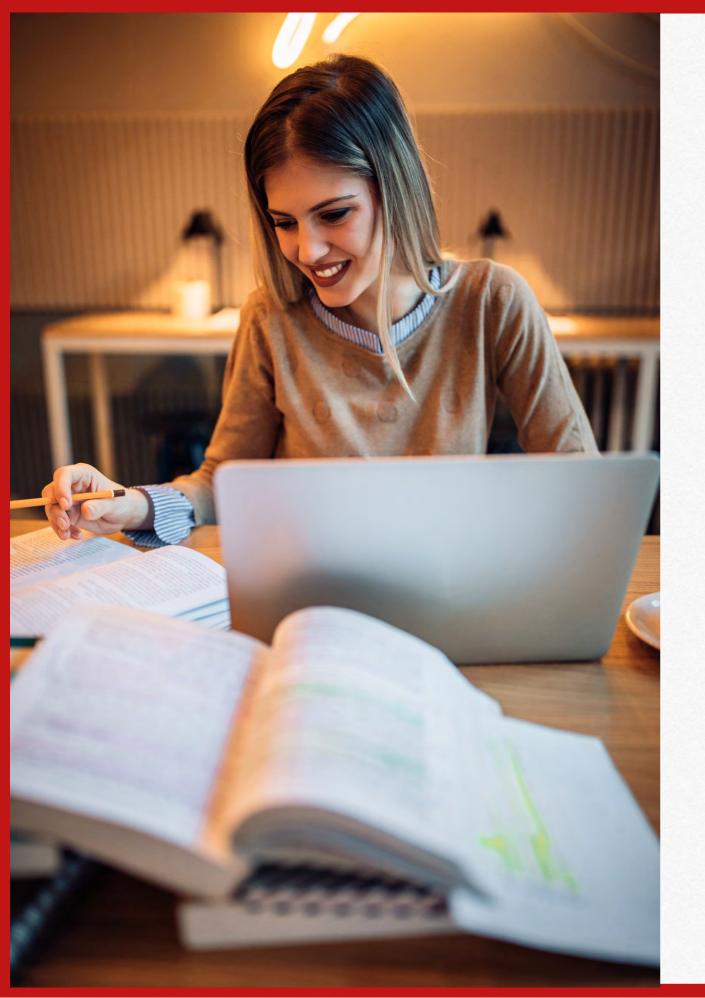
Write 5% as a fraction out of 100. Then you should see that both 5 and 100 can be divided by 5.



Answer: 1/20

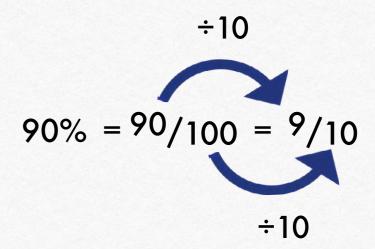
Therefore, to find 5% of something you will divide by 20 but we will also look at another way later on.





9 90%

Write 90% as a fraction out of 100. Then you should see that both 90 and 100 can be divided by 10.



Answer: 9/10

Therefore, to find 90% of something you will divide by 10 and multiply by 9.

10 100%

You do not need to write 100% as a fraction as you know that 100% of something is 100/100 or 1 whole

Answer: 1

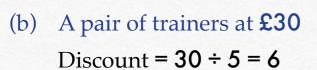
11 In a store all prices on Black Friday are reduced by 20%, what is the discount on these goods that are normally priced as follows:

20% is $^{1}/_{5}$ so you will divide by 5

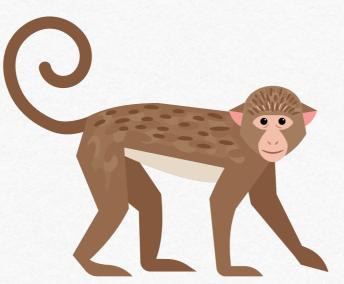
(a) A shirt at £15

Discount =
$$14 \div 5 = 3$$

Answer: £3



Answer: £6



(c) A television at £300

Discount =
$$300 \div 5 = 60$$

Answer: £60

12 In a store all prices on Cyber Monday are reduced by 25%, what is the discount on these goods that are normally priced as follows:

25% is 1/4 so you will divide by 4

(a) A mobile phone at £240

Discount =
$$240 \div 4 = 60$$

Answer: £60

(b) A tablet at £360

Discount =
$$360 \div 4 = 90$$

Answer: £90

(c) A pair of headphones at £32

Discount =
$$32 \div 4 = 8$$

Answer: £8

- 13 A fashion shop is closing down and everything is to be sold, so calculate these discounts:
- (a) 50% off a pair of shoes originally costing £35

50% is ½ so you will divide by 2

 $35 \div 2 = 17.5$

Remember your answer is money with 2 decimal places.

Answer: Discount = £17.50

(b) 40% off a shirt originally costing £25

40% is 4 /10 so you will divide by 10 and multiply by 4

$$25 \div 10 = 2.5$$

$$2.5 \times 4 = 10$$

Answer: Discount = £10

(c) 60% off a coat originally costing £125

60% is $^6/10$ so you will divide by 10 and multiply by 6

$$125 \div 10 = 12.5$$

$$12.5 \times 6 = 12.5 \times 2 \times 3 = 25 \times 3 = 75$$

Answer: Discount = £75

(d) 75% off a dress originally costing £48

75% is $\frac{3}{4}$ so you will divide by 4 and multiply by 3

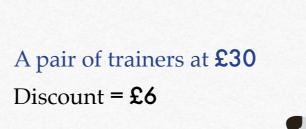
$$48 \div 4 = 12$$
.

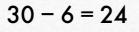
$$12 \times 3 = 36$$

Answer: Discount = £36

$$15 - 3 = 12$$

Answer: Reduced price = £12





Answer: Reduced price = £24

$$300 - 60 = 240$$

Answer: Reduced price = £240

- 15 In a store all prices on Cyber Monday are reduced by 25%, what is the reduced price on these goods that are normally priced as follows:
- (a) A mobile phone at £240

$$240 - 60 = 180$$

Answer: Reduced price = £180

(b) A tablet at £360

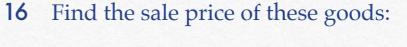
$$360 - 90 = 270$$

Answer: Reduced price = £270

(c) A pair of headphones at £32

$$32 - 8 = 24$$

Answer: Reduced price = £24



(a) A television normally costing £800 reduced by 10% in the sale.

10% is $^{1}/10$ so you will divide by 10

$$800 \div 10 = 80$$

$$800 - 80 = 720$$

Answer: Reduced price = £720

(b) A pair of socks normally costing £2.50 reduced by 10% in the sale.

10% is $\frac{1}{10}$ so you will divide by 10

$$2.5 \div 10 = 0.25$$

$$2.5 - 0.25 = 2.25$$



(c) A jumper normally costing £40 reduced by £25% in the sale.

25% is 1/4 so you will divide by 4

$$40 \div 4 = 10$$

$$40 - 10 = 30$$

Answer: Sale price = £30

(d) A dress normally costing £40 marked down by 20% in the sale.

20% is $^{1}/_{5}$ so you will divide by 5

$$40 \div 5 = 8$$

$$40 - 8 = 32$$

Answer: Sale price = £32



Again, these answers are examples of what you might do in your head but you can do things a bit differently if that works for you.



- 17 On the 15th February, all boxes of chocolates were reduced by 15%.
- (a) What is the reduction for a box of chocolates normally costing £5?

$$10\% \text{ of } £5 = 5 \div 10 = 0.5$$

$$5\% \text{ of } £5 = 0.5 \div 2 = 0.25$$

$$0.5 + 0.25 = 0.75$$

Answer: Reduction = 75p or £0.75

(b) What is the reduced price for a box of chocolates normally costing £7.20?

$$10\% \text{ of } £7.20 = 7.2 \div 10 = 0.72$$

$$5\%$$
 of £7.20 = $0.72 \div 2 = 0.36$

$$0.72 + 0.36 = 1.08$$

$$7.2 - 1.08 = 6.2 - 0.08 = 6.12$$

Answer: Reduced price = £6.12

18 My annual salary was £30,000 but I have been awarded a pay rise of 4%. What will my new salary be?

$$1\% \text{ of } 30,000 = 30,000 \div 100 = 300$$

$$4\%$$
 of $30,000 = 300 \times 4 = 1,200$

New salary =
$$30,000 + 1,200 = 31,200$$

Answer: New salary = £31,200

19 The value of my car has decreased by 15%. If I bought it for $\pounds 7,500$, what is it worth now?

$$10\% \text{ of } £7,500 = 7,500 \div 10 = 750$$

$$5\%$$
 of £7,500 = $750 \div 2 = 375$

$$Total = 750 + 375 = 1,125$$

New value =
$$7,500 - 1,125 = 6,375$$

Answer: New value = £6,375

20 A restaurant adds 15% service charge to the bill. If my bill comes to £36, what service charge will be added?

$$10\% \text{ of } £36 = 36 \div 10 = 3.6$$

$$5\%$$
 of £36 = $3.6 \div 2 = 1.8$

Service charge =
$$3.6 + 1.8 = 5.4$$

Answer: Service charge = £5.40

21 Shop A is selling a blue jacket for £40. Shop B is selling the same jacket for £48 but with a 10% discount. Which is better value?

Shop B selling for
$$£48 - 4.8 = £43.20$$

Answer: Shop A is better value



22 Deluxe Modes is selling a dress for £48 marked down by 25% in the sale whilst Cheap Tat is selling the same dress for £25. Which is better value?

$$25\%$$
 of $48 = 48 \div 4 = 12$

Deluxe Modes selling for 48 - 12 = £36

Answer: Cheap Tat is better value

I can buy a laptop online for £350 but with a 15% delivery charge added. I can buy the same laptop in store for £399. Which is better value?

$$10\% \text{ of } 350 = 350 \div 10 = 35$$

$$5\% \text{ of } 350 = 35 \div 2 = 17.5$$

$$15\% \text{ of } 350 = 35 + 17.5 = 52.5$$

Price with delivery charge = 350 + 52.5 = £402.50

Answer: Buying from the store is better value

24 Classy Clothes has a 30% off everything sale whilst Swanky Style has a 15% off everything sale. Before the sales, a jumper was £30 at Classy Clothes but the same jumper was priced at £24 at Swanky Style. Which is the better value in the sales?

Classy Clothes:

$$10\% \text{ of } £30 = £3$$

Sale price =
$$30 - 9 = £21$$

Swanky Style:

$$10\% \text{ of } £24 = £2.40$$

$$5\% \text{ of } £24 = £1.20$$

$$15\% \text{ of } £24 = £2.40 + £1.20 = £3.60$$

Sale price =
$$24 - 3.6 = £20.40$$

Answer: Swanky Style is better value

25 A shirt normally costing £19.99 is marked down 10% in the sale. What is the reduced price?

$$10\% \text{ of } £20 = £2$$

Reduced price =
$$20 - 2 = 18$$

Answer: £18

26 April and Beattie have a meal in the pizzeria that comes to £25.87. Service charge of 15% is then added to the bill. What is the new total?

$$10\% \text{ of } £26 = £2.60$$

$$5\% \text{ of } £26 = £2.60 \div 2 = £1.30$$

$$15\% \text{ of } £26 = £2.60 + £1.30 = 3.90$$

$$Total = 26 + 3.90 \approx 30$$

Answer: £30



27 In the end of season sale all prices are marked down by 20%. What will Charlie pay for a refrigerator that originally sold for £249.99?

$$10\% \text{ of } 250 = 250 \div 10 = 25$$

Sale price =
$$250 - 50 = 200$$

Answer: Charlie will pay £200

28 Donna is a doctor in training and her starting salary is £28,243. After two years, the salary should increase by 15%. Estimate what Donna's new salary will be?

$$10\% \text{ of } 28,000 = 28,000 \div 10 = 2,800$$

$$5\% \text{ of } 28,000 = 2,800 \div 2 = 1,400$$

$$15\% \text{ of } 28,000 = 2,800 + 1,400 = 4,200$$

New salary
$$\approx 28,000 + 4,200 = 32,200$$

Answer: Donna's new salary will be about £32,200

29 Edwina wants to buy a new dress for her best friend's wedding. She has found one online for £49.99 but then sees the same dress in a shop for £55.49 but marked down 15% in a sale. Which purchase is the best value?

$$10\% \text{ of } £55.50 = £5.55$$

$$5\% \text{ of } £55.50 = 5.55 \div 2 \approx 2.75$$

$$15\% \text{ of } £55.50 = 5.55 + 2.75 \approx 8.30$$

Sale price =
$$55.5 - 8.3 = £47.20$$

Answer: Better value in the shop

30 Fifi is a personal assistant with a salary of £25,150. Her boss earns £118,700 and has just awarded herself a 1% pay rise but tells Fifi that she is so pleased with her work that she is giving her a 4% pay rise. Who has the greater increase in pay?

Boss's pay rise

$$1\% \text{ of } £120,000 = 120,000 \div 100 = 1200$$

Fifi's pay rise;

$$1\% \text{ of } 25,000 = 25,000 \div 100 = 250$$

Answer: Fifi's boss has the greater increase in pay.



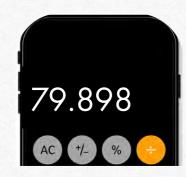
31 Jyoti wants to buy a laptop. She has chosen one that normally costs £399.49 but is now marked down by 20%. How much has she saved?

Jyoti has saved 20% of £399.49

Key in



The display reads:



Round this to the nearest penny: £79.90

Answer: Jyoti has saved £79.90

32 Kay is a trainee beautician and is paid the minimum wage of £8.91 per hour. In April, the minimum wage is to rise by 2.2%. How much will Kay be paid per hour from April?

Kay will be paid 100% plus 2.2% of £8.91 or 102.2% of £8.91

Key in



The display reads:



Round this to the nearest penny: £9.11

Answer: Kay will be paid £9.11 per hour

33 Louisa has saved £40. Everything in Modish Madame is marked down by 35% today. Has Louisa saved enough to buy 2 shirts that were each £14.95 and a skirt that was £24.49?

Louisa will need to spend 2 x 14.95 + 24.49 less 35%

Key in



The display reads:



As there is **35**% off, Louisa will be paying **65**% of this amount. Without changing the display key in:



The display reads:



Which is £35.35

Answer: Yes, Louisa has saved enough.

34 Mona has had a meal out with her friends. They order 5 set meals at £19.49 a head. Service at 12.5% is added to their bill. How much was the final bill including service?

Mona needs to work out 5 x 19.49 x 112.5%

Key in



The display reads:



Which is £109.63 to the nearest penny

Answer: the final bill will be £109.63.

35 Naomi is buying a new mobile phone. It would usually cost £179 but this CyberMonday there is a 17.5% discount. What does she pay for her new phone?

Naomi first works out that 100% - 17.5% = 82.5%

And then 82.5% of £179

Key in



The display reads:



Which is £147.68 to the nearest penny

Answer: Naomi pays £147.68 for her new phone.

36 Rai has spent a total of £65.42 in a shop where all the prices had been reduced by 25% in a sale. What would she have spent if she had bought the same items before the sale?

Rai needs to work out £65.42 ÷ 75%

Key in

The display reads:



Which is £87.23 to the nearest penny

Answer: Rai would have paid £87.23 before the sale.

37 Stella has bought stationery for her business and paid a total of £154.68. The amount includes Value Added Tax (VAT) of 20%. Stella needs to know the value of her stationery without VAT. What is the value before VAT was added?

Stella needs to work out £154.68 ÷ 120%

Key in



The display reads:



Which is £128.90 to the nearest penny

Answer: Before VAT the value is £128.90

38 Tania is selling home-made cakes on a market stall. She has priced her cakes so that she makes a 55% profit over her costs. If she sold all her cakes and took home £325.50, what were her costs? What was her profit for the day?

Tania needs to work out £325.50 ÷ 155%

Key in

The display reads:



Answer: Tania's costs were £210

To find her profit, deduct the costs from the sale income:

$$£325.50 - £210 = £115.50$$

Answer: Tania's profit was £115.50

39 Una works as a freelance journalist. She has just written an article on the changing face of the cosmetic industry and sold it for £1,292.50. If the £1,292.50 includes an agency commission of 17.5%, how much has Una earned before adding on the commission?

Una needs to work out £1,292.50 ÷ 117.5%

Key in

The display reads:



Which is £1,100

Answer: Una earned £1,100

40 A furniture supplier is offering a discount of 15% on all orders over £300. Verity buys 4 bookcases and pays a total of £316.20 once the discount has been deducted. What was the original cost of each bookcase?

$$100\% - 15\% = 85\%$$

4 book cases would have cost £316.20 ÷ 85%

Key in

The display reads:



Now divide that by 4 for the cost of one bookcase

Answer: The original cost of a bookcase was £93

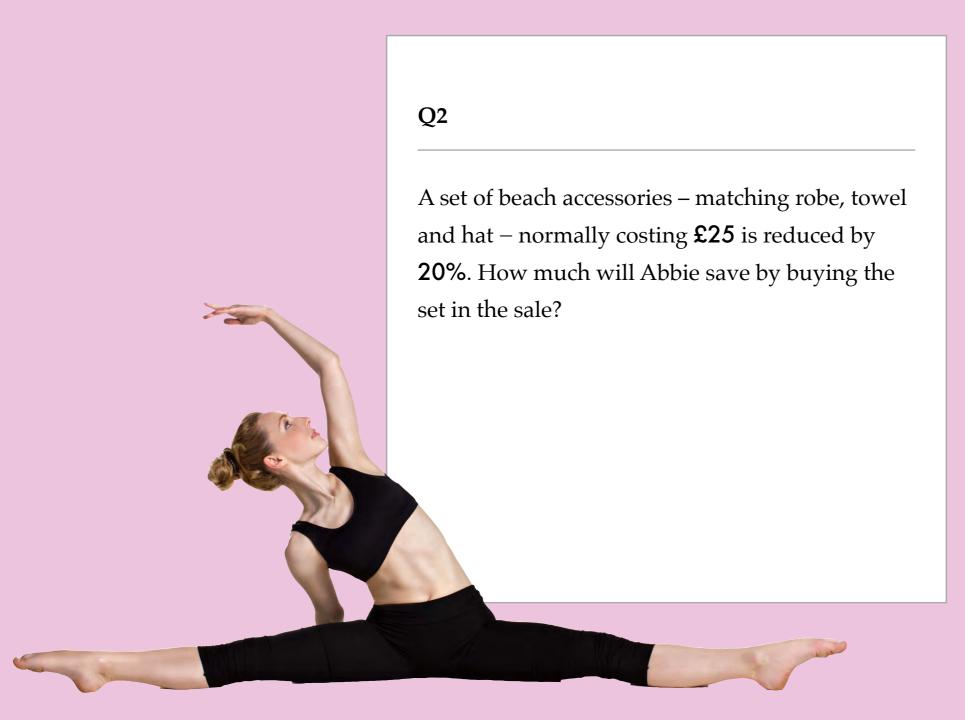
Do each calculation in your head and then check the answer to see if you were correct.

Q1

What is 25% as a fraction in its lowest terms?



Your Brain Workout 230



Your Brain Workout

Q3

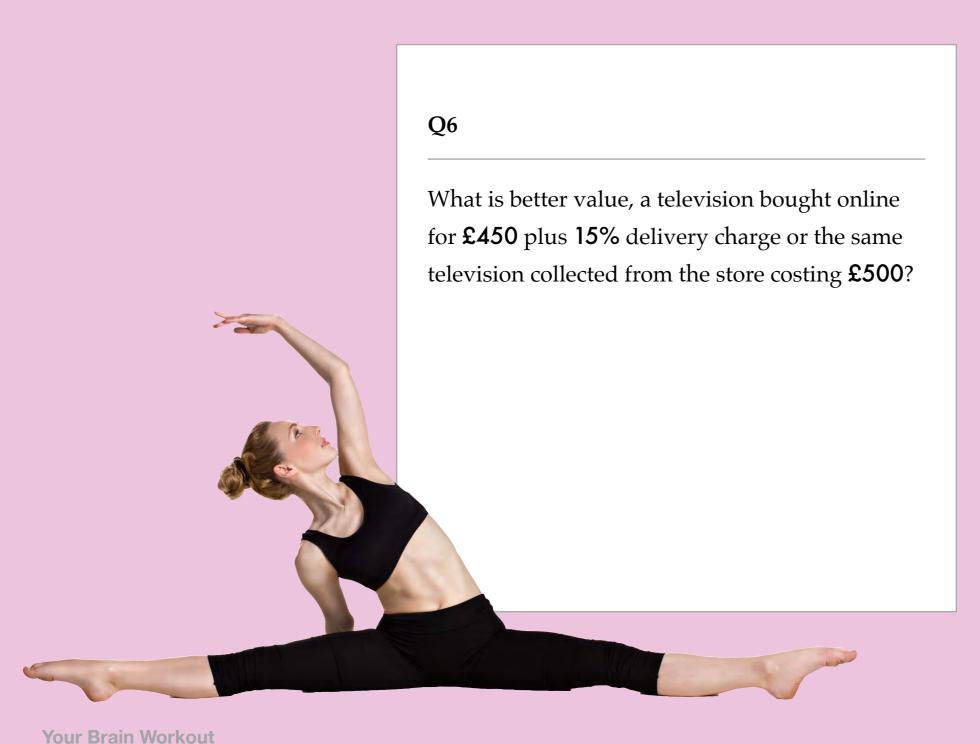
Charlotte buys a new set of bedclothes that were priced at £40 but have been reduced by 30% in the sale. What does Charlotte pay for her new bedding?

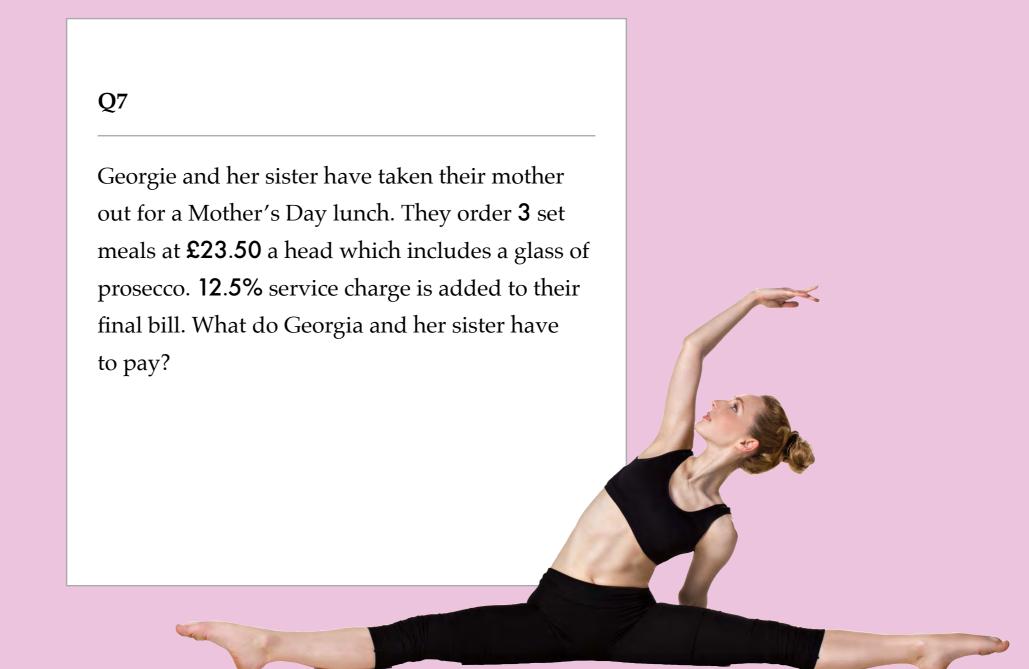


Your Brain Workout

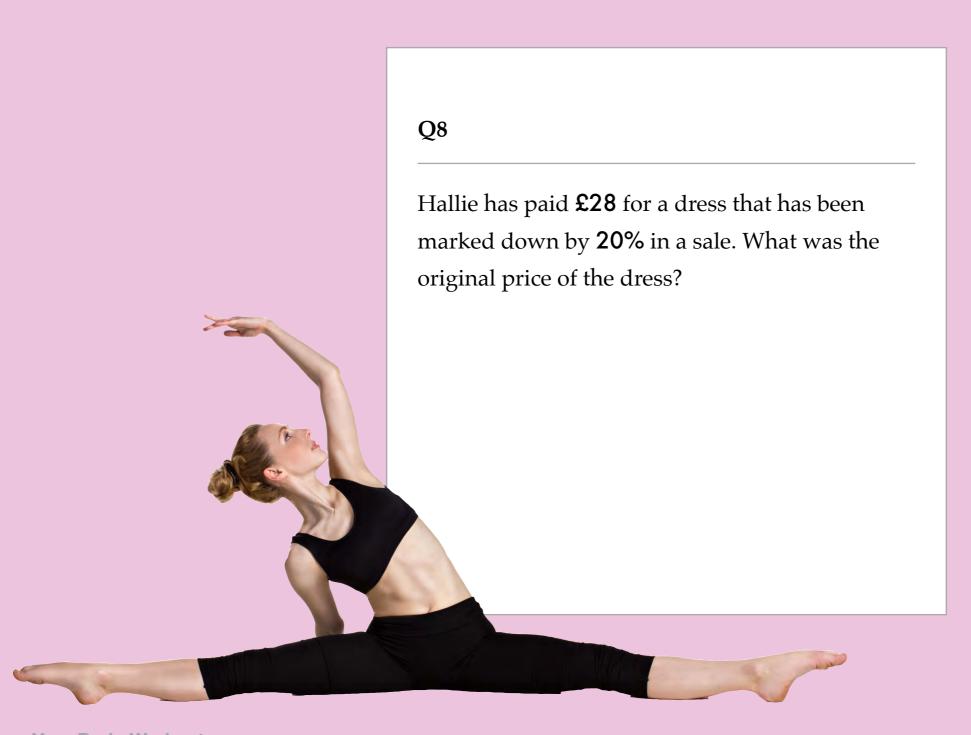


Your Brain Workout 234





Your Brain Workout 236



Your Brain Workout 237

Answers

Q1
$$25\% = \frac{25}{100} = \frac{1}{4}$$

Q2 20% of
$$25 = 25 \div 5 = £5$$

Q3 30% of
$$40 = 4 \times 3 = 12$$
 Charlotte pays $40 - 12 = £28$

Q4
$$15\% \text{ of } 36 = 3.60 + 1.80 = £5.40$$

Q5 1% of 32,000 = 320, 3% of 32,000 = 960 New salary = £32,960

Q6 15% of 450 = 45 + 22.5 = 67.7 In store is better value

Q7 $3 \times 23.50 \times 112.5\% = £79.31$

Q8 $28 \div 80\% = £35$



Shaken not Stirred

The term 'shaken not stirred' first appeared in Ian Fleming's novels about intelligence officer James Bond. In the film version of Casino Royale, Bond orders a martini from a barman when at the casino.

Have you ever wondered what goes into a martini? Bond tells the barman to mix **3** measures of gin, one measure of vodka and half a measure of Kinna Linnet (a French liqueur). He instructs him to shake it very well until ice cold and then pour it into a glass and add a thin slice of lemon.

'What' you ask, 'is a measure?'

A measure is a small cup that bartenders use to mix cocktails. Nowadays spirits are measured in measures of either 25, 35 or 50 millilitres, but this was not regulated in 1953. There is no record of the size of the measure that the bartender used to mix Bond's martini, so we do not know how large it was. However, the size of the cocktail does not

change the flavour. The important thing is that the ingredients are in the same ratio.

You can think of a **ratio** as being a **part**, so Bond's cocktail is **3** parts gin, **1** part vodka and half a part of Kinna Linnet or a total of $4\frac{1}{2}$ parts.



On investigation, scientists have found that shaking a drink introduces air bubbles into the mixture and the air then mixes with the ingredients. Shaking can also chip off small pieces from the ice cubes as they bounce around. These can both lead to a cloudy appearance and a different texture from a stirred drink.



Shaken not Stirred 240

Ratio

Let us remind ourselves about what we learnt about Ratio.

A simple ratio is a comparison of two quantities. For example, if there are **30** students and **3** teachers on a trip to an art gallery, the ratio of students to teachers is:

30 students to 3 teachers, which is written 30:3.

Just like a fraction, we generally write a ratio in its lowest terms, which means that it is simplified by **dividing** both numbers by the same common factor.

3 is the common factor of **30** and **3**, so **30** : **3** when simplified becomes:

$$(30 \div 3) : (3 \div 3) = 10 : 1$$

This means that on the trip there are ten students to every one teacher.

So, the ratio of students to teachers is 10 : 1.

This can be written the other way round:

The ratio of teachers to students is 1 : 10.

The colon: is read out as the word 'to' so the ratio of teachers to students is 1 to 10.

You use ratios all the time, although you might not realise it.

The instructions on this bottle of lime juice cordial tell you to dilute it ${\bf 1}:{\bf 8}$ with still or sparkling water.

You can think of that as 1 part cordial to 8 parts water and therefore 9 parts altogether.

The total number of parts is important because that tells you how much your total mixture will be.

If you are just mixing a glass of lime drink for your self then your parts will be small, but if you are mixing a jug for a party then your parts will be large.

Example

Anya wants to mix lime juice cordial with water into a **250 ml** glass. If the ratio of cordial to water is **1** : **8**, how much cordial does she use?

First think about pouring yourself a drink – you don't fill the glass!



When making real calculations about ratio and measures you can use **convenient quantities** that you work out by **rounding DOWN**.

So a drink that is 1 part cordial to 8 parts water is 9 parts.

One part = $250 \div 9$

Now do NOT reach for your calculator.

You know that 3 x 9 is 27 so 30 ml will be too much.

Rounding down:

Which will make 25 ml cordial

It is a good idea to check your answer by calculating the total amount of the drink:

Anya uses 25 ml of cordial





But if you wanted lots of ice in your glass you might round down further:

Which will make 20 ml cordial

$$20 \times 8 = 160 \text{ ml water}$$

Total 180 ml

It is always a good idea to carry a picture of a problem in your head. That helps you to work out if your answer is sensible.

Do you know what 20 ml looks like?

This is a normal measuring jug that you will probably find in your kitchen:



Unhelpfully, the scale does not go below 50 ml.

You may have a smaller measuring jug like this:



But if not, you will have to look for other inspiration.

You can find out how much is in a smaller container like this example:



Take an egg cup and fill it with water. Pour it into the measuring jug.

Repeat several times until the water level in the jug is 100 ml.

Divide 100 ml by the number of egg cups. If it took four cupfuls to reach 100 ml then the volume of the egg cup is:

$$100 \text{ ml} \div 4 = 25 \text{ ml}$$

If it took three then the volume is

Now you have a useful measure to use for mixing drinks and also a visual clue.

You might also like to check the volume of some of the glasses that you have around the place.





Food for Thought

Although most measures are in litres or millilitres, beer is still sold in pubs in pints.

If the calculation is rather difficult to do in your head, you could use a table to help.

Example

Beattie is making an orange drink by mixing orange cordial with water in the ratio 1 : 6. If she uses 20 ml of cordial, how much water will she need to add and what will be the total volume of her drink?

It is always a good idea to work with the total when calculating with a ratio as then you collect all the information in one go. If the ratio is 1:6 the total number of parts is 7

Write down the information you have like this:

С	W	Tot
1	6	7

Now you know you have **20 ml** of cordial, add the next line:

C	W	Tot
1	6	7
20		

You can see that you have multiplied 1 by 20 so you need to multiply the other numbers by 20 as well.

Your table will look like this

С	W	Tot
1	1 6	
20	6 x 20 =	7 x 20 =
20	120	140

It is good practice to check that the sum of the parts equals the total:

$$20 + 120 = 140$$

Now you can answer the question.

Answer: Beattie adds 120 ml water

The total volume of her drink is 140 ml

When you start, if you don't know the equivalent of one part, then you can use what is called the unitary method. This means adding an extra line where you work out the value of each quantity for 1 part.

Example

Chiara is making jugs of orange drink by mixing squash with water in the ratio **2** : **5**.

How many 1 litre jugs can she make up using **500 ml** of squash?

First Chiara writes her table:

S	W	Tot	
2	5	7	

Then she adds the next line for just one part squash by dividing by **2**, but she does not work out the answers yet:

S	W	Tot
2	5	7
1	5÷2	7÷2

She now adds the next line, where she multiplies by **500 g** to get the exact amounts:

S	W	Tot
2	5	7
1	5÷2	7÷2
500	5÷2x500	
300	=1,250 ml	=1,750ml

Check: 500 + 1,250 = 1,750

As you know 1,750 ml = 1.75 litres.

Answer: Chiara can make 1.75 jugs of diluted squash – which is really two jugs with about 800 ml in each and loads of ice!

The advantage of this method is that you can fill in the amount you are given, regardless of which part it is or even the total.

If the question had read:

Squash is mixed with water in the ratio **2** : **5**.

How much squash will I need to make **2** litres of orange drink?

The table would have looked like this:

S	W	Tot
2	5	7
		1
		2,000

S	W	Tot		
2	5	7	Your unit is in the total column	
2÷7	5÷7	1 🗸	and so you hav divided by 7	
2÷7x2,000 =570ml	5÷7x2,000 =1,430ml	2,000ml	With awkward	
			numbers it can be sensible to round	

to the nearest 10

Check: 570 + 1,430 = 2,000

Answer: I will need 570 ml of squash.

Exercises

- 1 Dahlia is making herself a glass of lime juice using **25 ml** of concentrate. If the ratio of lime juice concentrate to water is **1** : **5**, how much water should she add?
- 2 Evie is making a blackcurrant drink for her little brother. She is to mix the blackcurrant cordial with water in the ratio 1 : 4. If she is to make 250 ml in all, how much cordial will she need?

- 3 Fran is making up 5 two litres jugs using tropical cordial for a party. The instructions say to mix the cordial with water in the ratio 1 : 7. How much cordial and how much water will she use?
- 4 Gail is mixing herself a drink that is two parts orange juice and 3 parts lemonade. If she uses 100 ml of orange juice, how much lemonade will she need and what is the volume of her drink?
- 5 Hannah's favourite summer drink is a mix of **2** parts raspberry cordial with **5** parts sparkling water. How much cordial and how much water will she need for a **250 ml** glass?



Cocktails

While squashes and cordials are a mix of the concentrate and water, most cocktails have more ingredients than that.

Remember James Bond's cocktail:

3 parts gin, 1 part vodka and half a part of Kinna Linnet, a total of 4 ½ parts.

Writing this as a ratio we would double the amount and get rid of the $\frac{1}{2}$:

Gin : Vodka : $KL = 3 : 1 : \frac{1}{2}$ = 6 : 2 : 1



Food for Thought

As Kinna Linnet is no longer available, French vermouth is sometimes added to a martini instead.



Food for Thought

Spirits such as gin and vodka, as well as wine and beer are alcoholic. It is against the law for anyone under 18 to buy alcohol.



Food for Thought

It was very fashionable to drink alcoholic cocktails in the 1950s and 1960s.

Nowadays, it is very fashionable to drink non-alcoholic cocktails or 'mocktails'. You might like to invent some of your own. There are lots of interestingly flavoured juices, cordials and mixers available in the supermarket. Take some of your favourites and mix them together with plenty of sparkling water and ice, and see what you can come up with.



Example

MOCKITO RECIPE

Ingredients: Instructions:

4 parts sparkling water

Mix all together and pour over lots of ice. Serve with mint leaves and lime wedges.

3 parts apple juice

mint ie ial

2 parts lime juice cordial

I have a bottle with **500 ml** of lime juice cordial. How much apple juice and sparkling water should I add, and how many litres of mockito will this make?

In this example we will use the unitary method.

Put all the ingredients in a table:

Water	Apple	Lime	Tot
4	3	2	9

Add the next line with 1 part lime juice and then the next with 500. You will first divide the other columns by 2 and then multiply by 500:

Water	Apple	Lime	Tot
4	3	2	9
4÷2	3÷2	1	9÷2
	3÷2×500=750	500 ml	9÷2x500=2,250

Check: 1,000 + 750 + 500 = 2,250

Now you have all the parts of the ratio, you can answer the question:

Answer: I should add 1,000 ml (or 1 litre) of sparkling water and 750 ml of apple juice which will make a total of 2,250 ml (or 2.25 litres) of Mockito



Exercises

6 SHIRLEY TEMPLE RECIPE

Ingredients:

Instructions:

10 parts ginger ale

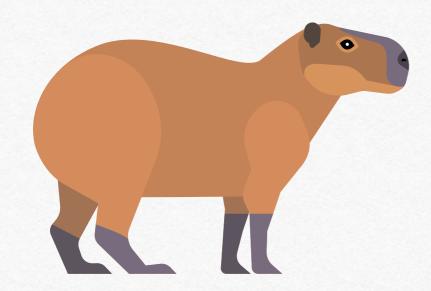
Mix all together .

3 parts grenadine (pomegranate cordial)

Serve with ice and a cherry.

2 parts lime juice cordial

I have 150 ml of grenadine. How much ginger ale and lime juice will I need to go with it, and how many litres of Shirley Temple will this make?





This cocktail was originally named for the actress Shirley Temple (1928 – 2014) in the 1930s.



Shirley Temple was a child star, beginning her film career in 1931 at the age of three. She made 43 films before retiring at the age of 22.

Although Shirley Temple did appear in some other subsequent roles, she embarked on a diplomatic career in 1959 under her married name of Shirley Temple Black.

She is reported as not having liked the cocktail, saying it was too sweet.

Cocktails 251

7 SEA BREEZE RECIPE

Ingredients: Instructions:

5 parts cranberry juice Mix all together.

3 parts sparkling water Serve with ice and with slices of cucumber.

2 parts grapefruit juice

I want to make 1.5 litres of sea breeze. How much of each ingredient will I need?

8 LIGHT AND SPICY RECIPE

Ingredients: Instructions:

6 parts ginger beer Mix all together.

3 parts pineapple juice Serve with ice and with

slices of lime.

1 part fresh lime juice

A hostess serves **250 ml** of this cocktail in a tall glass with ice. How much of each ingredient goes into her cocktail?

9 MOCK TODDY RECIPE

This is lovely when served hot on a cold winter's day:

Ingredients: Instructions:

7 parts Green Tea Serve in glasses of 250 ml with a

2 parts apple juice cinnamon stick per serving to stir.

1 part honey

How much of each ingredient will be needed for a big pot of **5** litres?

10 WIMBLEDON SWERVE RECIPE

Ingredients: Instructions:

10 parts iced tea Mix all together.

3 parts lemonade Serve with ice garnished

with strawberries.

2 parts strawberry

cordial

Strawberry cordial comes in bottles of **350 ml**. A caterer at Wimbledon expects to serve **150** litres of The Wimbledon Swerve in a day. How many bottles of strawberry cordial will she need?

Cocktails 252

Food for Thought



The tennis player Serena Williams won the women's singles title at Wimbledon in 2002, 2003, 2009, 2010, 2012, 2015 and 2016.

Cooking: A Bit of Science

Learning to cook is just as important as learning to manage your money, but some people are put off by the idea. They worry that their cooking will not turn out 'right'. Rather like mental arithmetic there is not always a 'right' way to cook.

In science, students carry out experiments though heating, mixing and blending chemicals. Much the same happens in the kitchen. Cookery is essentially applied science.



Cocktails 253

Think of cake mix. It goes into the oven as a thick liquid and comes out as a soft fluffy cake. It has had a change of state and a variety of chemical processes have transformed the ingredients.

Eggs are another example. You should know that you can do incredible things with an egg. It can simply be hard boiled, or you may prefer it fried or poached. Many people like their eggs scrambled. But there is more to an egg than that.

Eggs are a vital ingredient of certain sauces, such as mayonnaise and hollandaise. They also are used to make cakes. As an egg is heated it sets and that is what helps transform your gooey liquid into a sensational cake.

Have you tried beating an egg white? The addition of air turns it into a soft and fluffy mixture that is the basis for meringues and so much more.

The science of cookery is not magic, it follows very simple rules and as you expand the range of dishes that you cook, you will become better at it. If you do not know where to start, then pick up an egg!



Recipes

When you start to cook, you will begin by following a recipe.

Just like our mocktails, a recipe has ingredients and instructions. Unlike the mocktails, there will be dry ingredients as well as wet ingredients.

Look at this list of ingredients for pancakes:

100 g plain flour (dry)	300 ml milk (wet)
	<u>.</u>

2 large eggs (wet) 1 tbsp of oil (wet)

Most recipes start with you having to mix together any dry ingredients and then stir in the wet ingredients, often starting with a beaten egg.

You can see that one wet ingredient, milk, is measured in **ml**, so you will use a measuring jug. The oil is only a small amount and **tbsp** stands for tablespoon.

You can measure that out using an ordinary spoon but these do vary in volume, but this doesn't usually matter since the measurements concerned are quite small.

It is a good idea to get some special spoons where the measures are standardised.



Food for Thought

The recipe that you have for pancakes makes what is called a batter. A batter, and therefore a similar recipe, is also used for Yorkshire puddings, toad in the hole and fritters. You can cover fish or other seafood in batter before deep fat frying. The batter keeps the fish inside moist. Batters are found in various cultures such as tempura in Japan and pakora in India, although slightly different local ingredients may be used.

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In most batter recipes you will see that the flour and butter are measured in grams. Measure these using kitchen scales. There are all sorts around, but you do not need anything fancy. Just make sure that you can read the display and make any adjustments if you put a bowl on the scale first.

You may find yourself using an American recipe or one of your grandmother's where the quantities are given in imperial units, or cups, for which your measure spoon set will come in handy.

Your scales should be able to measure in pounds and ounces as well as grams and kilograms.



This book is not going to teach you to cook, but to help you to understand the numbers involved.

You can remind yourself of the various measurements and how to convert between them by looking back to Step 3.



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Small Parties / Big Parties

One of the joys of cooking is to be able to entertain your friends. One of the problems is that the recipe does not always cater for the number of friends that you want to invite. This section shows how to adjust your recipes.

First, small parties as there will be times when you want to cater for just one or two friends or even just for yourself.

Example

Indigo is having two friends round for supper. She is going to make a beef casserole and has this recipe:

For 4 people:	
Ingredients	
800 g stewing steak,	2 tbsp plain flour
2 onions	600 ml beef stock
3 carrots	Salt, pepper and herbs to season

How much steak, onions, carrots, flour and stock will Indigo need?

You could make a table as before but quite a lot of recipes have lots of ingredients and so your table could get quite wide. Have another look at the ingredients, they are all written down for you already, so all you have to do is to make notes on the list. (This is fine if it is a page printed out or your own cookery book, but you would not be so ill mannered as to write notes in someone else's book!)

Indigo is going to use the unitary method so she adds two columns to the right of the ingredients as well as a note of the number of servings:

4 servings	1	3
800 g stewing steak		
2 onions		
3 carrots		
2 tbsp plain flour		
600 ml beef stock		

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Next Indigo divides each quantity by **4** to find the amount for **1** serving and then she multiplies that by **3** to find the amount she needs for **3** servings:

4 servings	1	3
800 g stewing steak	200 g	600 g
2 onions	1/2	11/2
3 carrots	3/4	9/4 = 21/4
2 tbsp plain flour	1/2	11/2
600 ml beef stock	150 ml	450 ml

Indigo knows that she cannot buy ½ an onion or ¼ carrot but onions and carrots do not come in a unform size, so 2 small onions and 2 large carrots should do it.

Answer: Indigo will need 600 g steak, 2 onions, 2 carrots, $1\frac{1}{2}$ tbsp flour and 450 ml of beef stock.





When Indigo goes to the supermarket, she may well find that packets of steak are not sold in packs of **800 g**. She could go to the fresh meat counter and ask for the exact amount there, or she could buy a pack that is a bit larger.

She may also find the steak is a bit expensive, and she can adjust the recipe by using less steak and more carrots and perhaps adding some mushrooms as well.



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Exercises

11 Jana is cooking spaghetti Bolognese for her boyfriend. She has a recipe with this list of ingredients:

SPAGHETTI BOLOGNESE

- for **4** persons

Ingredients:

500 g lean beef mince

1 onion

2 carrots

500 g jar of Bolognese Pasta Sauce

250 g of spaghetti

How much of each ingredient will Jana need for her meal for two?



Food for Thought

Spaghetti Bolognaise and Corn on the Cob are two suggestions for you to serve to your boyfriend's ex-girlfriend when he invites her to dinner because it impossible to eat them without food sliding down your chin. Naughty girl!

12 Jana is going to serve pancakes for pudding. A recipe for 15 pancakes lists these ingredients:

100 g plain flour	300 ml milk
2 large eggs	50 g butter

If Jana wants to make **5** pancakes, how much of each ingredient will she need?

13 Kelly is attending a baby shower via an online video platform. Everyone is to make cup cakes with pink and blue icing and will eat them together as they chat online about the happy event. Kelly is using this recipe.

To make 12 cupcakes:

150 g softened butter

3 small eggs

150 g golden caster sugar 150 g self-raising flour

As Kelly lives on her own, she does not want 12 cupcakes. She decides to take one egg and beat it up then divide the beaten egg in two. She then proportions the rest of the ingredients to match half an egg. How many cupcakes does Kelly make and what quantity of the other ingredients does she need?



14 Kelly is also going to make herself some smoked salmon appetisers. Her recipe for 20 appetisers has these ingredients:

100 g smoked salmon200 g cream cheese1 teaspoon fresh lemon juice

Kelly will make enough for **6** appetisers. How much of each ingredient will she need?

You can see that for **small parties** you divide all the ingredients in a recipe by the same number.

For large parties you will need to multiply all the ingredients by the same number. When talking about larger quantities, remember that it will be useful to convert grams to kilograms and millilitres to litres for your shopping list.

It is also useful to know that a teaspoon is equal to **5 ml** and a tablespoon to **15 ml** so that you know how much to buy of these quantities.

Example

Lara is a freelance chef and has an order to make 120 cup cakes for a vegan tea shop. Her recipe has these ingredients:

For 12 cup cakes	
140 g self-raising flour	40 ml vegetable oil
100 g castor sugar	1 tsp baking powder
120 ml soya milk	

How much of each ingredient should Lara buy to make all her cupcakes?

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Lara is to make 120 cupcakes and the recipe is for 12, so Lara needs to multiply each ingredient by 10. She will then need to convert some of these to grams and litres by dividing by 1,000.

12 cupcakes	120
140 g self-raising flour	1,400 g = 1.4 kg
100 g caster sugar	1,000 g = 1 kg
120 ml soya milk	1,200 ml = 1.2 l
40 ml vegetable oil	400 ml
1 tsp baking powder	5 ml x 10 = 50 ml

Answer: Lara will need 1.4 kg flour, 1 kg of castor sugar, 1.2 l of milk, 400 ml of oil and 50 ml of baking powder

That example was straightforward because you simply had to multiply the ingredients by **10**. There are other times when you have to do the calculation in two stages.





Example

Moana is throwing a supper party for **14** friends (including herself). She is making a cheesecake for dessert.

These are the ingredients in her recipe:

For 10 servings:	
200 g digestive biscuits	200 g icing sugar
75 g butter	1 tsp vanilla extract
700 g cream cheese	

How much of each ingredient will Moana need?

Moana uses the unitary method. She divides by **10** and then multiplies by **14**.



Moana adds two columns to her ingredient list:

10 servings	1	14
200 g digestive biscuits	20 g	280 g
75 g butter	7.5 g	105 g
700 g cream cheese	70 g	980 g
200 g icing sugar	20 g	280 g
1 tsp vanilla extract	0.1 tsp	1.4 tsp

Answer: Moana will need 280 g digestive biscuits, 105 g butter, 980 g cheese, 280 g icing sugar and 1.4 teaspoons of vanilla extract.



Exercises

15 Moana is cooking salmon and green beans for a main course from a recipe with these ingredients:

For 4 servings:	
200 g green beans20 small cherry tomatoes	2 tablespoons extra virgin olive oil4 salmon fillets

What quantity of each ingredient will she need for **14** people?

16 Nula is serving brown rice to her 20 party guests. A 1 kg bag of brown rice tells her that it will serve 12 people and that she should add 1 cup of rice to 2 ½ cups of boiling water.

How much rice will Nula need and how much water should she put on to boil?



17 Olla is making brownies for a charity bake sale. She uses a recipe with these ingredients:

For 12 brownies

3 eggs 225 g dark chocolate

225 g caster sugar 100 g plain flour

225 g butter 100 g cocoa powder

How much of each ingredient will Olla need to make 100 brownies?

18 Pru is cooking a vegetable curry for 50 people using a recipe with these ingredients:

Serves 3

350 g potatoes

1 tbsp medium or hot curry powder

1 large carrot

1 x 227 g tin chopped

½ cauliflower

tomatoes

1 large onion

How much of each ingredient will she need?

19 Rula is cooking for a year group reunion. She is making an aubergine dish that will be suitable for vegetarians. How much of each of these ingredients will she need to make 50 servings:

Serves 4

1 large aubergine

100 g vegetarian cream cheese

140 g couscous

200 g cherry tomatoes

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20 Rula is also making coronation chicken. She is following a recipe with these ingredients:

Serves 6

175 g mayonnaise

3 tsp mild curry powder

2 tbsp mango chutney

50 g sultanas

500 g shredded cooked chicken

How much of each ingredient will Rula need for **80** servings?



Food for Thought

Coronation Chicken is a dish that is credited to the principals of the Cordon Bleu cookery school
Constance Spry and Rosemary Hume. It was invented as part of the banquet for the coronation of Queen
Elisabeth II in 1953 and has proved very popular since as a simple and flavourful dish to serve for large quantities of people. It also makes a good filling for a sandwich or a baked potato.

Other Ratios

So far in this chapter we have looked at recipes for drinks and for food. However, there are lots of other things that can be mixed up in ratios, or in parts.

Try the questions in these last exercises. You can use whichever method that you prefer.



Small parties / Big parties

Exercises

- 21 In Selima's school the teacher-to-pupil ratio is 1 : 8. If there are 30 teachers, how many pupils are there?
- 22 The following term 50 extra pupils join the school, how many extra teachers are needed to keep the teacher-to-pupil ratio the same?
- 23 In the village hall there are tables that seat 6 people. Thelma is organising a village party for 200 people. How many tables will she need?
- **24** Uma is making a bracelet using one big bead for every five little beads. She has a bag of **200** little beads. How many big beads should she buy?



- 25 Bronze is a metal made by mixing two other metals, tin and copper, in the ratio 1 : 7. Visha wants to make200 kg of bronze, how much copper and tin will she need?
- 26 Wei is fertilising her lawn. Fertiliser must be mixed with water before using it. The bottle says to mix 1 part of fertiliser to 8 parts of water. How much fertilizer should Wei mix with 1 litre of water?
- 27 Xiang is making up some cement mortar in order to repair her garage wall. She needs to mix water, cement and sand in the ratio 1:2:3.

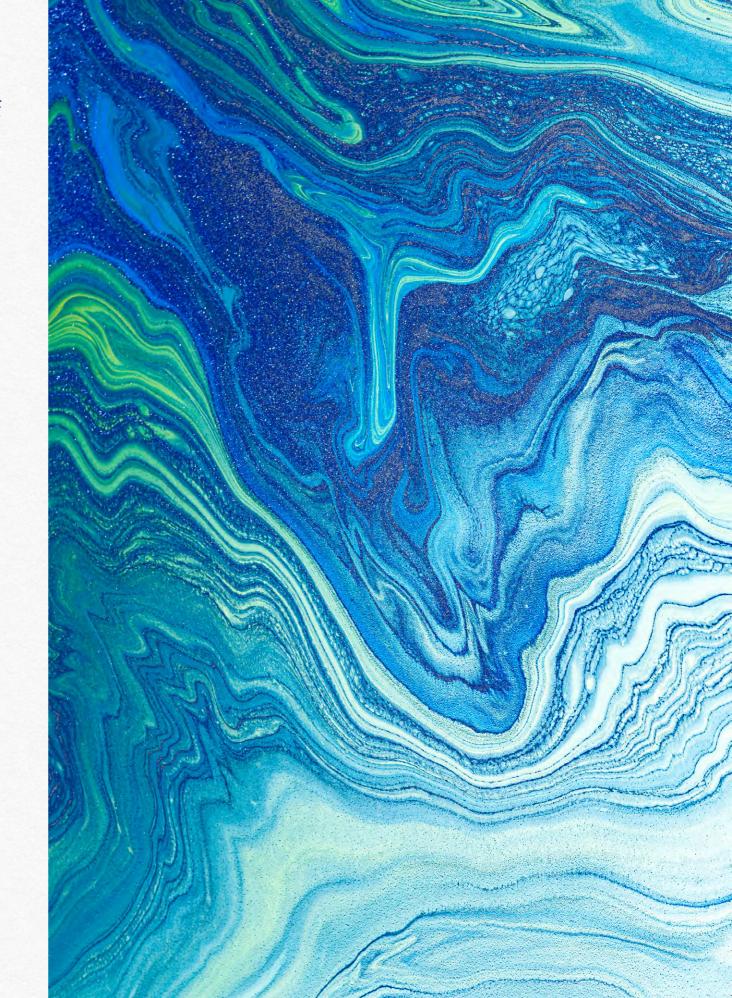
If she needs **5** litres of mortar, how much of each ingredient should Xiang use?

28 At the end of the season, the ratio of Yulia's netball team's results are as follows. The ratio of wins: draws: losses is 6:3:1. If they played 20 games, how many games do they win and how many did they lose?

29 Zefanie is making for her daughter a doll's house like their own home. The doll's house is to be built to a scale of 1:12. If Zefanie's house is 8 m high, what will be the height of the doll's house?



30 Zefanie's house has 8 windows, how many windows will the doll's house have?



Answers to Part 5

1 Dahlia is making herself a glass of lime juice using **25** ml of concentrate. If the ratio of lime juice concentrate to water is **1** : **5**, how much water should she add?

Dahlia can probably do this in her head, but these are the stages of thinking that her head might be doing:

Dahlia draws up a table like this:

LJC	W	Tot
1	5	6

Then she adds a second line showing the **25** ml of lime juice concentrate

LJC	W	Tot
1	5	6
25		

Dahlia can see that the 1 has been mutiplied by **25** to get **25 ml** and so she multiplies the next two columns by **25** also:

LJC	W	Tot
1	5	6
25	125	150

Check
$$25 + 125 = 150$$

Now that she has all the quantities calculated Dahlia can answer the question.

Answer: Dahlia should add 125 ml of water



2 Evie is making a blackcurrant drink for her little brother. She is to mix the blackcurrant cordial with water in the ratio 1 : 4. If she is to make 250 ml in all, how much cordial will she need?

Evie does exactly the same as Dahlia but this time fills **250** ml in the total column:

ВС	W	Tot
1	4	5
		250

Evie can see that $5 \times 50 = 250$ and so she multiplies her other two columns by 50:

ВС	W	Tot
1	4	5
50	200	250

Check
$$50 + 200 = 250$$

Now that she has all the quantities calculated, Evie can answer the question.

Answer: Evie will need 50 ml of cordial

3 Fran is making up 5 two litres jugs using tropical cordial for a party. The instructions say to mix the cordial with water in the ratio 1 : 7. How much cordial and how much water will she use?

5 two litre jugs will be a total of **10** litres. Fran's table will initially look like this:

TC	W	Tot
1	7	8
		10

Using a calculator Fran can see that $10 \div 8 = 1.25$

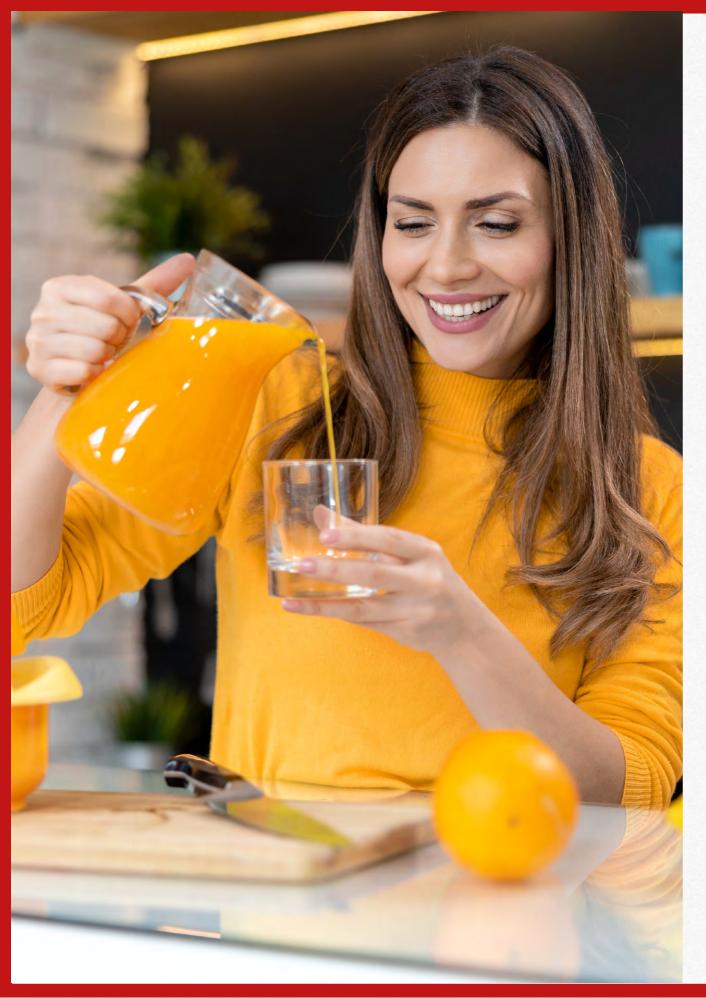
She therefore multiplies her other 2 columns by 1.25

TC	W	Tot
1	7	8
1.25	8.75	10

Check 1.25 + 8.75 = 10

Now she has all the quantities calculated Fran can answer the question.

Answer: Fran will need 1.25 | of cordial and 8.75 litres of water



4 Gail is mixing herself a drink that is two parts orange juice and 3 parts lemonade. If she uses 100 ml of orange juice, how much lemonade will she need and what is the volume of her drink?

Gail's table will initially look like this:

OJ	L	Tot
2	3	5
100		

Gail can see that $2 \times 50 = 100$

She therefore multiplies her other 2 columns by 50

OJ	L	Tot
2	3	5
100	150	250

Check 100 + 150 = 250

Now she has all the quantities calculated, Gail can answer the question.

Answer: Gail will need 150 ml of lemonade and the drink will be 250 ml in all

5 Hannah's favourite summer drink is a mix of 2 parts raspberry cordial with 5 parts sparkling water. How much cordial and how much water will she need for a 250 ml glass?

Hannah's table will initially look like this:

RC	W	Tot
2	5	7
		250

Using a calculator, Hannah can see that $250 \div 7 = 35.714...$ which she rounds down to 35, so that she doesn't end up with more than can fit into her glass.

Hannah therefore multiplies her other 2 columns by 35

RC	W	Tot
2	5	7
70	175	250

Check: 70 + 175 = 245

Hannah knows her check will not be exactly right because she rounded the **35.714** to **35**.

Now she has all the quantities calculated Hannah can answer the question.

Answer: Hannah will need 70 ml of raspberry cordial and 175 ml of water



6 SHIRLEY TEMPLE RECIPE

Ingredients: Instructions:

10 parts ginger ale Mix all together.

Serve with ice and

3 parts grenadine (pomegranate cordial) with a cherry.

2 parts lime juice cordial

I have 150 ml of grenadine. How much ginger ale and lime juice will I need to go with it, and how many millilitres of Shirley Temple will this make?

My table will initially look like this:

GA	Gr	LJC	Tot
10	3	2	15
	150		

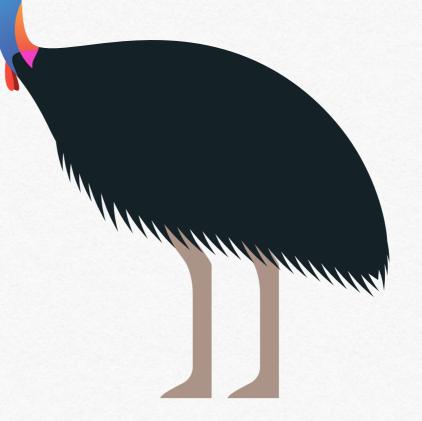
I can see that $3 \times 50 = 150$ and therefore I multiply the other columns by 50:

GA	Gr	LJC	Tot
10	3	2	15
500	150	100	<i>7</i> 50

Check: 500 + 150 + 100 = 750

Now I have all the quantities, I can answer the question.

Answer: I will need 500 ml of ginger ale and 100 ml of lime juice cordial, this will make 750 ml of cocktail



7 SEA BREEZE RECIPE

Ingredients: Instructions:

5 parts cranberry juice Mix all together.

Serve with ice and

3 parts sparkling water

slices of cucumber.

2 parts grapefruit juice

I want to make 1.5 litres of sea breeze. How much of each ingredient will I need?

My table will initially look like this:

CJ	SW	GJ	Tot
5	3	2	10
			1,500

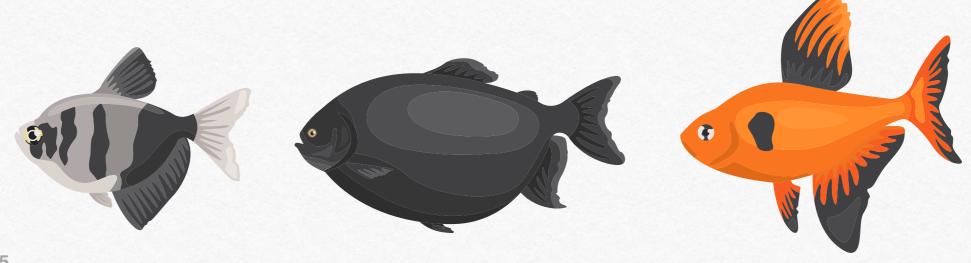
I can see that $10 \times 150 = 1,500$ and therefore I multiply the other columns by 150:

CJ	SW	GJ	Tot	
5	3	2	10	
<i>7</i> 50	450	300	1,500	

Check: 750 + 450 + 300 = 1,500

Now I have all the quantities, I can answer the question.

Answer: I will need 750 ml of cranberry juice, 450 ml of sparkling water and 300 ml of grapefruit juice.



8 LIGHT AND SPICY RECIPE

Ingredients: Instructions:

6 parts ginger beer Mix all together.

3 parts pineapple juice Serve with ice and with slices of lime.

1 part fresh lime juice

A hostess serves **250 ml** of this cocktail in a tall glass with ice. How much of each ingredient goes into her cocktail?

The hostess's table will initially look like this:

GB	PJ	LJ	Tot
6	3	1	10
			250

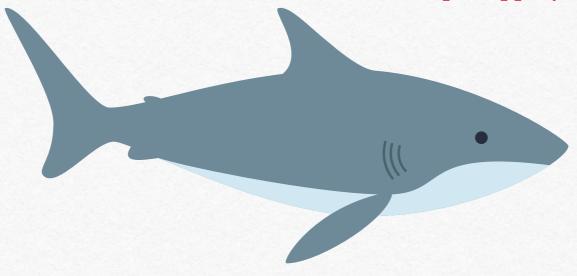
The hostess can see that $10 \times 25 = 250$ and therefore she multiplies the other columns by 25:

GB	PJ	LJ	Tot
6	3	1	10
150	75	25	250

Check: 150 + 75 + 25 = 250

Now she has all the quantities, the hostess can answer the question.

Answer: The hostess will need 150 ml of ginger beer, 75 ml of pineapple juice and 25 ml of lime juice.



9 MOCK TODDY RECIPE

This is lovely when served hot on a cold winter's day:

Ingredients: Instructions:

7 parts Green Tea Serve in glasses of 250 ml with a

2 parts apple juice cinnamon stick per serving to stir.

1 part honey

How much of each ingredient will be needed for a big pot of **5** litres?

My table will initially look like this:

GT	AJ	Н	Tot
7	2	1	10
			5,000

I can see that $10 \times 500 = 5,000$ and then I multiply the other columns by 500:

GT	AJ	Н	Tot	
7	2	1	10	
3,500	1,000	500	5,000	

Now I have all the quantities, I can answer the question once I have worked out how many glasses I can fill:

$$5,000 \div 250 = 20.$$

Answer: I will need 3.5 | of green tea, 1 | of apple juice, 500 ml of honey and 20 cinnamon sticks.







10 WIMBLEDON SWERVE RECIPE

Ingredients: Instructions:

10 parts iced tea Mix all together.

3 parts lemonade Serve with ice and

with strawberries.

2 parts strawberry

cordial

Strawberry cordial comes in bottles of **350 ml**. A caterer at Wimbledon expects to serve **150** litres of The Wimbledon Swerve in a day. How many bottles of strawberry cordial will she need?

The caterer's table will initially look like this:

IT	L	SC	Tot
10	3	2	15
			150

The caterer can see that $15 \times 10 = 150$ and then she multiplies the other columns by 10:

IT	L	SC	Tot
10	3	2	15
100	30	20	150

Check: 100 + 30 + 20 = 150

Now check the question.

The caterer needs to know how many bottles of strawberry cordial she needs to buy. She now knows that she needs **20** litres. She will need to divide the total amount of strawberry cordial; **20** l or **20,000** ml by the amount in one bottle **350** ml:

$$20,000 \div 350 = 57.1428...$$

Answer: The caterer will need 58 bottles of strawberry cordial (57 wouldn't be quite enough)

Catering on a large scale needs lots of maths. Assuming the Wimbledon Swerve is served in **250 ml** servings then **150 l** is **600** servings.

It might be sensible for the caterer to put everything on a spreadsheet.

If the caterer makes her own iced tea from tea bags, honey and lemon juice then her shopping list might look like this:

	Α	В	С	D	E	F	G
1					Tot		per serving
2	100	tea bags	£3.50		£3.50		£0.01
3	10	jars of honey	£1.80		£18.00		£0.03
4	20	lemons	£0.30		£6.00		£0.01
5	30	lemonade	£0.50		£15.00		£0.03
6	58	strawberry cordial bottles	£5.00		£290.00		£0.48
7	30	bags of ice	£1.00		£30.00		£0.05
8				Total	£359.50		£0.60

Not all her costs are yet included as she will need glasses, hopefully not disposable ones, but perhaps the reusable kind where you pay £1 deposit. She also will need to pay for her space in the grounds and a licence to sell.

How much do you think you would pay for a cool and refreshing glass of Wimbledon Swerve?

11 Jana is cooking spaghetti bolognese for her boyfriend. She has a recipe with this list of ingredients:

SPAGHETTI BOLOGNESE

- for **4** persons

Ingredients:

500 g lean beef mince **500 g** jar of Bolognese

1 onion Pasta Sauce

2 carrots 250 g of spaghetti

How much of each ingredient will Jana need for her meal for two?

Jana does not need to use the unitary method. She can simply divide the original ingredients by **2**:

4 portions	2
500 g lean beef mince	250 g
1 onion	½ onion
2 carrots	1 carrot
500 g jar of Bolognese Pasta Sauce	½ jar
250 g of spaghetti	125 g

Answer: Jana will need 250 g of lean beef mince, $\frac{1}{2}$ onion, 1 carrot, $\frac{1}{2}$ jar of Bolognese sauce and 125 g of spaghetti.

12 Jana is going to serve pancakes for pudding. A recipe for 15 pancakes lists these ingredients:

100 g plain flour	300 ml milk
2 large eggs	50 g butter

If Jana wants to make **5** pancakes, how much of each ingredient will she need?

Again, Jana does not need to use the unitary method. She can simply divide the original ingredients by **3**. Not all the quantities divide exactly by **3** so Jana does some sensible rounding and uses a smaller egg:

15 pancakes	5
100 g plain flour	35 g
2 large eggs	² /3 or 1 medium egg
300 ml milk	100 ml
50 g butter	17 g

Answer: Jana will need 35 g flour, 1 medium egg, 100 ml milk and 17 g of butter.

13 Kelly is attending a baby shower via an online video platform. Everyone is to make cup cakes with pink and blue icing and will eat them together as they chat online about the happy event. Kelly is using a recipe with these ingredients:

To make 12 cupcakes:

150 g softened butter 3 small eggs

150 g golden caster sugar 150 g self-raising flour

As Kelly lives on her own, she does not want 12 cupcakes. She decides to take one egg and beat it up then divide the beaten egg in two. She then proportions the rest of the ingredients to match half an egg. How many cupcakes does Kelly make and what quantity of the other ingredients does she need?

As Kelly's recipe needed **3** eggs and she is to use half an egg, she divides all the quantities by **6**.

12 cupcakes	2
150 g softened butter	25 g
150 g golden caster sugar	25 g
3 small eggs	½ egg
150 g self-raising flour	25 g

Answer: Kelly will make 2 cupcakes
She will need 25 g each of butter, sugar and self raising flour.



14 Kelly is also going to make herself some smoked salmon appetisers. Her recipe for 20 appetisers has these ingredients:

100 g smoked salmon	1 teaspoon fresh lemon juice
200 g cream cheese	

Kelly is going to make enough for **6** appetisers. How much of each ingredient will she need?

Kelly is going to use the unitary method. She will first divide by **20** and then multiply by **6**:

20 appetisers	1	6
100 g smoked salmon	5 g	30 g
200 g cream cheese	10 g	60 g
1 teaspoon fresh lemon juice	1/20	$6/20 \approx 1/4 \text{ tsp}$

Answer: Kelly will need 30 g smoked salmon, 60 g cream cheese and ½ teaspoon of lemon juice

15 Moana is cooking salmon and green beans for a main course from a recipe with these ingredients:

For 4 servings:	
200 g green beans20 small cherry tomatoes	2 tablespoons extra virgin olive oil4 salmon fillets

What quantity of each ingredient will she need for **14** people?

Moana will use the unitary method and will divide by 4 and multiply by 14

4 servings	1	14
200 g green beans	50 g	700 g
20 small cherry tomatoes	5	70
2 tablespoons extra virgin olive oil	$\frac{2}{4} = \frac{1}{2}$	7 tbsp
4 salmon fillets	1	14

Answer: Moana will need 700 g green beans, 70 cherry tomatoes, 7 tablespoons of olive oil and 14 salmon fillets.

16 Nula serves brown rice to her 20 party guests. A 1 kg bag of brown rice tells her that it will serve 12 people and that she should add 1 cup of rice to 2 cups of boiling water.

How much rice will Nula need and how much water should she put on to boil?

Nula uses the unitary method:

Rice	People
1 kg	12
1/12	1
$^{1}/12 \times 20$	20
= 1.67 kg	

Answer: Nula will need 1.67 kg brown rice

To calculate the amount of water needed, Nula first needs to know how many cups of rice she will be using. She weighs out a metric cup (250 ml) of rice and finds it weighs 201 g \approx 200 g. Therefore she will be using: 1,670 ÷ 200 = 8.3 cups of rice.

To calculate the amount of water needed, Nula multiplies the number of cups of rice by **2**.

$$2 \times 8.3 = 16.6 \text{ cups of water}$$

A metric cup is **250 ml**, therefore:

$$16.6 \text{ cups } \times 250 = 4,150 \text{ ml}$$

Answer: Nula needs 4.15 litres of water



In real life, calculations often do not work out to an exact number or quantity. Just round off the answer sensibly. Generally, with a recipe it is best to slightly round up basic ingredients but round down seasonings – you can always add more of them later.

Answers to Part 5

17 Olla is making brownies for a charity bake sale. She uses a recipe with these ingredients:

For 12 brownies

3 eggs 225 g dark chocolate

225 g castor sugar 100 g plain flour

225 g butter 100 g cocoa powder

How much of each ingredient will Olla need to make 100 brownies?



Olla will divide by **12** and then multiply by **100** and change large quantities from **g** to **kg**, Olla dos not need to calculate every quantity because several of them are the same.

12 brownies	1	100
3 eggs	$^{3}/_{12} = ^{1}/_{4}$	1/4 x 100 = 25 eggs
225 g castor sugar	225/12	$^{225}/_{12} \times 100 = 1,875 \text{ g}$
225 g butter		1,875 g or 1.875 kg
225 g dark chocolate		1,875 g or 1.875 kg
100 g plain flour	100/12	$^{100}/_{12} \times 100 = 833 \text{ g}$
100 g cocoa powder		833 g

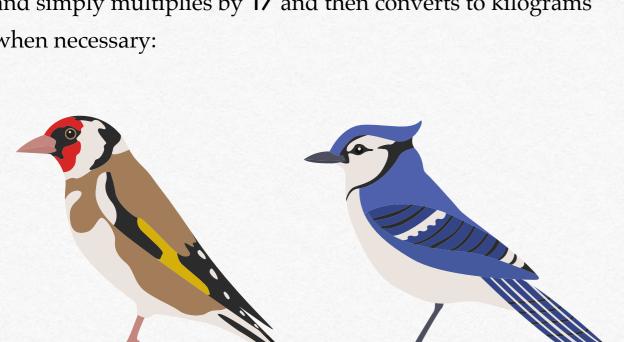
Answer: 25 eggs, 1.9 kg each of castor sugar, butter and dark chocolate, 850 g each of plain flour and cocoa powder.

18 Pru is cooking a vegetable curry for 50 people using a recipe with these ingredients:

Serves 3	
350 g potatoes	1 tbsp medium or hot curry
1 large carrot	powder
½ cauliflower	1 x 227 g tin chopped tomatoes
1 large onion	

How much of each ingredient will she need?

Pru could use the unitary method and divide by 3 and then multiply by 50 but that will give her some awkward numbers. Instead she works out that $50 \div 3$ is roughly 17 and simply multiplies by 17 and then converts to kilograms when necessary:



Serves 3	Serves about 50
350 g potatoes	5,950 g ≈ 6 kg
1 large carrot	17
½ cauliflower	8.5 or 9
1 large onion	17
1 tbsp medium	17 x 15ml = 255 ml
hot curry powder	
1 x 227 g tin chopped	3,860 g ≈ 4 kg
tomatoes	(or 10 x 400g tins)

Answer: Pru will need 6 kg of potatoes, 17 large carrots, 9 cauliflowers (you cannot buy ½ cauliflower), 17 large onions, 255 ml curry powder and 10 larger (400g) tins of tomatoes.



Answers to Part 5

19 Rula is cooking for a year group reunion. She is making an aubergine dish that will be suitable for vegetarians. How much of each of these ingredients will she need for 50 servings:

Serves 4	
1 large aubergine	200 g cherry tomatoes
140 g couscous	100 g vegetarian cream cheese

Rula uses the unitary method and divides by **4** then multiplies by **50**:

4 servings	1	50
1 large aubergine	1/4	1/4 x 50 ≈ 13 aubergines
140 g couscous	¹⁴⁰ /4 g	$140/4 \times 50 = 1,750 \text{ g}/1.75 \text{ kg}$
200 g cherry tomatoes	50 g	$50 \times 50 = 2,500 \text{ g/}2.5 \text{ kg}$
100 g veggie cream cheese	25 g	25 x 50 = 1,250 g/1.25 kg

Answer: Rula will need 13 aubergines, 1.75 kg couscous, 2.5 kg cherry tomatoes and 1.25 kg cream cheese

20 Rula is also making coronation chicken. She is following a recipe with these ingredients:

Serves 6	
175 g mayonnaise	50 g sultanas
3 tsp mild curry powder	500 g shredded cooked
2 tbsp mango chutney	chicken

How much of each ingredient will Rula need for **80** servings?

Rula will use the unitary method and divide by **6** then multiply by **80**:

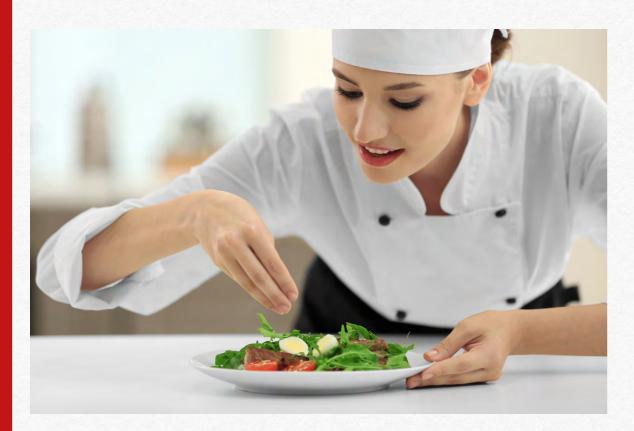
6 servings	1	80
175 g mayonnaise	175/6	$175/6 \times 80 \approx 2,333 \text{ g}$
3 tsp mild curry powder	1/2	40 tsp
2 tbsp mango chutney	1/3	¹ /3 x 80 ≈ 26 tbsp
50 g sultanas	50/6	⁵⁰ /6 x 80 ≈ 650 g
500 g shredded cooked chicken	500/6	⁵⁰⁰ /6 x 80 ≈ 6.5 kg

Rula has rounded down the chutney as that is a seasoning and rounded up main ingredients. She now needs to convert the large numbers of teaspoons and tablespoons to ml or l.

$$40 \text{ tsp} = 40 \times 5 \text{ ml} = 80 \text{ ml}$$

 $26 \text{ tbsp} = 26 \times 15 \text{ ml} = 390 \text{ ml} \approx 400 \text{ ml}$

Answer: Rula will need 2.3 kg mayonnaise, 80 ml of curry powder, 400 ml of mango chutney, 650 g of sultanas and 6.5 kg of cooked chicken.



21 In Selima's school the teacher-to-pupil ratio is 1 : 8. If there are 30 teachers, how many pupils are there?

Teacher	Pupil	
1	8	
30	240	You must then multiply 8 by 30

Answer: There are 240 pupils.



22 The following term 50 extra pupils join the school, how many extra teachers are needed to keep the teacher-to-pupil ratio the same?

Teacher	Pupil	Now th
1	8	1 ÷ 8 x 2 Round teachers
1 ÷ 8	1	
1 ÷ 8 x 290 ≈ 37	290	

Now there are 290 pupils.

 $1 \div 8 \times 290 = 36.25$. Round up to 37 as 36 seachers will be too few.

Now answer the question.

Answer: The school must employ 7 extra teachers.

23 In the village hall there are tables that seat 6 people. Thelma is organising a village party for 200 people. How many tables will she need?

Thelma uses the unitary method and makes a table:

People	Table	
6	1	
1	1÷6	
200	1÷6×200=33.33	

Answer: Thelma will need 34 tables

24 Uma is making a bracelet using one big bead for every five little beads. She has a bag of **200** little beads. How many big beads should she buy?

Uma makes a table:

Big	Little
1	5
40	200

Uma can see $5 \times 40 = 200$, so multiply 1 by 40

Answer: Uma will need to buy 40 big beads



25 Bronze is a metal made by mixing two other metals, tin and copper, in the ratio 1 : 7. Visha wants to make 200 kg of bronze, how much copper and tin will she need?

Visha makes a table:

Tin	Copper	Tot
1	7	8
?	?	200

Visha can see that $8 \times 25 = 200$ and therefore multiplies the other columns by 25:

Tin	Copper	Tot
1	7	8
25	175	200

Check: 25 + 175 = 200

Answer: Visha needs 25 kg of tin and 175 kg of copper.

26 Wei is fertilising her lawn. Fertiliser must be mixed with water before using it. The bottle says to mix 1 part of fertiliser to 8 parts of water. How much fertilizer should Wei mix with 1 litre of water?

First Wei converts 1 litre to 1,000 ml

	Fertiliser	Water	Total
	1	8	9
$1,000 \div 8 = 125$ Multiply 1 by 12	125	1,000	1,250

Answer: Wei should mix 125 ml fertiliser with 1 litre of water.



27 Xiang is making up some cement mortar in order to repair her garage wall. She needs to mix water, cement and sand in the ratio 1:2:3.

If she needs **5** litres of mortar, how much of each ingredient should Xiang use?

Xiang makes a table:

W	С	S	Tot
1	2	3	6
?	?	?	5 litres

Xiang can see that by dividing **5,000 ml** by **6** she will get some awkward numbers so she uses the unitary method:

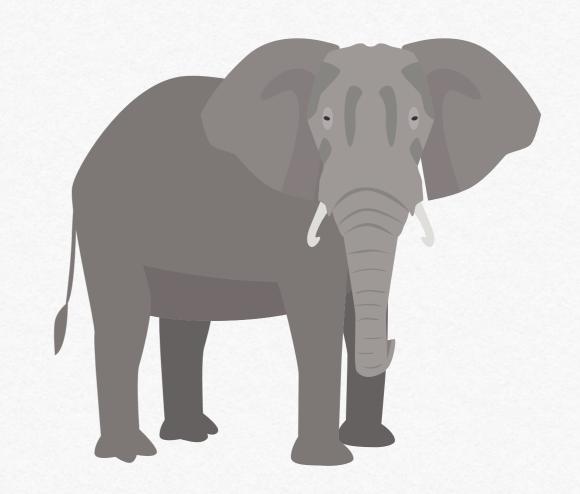
W	С	S	Tot
1	2	3	6
1 ÷ 6	2 ÷ 6	3 ÷ 6	1
1 ÷ 6 x 5,000 = 833	2 ÷ 6 x 5,000 = 1,667	3 ÷ 6 x 5,000 = 2,500	5,000

Xiang rounds the awkward amounts to the nearest **50 ml**:

833 ml \approx 850 ml, 1.666 ml \approx 1.65 litres

Check: 0.850 + 1.65 + 2.5 = 5

Answer: Xiang will need 850 ml of water, 1.65 litres cement and 2.5 litres of sand.



28 At the end of the season the ratio of Yulia's netball team's results are as follows. The ratio of wins: draws: losses is 6:3:1. If they played 20 games, how many games do they win and how many did they lose?

Yulia makes a table:

W	D	L	Tot
6	3	1	10
?	?	?	20

Yulia can see that $10 \times 2 = 20$ and multiplies all columns by 2

W	D	L	Tot
6	3	1	10
12	6	2	20

Check: 12 + 6 + 2 = 20

Now answer the question:

Answer: Yulias team won 12 matches and lost 2 matches.

29 Zefanie is making for her daughter a doll's house that looks like their own home. The doll's house is to be built to a scale of 1 : 12. If Zefanie's house is 8 m high, what will be the height of the doll's house?

Convert the 8 m to cm

Ratio model to house

= 1 : 12

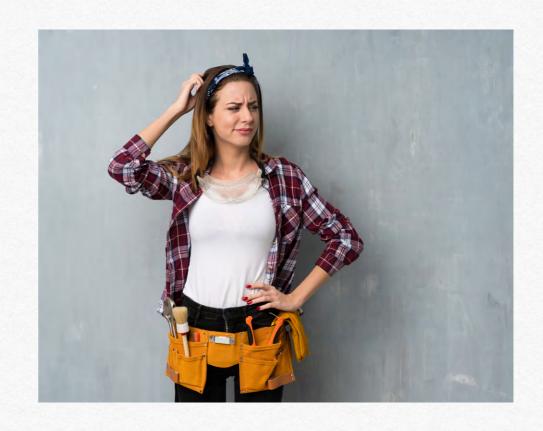
 $800 \div 12 = 66.66...$ So $12 \times 66.66... = 800$

= ? : 800

= 66.67: 800

Multiply 1 by 66.66...

Answer: The doll's house will be 66.7 cm high.





Food for Thought

The scale 1:12 is a traditional one for model toys. That is because until the 1960s the UK used the imperial system of measurement (and the USA still does.)

As there are 12 inches in 1 foot, the scale makes a lot of sense.

If Zephanie's house had been measured in feet it might have been found to be **27** feet and the model would therefore be **27** inches, or **2** feet **3** inches.

30 Zefanie's house has 8 windows, how many windows will the dolls house have?

Answer: You obviously spotted that this is a trick question. The doll's house will also have 8 windows!



Answer each question in your head and then check to see if your answer is correct.

Q1

In a school the ratio of girls to boys is **7** : **8**. If there are **210** girls, how many boys are there?

Q2

To make an elderflower drink, 1 part elderflower cordial should be diluted with 9 parts water.

How much elderflower cordial is needed to fill a 2 litre jug of elderflower drink?

Your Brain Workout 291

Q3

'Mimms' is a mix of 1 part orange barley water, 3 parts iced tea and 8 parts lemonade. How much iced tea will I need for a 240 ml glass of 'Mimms'?

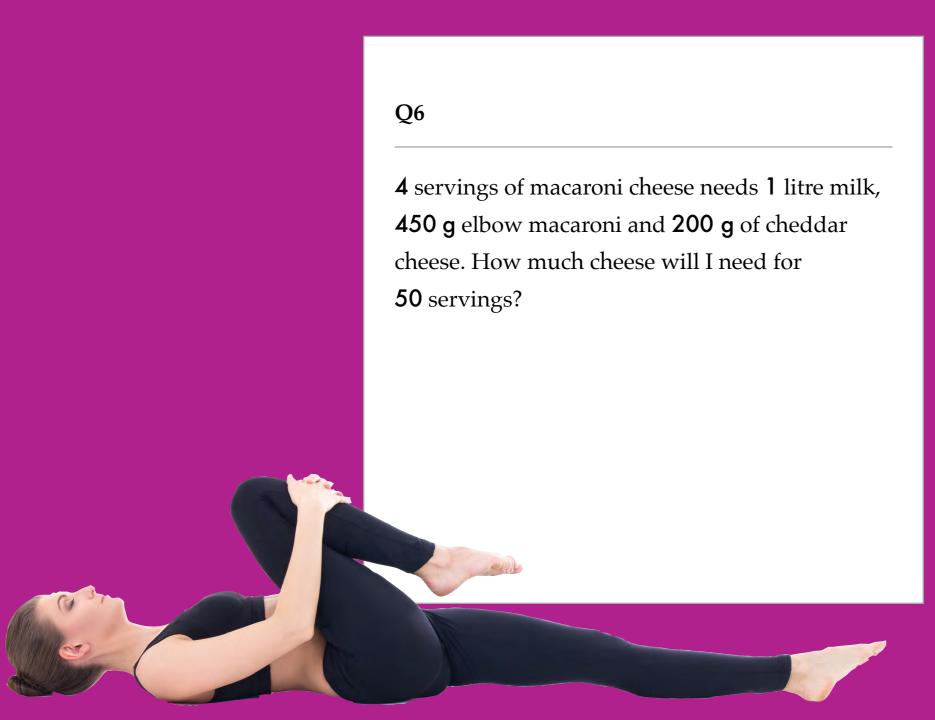


Q4 25 peanut butter cookies are made using 250 g peanut butter, 100 g sugar and 1 large egg. How much sugar will I need to make 10 peanut butter cookies?

Q5

4 servings of banana oat smoothie are made from mixing 200 g rolled oats, 1 litre of oat milk and 3 bananas in a blender. How many bananas will I need to make enough banana oat smoothies for a party of 10?

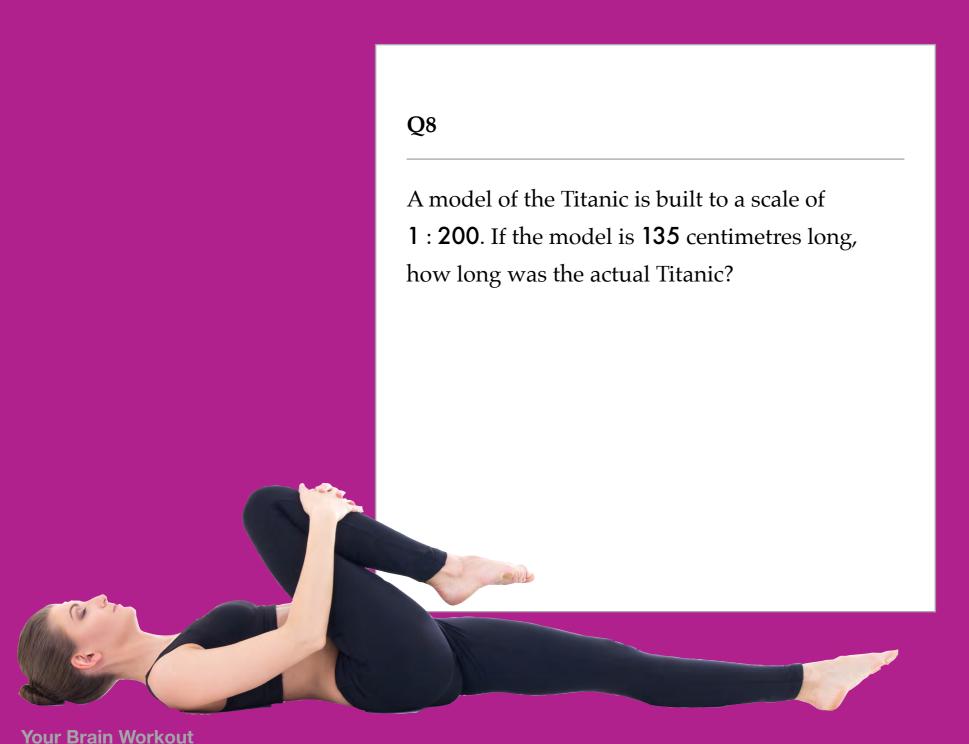




Your Brain Workout

Q7

White gold is an alloy of gold and nickel in the ratio **7** : **3**. How much nickel will a jeweller need to make **1 kg** of white gold?



Answers

- Q1 $210 \div 7 = 30, 8 \times 30 = 240 \text{ boys}$
- Q2 10 parts, $2,000 \text{ ml} \div 10 = 200 \text{ ml}$
- Q3 12 parts, $240 \div 12 \times 3 = 60 \text{ ml}$
- Q4 $100 \div 25 \times 10 = 40 \text{ g}$
- Q5 $3 \div 4 \times 10 = 7.5$ so 8 bananas needed
- Q6 $200 \div 4 \times 50 = 2,500 \text{ g or } 2.5 \text{ kg}$
- Q7 10 parts, 1,000 g ÷ 10 x 3 = 300 g
- Q8 $200 \times 135 = 27,000 \text{ cm or } 270 \text{ m}$

Money Stuff MATHS PLAN STEP 5

I judge myself competent in the following:

MENTAL ARITHMETIC

Addition, subtraction, multiplication & division

Rounding

WRITTEN WORK All the above plus

Working with money

Using a spreadsheet

Squares, cubes & prime numbers

Fractions, decimals & percentages

Calculating discounts

Using a science calculator

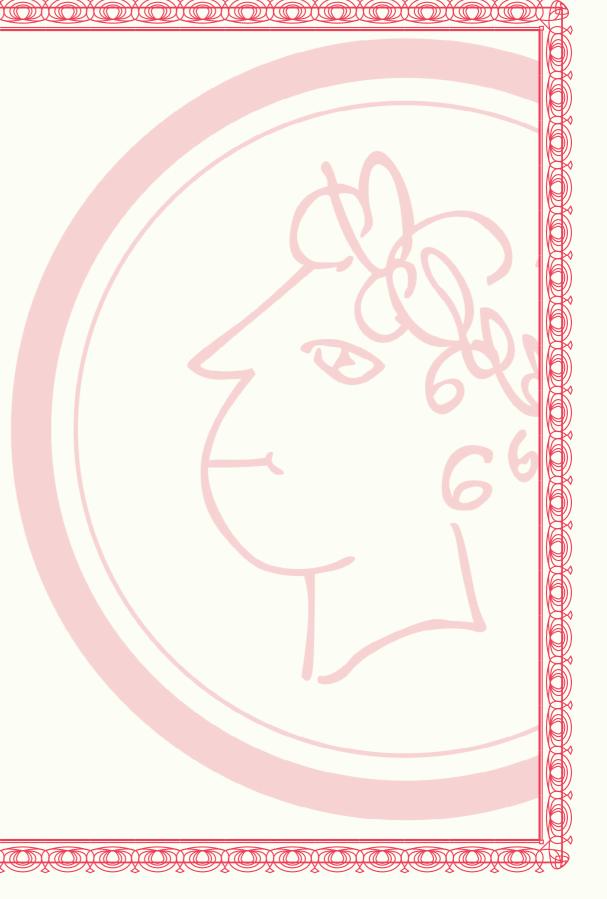
Number puzzles

Ratios, including in cooking & cocktails

Signed

Date





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For their voluntary work in checking text and maths, then revising and re-checking and again, revising and re-checking, we are most grateful to our longstanding supporters, Keith Pledger, BSc, PGCE, former Chair of Examiners at Edexcel and Georgina Godley, MA, IE Post Graduate in Fine Arts.

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