BY INTERNATIONAL AUTHOR OF SUPERWOMAN **DAME SHIRLEY CONRAN**

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THE DO-IT-YOURSELF
6 STEP MATHS PLAN
FOR REAL LIFE

THIS IS STEP 4

Money Stuff

MATHEMATICS WILL GIVE YOU GIRL-POWER

One more step and you can be
on top of your world.

HOW TO NAVIGATE MONEY STUFF IS AT THE START OF STEP 1.

MONEY STUFF is a new way to learn, and each Part is built on the previous Part so - even if you know the basics - PLEASE START AT THE BEGINNING, with STEP 1. Some older testers started in the middle, got in a muddle and had to go back to the start. Don't risk it.

HOW TO GET LUCKY

How to Get Lucky

"I have seen time and time again that life rewards the ***persistent****… this combination of cheerfulness and determination is one of life's secrets to success, yet not many people realise this… It is a law of the universe. The more you try, the luckier you will get."* TV adventurer and Chief Scout, **Bear Grylls** in his book, 'Great Outdoor Adventures'.

Actress and campaigner **Joanna Lumley** showed proof of the power of **persistence, determination and cheerfulness** by the success of the Gurkha Justice Campaign. For years, Joanna and her team had battled, so that Gurkha soldiers who had fought in the British army and had been prepared to die for Britain, should have the right to live in Britain.

Sometimes in life, you need to be as brave as a Gurkha in **small** matters. You need to **persist**, as Joanna Lumley did. As in learning to ride a bike, once you can do it … you can't imagine why you thought it was so difficult.

Before I started to write this final STEP, I talked to many famous and successful people, and I've included what they said had been useful to them. What they all stressed was the importance of **persistence**.

You're nearly at the end of MONEY STUFF and already you've achieved a great deal. But it's never easy to complete that last lap, to get to the top of the hill, to produce the last spurt that will get you past the post - as the lucky winner.

So take a deep breath, clear your head, look at your success skyscraper and your push-ahead pyramid and **GO FOR IT**.

As any sports star or stage star will tell you, success doesn't just drop into your lap: you need patience, persistence and **a lot of practice** to get into the spotlight and stay there.

Perhaps you don't want to be famous or rich... just a bit better off, so you'll never have any money worries. Whatever you hope for... read on.

Because that's how you will **GET LUCKY**.

OUR MOTTO

Life is too short

to be short of money

!!! Watch out for prices !!!

(Another warning)

The cost of living has been zig-zagging upwards for hundreds of years. In the sixteenth century, Queen Elizabeth I worried about the increasing costs of feeding and equipping her army and navy. Today, you can still expect prices to rise unsteadily in the unforeseeable future.

What causes prices to rise? Many reasons, including bad weather, which increases farmers' food prices. So workers need higher wages, which means that the cost of the goods they make will increase. If the prices of bricks, cement and steel increase then so will the cost of housing and rents.

Sometimes the price rises are so small you don't notice them – but you will certainly notice if your home energy bill shoots up in a few months and mum starts switching off the lights and heating.

These rising prices are called **inflation**.

When I started to write this maths course, the prices I used in the exercises were the same as the prices in the shops – but by the time I had finished **Step 1**, the shop prices had risen - so the exercise prices were out-of-date. That is why the prices in **MONEY STUFF** are not current prices; they are historically correct prices, paid by your grandmother and mother in the early **21st** century.

In maths, as in life, people have different ways to writing numbers. For example, you can write a fraction as either ½ with a diagonal line, as we do, or as $\frac{1}{2}$ which you may also see. Whichever you use, the meaning is the same. Likewise, some people write **1,000** or **1,000,000** as we do, with commas to break up the digits, others prefer just to leave a space, like this **1 000** or **1 000 000**. The choice is yours – that's the joy of maths!

Shop prices will alter throughout your life.

But the maths you need to shop will never alter.

Dame Shirley Conran

CONTENTS

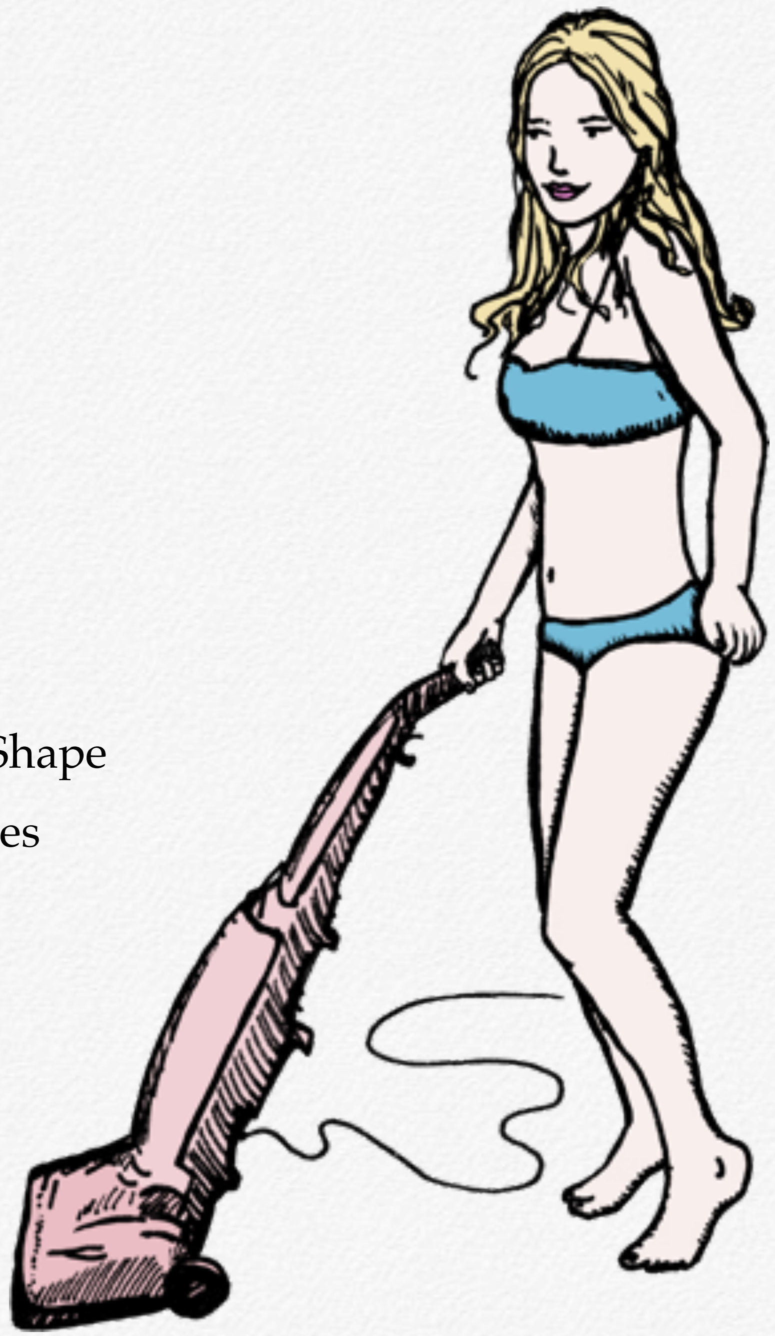
Concentration

Sinead Kerr
and John Kerr.
Paris, France.

Contents

To throw a ball accurately, you need practice, not luck.
Handball tournament,
Hungary, Siofok v. Kadarkut.

Keep going!
You've nearly finished MONEY STUFF.



Inflation is prices...going up.

PART 26
PROBABILITY

Probability of engine flooding?

Quick Quiz

Question 1 of 4.

24, 36, 48, 60, 72 is a sequence from which times table?

- **A.** x 11
- **B.** x 9
- **C.** x 12
- **D.** x 8

Quick Quiz

Question 2 of 4.

What is the answer to:

$15 - 3 \times 2 + 4 =$

- A. 28
- B. 72
- C. 13
- D. 5

Quick Quiz

Question 3 of 4.

How many inches in a foot?

- **A.** 10
- **B.** 12
- **C.** 14
- **D.** 16

Quick Quiz

Question 4 of 4.

The weather forecast is for a light frost and a temperature of ...

- **A.** 100°C
- **B.** 100°F
- **C.** 0°F
- **D.** 0°C

Quick Quiz

Answers

Q1. x12

Q2. 13

Q3. 12

Q4. 0˚C

Probability

Until someone invents a crystal ball that really will foretell the future, no-one can predict with certainty what will happen next; if they could, bookmakers would go out of business.

However, **using probability** can improve your ability to predict the future in certain areas, which range from horse racing to a declaration of war. Here's **how to calculate probability**, to clarify the risks involved in making decisions.

Possibility is a vague word referring to anything that has a chance of happening.
Probability is a measure of how likely it is that an event will occur.

Probability is measured on a scale from 0% (impossible) to 100% (certainty).
Probability of 0% = **impossible**, it will never happen. (Pigs might fly.)
Probability of 100% = **certain**, it will definitely happen. (The sun will rise tomorrow.)
Probability of **even chance** is 50%. (Will it be a boy or a girl?)

To **quantify** is to give a probability a precise numerical value on the scale of 0% (won't happen) to 100% (dead cert).

Note

A probability can be given as a fraction, decimal or percentage.

	Impossible	Even chance	Certain
Percentages:	0%	50%	100%
Fractions:	0	$\frac{1}{2}$	1
Decimals:	0	0.5	1.0

In Real Life, probability is usually worked out as a fraction. Decimals are hardly ever used. You can convert the fraction to a percentage, if that gives you an easier comparison. (For a reminder of how to change between fractions, percentages and decimals see STEP 2).

Example of percentage

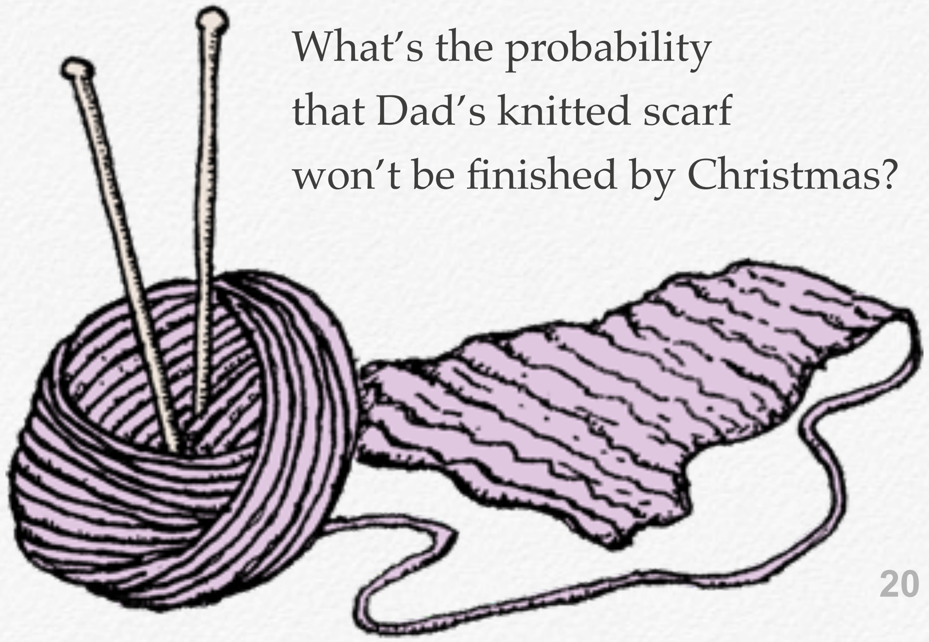
The weather girl says there is a **75%** chance of rain tomorrow.

Example of fraction

There's a **3** in **4** chance it will rain on sports day. **3** in **4** = $\frac{3}{4}$.

Example of decimals

There's a **0.75** chance that it will rain tomorrow. Goodbye picnic!



What's the probability that Dad's knitted scarf won't be finished by Christmas?

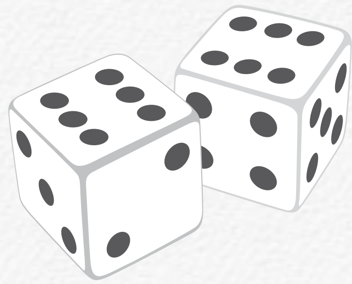
Gone shopping, need maths.

How to Calculate Probability

Basic gambling information:

This is a die:

This is a pair of dice (or two dice):



When working out a probability, consider all the possible outcomes.

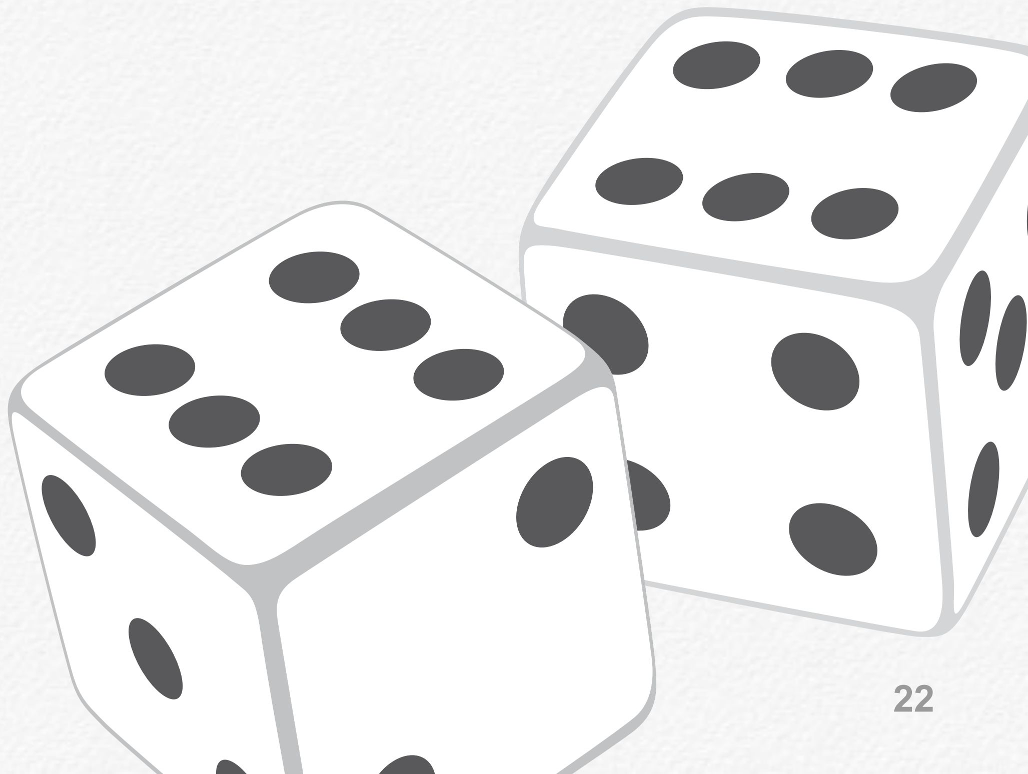
There are **6** sides to a die, each numbered differently from **1** to **6**, so there are **6 possible outcomes** to each throw.

There is one **2** on the die, so the chance of throwing a **2** is one chance in six, $= \frac{1}{6}$

There is one **6** on the die, so the chance of throwing a **6** is one chance in six, $= \frac{1}{6}$

There is one **3** on the die, so the chance of throwing a **3** is one chance in six, $= \frac{1}{6}$

The probability of throwing a **3** again is **still one chance in six**, $= \frac{1}{6}$



There are **2** sides to a coin – head and tail – so there are **2 possible outcomes** to each throw.

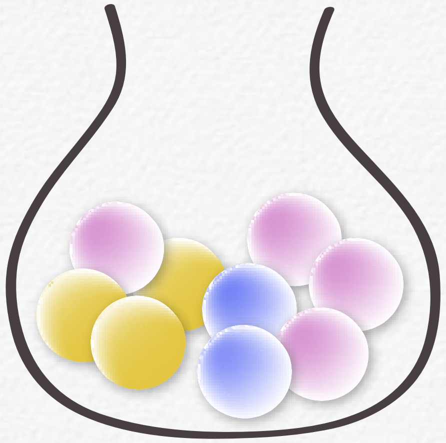
The chance of throwing a head is a **1** in **2** chance $= \frac{1}{2}$ every time the coin is tossed. The probability is $\frac{1}{2}$.

If you tossed a head fourteen times in a row, the probability of throwing a head next time is **still one in two**, $= \frac{1}{2}$.

A price reduction is only a bargain if it's on your shopping list.

Example

The drawstring bag contains lottery balls for the village fair; there are three yellow balls, two blue and four pink.



If a ball is picked at random, there is an equal chance that any one of the **9** balls will be chosen.

To find the probability of a pink ball being chosen, count how many pink balls are in the bag. **4** pink balls could be chosen from a total of **9** possibilities, so the probability that a pink ball will be picked is **4** out of **9** or $\frac{4}{9}$.

As there are **3** yellow balls, the probability of pulling out a yellow ball would be **3** out of **9** or $\frac{3}{9}$. The probability of pulling out one of the **2** blue balls would be **2** out of **9** or $\frac{2}{9}$.

Exercises

1) Design student Jane has pulled on one red-striped sock. Her drawer is a confusion of socks: **5** plain black, **4** green and **1** red-striped. What is the probability of Jane's pulling out the matching red-striped sock without looking in the drawer?

2) Before the card game starts, Jenny bets **£10** that her boyfriend Pete can't pick the Queen of Spades from a normal pack of **52** cards (no jokers included). Also, if Pete picks a queen of any of the other three suits, Jenny will give him **£5**, but if Pete picks any other card, he must give **£1** to Jenny.

a) What is the probability that Pete will win **£10** by pulling out the Queen of Spades when he selects a card at random?

b) Starting again with **52** cards and no joker, what is the probability that Pete wins **£5** by picking out a queen of any other suit when he picks out a card at random?

c) Starting again with **52** cards and no joker, what is the probability that Pete will pay Jenny **£1** when he picks a card that is not a queen?

d) Is Jenny likely to make a profit if she persuades more people to play her game? (Hint: calculate the amount of money Jenny gains and Jenny loses if all of the **52** possibilities occur.)

3) Annabel longs to win the candlelit dinner for two at the Unicorn Restaurant, which is offered in the church raffle. Annabel rashly buys **20** tickets, and she knows that only **300** tickets were sold. What's Annabel's chance of winning?

Probability of Something NOT Happening

Strangely enough, if you know the probability of something happening, the probability of it **not** happening is easy to work out, and can be an equally useful bit of information.

> Rule for percentages:
>
> **Probability of an event NOT happening = 100% minus the percentage probability of the event happening**

Example using percentages

Given the probability that it will rain tomorrow is 45%,
then the probability that it will **not** rain tomorrow is 100% – 45% = 55%.

Second example

Given the probability of an event happening is 30%,
then the probability of it **not** happening is 100% – 30% = 70%.

Emerging market – Asia.
Many Asians like to gamble.

If, instead of using percentages, you use decimals or fractions, simply change the rule from using 100% to using the number 1.

Rule for fractions and decimals:

Probability of an event NOT happening = 1 minus the probability of the event happening

First example

If the probability of the tiny, Mediterranean ferry running aground sometime during the winter is 0.2, then…

The probability of the ferry **not** running aground $= 1 - 0.2$

$= 0.8$

Similarly, if you're using fractions, instead of 100% probability, use the number 1

(see preceding rule box for fractions and decimals).

Second example

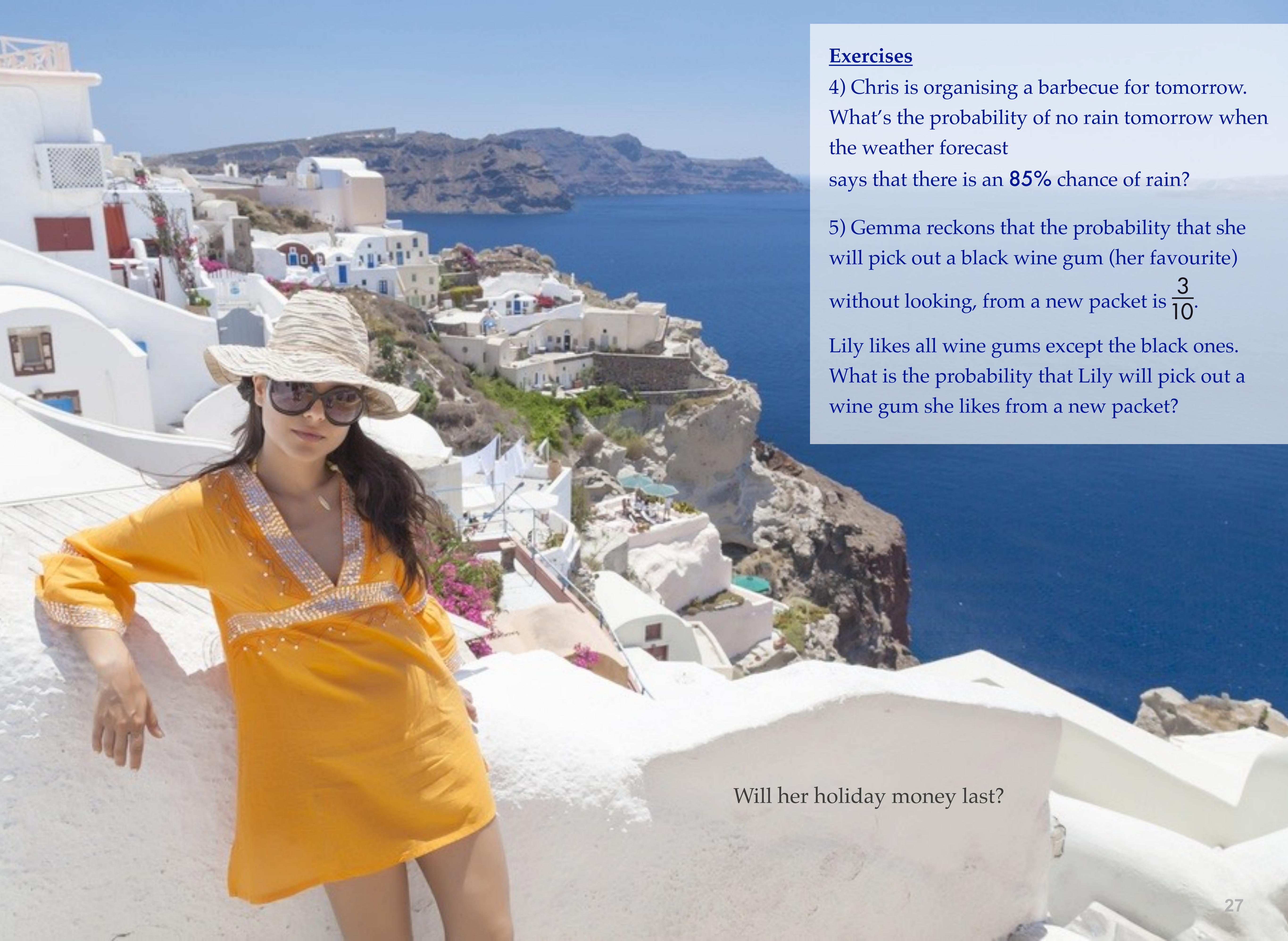
If the probability of picking a pink ball from the bag is $\frac{4}{9}$

then…

The probability of **not** getting a pink ball $= 1 - \frac{4}{9}$

$= \frac{9}{9} - \frac{4}{9}$

$= \frac{5}{9}$



Exercises

4) Chris is organising a barbecue for tomorrow. What's the probability of no rain tomorrow when the weather forecast
says that there is an **85%** chance of rain?

5) Gemma reckons that the probability that she will pick out a black wine gum (her favourite) without looking, from a new packet is $\frac{3}{10}$.

Lily likes all wine gums except the black ones. What is the probability that Lily will pick out a wine gum she likes from a new packet?

Will her holiday money last?

Holiday destination.
Santorini, Greece.

Guesstimating (estimating probability)

Probability can help you with almost any decision in Real Life and is especially useful with financial investment and risk assessment.
Probability can be guesstimated from previous experience.

First example

If you know there has been a traffic jam on 9 out of the last 10 times you drove on the M1, then the probability that you'll meet another hold up on that same route is pretty high:
$\frac{9}{10}$ or 90%, is a sensible guess.

Second example

Trekking in the Himalayas, one side of the narrow and knobbly Perilous Path has a sheer drop of **500 ft** to the crimson rhododendrons below.

On average, trekkers with a team of **8** pack ponies, use the path **20** times a year. On average, two pack ponies a year miss a footing and crash to their death. (I'm glad they never told me until I'd reached the far side.)

What is the probability of losing Mirba, your favourite pack pony, on your next trek?

2 pack ponies die each year from a total of **160** ponies.
(A pack pony team using the path **20** times a year = **8 x 20**)

2 out of **160** ponies die each year $= \frac{2}{160} = \frac{1}{80} = 1$ in **80**.

Answer: **The probability of Mirba, your pack pony, plunging to his death on your next trek is $\frac{1}{80}$ or 1 in 80 (fraction).**

To change this fraction probability to a **percentage** (which is most often used when talking), multiply by **100**:

$$\frac{1}{80} \times 100 = \frac{100}{80} = 100 \div 80$$
$$= 1.25\%$$

Answer: **The probability of Mirba crashing on the rhododendrons is 1.25%.** This is a low probability, so the rhododendrons are almost definitely safe, and so are you and your pack pony.

NOTE: When talking, you might round to, "less than **2%**".

To change this fraction probability to a **decimal**, divide **1** by **80**:

$$\begin{array}{rl} & 0.0125 \\ 80 & \overline{)\,1.0000} \\ - & 80 \\ & \;\;200 \\ - & \;\;160 \\ & \;\;\;\;400 \\ - & \;\;\;\;400 \\ & \;\;\;\;\;\;\;0 \end{array}$$

Answer: **The probability of losing Mirba is 0.0125.**

Exercises

6) Helena is the editor in chief of a successful magazine 'Part Timer'. In the past **20** years Helena calculates that she has employed about **90** people to work in-house on the magazine. Helena says that **5** of these people have turned out to be brilliant employees, real assets to the company. Helena thinks she needs to find one more employee like this, before she can retire. What is the probability that each new employee recruited will turn out to be brilliant? Give your answer as a fraction and percentage.

In everyday speech, percentages are generally used, because they are easier to understand.

Trekking in the Himalayas.

Risk Assessment

You will certainly need **risk assessment** if you run a business or are organising an event such as a wedding or a conference. **Here's how to identify risk**.

This mathematical method is used a) to evaluate where potential safety hazards lie, b) to assess which of those hazards needs making safe. A **risk assessment** weighs up the probability of a hazard occurring and the severity of that hazard. In other words:

a) How likely is it that the nasty event will happen?

b) How **serious** might it be?

This helps you decide which risks need safety measures applied to them, and which do not.

Self-starter Emma Bridgewater designs for her own kitchen shops. Needs to know her numbers to run her international business.

First Example

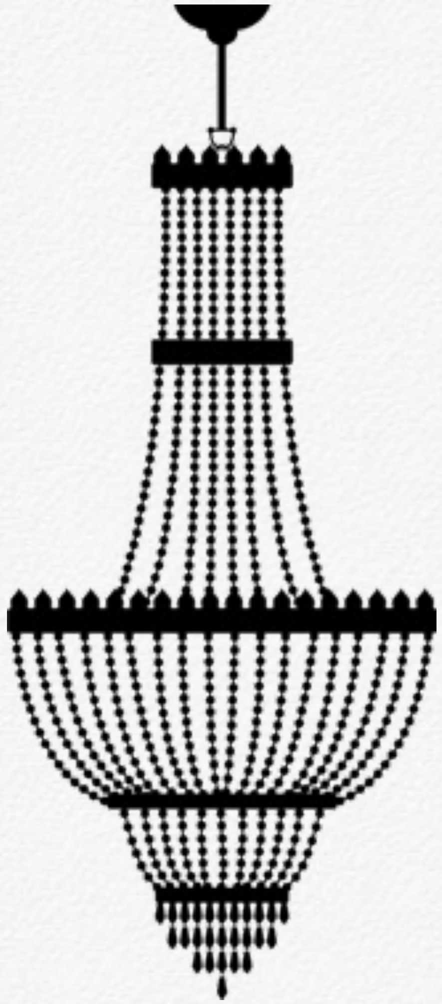
Start by drawing up your blank table as follows. This example will explain how to fill it in, step by step.

Example Table 1

No	Hazard	Consequences (1-5) (Seriousness)	Probability Score (1-5) (Likelihood)	Risk Rating	Action
1					
2					

Next, list each possible hazard, such as an unexpected step in a dark corridor with a polished floor, or a big heavy chandelier that's been hanging for years in a reception room.

Now, decide the **likely consequences** of these hazards, according to the following categories.



Likely consequences scores

1 = **Negligible:** Trivial injury, requiring minor first aid

2 = **Minor**: One minor injury

3 = **Serious:** Single severe injury and/or multiple minor injuries
(falling over the unexpected step might result in a bridesmaid's broken ankle)

4 = **Critical:** Single fatality and/or multiple severe injuries.
(Grandad standing under chandelier when ceiling falls)

5 = **Catastrophic:** Multiple fatalities

Now on your table, fill in the **consequences** column and then decide on the **probability** of the hazard occurring, according to the following categories (see example table 2 that follows).

Probability scores

1 = **Improbable** / **Rare**: probability is close to zero
(the well-secured chandelier is unlikely to fall)

2 = **Unlikely:** but just conceivable

3 = **Occasional:** could occur

4 = **Likely:** occurs repeatedly / an event to be expected
(tripping on that unexpected step in the dark corridor)

5 = **Certain:** will definitely occur

In **risk assessment** the **probability score** is estimated on a scale of 1 to 5.

Now, on your table, fill in the **probability score** column (see example table 2 that follows).

Example Table 2

No	Hazard	**Consequences** (1-5) (Seriousness)	**Probability Score** (1-5) (Likelihood)	**Risk Rating**	**Action**
1	Unexpected step in dark corridor	3 = Serious	4 = likely to occur	3 x 4 = 12	
2	Chandelier falling	4 = Critical	1 = improbable because the chandelier was recently tested	4 x 1 = 4	

To find the risk rating, simply multiply the **consequences** score by the **probability score** and stick it in the **risk rating** column (see example table 2).

Next, consider what action is necessary, by checking table 3 below, before filing it in your action column on table 2.

Example Table 3

Risk Rating (consequence x prob score)	Action Required
1 – 5	**Tolerable:** No additional controls required.
6 – 10	**Low Risk:** Probably require a written Safe System of Work (SSW).
11 – 15	**Medium Risk:** Written SSW required until risk is eliminated.
16 – 20	**High Risk:** Permit to work is advised.
21 – 25	**Intolerable Risk:** Cease activity.

The risk rating of the **dark step in the corridor** is 12, which lies between 11 – 15, this is a Medium Risk so it should be dealt with immediately, perhaps by fitting a floor level light beside the step.

The **chandelier** risk rating is 4, which lies between 1 – 5, so no action is required.

A risk rating list is easy to use: you simply look on your table to find the risk rating for your hazard and decide the action required.

Elizabeth, MONEY STUFF's maths consultant, uses this risk assessment scoring method to eliminate hazards when she organises children's summer camps in Shropshire.

Woollen socks slip easily on polished floors.

Every step involves a probability assessment.

Situation: There are 10 slippery, weed- and lichen-covered stone paths in the garden of a block of flats. The paths haven't been properly cleaned for years. Recently, from one flat alone, three people fell on different days: a 4-year-old boy, his father and his grandfather. Craig, the father, did the risk assessment below, to urge the managing agent to get the paths properly cleaned. (This really happened, by the way.)

Hazard	**Consequences** (1-5) (Seriousness)	**Probability Score** (1-5) (Likelihood)	**Risk Rating**	**Action**
10 Slippery paths used daily.	3 = **Serious.** Likelihood of broken bones or other injury.	5 = **Certain.** Will definitely occur sometime.	3 x 5 = 15	Until the risk is eliminated, official safety warning signs should be exhibited at all entrances to the estate and also in all common areas.

Note

Should an injury occur, it would be the responsibility of the estate to cover the claim of any injured party. The estate management should immediately check the position of the estate's 3rd party liability insurance to ensure that it is covered in case of an accident, which is due to the negligence of the estate.

Note

For insurance purposes, it's useful to have a dated risk assessment to show that the risk has been identified, acted on and addressed before an accident occurs.

Note

You may find that your own answers are slightly different from the answers given at the end of this chapter.
This is because your risk assessment is always going to be influenced by your own judgement to some extent. This is normal.

Exercises

7) Complete this risk assessment form for Jade's ballerina shoe shop.

(Hint: you will need to refer back to: Likely Consequences scores, Probability scores and Risk Rating table.)

Hazard	**Consequences**	**Probability Score**	**Risk Rating**	**Action**
Customers with bare feet.				
Staff climbing ladder to get shoes from high shelves.				
Car crashing into shop window from road.				

8) Complete this risk assessment form for Elizabeth's next summer camp for children.

Hazard	Consequences	Probability Score	Risk Rating	Action
Car backing into tent.				
Child is burned by camp fire.				
Child drowns in Dingle's pond.				

Warning!

The risk assessment scoring method, as described above, is based on human judgement and therefore liable to error. Always use your common sense to check a risk assessment made by you or somebody else.

FACT!

Employers stress that they need their staff to have some basic understanding of risks, odds and probabilities.
This is so that you can make a realistic assessment of risk, rather than treating every risk as equally likely to happen.

Ref: Confederation of British Industry (CBI) Report 'Working on the Three R's: Employers' Priorities for Financial Skills in Maths and English.' Published 2006

Gambling: Calculating the Odds

Bookies

You can be confident that a bookmaker is better than anyone else at calculating probability. Bookmakers set the odds on the outcome of an event (usually a sports competition) that has not yet happened. They use all the research and knowledge available to them to calculate the probability of all the possible outcomes and then set their odds accordingly.

Betting odds are calculated to let you win a little, but not much, if you gamble on the most likely outcome.

Inspiration

J.K. Rowling went from living on social security to multi-millionaire within five years, as author of Harry Potter books and films.

What were her chances? Less than one in a million.

Example

The odds on the favourite horse 'Sugarbabe' in the Grand National is **4-7** (read aloud as "four to seven"), which means that for every **£7** you bet, if your horse wins you'll get your **£7** back plus **£4** winnings.

However, if you place your **£7** on 'Silly Me', an outsider with odds of **15-1**, you'll get **£15** back for every **£1** you bet on that horse **plus** your original stake, if she wins.

So if 'Silly Me' wins the race, your winnings would be **15 x £7 = £105**, **plus** your original bet of **£7**, a total of **£112**.

But 'Silly Me' has been given the odds of **15-1** because she is **unlikely** to win, so you are likely to lose your **£7**.
A win for the bookies.

Here's an easy way to remember that the winning odds are always written first. If Black Beauty wins the Derby with odds at **100-1** ("one hundred to one"), that means that if the gambler has bet **£1** on Black Beauty, the bookmaker has to pay **£100** to the gambler, plus the original bet of **£1**.

Sometimes you may know more than the bookmakers. Weatherman Piers Corbyn made money by betting on the weather. The bookies used the latest Meteorological Office information, whereas Piers Corbyn had his own. However, this was an exceptional case. Statistically speaking, the ultimate winner is **always the bookmaker**.

Similarly, casinos and slot machines are always the eventual winners. Very occasionally, some lucky person cleans up at a casino, but this is rare. The casino and slot machine owners also carefully calculate their odds. Their calculations will allow the occasional small win and a rare big win for its publicity value, which brings in more gamblers. But the casino will **always** make more money than it loses. If you hit the jackpot, your instant problem will be getting your winnings safely home. Due to human nature, the sheer thrill of winning – because Lady Luck seems to be on your side – and sometimes simple greed stops you from stopping betting.

Chances are that the winner will gamble his winnings until, eventually he loses them and the casino gets its money back.

A casino can project how much money it takes **every hour**, but even casinos can lose heavily and unexpectedly. I met a nice, quiet man in Palm Beach who had the sort of yacht you look up at: he ran a famous Las Vegas casino. “What’s the most the casino ever lost?” I asked, not really expecting an answer. Sorrowfully, he said, “The night that Cher sang. Her act took **2** hours – everybody watched it – and we were down thirteen million in that time. We didn’t ask her back”.

For the probability of winning the **lottery**, see later: you’ll find that the probability of winning the lottery with one lottery ticket is almost the same as the probability of winning the lottery without a ticket.

You need numbers to be an architect or a designer.
The Great Mosque, Cordoba, Spain.

Taking Chances

One day you may stand onstage in the spotlight. Or you may choose to succeed in some less public area. Whatever you decide to do, **you can improve your chances of success** if you do it with **care**.

What does 'care' mean? Care means paying serious attention to what you're doing: being cautious; taking all possible facts into consideration; never thinking, "That won't happen to **me**."

1. **Be realistic**. No use wanting to be a vet if you are allergic to fur. You can never be a champion tennis player if you have weak wrists.
Recognise **what will diminish your chances of success**, such as partying all night before an exam.

2. **Don't be careless**. Don't type a text message when you're walking downstairs. In a café, don't hang your bag on the back of your chair, where you can't see it but a thief can dip into it.

3. **Cut out unnecessary risks**. Don't nip across the road in the path of a seemingly distant lorry. Don't leave a saucepan handle sticking out over the hob.

It's surprising how many people take a lot of simple risks every single day. To prove this, **for one day only, count how many simple risks you take**. For instance, not allowing extra time to get to work, if it rains; or leaving the bathroom while the bath taps are running.

If you're careful in simple things, you will increase your chance of success in every area of your life…
especially with money.

Who wants to be powerful?
Not you?
Are you sure?

In the nursery,
Power is called
“Getting your own way.”

You need power to improve anything.

Women who improve their lives,
and your life and mine,
Women who make a difference,
Are women with power

You have seen powerful women
In MONEY STUFF.

So maybe think again?

Who wants to be powerful?

Queen Elizabeth I – the most powerful person in England – took a huge gamble when she ordered a little British fleet to fight the mighty ships of the invading Spanish Armada. (Photo by kind permission of Woburn Abbey.)

Answers to Part 26

1) Design student Jane has pulled on one red-striped sock. Her drawer is a confusion of socks: **5** plain black, **4** green and **1** red-striped. What is the probability of Jane pulling out the matching redstriped sock without looking in the drawer?

There are $5 + 4 + 1 = 10$ socks altogether in the drawer.

1 of the socks is red-striped.

Probability of getting a red-striped sock = 1 out of 10, or = 0.1 = 10%.

Answer: **Probability that Jane picks out the matching** red-striped **sock without looking = 10%.**

In fact, she picked out a green sock, the David Hockney look, she explained at breakfast. Karen looked up from her letter and shrieked, "Stuff your socks! I've passed my finals!" Jane grabbed her own envelope, "So have I!!".

2) Before the card game starts, Jenny bets **£10** that her boyfriend Pete can't pick the Queen of Spades from a normal pack of **52** cards (no jokers included). Also, if Pete picks a queen of any of the other three suits, Jenny will give him **£5**, but if Pete picks any other card, he must give **£1** to Jenny.

a) What is the probability that Pete will win **£10** by pulling out the Queen of Spades when he selects a card at random?

There is a total of **52** cards to choose from.

There is only one Queen of Spades.

Probability of getting a Queen of Spades = 1 out of 52, or $\frac{1}{52}$.

Answer: **Probability of Pete winning £10 by picking a Queen of Spades =** $\frac{1}{52}$.

b) What is the probability that Pete wins **£5** by picking out a queen of any other suit when he picks out a card at random?

There is a total of **52** cards to choose from.

There are **3** queens of suits other than spades in the pack.

Probability of getting a queen that is not a spade

= **3** out of **52**, or $\frac{3}{52}$.

Answer: **Probability of Pete winning £5 by picking a queen that is not spade =** $\frac{3}{52}$.

c) What is the probability that Pete will pay Jenny **£1** when he picks a card that is not a queen?

There is a total of **52** cards to choose from.

There are **4** queens in the pack, so **48** cards that are not queens in the pack.

Probability of getting a card that is not a queen

= **48** out of **52**, or $\frac{48}{52} = \frac{24}{26} = \frac{12}{13}$.

Answer: **Probability of Pete paying £1 by picking a non-queen =** $\frac{12}{13}$.

d) Is Jenny likely to make a profit if she persuades more people to play her game?

(Hint: calculate the amount of money Jenny gains and Jenny loses if all of the **52** possibilities occur.)

Jenny is more likely to be the winner each time because the probability is $\frac{48}{52}$ (more than half).

If Pete selects all **52** cards in the pack - one after the other - is Jenny likely to make a profit?

After **48** goes Jenny wins **£1 x 48 = £48** total.
After **3** goes Jenny has to pay Pete **£5 x 3 = £15** total lost.
After **1** go Jenny has to pay Pete **£10 x 1 = £10** total lost.

Answer: **Jenny should still make a profit as she would win £23 more than she would lose if all the possible outcomes occur**.

That evening, Jenny won **£31** with her game, but she also won a reputation as a tough card shark.
She never saw Pete again.

3) Annabel longs to win the candlelit dinner for two at the Unicorn Restaurant, which is offered in the church raffle. Annabel rashly buys **20** tickets, and she knows that only **300** tickets were sold. What's Annabel's chance of winning?

Annabel has **20** tickets out of **300** tickets,
so a **20** out of **300** chance = $\frac{20}{300}$

$\frac{20}{300}$ can be cancelled down to $\frac{2}{30}$ and then to $\frac{1}{15}$.

Answer: **The probability of Annabel winning the raffle is $\frac{1}{15}$ (or $\frac{20}{300}$).**

Sadly, Annabel didn't win, but her boyfriend took her to the Unicorn on her birthday.

Birthday bouquet for Annabel.

4) Chris is organising a barbeque for tomorrow. What's the probability of no rain tomorrow when the weather forecast says that there is **85%** chance of rain?

Probability of it not happening = 100% **minus probability of it happening**

= 100% – 85%

= 15%

Answer: **The probability that it won't rain tomorrow is only 15%.**

5) Gemma says the probability that she will pick out a black wine gum (her favourite) without looking, from a new packet is $\frac{3}{10}$. Lily likes all wine gums except the black ones. What is the probability that Lily will pick out a wine gum she likes from a new packet?

Probability of it not happening = 1 **minus probability of it happening**

$= 1 - \frac{3}{10}$

$= \frac{10}{10} - \frac{3}{10}$

$= \frac{7}{10}$

Answer: **The probability that Lily will pick out a wine gum she likes is** $\frac{7}{10}$.

6) Helena is the editor in chief of a successful magazine 'Part Timer'. In the past **20** years Helena calculates that she has employed about **90** people to work in-house on the magazine. Helena says that **5** of these people have turned out to be brilliant employees, real assets to the company. Helena thinks she needs to find one more employee like this, before she can retire. What is the probability that the next new employee recruited will turn out to be brilliant? Give your answer as a fraction and percentage.

5 brilliant employees found among **90** people recruited by Helena = $\frac{5}{90} = \frac{1}{18}$ = **1** in **18**.

Answer as a fraction: **There is a 1 in 18 ($\frac{1}{18}$) chance that the next new recruit will be the brilliant employee that Helena is looking for**.

Typical small, fashion magazine conference.

To change this fraction probability to a **percentage** (which is most often used when talking), multiply by **100**:

$$= \frac{1}{18} \times 100 = \frac{100}{18} = 100 \div 18$$

$$= 5.6\% \text{ (rounded up)}$$

Answer as a percentage: **The chance that the next new recruit will be the brilliant employee that Helena is looking for is 5.6% so that is unlikely to happen**.

7) Complete this risk assessment form for Jade's ballerina shoe shop.

Hazard	Consequences	Probability Score	Risk Rating	Action
Customers with bare feet.	Stepped on **1** = Trivial	**2** = Unlikely	1 x 2 = **2**	1–5 Tolerable: No action required.
Staff climbing ladder to get shoes from high shelves.	Fall **3** = Single severe injury	**3** = Occasional	3 x 3 = **9**	6–10 Low Risk: Requires written Safe System of Working.
Car crashing into shop window from road.	**5** = Catastrophic, multiple fatalities	**1** = Rare	5 x 1 = **5**	1–5 Tolerable: No action required.

8) Complete this risk assessment form from Elizabeth's next summer camp for children.

Hazard	Consequences	Probability Score	Risk Rating	Action
Car backing into tent.	Child is run over **4** = Single fatality	**3** = Could occur	4 x 3 = **12**	11–15 Medium Risk: Notice to say cars only allowed in car park. Barriers around tent area.
Child is burned by camp fire.	**2** = Minor injury	**3** = Could occur	3 x 2 = **6**	6–10 Low Risk: Children always supervised around the camp fire.
Child drowns in Dingle's pond.	**4** = Single fatality	**2** = Unlikely to occur	4 x 2 = **8**	6–10 Low Risk: Life belt at Dingle's Pond plus children always to be supervised at pond.

Note

You may find that your own answers are slightly different from the answers above. This is because your risk assessment is always going to be influenced by your own judgement to some extent. This is normal.


Camp fire - risk?

YOUR BRAIN WORKOUT

Question 1 of 10.

What is the probability of finding the only heart-shape in a bag of 10 chocolates?

- **A.** $\frac{1}{11}$
- **B.** $\frac{1}{10}$
- **C.** $\frac{2}{10}$
- **D.** $\frac{1}{5}$

YOUR BRAIN WORKOUT

Question 2 of 10.

What is the likelihood of catching the wedding bouquet when you are one of a group of **5** eager bridesmaids?

- **A.** $\frac{5}{10}$
- **B.** $\frac{1}{10}$
- **C.** $\frac{2}{5}$
- **D.** $\frac{1}{5}$

YOUR BRAIN WORKOUT

Question 3 of 10.

What is the probability of picking a bad apple from a bowl containing **4** good apples and **3** bad apples?

- A. $\frac{3}{4}$
- B. $\frac{4}{3}$
- C. $\frac{3}{7}$
- D. $\frac{4}{7}$

YOUR BRAIN WORKOUT

Question 4 of 10.

What is the probability of winning the raffle prize if you have bought 10 tickets and only 60 were sold?

- A. $\frac{1}{4}$
- B. $\frac{1}{6}$
- C. $\frac{10}{70}$
- D. $\frac{50}{60}$

YOUR BRAIN WORKOUT

Question 5 of 10.

What is the probability of **not** winning the raffle prize if you have bought 10 tickets and only 60 were sold?

- **A.** $\frac{1}{4}$
- **B.** $\frac{1}{6}$
- **C.** $\frac{10}{70}$
- **D.** $\frac{50}{60}$

YOUR BRAIN WORKOUT

Question 6 of 10.

If four toys in the lucky dip are for girls and six toys are for boys, what is the probability of Aimee choosing a boy's toy?

- A. $\frac{6}{10}$
- B. $\frac{4}{10}$
- C. $\frac{4}{6}$
- D. $\frac{6}{40}$

YOUR BRAIN WORKOUT

Question 7 of 10.

If four toys in the lucky dip are for girls and six toys are for boys, what is the probability of Aimee choosing a girl's toy?

- **A.** $\frac{6}{10}$
- **B.** $\frac{4}{10}$
- **C.** $\frac{4}{6}$
- **D.** $\frac{6}{40}$

YOUR BRAIN WORKOUT

Question 8 of 10.

What is the likelihood of most of the leaves being blown from a tree in an autumnal storm?

- **A.** Impossible
- **B.** Even chance
- **C.** Likely
- **D.** Certain

YOUR BRAIN WORKOUT

Question 9 of 10.
What is the likelihood of the first lottery number being an odd number in a big lottery?

- **A.** Impossible
- **B.** Almost even chance
- **C.** Likely
- **D.** Certain

YOUR BRAIN WORKOUT

Question 10 of 10.

What is the likelihood of picking a green sock from a drawer containing red and blue socks?

- **A.** Impossible
- **B.** Even chance
- **C.** Likely
- **D.** Certain

YOUR BRAIN WORKOUT

Answers

Q1. $\frac{1}{10}$

Q2. $\frac{1}{5}$

Q3. $\frac{3}{7}$

Q4. $\frac{1}{6}$

Q5. $\frac{50}{60}$

Q6. $\frac{6}{10}$

Q7. $\frac{4}{10}$

Q8. Likely

Q9. Almost even chance

Q10. Impossible

Dancers count the rhythm.

PART 27
FURTHER PROBABILITY

Quick Quiz

Question 1 of 4.

What is 30 x 400?

- **A.** 12
- **B.** 1,200
- **C.** 12,000
- **D.** 120,000

Quick Quiz

Question 2 of 4.

What is the remainder when 37 is divided by 6?

- **A.** 1
- **B.** 2
- **C.** 3
- **D.** 5

Quick Quiz

Question 3 of 4.

Which is the correct calculation to work out the price of six roses which cost **£3** each?

- **A.** 6 ÷ 3 =
- **B.** 3 x 6 =
- **C.** 3 + 6 =
- **D.** 6 – 3 =

Quick Quiz

Question 4 of 4.

Which of the following is greater than half?

- **A.** 0.3
- **B.** 44%
- **C.** $\frac{3}{10}$
- **D.** $\frac{3}{5}$

Quick Quiz

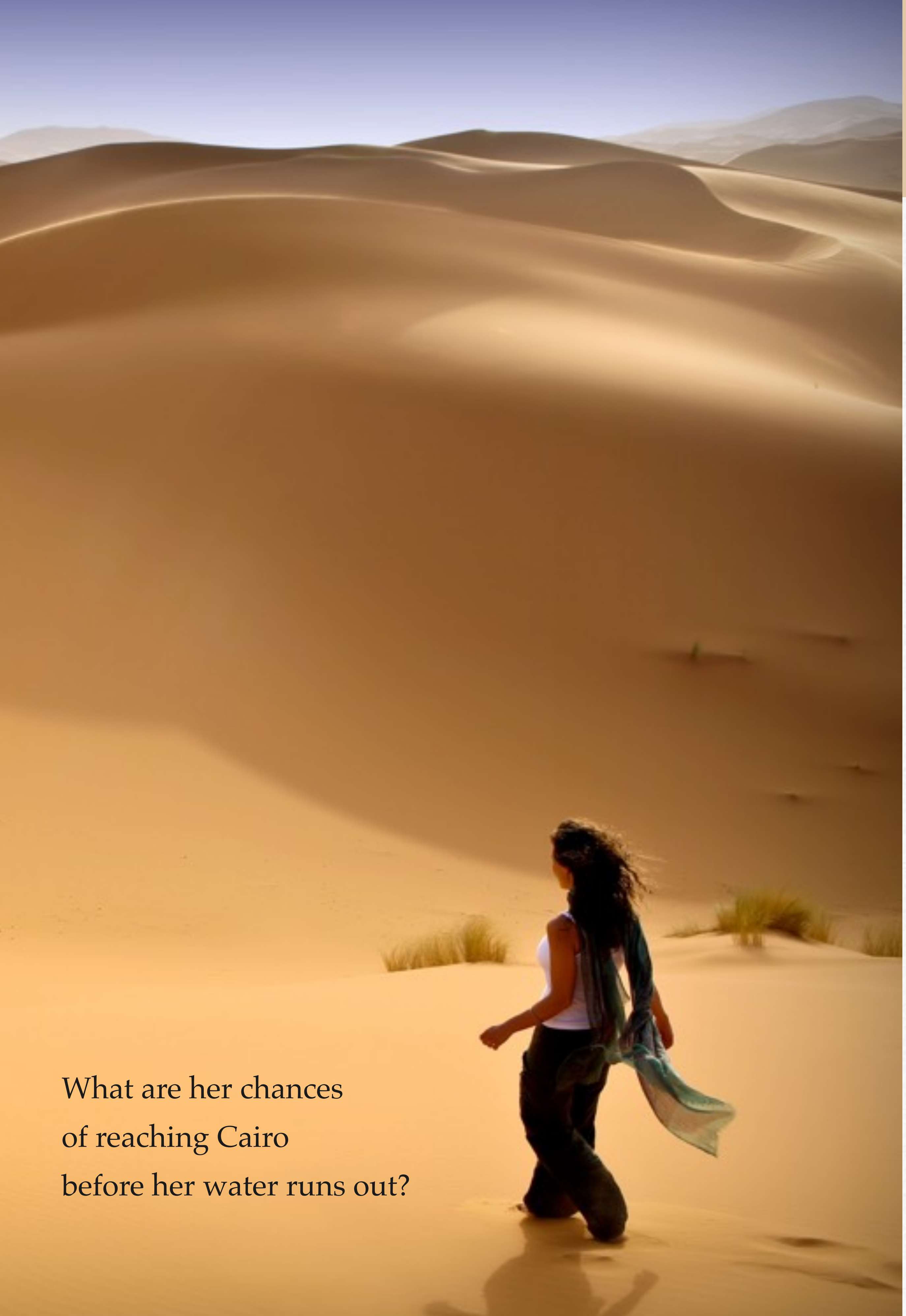
Answers

Q1. 12,000

Q2. 1

Q3. 3 x 6

Q4. $\frac{3}{5}$



What are her chances
of reaching Cairo
before her water runs out?

Decisions, Decisions...

In Real Life, one decision often leads to another. For instance, do you want to take your holiday in summer or winter? If you take a two week summer holiday, how will it affect your job, as a tour guide in London, working on commission? You earn most of your money in the summer. What do you decide?

For financial reasons, you can only take a winter holiday but you'll have to travel alone because all your friends take their holiday in the summer. You **might** decide to go on a group holiday in winter and make new friends, or you **might** decide to take a shorter holiday with a friend in the summer.

Whatever you decide, it will lead to another decision that needs to be taken… and so on. In Mathspeak, this is called **multiple event probability**. Very dull, you might think. However it shows you how to clarify your decision-making in Real Life, which is very useful.

Probability of Two Events or More

This is similar to working out the probability for one event. When working out the probability for two or more events, consider how many different possible outcomes there are.

First Example

When you flip a coin and throw a die at the same time, what is the probability of getting heads on the coin at the same time as a six on the die?

First, consider all possible outcomes. Below are listed all the possible outcomes. These are equally likely.

Heads on the coin **+ 1** on the die	**Tails** on the coin **+ 1** on the die
Heads on the coin **+ 2** on the die	**Tails** on the coin **+ 2** on the die
Heads on the coin **+ 3** on the die	**Tails** on the coin **+ 3** on the die
Heads on the coin **+ 4** on the die	**Tails** on the coin **+ 4** on the die
Heads on the coin **+ 5** on the die	**Tails** on the coin **+ 5** on the die
Heads on the coin **+ 6** on the die	**Tails** on the coin **+ 6** on the die

As you see, there are 12 possibilities altogether. Only one of these possibilities is **heads** on the coin and **six** on the die. The probability of throwing heads and a six is 1 out of 12 throws.

1 out of 12 is written as $\frac{1}{12}$.

Answer: **The probability of together throwing heads on a coin and a six together on a die is** $\frac{1}{12}$.

Second Example

To calculate the probability of throwing **tails** and a **six**, use the same table in the first example. Out of **12** possibilities, there is one way of throwing a tails and a six, so the probability is also **1** out of **12**, or $\frac{1}{12}$.

Third Example

Using a coin with a die as in the previous examples, what is the probability of throwing tails together with an odd number on the die?

You can still use the same list, which has a total of **12** possibilities.

There are **three possible odd numbers** in the tails column: Tails+**1**, Tails+**3** and Tails+**5**.

So the probability is **3** out of **12** = $\frac{3}{12}$.

To cancel this down, divide top and bottom of the fraction by **3**, so $\frac{3}{12} = \frac{1}{4}$.

Answer: **The probability of throwing tails and an odd number together is 1 in 4.**

Counting up all the possible outcomes is only useful for situations with relatively few possible outcomes. Here's a method for less simple cases.

The AND Method

When calculating probability, the **word AND means 'to multiply'**. When considering a problem, you may have to rewrite the facts to see if you can insert **AND** without altering the meaning of the sentence.

First Example

When you flip a coin and throw a die at the same time, what is the probability of getting heads on the coin at the same time as a six on the dice?

Step 1: See if you can rewrite the sentence using **AND**, as in: "What's the probability of throwing heads on the coin **AND** a six on the die together?"

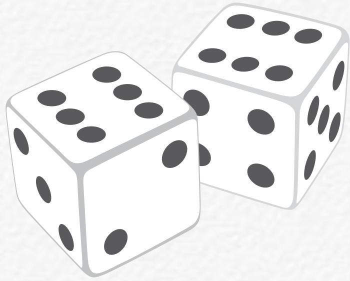
Step 2: Replace **AND** with the multiplication sign.

The probability of throwing heads on the coin and a six on the die	=	the probability of throwing heads on the coin	x	the probability of throwing a six on the die
	=	$\frac{1}{2}$ (coin)	x	$\frac{1}{6}$ (die)

$$= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

If you've forgotten how to multiply fractions, turn back to STEP 2.

Answer: **The probability of together throwing heads on a coin and 6 on a die is $\frac{1}{12}$.**



Exercise

1) While waiting for a poker game to start, Martina bets her new date, Hugh, that he can't throw a double six with one throw of two dice. What is the probability of Hugh throwing a double six?

Second Example

Absent-minded Chris, who lives in Manchester, forgets her umbrella every other day (**50%** of the time). The probability that it will rain tomorrow is **80%**. What is the probability that Chris will get wet tomorrow?

The probability that Chris will get wet tomorrow depends on whether Chris forgets her umbrella (50%) AND it rains (80%).

The probability is... 50% AND 80%

To write this mathematically, swap the AND for the multiplication sign, and change the percentages to fractions:

$$\text{The probability} = \frac{50}{100} \times \frac{80}{100}$$

$$\text{This calculation can be reduced to } \frac{5}{10} \times \frac{8}{10}$$

$$= \frac{40}{100} = 40\%$$

Answer: **Tomorrow, it is 40% probable that Chris will get wet**.

She'd better buy an old-fashioned shower-cap, and keep it in her coat pocket.

Exercise

2) In Paris Fashion Week, super-model Naomi panics when she hears that at the Yves St Laurent show one of the **15** super-models will need to wear **8″** high platform gold sandals for the moonlight finale. Naomi panics because she has only modelled high platforms four times, and fell painfully on one occasion. What are Naomi's chances of being chosen to wear the gold sandals, stumbling on the catwalk, and falling flat on her face?

Expect daytime high temperatures when hiking in Arizona wilderness.

Time for a Prenup?

Dependent Events

In probability calculations you may also need to consider whether an event is **dependent** or **independent**. In other words, will the outcome of the first event affect the probability of the second event? If so, the events are **dependent**. If the events have no bearing on each other they are **independent**. If you visualise each situation you may find it easier to understand.

Throwing two dice is one example of **independent events**: the outcome of one die will not affect the outcome of the other die. In fact, so far in this Section, all the examples and exercises have been examples of independent events.

Examples of **dependent events** are: drawing national lottery numbers (balls) and raffle tickets, because **the items taken out are NOT replaced**. Each time one item is removed, there is one less item in the box, so the **probability** for picking the next item **changes**.

Third Example

A box of 20 Belgian chocolates contains 8 chocolates with hazelnuts. The probability of your picking a hazelnut in your first chocolate is $\frac{8}{20}$. What is the probability that the first two chocolates Gemma takes from the box contain her favourite hazelnuts?

When a chocolate is removed, there is one chocolate less in this box.

Assume the first chocolate contains a hazelnut which leaves only 7 hazelnuts in the box.

Gemma is picking two chocolates. Can this sentence be rewritten to include **AND**? Yes:

"Gemma picks a hazelnut chocolate **AND** then she picks another hazelnut chocolate".

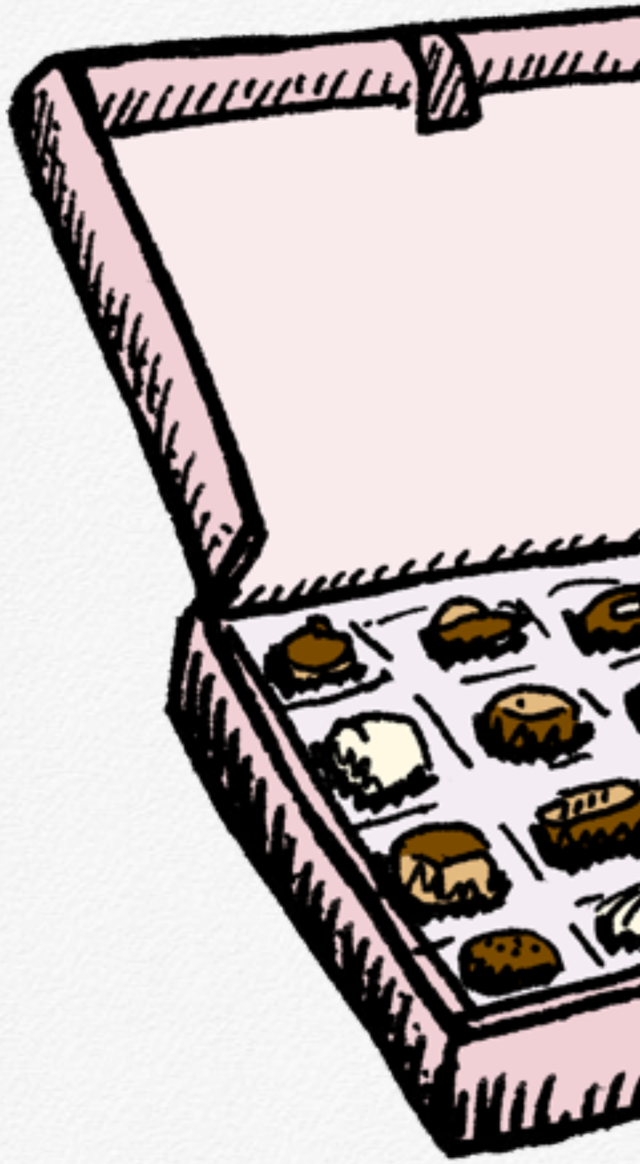
Probability that the **1st** chocolate is hazelnut = 8 out of 20 = $\frac{8}{20}$

Probability that the **2nd** chocolate is hazelnut = 7 out of 19 (one less hazelnut chocolate) = $\frac{7}{19}$

Probability of Gemma picking 2 hazelnut chocolates	=	probability 1^{st} chocolate is hazelnut	x	probability 2^{nd} chocolate is hazelnut
	=	$\frac{8}{20}$	x	$\frac{7}{19}$

$$= \frac{8}{20} \times \frac{7}{19} = \frac{56}{380}$$

This cancels down to $= \frac{14}{95}$



Answer: **The probability that Gemma picks two hazelnut chocolates, consecutively, is $\frac{14}{95}$.**

Fourth Example

There are 12 chemists at the laboratory Christmas party. The name of each chemist is written on a card which is put into a hat. Two lucky winners will get a meal at chic Spanish tapas bar, Casa Blanca. For months, Harry and Claire have been shyly admiring one another across the test tubes: what is the probability that they will both win a paella?

First draw: There are two people, one of whom we hope to pick in the first draw (Harry or Claire). So the probability of one of the shy couple being chosen in the first draw is **2** out of **12** $= \frac{2}{12} = \frac{1}{6}$.

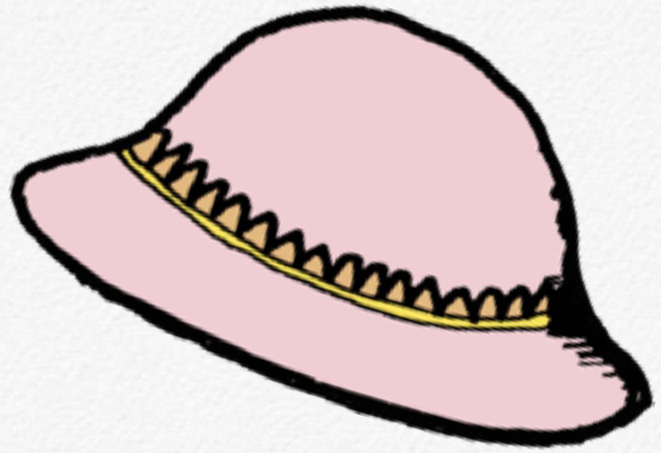
Second draw: Next, assume that either Harry or Claire is the first winner. Because there are now only **11** names in the hat, the probability of the other shy admirer being chosen at the second draw is then **1** out of **11** $= \frac{1}{11}$. This fraction cannot be cancelled down.

Now determine if it is an **AND** problem: "One of the couple is chosen **AND** then the other."

So replace the **AND** with the **x** sign.

The probability of both Harry's & Claire's names being pulled from the hat.	=	Probability of one of the shy couple being chosen in the first draw.	x	Probability of the other shy person being in the second draw
	=	$\frac{1}{6}$	x	$\frac{1}{11}$

$$= \frac{1}{6} \times \frac{1}{11} = \frac{1}{66}$$



Answer: **Unfortunately the odds are not good. There is only a 1 in 66 chance of both Harry and Claire being picked for the dinner for two**. What happened? Alice, who organized the draw and picked the winning cards, cunningly played Cupid by folding Harry and Claire's cards so that she could easily feel them when she drew the winner from the hat.

She’s thinking, “This mix might
save millions of lives. Also, might
make me late for my date...”

Fifth Example

What is the probability that **YOU WIN THE NATIONAL LOTTERY** this Saturday? You've been told there's a 1 in 14 million chance, but is this true?

The question is, 'What is the probability of all 6 of your numbers coming up in the lottery draw on Saturday?'

There are **49** numbers to choose from and **49** balls in the lottery draw. Assume you choose the numbers **1**, **2**, **3**, **4**, **5** and **6** for your lottery ticket (any numbers will do).

The probability that the **first ball** will be one of your six numbers is **6** out of **49** (the total number of balls) = $\frac{6}{49}$.

There are now only **48** balls left. **As always in probability calculations**, assume that the first ball was one of your chosen numbers, which means that **5** of your numbers remain to be chosen.

The probability that the **second ball** is one of your five remaining numbers is **5** out of **48** remaining balls = $\frac{5}{48}$.

Every time a ball pops out with a winning number, **the probability alters**: the top number and bottom number of the fraction will decrease by one:

The probability that the **third ball** which pops up is one of your four remaining numbers is **4** out of **47** remaining balls = $\frac{4}{47}$.

The probability that the **fourth ball** is one of your three remaining numbers is **3** out of **46** = $\frac{3}{46}$.

The probability that the **fifth ball** is one of your two remaining numbers is **2** out of **45** = $\frac{2}{45}$.

The probability that the **sixth ball** is your one remaining number is **1** out of **44** = $\frac{1}{44}$.

Because you need all six of your numbers to win, this is an AND situation; you need the numbers on the balls drawn to be 1 **AND** 2 **AND** 3 **AND** 4 **AND** 5 **AND** 6.

Next multiply all your probabilities.

$$\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44} = \frac{720}{10{,}068{,}347{,}520}$$

(divide top and bottom by 720 to cancel down)

$$= \frac{1}{13{,}983{,}816}$$

Answer: **You have a 1 in 13,983,816 chance of winning the National Lottery with one lottery ticket; rounded up, this is a 1 in 14 million chance**. Sadly not much hope.

Remember, that in probability calculations, the word

AND means to multiply

AND = X

Exercise

3) At the family Christmas meal, two of Annabel's twelve Christmas mince pies contain a gold charm. What is the probability that little Daisy will get both charms when she takes two mince pies?


Little Daisy.

The OR Method

When calculating probability, the **word OR means 'to add'**.

Example of the OR method

What is the probability that Lily, financed by her dad, will win a goldfish at the fairground game if, to win, she needs to throw an odd number or a 6 on a normal die?

It's easy to see that there are 3 odd number possibilities plus one 6. (1, 3, 5 and 6) which gives a total of 4 out of 6 = $\frac{4}{6}$.

Here's how to do it using the **OR method**, because not all your future calculations will be as easy as that example. First, notice that this question contains a clue, the word **OR**, which means that the calculation will be an **addition**.

The probability of throwing an odd number = $\frac{3}{6}$

The probability of throwing a 6 = $\frac{1}{6}$

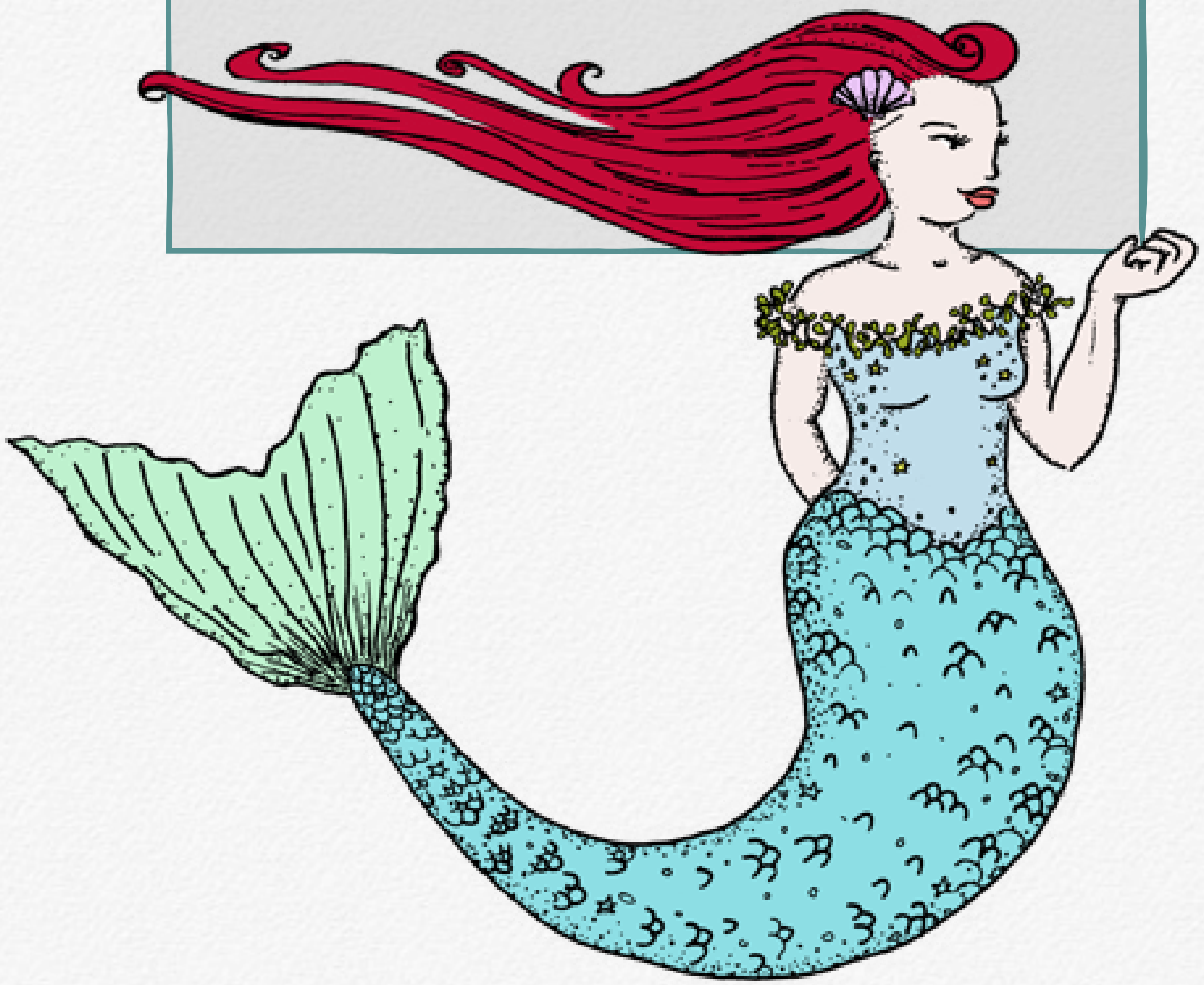
So the probability of throwing an odd number OR a 6 = $\frac{1}{6} + \frac{3}{6}$

= $\frac{4}{6}$ This cancels down to $\frac{2}{3}$

Answer: **The probability of Lily winning the goldfish is** $\frac{2}{3}$ This may seem like good odds but don't be fooled: you can probably buy a goldfish at the local pet shop for less than the price you pay to play the game. Of course, you'll probably need to buy the fish-food and a fish bowl.

Lily's goldfish game is an example of **mutually exclusive** events: it is obviously impossible to throw the die **and** get a single number that is both a 6 and an odd number, because 6 is an even number.

Remember, that **in probability** the word
OR means to **add**
OR = +



Is the fairground mermaid genuine? What's the probability?

Example of Events that are NOT mutually exclusive

At the next fairground stall, there is a top hat full of cards numbered 1 to 20. Lily will win a teddy bear if she picks a card with an odd number or a card less than 10.

What is Lily's chance of winning the teddy bear if her dad pays for only one go?

Step 1:

Rewrite the important bits of the sentence, using **AND** or **OR**:

"Lily needs a card numbered less than 10 **OR** a card with an odd number."

So this is an **OR** problem.

In the hat with twenty numbered cards, there are:

9 numbers less than 10, so the probability of a card numbered less than 10 is $\frac{9}{20}$.

10 odd numbers from 1 to 20, so the probability of an odd numbered card is $\frac{10}{20}$.

But hey – there's an **overlap** of 5 numbers which are both odd **and** under 10: these are 1, 3, 5, 7 and 9. The probability of one of the overlap cards being chosen is $\frac{5}{20}$.

Because there is an overlap, this problem contains **NOT mutually exclusive events**.

Since it's an **OR** problem for **NOT mutually exclusive events**, it's not a simple addition, so you need to carefully clarify the problem, as follows.

Add the probabilities, but then deduct the probability of an overlap.

$$\text{The probability of Lily winning} = \text{Probability of picking a number less than 10} + \text{Probability of picking an odd number} - \text{Probability of picking one of the overlaps}$$

$$= \frac{9}{20} + \frac{10}{20} - \frac{5}{20}$$

$$= \frac{14}{20} = \frac{7}{10} \text{ (when cancelled down)}$$

Answer: **Lily has a 7 in 10 chance of winning the teddy bear.**

Exercises

4) At the village fete, there are 50 plastics ducks which will race along the river. 25 of the ducks are yellow, 15 are blue and the rest are pink.

a) What is the probability that the winning duck is blue or pink?

b) Every duck is numbered in the race. The yellow ducks are numbered 1 to 25, the blue ducks are numbered 26 to 40 and the pink ducks are numbered 41 to 50.

Rachel thinks a yellow duck or a duck with an even number on it will win the race. What is the probability that Rachel is correct? Hint: can you see an overlap?

Rachel at the village fair eating candy floss.

Probability Summary for 2 or More Events	
AND method	**If you can rewrite the problem with the word AND, then multiply:** **AND = x**
OR method	**If you can rewrite the problem with the word OR, then add:** **OR = +**
Watch out for:	**Dependent events**, when you will need to alter the probability of each event. **NOT Mutually exclusive events**, when you will need to subtract the probability of any overlap.

What is the probability that a cheap belt will fall apart, fast?

Algorithms

An **algorithm** is basically a precise set of instructions, such as how to operate a dishwasher, or cook a recipe.

An **algorithm** can also replace a lot of words with an easy-to-follow diagram. It is a clear step-by-step **flow chart** used to describe a logical sequence of actions in order to:

a) solve a mathematical problem,
b) complete a task,
c) clarify responsibilities,
d) help you to make a choice.

Algorithms are also used to program computers.

Examples of algorithms used to complete a task are:

- **directions** for getting from one town to another,
- cookery **recipes**,
- assemble-it-yourself **instructions**.

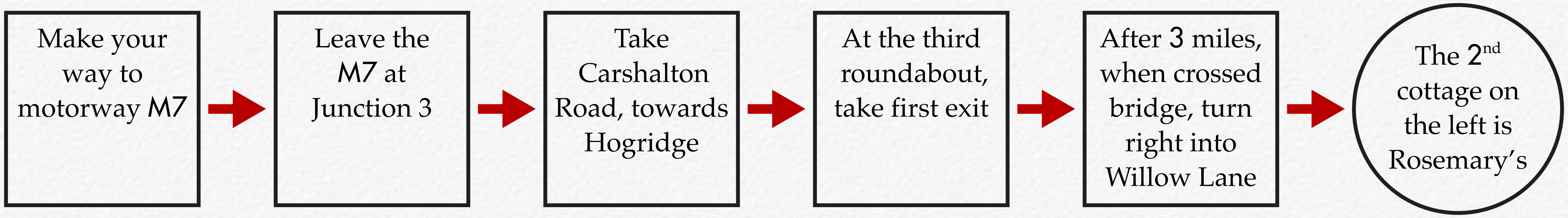
If you follow the instructions correctly – in the right order – you will reach your friend's country cottage, you'll serve the perfect soufflé, and your bookcase won't collapse.

Military commanders use algorithms.
Homecoming photo, courtesy of Mary Haft.

Rosemary's online, home-delivery, party-dress business has made enough profit for a down payment on a dream, country cottage. Here are the **directions** to reach Rosemary's country cottage: "My cottage is near the M7, so get on the M7. Take the motorway exit marked junction **3**. You'll then be on the Carshalton Road; drive along this towards Hogridge. At the third roundabout, turn left. Follow the winding road for about 3 miles and you'll come to a bridge. After the bridge, turn right into my road which is called Willow Lane. You'll find a cool drink waiting at the **2nd** cottage on the left."

Here are the same instructions, drawn as an algorithm.

Diagram A



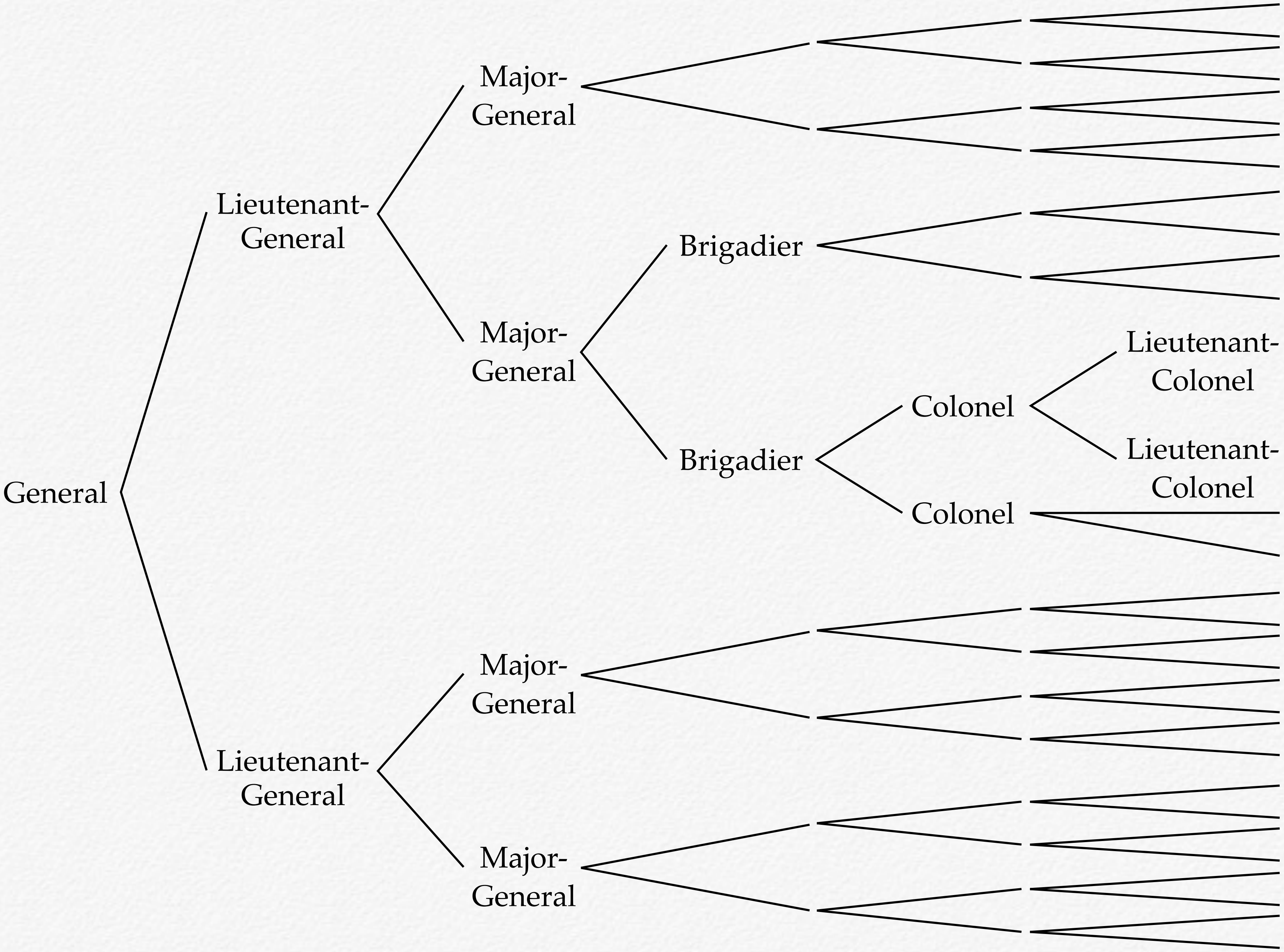
The structure of an algorithm can be adapted to clarify responsibilities – organisation charts – and to help you make a choice – decision trees.

Incidentally, an algorithm can be drawn in any direction – up, down or sideways.

Rosemary's not-so-little cottage.

Diagram B

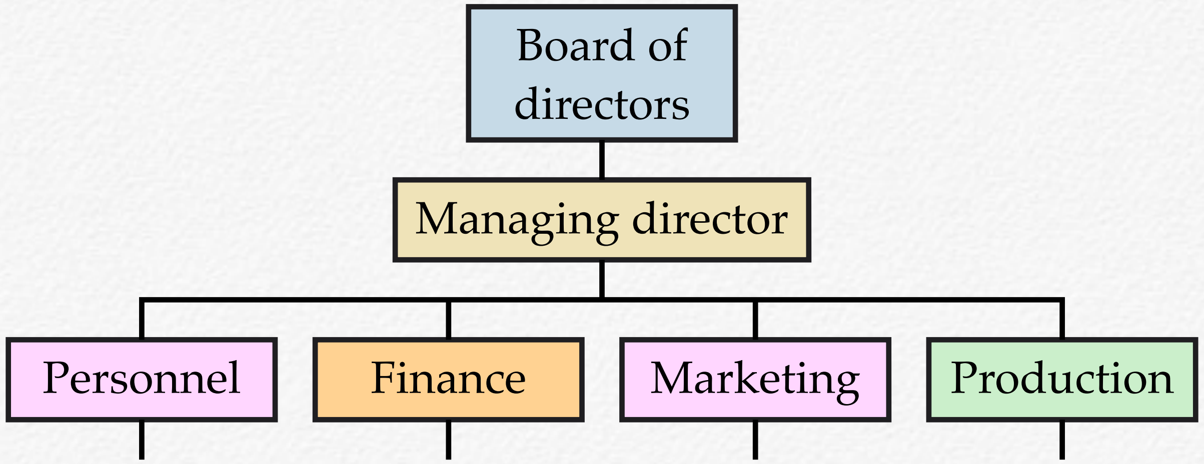
A flow chart that is a **plan of command**. Just think how long it would take to convey this information in writing, rather than in a diagram.



Underneath the lieutenant-colonels, in order of rank, come the majors, captains, lieutenants and second lieutenants.

Diagram C

An office chain of responsibility, is like an army's plan of command.



In a big business, everyone needs to know their place in the chain of responsibility. A diagram makes this easier to understand. For instance, all the people in marketing are responsible to the head of marketing and she is responsible for giving her reports of the department's progress to the managing director.

Algorithms are not only used to **program computers**, but the choices you make when using a computer – whether you are filing your documents or surfing the web – are also **algorithmic progressions**.

Love.

Example

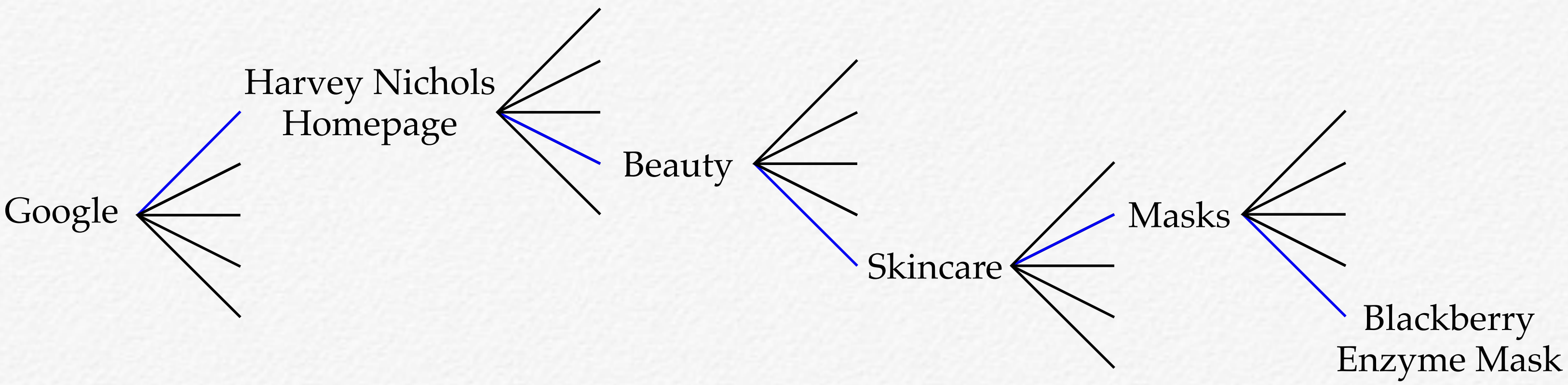
Chris wants to try a new face mask. She googles Harvey Nichols, to find their website (1st progression).

From their home page, she chooses beauty (2nd progression).

From the drop-down menu she chooses skincare (3rd progression). From the skincare page she chooses masks (4th progression). Finally Chris chooses a Blackberry Enzyme Mask (5th progression). At each step, Chris was offered a **choice** of progression which led to her final selection – an expensive blackberry enzyme face mask. (See the progression drawing in the following diagram).

Diagram D

Harvey Nichols Online Shopping, showing Chris' choice of Algorithmic Progression.



In business, algorithmic progressions are used to simplify **decision making**.

Love.

First Example

To clarify my own options and make a decision, here's how I used an algorithmic **decision tree** to decide my Easter Break.

Diagram E

Decision Tree

Because I get bad jet lag, I need a country in a similar time zone to mine.

- Europe = weather too unreliable in March and April ✗
- Africa
  - Activity based holiday (safari/Nile cruise) = Too active, doctor prescribed REST ✗
  - Beach resort
    - Tunisia
      - Package Holiday = Remember ending up in big hotel filled with noisy holiday makers, in room next to the Karaoke, which stopped at 2am ✗
      - Private booking = Can choose small, quiet hotel where I have been before so I know what to expect. ✓
    - Kenya = Weather unreliable, end of monsoon. ✗

That's an example of a decision tree broken down into separate decisions. The final decision is easy to reach. If you end up with two alternatives from which you can't seem to choose, a psychologist once told me that, after having carefully considered two options without making up your mind, it doesn't really matter which you choose, so you might as well flip a coin. Personally, if I flip a coin, get heads and feel disappointed, then I take the tails option. So flipping the coin has finally helped me decide what I really want most.

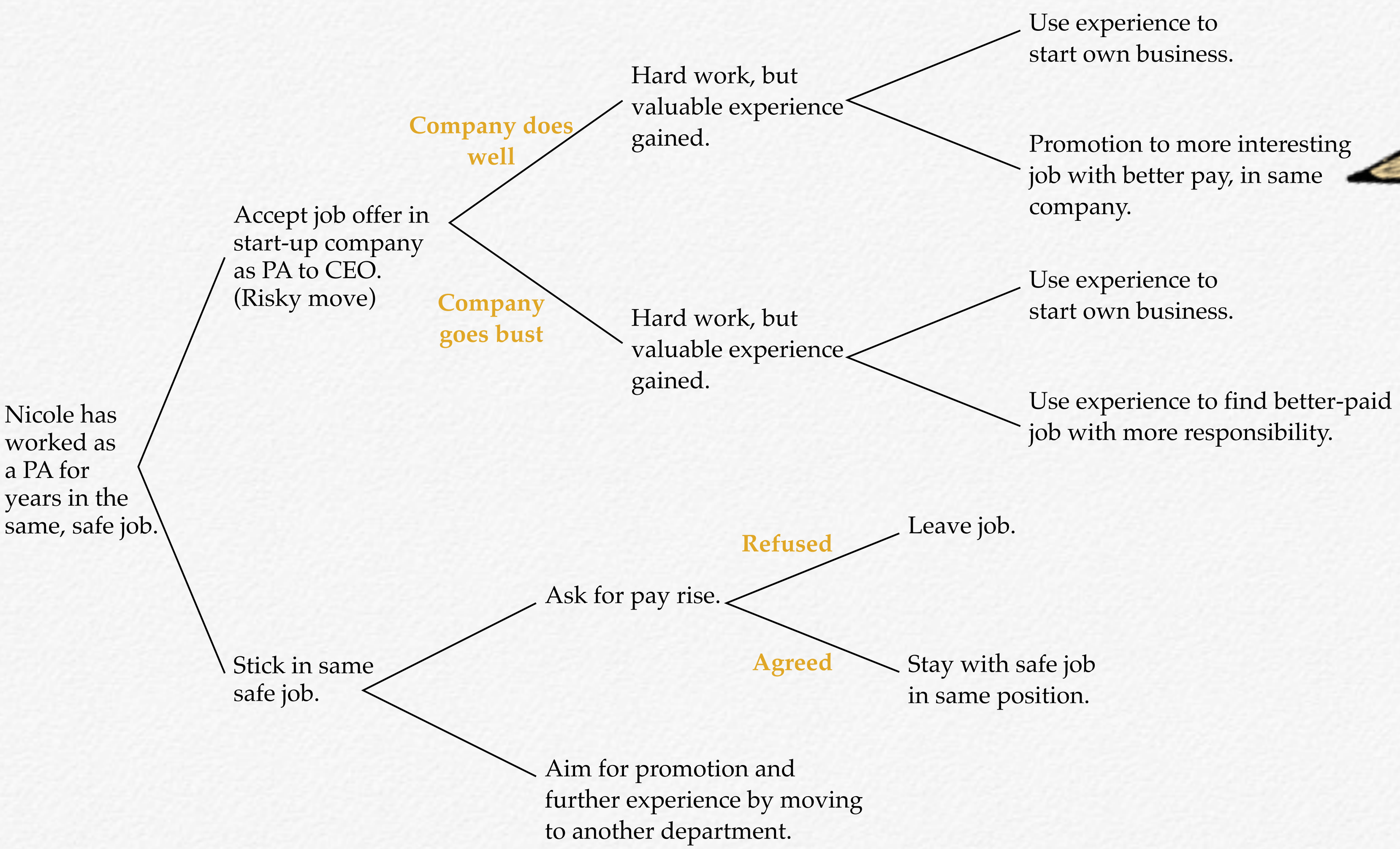
Second Example

Decision tree for **career planning**.

Whatever happens in your life, you'll sometimes need to make decisions; whether to turn left or right at the unmarked crossroads; whether to take the unexpected offer of drama school or go to art school, as you had planned; whether to take the more interesting job or the better paid one.

25 year old Nicole is an empathetic, caring person. She has completed a secretarial business course and has a degree in marketing. Photography is her main hobby. She is mildly ambitious, adventurous and impatient.

Nicole focuses on her future career by drawing a decision tree of her options, which will change with time, but the algorithmic progression will help her to decide where she wants to get to… and roughly how to get there. See diagram F below.



Note

Anyone can use a similar career-planning algorithm.

Diagram F

Exercise 5

Plan your own imaginary future career. Make sure you aim at success, whether you want to breed pink flamingos or marry the King of Sweden.

Probability Tree Diagrams

A **probability tree diagram** can also be used to solve a problem with two or more events.

Probability tree diagrams are not included in this STEP because you are unlikely to need them in Real Life.

Contrary to what you may think, these are officially pink flamingos.

Answers to Part 27

1) While waiting for a poker game to start, Martina bets her new date, Hugh, that he can't throw a double six with one throw of two dice. What is the probability of Hugh throwing a double six?

First, rewrite the question to check whether you can include an AND, while keeping the same meaning: for instance, "What's the probability of throwing a six **AND** six together?"

The probability of throwing a double six = Probability of six **AND** probability of six

$$= \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Answer: **The probability of Hugh throwing a double six is 1 in 36, or $\frac{1}{36}$**

Can you afford a holiday in Paris?

2) In Paris Fashion Week, super-model Naomi hears that at the Yves St Laurent Show one of the 15 super-models will need to wear 8″ high platform gold sandals for the finale. Naomi panics because she's only modelled high platform shoes four times, and fell painfully on one occasion. What are Naomi's chances of being chosen to wear the gold sandals, stumbling on the catwalk, and falling flat on her face?

The probability of Naomi being chosen to wear the gold sandals is 1 out of 15, $\frac{1}{15}$.

The probability of Naomi stumbling in the platforms is 1 out of 4, $\frac{1}{4}$.

Whether Naomi falls on the catwalk depends on her being chosen to wear the platforms **AND** her falling.

The probability of Naomi falling on the catwalk	=	The probability of Naomi being chosen to wear platforms	**AND**	The probability of Naomi falling
	=	$\frac{1}{15}$	x	$\frac{1}{4}$

$$= \frac{1}{15} \times \frac{1}{4} = \frac{1}{60}$$

Answer: **The probability that Naomi will fall flat on her face is 1 in 60**, nevertheless, the bookies decide to call it at 15 to 1
which makes Naomi even more nervous, so more likely to fall (she didn't).

3) At the family Christmas meal, two of Annabel's twelve Christmas mince pies contain a gold charm.

What is the probability that little Daisy will get both charms when she takes two mince pies?

For a gold charm to be in Daisy's first mince pie, the probability is **2** out of **12**, $\frac{2}{12}$

For a gold charm to be in Daisy's second mince pie, the probability is **1** out of **11**, $\frac{1}{11}$

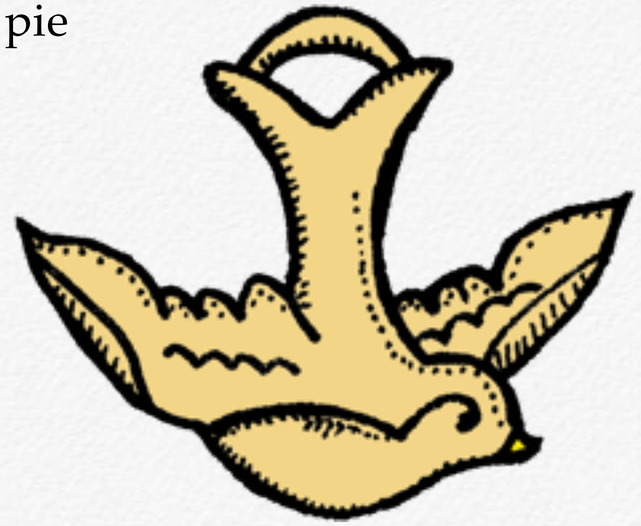
Next, rewrite the question to see if you can include an AND, while keeping the same meaning:

What is the probability that Daisy will get one charm **AND** the second charm in her Christmas mince pies?
Yes, AND fits in that sentence.

Probability that Daisy gets both charms	=	Probability Daisy gets a charm in her first pie	**AND**	Probability Daisy gets a charm in her second pie
	=	$\frac{2}{12}$	x	$\frac{1}{11}$

$$= \frac{2}{12} \times \frac{1}{11} = \frac{2}{132}$$

which cancels down to $= \frac{1}{66}$



Answer: **The probability of little Daisy getting both gold charms is 1 out of 66**. However, the chances that Grandad will give little Daisy his gold charm – if he gets one – are very high.

4) At the village fete, there are 50 plastics ducks which will race along the river.
25 of the ducks are yellow, 15 are blue and the rest are pink.

a) What is the probability that the winning duck is blue or pink?

This is an OR problem: What is the probability that the winning duck is blue **OR** pink?

There are: 25 yellow ducks
15 blue ducks
10 pink ducks (25 + 15 = 40 yellow and blue ducks. 50 – 40 = 10 pink ducks)

The probability of a blue duck winning is 15 out of 50 = $\frac{15}{50}$

The probability of a pink duck winning is 10 out of 50 = $\frac{10}{50}$

The probability that the winning duck is blue **OR** pink	=	Probability of a blue duck winning	+	Probability of a pink duck winning
	=	$\frac{15}{50}$	+	$\frac{10}{50}$
	=	$\frac{25}{50} = \frac{1}{2}$		

Answer: **The probability that the winning duck is blue or pink is** $\frac{1}{2}$

The probability of Rachel having an ice cream at the fair is very high.

b) Every duck is numbered in the race. The yellow ducks are numbered 1 to 25, the blue ducks are numbered 26 to 40 and the pink ducks are numbered 41 to 50.

Rachel thinks a yellow duck or a duck with an even number on it will win the race. What is the probability that Rachel is correct?

This is an OR problem: What is the probability that a yellow duck **OR** an evenly numbered duck will win?

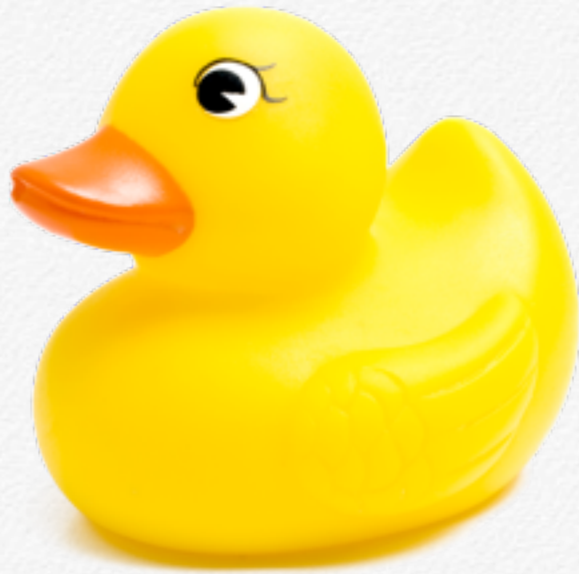
The probability that a yellow duck wins is $\frac{25}{50}$ (25 yellow ducks out of 50).

The probability that a evenly numbered duck wins is $\frac{25}{50}$ (there are 25 evenly numbered ducks out of 50).

But some yellow ducks are evenly numbered, so there is an overlap. The events are NOT mutually exclusive.

There are 12 ducks that are both yellow and evenly numbered (2, 4, 6, …. 24), so the probability of a duck in the overlap winning is 12 out of 50 = $\frac{12}{50}$

The probability of Rachel being correct	=	Probability of a yellow duck winning	+	Probability of an even duck winning	–	Probability of the overlap
	=	$\frac{25}{50}$	+	$\frac{25}{50}$	–	$\frac{12}{50}$
	=	$\frac{50}{50} - \frac{12}{50}$				
	=	$\frac{38}{50}$				
which cancels down to	=	$\frac{19}{25}$				



Answer: **The probability that Rachel is correct is $\frac{19}{25}$**

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 1 of 10.

Two red cards are
picked from the
same pack of cards.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 2 of 10.

Two red cards are picked from two different packs of cards.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 3 of 10.

Two sixes are thrown
with two different dice.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 4 of 10.

Two sixes are thrown
one after the other
on the same die.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 5 of 10.

The train arriving on time and the plane departing on time.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 6 of 10.

Picking two black socks
from the sock drawer.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 7 of 10.

Two bad apples being picked from the tree.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 8 of 10.

Two boys' toys being
picked from the lucky dip.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 9 of 10.

You win the lottery and the slot machine jackpot on the same day.

YOUR BRAIN WORKOUT

Decide if the following are
dependent or independent events.

Question 10 of 10.

Your bicycle gets a puncture and it begins to rain.

YOUR BRAIN WORKOUT

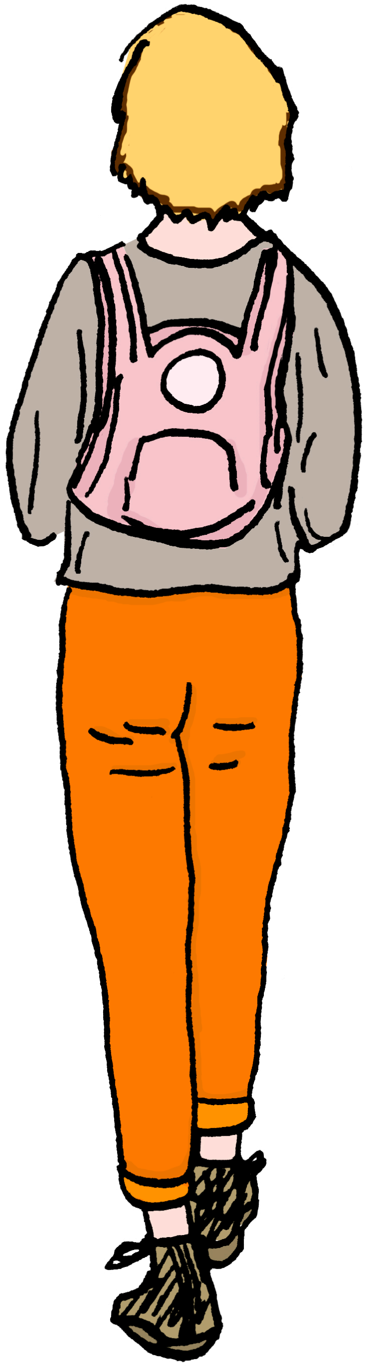
Answers

Q1. Dependent
Q2. Independent
Q3. Independent
Q4. Independent
Q5. Independent
Q6. Dependent
Q7. Dependent
Q8. Dependent
Q9. Independent
Q10. Independent

Determination.
Team of cancer survivors at International Dragon Boat Festival, Malaysia.

PART 28
ALGEBRA

Quick Quiz

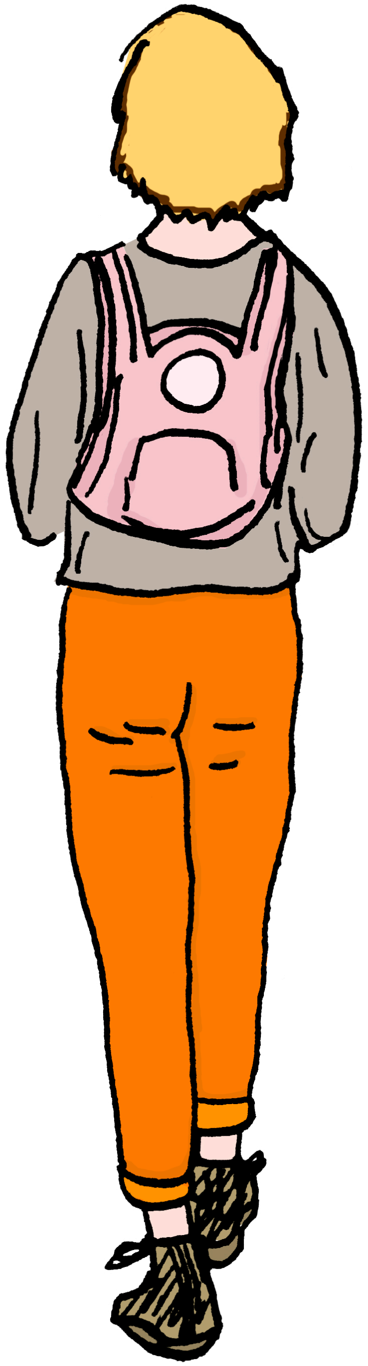


Question 1 of 4.

What is 400,000 ÷ 50?

- **A.** 8,000,000
- **B.** 800,000
- **C.** 80,000
- **D.** 8,000

Quick Quiz

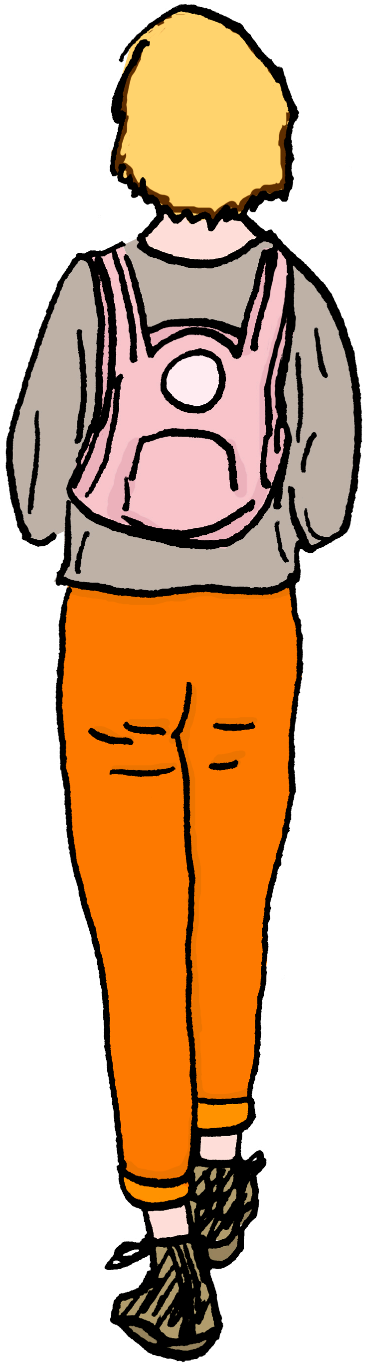


Question 2 of 4.

Which of the following numbers is **not** a prime number?

- **A.** 9
- **B.** 11
- **C.** 13
- **D.** 17

Quick Quiz

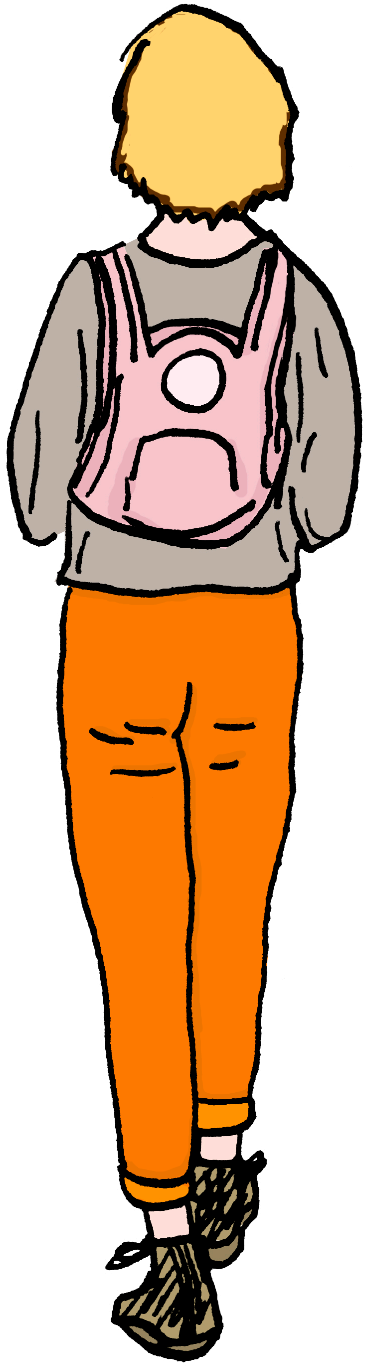


Question 3 of 4.

What is 10.99 rounded to the nearest whole number?

- **A.** 9
- **B.** 10
- **C.** 11
- **D.** 12

Quick Quiz

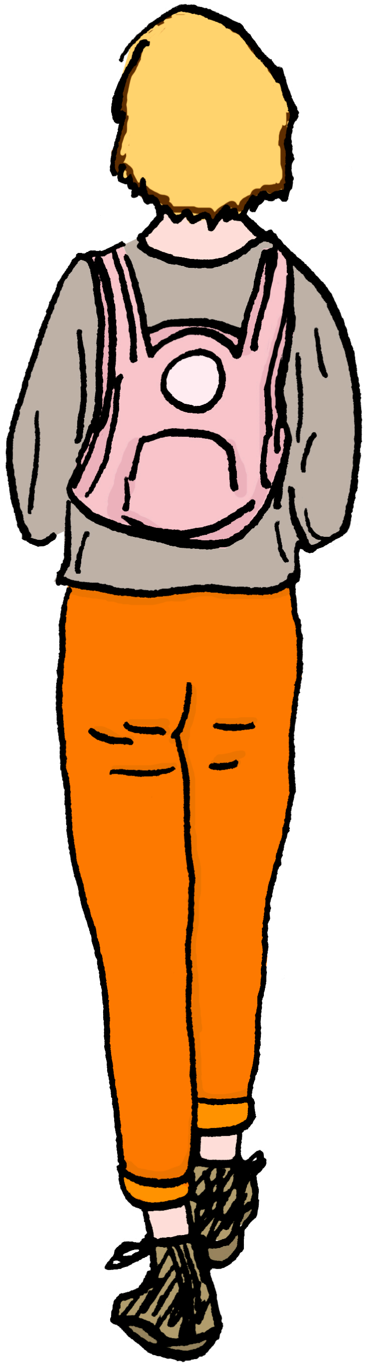


Question 4 of 4.

Which of the numbers below is the biggest?

- A. 0.43
- B. 0.34
- C. 0.304
- D. 0.44

Quick Quiz



Answers

Q1. 8,000

Q2. 9

Q3. 11

Q4. 0.44

Algebra

What is algebra? And why should I bother about it?

Algebra solves problems

Algebra can simplify a problem, in order to find the answer as quickly and easily as possible.

A bit of algebra gives you a useful shorthand system for **stating** a Real Life problem (write it down); **clarifying** a Real Life problem and **solving** a Real Life problem.

Some people are alarmed by THE LOOK of a number combined with a letter, such as $4a$ or $8c$...

Algebra uses letters in place of unknown numbers, in order to work out what the unknown numbers are. Here are two examples, using simple problems that you can probably solve in your head, in order to show you a method using algebra. It may look long-winded, but the algebra method solves complicated problems that you **can't** do in your head.

First Example

In Brighton, retired actress Sally takes the day off to sun on the beach. She buys a beach bag and a pair of flip flops for **£17.50**. Sally knows that the beach bag cost **£12**, how much did the flip flops cost?

Let the letter f represent the cost of the flip flops. Your aim is to isolate the f, in order to work out what f represents.

Total spent	=	cost of beach bag	+	cost of flip flops
£17.50	=	£12.00	+	£f

The calculation is then simplified to read:

$17.50 = 12 + f$

Isolate f by taking away 12 from both sides of the equals sign:

$17.50 - 12 = 12 + f - 12$

$5.50 = f$ (the cost of the flip flops)

Answer: **So $f = 5.50$, so the price of the flip flops is £5.50**.

Second Example

Sally gives her daughter, Michelle, a **£10** note to go and buy ice creams. Michelle buys **3** cones and brings back **£6.70** change. How much did one ice cream cost?

Let the letter **y** represent the cost of one ice cream. Your aim is to find the cost of one ice cream, and so you need to isolate **y**.

Cost of **3** ice creams + the change = **£10**.

1 ice cream = y, then **3** ice creams = **3y**.

$$3y + 6.70 = 10$$

Isolate **3y** by taking **6.70** from both sides of the equals sign:

$$3y = 10 - 6.70$$

$$3y = 3.30$$

Isolate y by dividing both sides by **3**:

$$3y \div 3 = 3.30 \div 3$$

$$y = 1.10$$

Answer: y = 1.10, **so each ice cream cost £1.10.**

Those two examples both use an **equation** to work out the answers. To equate means to balance.

Think of a pair of old fashioned grocer's scales, with a pan on both sides. Imagine that pan **A** contains one orange which weighs exactly as much as four bananas in pan **B**. When the weight in pan **A** is exactly equal to the weight in pan **B**, the weights are **equal**.

An **equation** is a statement in which two groups of figures are equal: **A = B.**

Weight **A** (the orange) is equal to weight **B** (bananas).

Note: An **equation** always includes an **equals sign**.

Examples

$2 + 2 = 4$ $\quad$ $4 + 3 = 7$ $\quad$ $6 + 2 = 3 + 5$

Keep your figures neat and in line.

Getting Started with Algebra: The Five Basic Rules

Absolute beginners might study only one rule each day and make up more examples. By the end of the week, she should know enough algebra for Real Life.

Remember that in algebra, letters represent missing numbers.

Every fashion design in every country involves maths.

Rule 1

If a letter representing a number is next to another letter or number, with nothing written between them, then they are always **multiplied** together.

Examples:

'2a' means '$2 \times a$'

'3ab' means '$3 \times a \times b$'

If $a = 2$, $b = 5$, here's how to find the value of **3ab**:

$$\begin{aligned} 3ab &= 3 \times a \times b \\ &= 3 \times 2 \times 5 \\ &= 3 \times 10 \\ &= \mathbf{30} \end{aligned}$$

Here's another example, using the same values: $a = 2$, $b = 5$

$$\begin{aligned} 12ab &= 12 \times a \times b \\ &= 12 \times 2 \times 5 \\ &= 12 \times 10 \\ &= \mathbf{120} \end{aligned}$$

Note

When two digits are next to each other, treat them as making a normal number. In the last example, 12ab means '$12 \times a \times b$' **not** '$1 \times 2 \times a \times b$'.
Similarly, **25** does **not** mean 2×5, it means 'twenty-five'.
173y means '$173 \times y$' **not** '$1 \times 7 \times 3 \times y$'.

In algebra, a **term** is any group that represents a number.
Examples: **3a** is a term, **4pq** is a term, **3** by itself is a term.

An **expression**, in algebra, is a string of terms, linked by + or – signs.

Examples

$3a + 4pq - 3$ is an expression.

$7y - 4k + 2m$ is an expression.

Rule 2

The **+** or **–** sign **before** a term, **belongs to that term** and is never separated from it. In the expression $2a - 4d$, the minus sign belongs to $4d$, not $2a$. In the expression $-6 + 8y$, the minus sign belongs to the 6 and the plus sign belongs to $8y$.

If a number has no sign, it automatically has an invisible plus sign in front of it. So 7 is $+7$, 12 is $+12$, $3x$ is $+3x$, $8D$ is $+8D$.

Example

$4a + 2b - 3c = z$

If you scribble a circle round each term, you won't get the signs muddled.

Make sure you include the signs in your circles:

$+4a + 2b - 3c = z$

You can place these terms in any order on the left side of the equals sign (=) and you won't alter their value: the answer will still be the same.

$+2b - 3c + 4a = z$

If $a = 3$, $b = 4$ and $c = 2$, then the original equation is:

$$
\begin{aligned}
4a + 2b - 3c &= z \\
(4 \times 3) + (2 \times 4) - (3 \times 2) &= z \\
12 + 8 - 6 &= z \\
20 - 6 &= z \\
\mathbf{14} &= \mathbf{z}
\end{aligned}
$$

(Remember **BIDMAS** tells you the order of your calculation: **B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction)

Scale patterns.

And with the rearranged equation:

$$
\begin{aligned}
+2b - 3c + 4a &= z \\
+(2 \times 4) - (3 \times 2) + (4 \times 3) &= z \\
8 - 6 + 12 &= z \\
2 + 12 &= z \\
\mathbf{14} &= \mathbf{z}
\end{aligned}
$$

Exercises

1) Write two different ways of placing the terms on the left hand side only of the following equation:

$6 + 5p - 2q = a$

2) Write the following equation in an alternative way:

$11a - 5b = y$

Maths is needed to convert cargo containers into human accommodation.

Before going on, remember:

Rule 1

If a letter is next to a number they are multiplied together:

$4a = 4 \times a$

Rule 2

The term and its sign are always super-glued together:

$+4a - 2b = - 2b + 4a$

Small structures need maths to stay standing…

…so do big ones.

New York: the reality.

Rule 3

Only terms containing the same letters can be combined by being added or subtracted from each other, as in $6n - 2n = 4n$.

It's the same in Real Life. You won't find a sum like this:

6 oranges + 3 oranges – 2 lemons + 1 orange = 8 oranges

__Examples__

$3a + 5a = 8a$ Here, the terms contain the same letter and so are simplified to $8a$.

$4b - 3b = 1b$ $1b$ is written simply as 'b'.

In the following examples, the terms cannot be combined because they do not contain the same letter.

$4a + 2b$ cannot be added together.
$4a + 2b$ is not equal to $6ab$.

$7d - 2h$ is **not** equal to $5dh$.

$3f + 8$ is **not** $11f$.

$5t - 3$ is **not** $2t$.

Below is an example. Before working out your equation, group the same terms together, in brackets. Remember the signs are super-glued to their terms.

$7t + 3k - 2t + 4 - 5k + 2 = y$

$(+7t - 2t) + (+3k - 5k) + (+4 + 2) = y$

Now **simplify** this equation by doing those little sums in brackets, so that the equation looks like this:

$+5t + - 2k + +6 = y$

Remember: a plus and a minus next to each other make a minus, two pluses next to each other make a plus (see How to Be a Weather Girl in STEP 1).

Answer: $5t - 2k + 6 = y$.

Geometric building patterns in Old Havana.

Rule 4

When working on an equation, **whatever you do to one side of the = sign, you must do to the other side**.
You can add, subtract, multiply or divide by anything, as long as you add, subtract, multiply and divide by the same number on both sides of the = sign. The equation will then stay balanced.

Example

Here is an equation: **4q + 5 = 13** (see scales in Diagram 1).

Both sides of the = sign are equal to each other: you can imagine them as equally balanced on scales:

Diagram 1

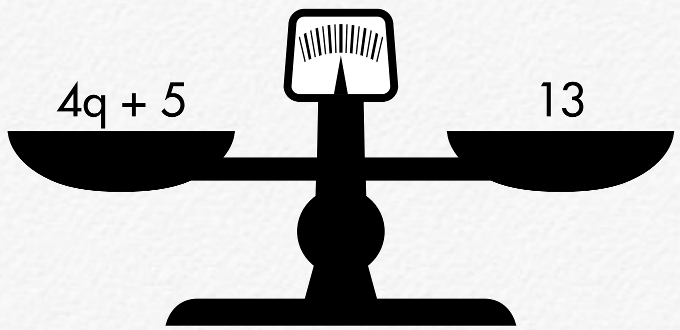
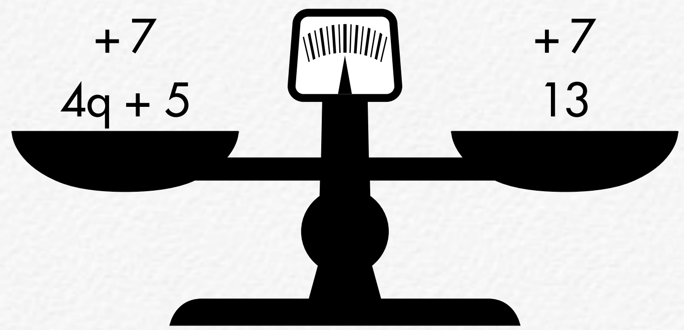


Diagram 2

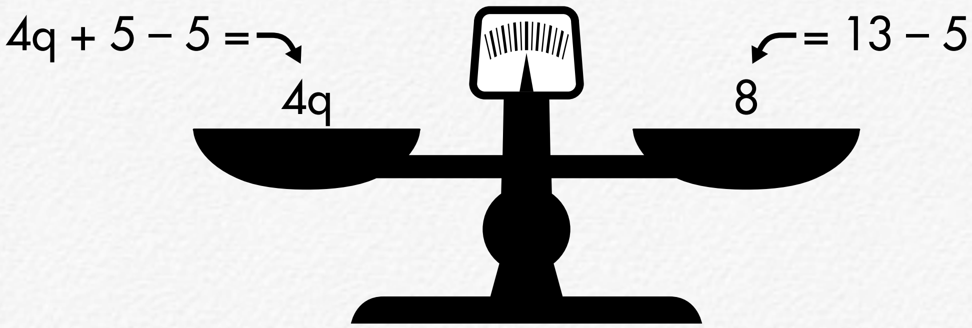


As long as you do the same thing to **both sides** of the scales, they will stay balanced (see Diagram 2 where **7** is added to **both sides** of the scales).

Similarly, if you remove (subtract) 5 from both sides of the scales (Diagram 3), the equation will still be balanced.

The left hand side will read: **4q + 5 – 5 = 4q**
The right hand side will read: **13 – 5 = 8** (see Diagram 3 on the next page).

Diagram 3

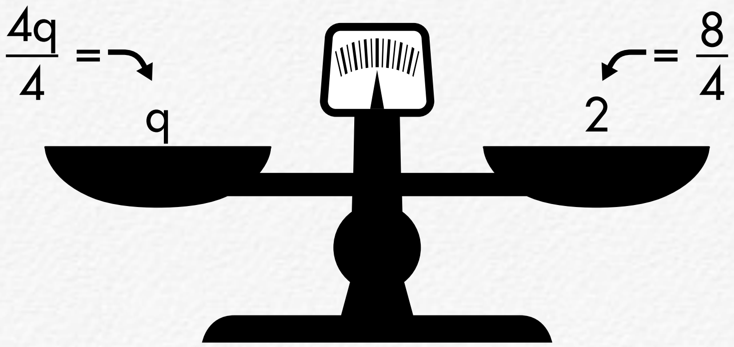


The equation that remains on the scales of Diagram 3 now reads: $\mathbf{4q = 8}$

Notice that both sides can be divided by $\mathbf{4}$

The left hand side will then read: $\mathbf{4q \div 4 = 1q}$
The right hand side will read: $\mathbf{8 \div 4 = 2}$ (see Diagram 4)

Diagram 4



Eureka! You have discovered that $\mathbf{q = 2}$

Finding what the puzzle letter represents is called **solving the equation**, (more later).

Determination on the prairie: covered wagon, 1890.

How to Use a Formula

A **formula** is a set of instructions used to work out the answer you are looking for.

First Example

A formula for the time it takes to roast a leg of lamb is **30 mins** for every pound in weight, plus an extra **20** minutes.

So let's say the **weight** of your leg of lamb is '**w**' pounds, the formula is summarised as:

30w + 20 = **Cooking time for a leg of lamb (in minutes).**

How long should it take to cook a leg of lamb weighing 4lb?

Here, w = 4, so:

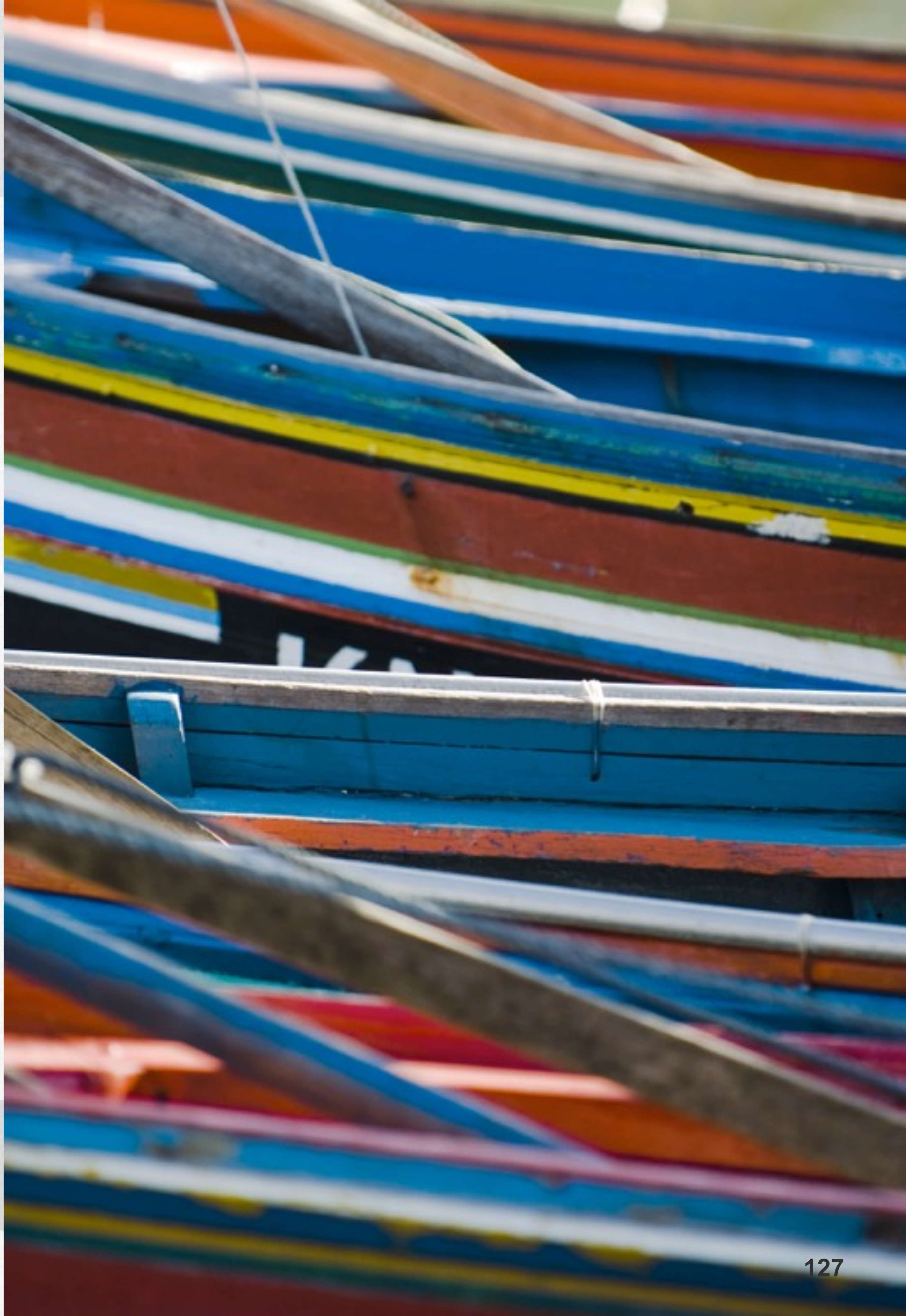
30 x 4 + 20 = **Cooking time for leg of lamb (in minutes)**

120 + 20 = 140 **minutes**

Answer: **Cook a 4 lb leg of lamb for 140 minutes** (**2 h 20 mins**).

Remember to sprinkle with rosemary before roasting.

You need maths to build a boat.



Second Example

You might want to know upon what charges your local plumber has based his astronomical bill for his visit. He probably used his own formula, such as this: Plumber's bill **=** call-out fee **+** (hourly work rate **x** number of hours spent fixing your boiler and chatting on his mobile) **+** total cost of spare parts.

Number of hours spent fixing your boiler and chatting on his mobile = A

Call-out fee = £50 Hourly work rate = £80 Total cost of spare parts = £20

Plumber's bill = call-out fee + (hourly work rate x A) + cost of spare parts

Plumber's bill (in £) = 50 + (80A) + 20

The plumber tells you that he spent **3** hours on your boiler and used **2** new widgets.

Plumber's bill (in £) = 50 + (80 x 3) + 20

Plumber's bill (in £) = 50 + 240 + 20

Plumber's bill (in £) = 310

Answer: **The total cost of your plumber's 3-hour visit was £310.**

Plus VAT, the plumber will helpfully remind you. Total cost = **£310** plus VAT.

Thick socks, until the boiler is working again.


Keep your figures neat and in line.

How to Make Your Own Algebra Formula

Take an unknown number; call it w, double it $(2w)$, then add 8 $(+ 8)$

In algebra this is written as $2w + 8$

You need maths to fit a shelf. Traditional Korean house.

Examples

Translate each of the following into a formula:

a) Take your unknown number, add **7**, double the result:

First, add **7** to your unknown number $= w + 7$

then double the result $= 2(w + 7)$

Answer: $2(w + 7)$

b) Multiply your unknown number by **4** and add **6**:

First, multiply w by $4 = 4w$, then add 6

Answer: $4w + 6$

c) Multiply your unknown number by **5**, add **3**, then divide the answer by **2**:

First, multiply w by $5 = 5w$, then add $3 = 5w + 3$

Finally, divide by $2 = \dfrac{5w + 3}{2}$

Answer: $\dfrac{5w + 3}{2}$

d) Multiply the unknown number by itself:

Answer: $w \times w$ or w^2 (for square numbers see STEP 2)

If you are told that the unknown number is **5**,
substitute 5 for w in each of the following formulae.
Next, work out the values of the same four formulae.

a) $2(w + 7) = 2(5 + 7)$
$= 2(12)$
$= \mathbf{24}$

b) $4w + 6 = 4 \times 5 + 6$
$= 20 + 6$
$= \mathbf{26}$

c) $\frac{5w + 3}{2} = \frac{5 \times 5 + 3}{2}$
$= \frac{25 + 3}{2}$
$= \frac{28}{2}$
$= \mathbf{14}$

d) $w^2 = 5^2$
$= 5 \times 5$
$= \mathbf{25}$

Exercises

3) Use the examples a, b, c and d,
to do the same calculations when **w** = 7

4) Use the examples a, b, c and d,
to do the same calculations when **w** = 12

Real Life problems can also be translated into **formulae**.
Here are some examples:

First Example

In a soothingly dark, city bar, a fruit cocktail costs **£1.50** more than a bowl of olives. Write a formula for the cost of a fruit cocktail and a bowl of olives.

Start with the cheaper item, olives: say the cost of the olives = **v**

If the olives cost **v**, then **the cost of a fruit cocktail** = v + 1.50

The cost of a fruit cocktail and a bowl of olives
= v + 1.50 + v
= 2v + 1.50

Answer: Where **v** is the cost of a bowl of olives, **the cost of a fruit cocktail and a bowl of olives = £[2v + 1.50]**

Second Example

For student card holders, there is a reduction of **£3** on the price of a cinema ticket. Gemma, Lily and two older friends are going to the cinema: three of them have student cards. Write a formula for the total cost of the cinema tickets, using **c** as the cost of one normal ticket.

Each normally priced ticket = c Each student ticket = $c - 3$

There is one non-student ticket which costs c and 3 student tickets which cost $3(c - 3)$.

What do you do with the brackets? Usually, you start by calculating the sum inside the bracket, but that won’t work here because the figures within the brackets are not **like terms**. (Remember Algebra Rule 1.)

When multiplying a number that contains algebra terms inside brackets…you multiply the number before the bracket with each term within the bracket, one at a time. So in this example, you calculate $3(c - 3)$ as follows:

$3 \times c = 3c$ and $3 \times -3 = -9$ The result is $3c - 9$

$$\begin{aligned} \text{Total cost of cinema tickets } &= c + 3(c - 3) \\ &= c + 3c - 9 \\ &= \mathbf{4c - 9} \end{aligned}$$

Answer: **Total cost of cinema tickets = £[4c – 9]** is the formula.

In algebra stick to BIDMAS wherever possible: notice that it is not always possible.

How to Solve a Simple Equation

To **solve** a simple **equation** means to find the value of the only letter in the equation.

An **equation** consists of two mathematical statements that are **equal**. The equals sign (=) between them shows you a) that the two sides are equal and b) that it is an equation.

You already know that whatever you do to one side of the equals sign, you must do to the other, in order that both sides stay equal.

The key to solving an equation is to get **all the terms containing puzzle letters on one side** of the **=** sign and **all the numbers without letters on the other side** of the **=** sign.

Then isolate the puzzle letter. Go slowly, step by step: add, subtract, divide or multiply **both** sides by the same numbers or letters.

Both sides of an equation are equal.

Do the opposite **of what is done to the puzzle letter,** using numbers already in the equation (see below).

Rule 5

+ and – are opposite actions.

First Example

Your equation is **y + 4 = 9**. Your aim is to isolate **y**.

Because **y** has **4 added** to it, you **subtract 4** on **both sides** of the = sign.

Your equation then becomes: $y + 4 \mathbf{- 4} = 9 \mathbf{- 4}$

Simplify your equation so that: $y + \cancel{4} - \cancel{4} = 9 - 4$

$y = 9 - 4$

$y = 5$

Answer: **y = 5**

Time for a reward?

Second Example

Your equation is **z – 17 = 9**. Your aim is to isolate **z**.

In this case, **z** has **17 subtracted** from it, so you **add 17** to **both sides** of the = sign.

Your equation then becomes: $z - 17 \mathbf{+ 17} = 9 \mathbf{+ 17}$

Simplify your equation so that: $z - \cancel{17} + \cancel{17} = 9 + 17$

$z = 9 + 17$

$z = 26$

Answer: **z = 26**

Exercises

Solve the following equations:

5) **d + 6 = 11**

6) **f – 3 = 5**

7) **10 = g – 8**

8) **4 = h + 10 – 3**

The rest of Rule 5 is that multiplication and division are also opposite actions.

First Example

Your equation is $4b = 28$.

Because b is multiplied by 4, you divide by 4 on both sides.

Your equation then becomes: $\frac{4b}{4} = \frac{28}{4}$

Next, simplify your equation: $\frac{\cancel{4}b}{\cancel{4}} = \frac{28}{4}$

$$b = \frac{28}{4}$$

$$b = 7$$

Answer: $b = 7$

Second Example

Your equation is $\frac{c}{5} = 3$.

Because c is **divided by** 5, you **multiply by** 5 on both sides.

Your equation becomes: $\frac{c}{5} \times 5 = 3 \times 5$

Next, simplify your equation: $\frac{c}{\cancel{5}} \times \cancel{5} = 3 \times 5$

$$c = 3 \times 5$$

$$c = 15$$

Answer: $c = 15$

Exercises

Solve the following equations:

9) $6y = 42$

10) $\frac{k}{7} = 5$

11) $10 = 5m$

12) $13 = \frac{n}{2}$

SUMMARY
OF THE FIVE BASIC RULES OF ALGEBRA

Rule 1

If a letter is next to a number
they are multiplied together: $4a = 4 \times a$

Rule 2

The term and its sign are always
super-glued together: $+4a - 2b = -2b + 4a$

Rule 3

Only terms containing the same letters
can be added to or subtracted
from each other, as in $6n - 2n = 4n$.

Rule 4

Whatever you do to one side of the = sign,
you must do to the other.

Rule 5

$+$ and $-$ are opposite actions
$\times$ and $\div$ are also opposite actions.

Examples of solving equations that have more than one action.

First Example

Solve the equation $4w + 6 = 42$, in order to find w.

The letter, w, needs to be isolated. $4w$ has 6 **added** to it, so **subtract** 6 from both sides:

$$4w + 6 - 6 = 42 - 6$$

Simplified this reads: $4w = 36$

The letter w has been multiplied by 4, so divide by 4 on both sides:

$$\frac{4w}{4} = \frac{36}{4}$$

Simplified this reads: $w = 9$

Answer: $w = 9$

A PRACTICAL TREAT?

Second Example

Solve the equation $23 = 2p - 3$, to find the value of p.

$$23 = 2p - 3$$

The letter p needs to be isolated. $2p$ has 3 subtracted from it, so add 3 to both sides:

$$23 + 3 = 2p - 3 + 3$$

Simplified this reads: $26 = 2p$

p has been multiplied by 2, so divide by 2.

$$\frac{26}{2} = \frac{2p}{2}$$

Simplified this reads: $13 = p$

Answer: $p = 13$

Note

When solving an equation, to keep it simple, **always add or subtract before you divide or multiply**. Try to avoid multiplying and dividing until you have lone terms on each side of the equation.

To see why, look back to the last example: **23 = 2p – 3**

Let’s see what happens if you make the mistake of dividing before you add.

If you divide by **2**, you will have:

$$\frac{23}{\mathbf{2}} = \frac{2p - 3}{\mathbf{2}}$$

$$\frac{23}{2} = \frac{2p}{2} - \frac{3}{2}$$

$$11\frac{1}{2} = p - 1\frac{1}{2}$$

Isolate **p** by adding $1\frac{1}{2}$ to both sides:

$$11\frac{1}{2} + \mathbf{1\frac{1}{2}} = p - 1\frac{1}{2} + \mathbf{1\frac{1}{2}}$$

$$13 = p$$

Here’s the easy way. **Remember to add before you divide.**

$$23 = 2p - 3$$

$$23 + \mathbf{3} = 2p - 3 + \mathbf{3}$$

$$26 = 2p$$

$$\frac{26}{\mathbf{2}} = \frac{2p}{\mathbf{2}}$$

$$13 = p$$

Both are mathematically correct methods but if you divide before you add, your sum will be more complicated, so you’re more likely to make a mistake.

Keep it simple with maths. California.

Exercises

Remember to avoid multiplying and dividing until you have lone terms on each side of the equation.

Choose the method you find easier to do the following exercises:

13) Solve the equation: $7b + 8 = 29$

14) Solve the equation: $6 = \frac{a}{2} - 4$

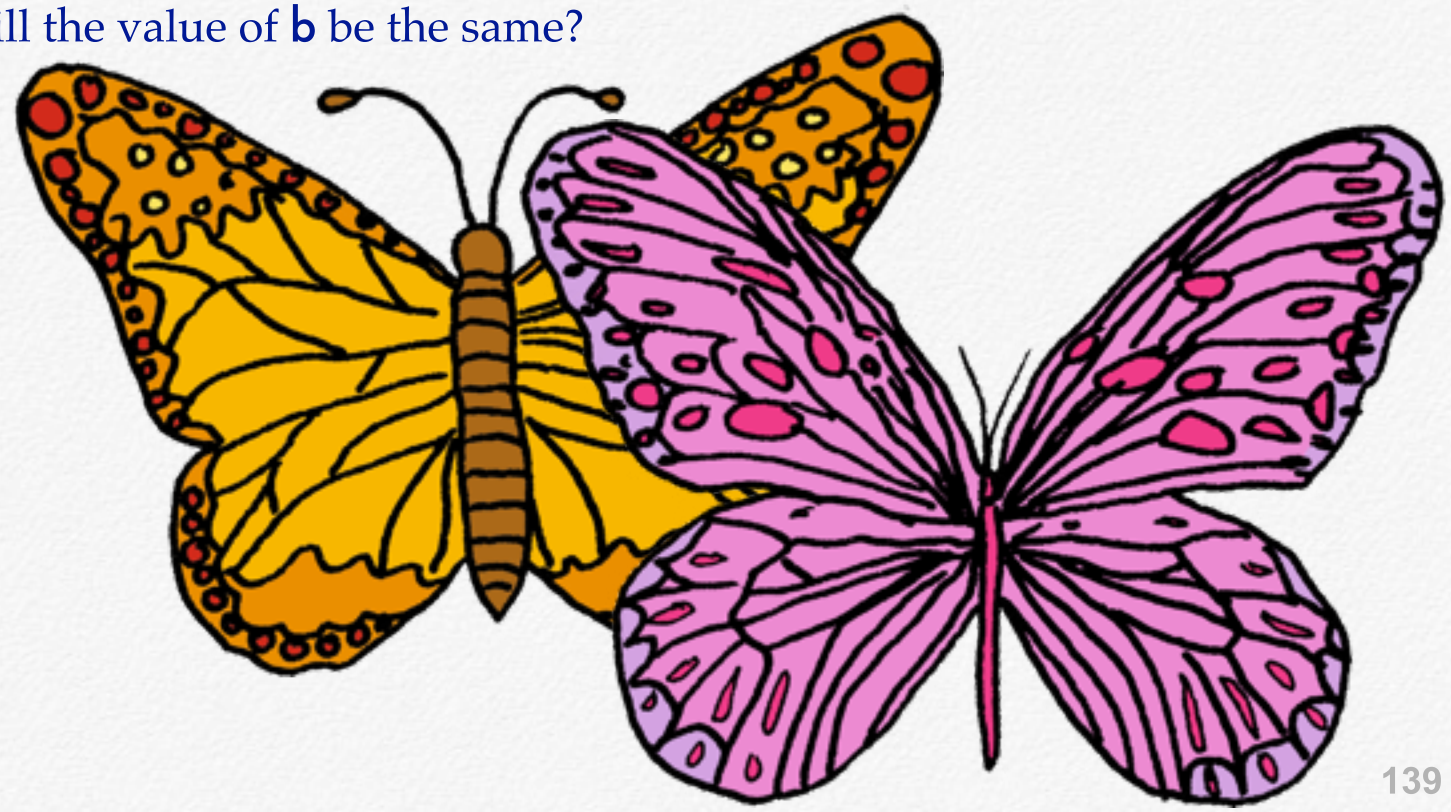
15) Find the value of **q** in the following equation: $7 + \frac{q}{3} = 25$

16) Solve the following equation to find **w**: $\frac{5w + 3}{2} = 9$

Hint: The whole of the left hand side of the equation is divided by **2**, so here it will be easiest to multiply first.

17) Take the equation $7b + 8 = 29$ from question 13.

If you double all the numbers in the entire equation, will the value of **b** be the same?



What if the puzzle letter is in more than one term of the equation?

First add or subtract one of those terms:

Example

Solve the equation **5p + 7 = 21 – 2p** in order to find **p**.

Your aim is to isolate **p**. So eliminate one of the terms containing **p**. Which one? You can either subtract **5p** from both sides, or add **2p** to both sides. **Always choose addition instead of subtraction, if you have a choice**.

$$5p + 7 \mathbf{+ 2p} = 21 - 2p \mathbf{+ 2p}$$

Simplified this reads: $7p + 7 = 21$

Now subtract **7** from both sides: $7p + 7 \mathbf{- 7} = 21 \mathbf{- 7}$

$$7p = 14$$

Divide both sides by **7**: $\dfrac{\cancel{7}p}{\cancel{7}} = \dfrac{14}{\mathbf{7}}$

$$\mathbf{p = 2}$$

Answer: **p = 2**

Another Example

Solve the equation $18 - 4q = 14 - 3q$ in order to find q.

Your aim is to isolate q. So eliminate one of the terms containing q. Which one? You can either add $4q$ to both sides, or add $3q$ to both sides. Add the bigger term, to avoid having a negative q term.

$$18 - 4q + 4q = 14 - 3q + 4q$$

(Remember that $-3q + 4q$ is the same as $4q - 3q$)

Simplified this reads: $18 = 14 + q$

Now subtract 14 from both sides: $18 - 14 = 14 + q - 14$

$$4 = q$$

Answer: $q = 4$

On site.

How to Solve Problems with Algebra Equations

Here's a simple mental arithmetic puzzle to amaze 10-year-olds. You can vary it, so long as you keep track of each stage of the ongoing sum in your head or on your fingers. Stick with single-digit numbers.

1st game

You say, "Think of a number and don't tell me what it is. Now, add 9 (pause), deduct 3 (pause), add 1 (pause), deduct 4 (pause), take away 2 (pause), add 5 (pause), and deduct 6." Then tell him or her that the answer is the number he first thought of.

Astonished child, "Yes!"

How do you do it? In your mind, you think of the unknown number as z, and ignore it until you get to the end of the teaser.

Whatever positive numbers or negative numbers you use, provided you aim to make your mental arithmetic sum add up to zero, then the remainder will be the child's secret number. To put this arithmetically:

$$z + 9 - 3 + 1 - 4 - 2 + 5 - 6 = z + 0$$

Sure, make the most of your looks but also spend time on developing other assets, such as maths.

2nd game

Next, you say to the **10**-year-old: "Think of a number. Add **8**, deduct **3**, deduct **4** and add **5**. What number do you have?"

The **10**-year-old, "Fourteen."
You then tell him that the number he first thought of was **8**.
Astonished **10**-year-old, "Yes! Show me how to do it!"

Here, you aim to end your mental arithmetic sum with $z + 6$.

$$z + 8 - 3 - 4 + 5 = z + 6$$

The answer the **10**-year-old gives is 6 more than his secret number, which in this case is **8**.

His answer was **14**, so
$$\begin{aligned} z + 6 &= 14 \\ z &= 14 - 6 \\ z &= 8 \end{aligned}$$


Note

Those mental puzzles are included here to show that, when doing algebra, you needn't be alarmed by a long string of numbers. Just keep calm.
Part of the fascination of algebra is the satisfaction of being able to simplify a long string of numbers.

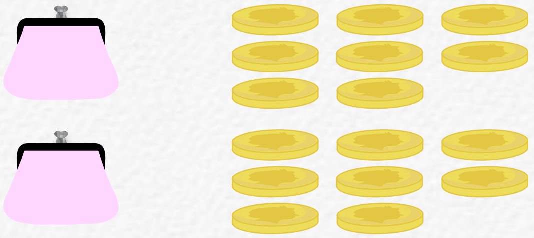

Every girl needs maths.

3rd game

The game below is included so that you can see, from the final equation, that **one line of algebra shorthand symbols can replace a lot of words, numbers and stages of a spelled-out problem**.

Tell your 10-year-old to think of a secret number. Tell him to add 8, double the answer and take away 10. Then divide his answer by 2. Finally take away the number he first thought of. Tell him his answer is 3.

To visualise the unknown number, **c**, imagine it as the number of coins in a purse.

	Words	Algebra	Picture
1	Think of a number or amount of money.	c	
2	Add £8.	$c + 8$	
3	Double it.	$2(c + 8) = 2c + 16$	
4	Take away £10.	$2c + 16 - 10 = 2c + 6$	
5	Divide by 2.	$\frac{2c + 6}{2} = c + 3$	
5	Take away the amount you first thought of.	$c + 3 - c = 3$	
7	You are left with £3	$= 3$	

In its simplest form, the algebra would look like the following equation; **c** stands for the 10-year-old's secret number:

$$\frac{2(c + 8) - 10}{2} - c = 3$$

Puzzle

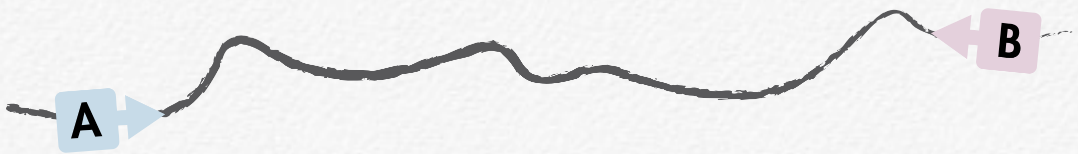
Imagine the sun setting behind a winding, narrow, country lane. At one end of the lane, our handsome hero has just left his romantic cottage to speed in his silver sports car to surprise his girlfriend by meeting her train.

At the same time, six miles away, at the opposite end of the lane, the girlfriend, who caught an earlier train, cautiously drives towards her boyfriend's cottage in a hired scarlet mini.

You start to feel anxious. A collision seems inevitable. Where will the two cars crash?

Method

Scribble a diagram of the scene with two arrows, A represents the silver sports car and B represents the scarlet mini.



You have just drawn an abstract diagram of a possible disaster. That is, you have simplified to the minimum, the paragraph of information you were given: all details have been pushed aside.

Question

A is exceeding the speed limit at **60mph** and B is driving cautiously at **30mph**. As the lane is **6** miles long, at what point will they smash into each other?

How to answer this question with algebra.

The sports car is travelling at twice the speed of the mini.

Therefore the sports car will cover twice the distance that the mini covers.

If the mini travels **d** miles,
then the sports car travels **2d** miles.

The lane is **6** miles long, so when they meet, the total of the miles driven by both cars together will be **6** miles.

Mini's distance (d) + Sports car's distance (2d) = 6 miles

$$d + 2d = 6$$

$$3d = 6$$

$$\frac{3d}{3} = \frac{6}{3}$$

$$d = 2$$

So at the point of impact the red mini will have travelled, **d = 2** miles.

The sports car will have travelled **2d = 2 x 2 = 4** miles.

Answer: **The sports car hits the red mini at a point 4 miles from A and 2 miles from B.**

What really happened? At dusk, the girl and the handsome hero both switched their car headlights on, so when each saw the other's lights, they slowed down. When they saw each other, they hit the brakes. The girl then reversed two miles, followed by the silver sports car. The hired, red mini was returned. Then they both drove off to his cottage in the silver sports car. So impressed was he by a girl that could reverse two miles down a winding road at dusk, that he proposed that evening.

Maths is needed for every aspect of fashion design, such as these traditional Thai clothes.

Real Life Example

Jane and Karen (see left) now run a mail order business for ski clothes, called Topski. Karen designs and Jane manages the business. In the end-of-year notes that Jane presented to her accountant, she describes how the sales (items sold) in October varied:

"Sales in the first week of October were about normal for the time of year. The second week showed a significant drop (30 sales less than the previous week) due to a factory delivery problem.

During the third and fourth weeks of October, sales greatly improved, with 80 more sales in week 3 than the first week and three times as many sales in week 4 than week 1."

Topski's accountant knows that the total sales, for the four weeks in October, was 350.

How can the accountant calculate the number of sales that Topski made during the first week of October?

First, simplify the information that Jane gives:

Week 1 =	normal number of sales (call this n)	= n
Week 2 =	30 sales less than week 1	= $n - 30$
Week 3 =	80 sales more than week 1	= $n + 80$
Week 4 =	3 times as many as week1	= $3n$

Total Sales for October = $n + n - 30 + n + 80 + 3n$

The accountant also knows that the total sales for October was **350**.

Therefore: $n + n - 30 + n + 80 + 3n = 350$

$$6n + 50 = 350$$

$$6n + 50 \mathbf{- 50} = 350 \mathbf{- 50}$$

$$\frac{6n}{\mathbf{6}} = \frac{300}{\mathbf{6}}$$

Simplified this reads: $n = 50$

n = number of sales in week 1= 50.

Answer: **The number of Topski sales in the first week of October was 50.**

Exercises

18) Najma runs a small accountancy partnership. She asks her bank for a business loan of **£3,000** to buy four identical computers and a new printer. The printer costs **£300**.
Use algebra to work out the maximum Najma can spend on each computer.

Fashion show audience.

19) Anna imports hand-embroidered lingerie from Portugal then sells it at her boutique in Edinburgh. She makes **£8** more profit on a bra than on a pair of knickers. On Valentine's Day, Anna made **£480** profit on her bras and knickers. She sold **5** pairs of knickers and **8** bras.

a) If the profit for one pair of knickers **= k**, what would the profit for a bra be in terms of **k**?

b) What is the profit for **5** knickers in terms of **k**?

c) What is the profit for **8** bras in terms of **k**?

d) What is the total profit for **5** knickers and **8** bras in terms of **k**?

e) If the total profit is **£480**, work out the value of **k**.

f) What is Anna’s profit on a matching knicker-and-bra set?

Triumphant designer Diana Simaan walks the runway at end of her show. New York, USA.

Solving Simple Simultaneous Equations

Simultaneous equations occur when you have **two or more** different unknown quantities.

You need two different equations in order to find the values of the two unknown quantities.

Fashion designers need maths.

First Example

Once, long ago, housewives (as they were then called) swapped gossip over the garden fence or met in a tea shop.

On an afternoon trip to a teashop, Betty and Patricia together consumed **2** ginger tarts and a cream meringue. Their food bill was **11p**.

On the following afternoon, they met again at Peggy's Pantry, where they ate **3** ginger tarts and **2** cream meringues. The food bill was **19p**.

What was the price of a ginger tart and what was the price of a cream meringue?

To simplify your information to the minimum, first, jot down the relevant facts. Let **t** represent the cost of a ginger tart and **m** represent the cost of a cream meringue.

Bill 1

2 ginger tarts and 1 cream meringue = 11p.
(in algebra shorthand: $2t + m = 11$).

Bill 2

3 ginger tarts and 2 cream meringues = 19p.
(in algebra shorthand: $3t + 2m = 19$).

Your next aim is to knock out either **t** or **m**, it doesn't matter which (but one may be easier to do than the other). Once you have deduced the cost of one item, it's easy to discover the cost of the other.

You are able to knock out by **addition or subtraction**, the terms which are **exactly** the same in both equations. Here, no two terms are exactly the same: $2t$ and $3t$ are different; m and $2m$ are different.

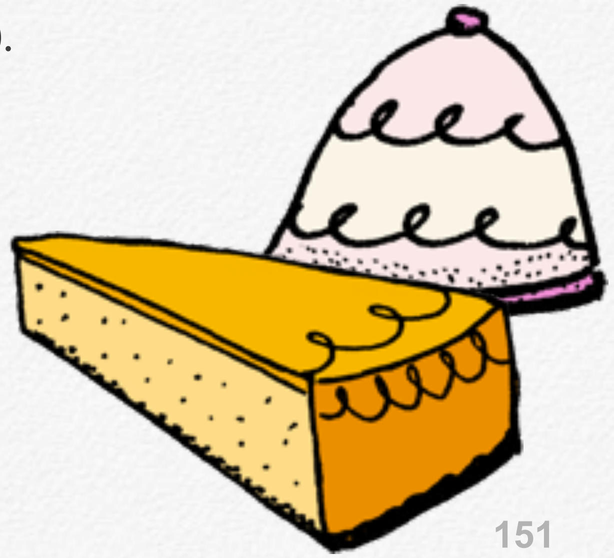
So multiply the entire **Bill 1** by **2**. The result will be that **Bill 1** contains **2m**. This matches **Bill 2**.

Bill 1 $2t + m = 11$ (x 2) becomes $4t + 2m = 22$.
Call this **Equation 3**.

Now $2m$ can be eliminated by subtracting Bill 2 from Equation 3 ($2m - 2m$ = nothing).

Equation 3		$4t + 2m = 22$
Bill 2	–	$3t + 2m = 19$
		$t \quad = 3$

So a ginger tart (t) costs 3p.



Look again at **Bill 1**. Now you can solve it.

Bill 1

$$
\begin{aligned}
2t + m &= 11 \\
2 \times 3 + m &= 11 \\
6 + m &= 11 \\
6 + m \mathbf{- 6} &= 11 \mathbf{- 6} \\
m &= 5
\end{aligned}
$$

So a ginger tart costs **3p** and a cream meringue costs **5p**.

If your figures add up in **Bill 1**, next check them in **Bill 2**, which will show any errors.

Bill 2

$$
\begin{aligned}
3t + 2m &= 19 \\
3 \times 3 + 2 \times 5 &= 19 \\
9 + 10 &= 19
\end{aligned}
$$

This is true, so the figures are correct.

Answer: **So, in Peggy's Pantry, a ginger tart costs 3p and a cream meringue costs 5p**. It's a different story in Starbucks today.

Fashion designers need maths.

Second Example

Natalia has just started her own translation agency. After a profitable six months, she moves into a Dickensian attic office in a smarter part of town. She buys wooden chairs and desks from a second-hand furniture shop.

Natalia purchases **11** items of furniture. The desks cost **£32** each and the chairs **£18**. Natalia spent **£240**. How many desks and how many chairs did she buy?

First, simplify the information to a minimum.
Let **d** represent the **number of desks** bought and **c** the **number of chairs** bought.

Natalia bought **11** items of furniture, so the number of desks + chairs = 11, or … $d + c = 11$.
Call this **Equation 1**.

One desk cost **£32**; two desks cost **£32 x 2**, so **d** desks would cost **£32 x d**.

One chair costs **£18**; two chairs cost **£18 x 2**, so **c** chairs would cost **£18 x c**.

The total cost of the desks and chairs is **£240**, so….
$32d + 18c = 240$. Call this **Equation 2**

Equation 1 $\quad d + c = 11$
Equation 2 $\quad 32d + 18c = 240$

You now have two equations. Now, try to eliminate one of the terms; choose either **c** or **d**.

Let's say you decide to eliminate **c**. To do this, you need to get the **c** in **Equation 1** equal to **18c**.
Remember, in equations you need to treat both sides in the same way, to keep the equation balanced.
So you multiply the **entire Equation 1** by **18**.

Equation 1 $d + c = 11$ $(\times 18)$ becomes $18d + 18c = 198$.
Call this **Equation 3**.

Take Equation 3 from **Equation 2** (the smaller from the bigger):

$$\begin{array}{lrcl} \textbf{Equation 2} & 32d + 18c & = & 240 \\ \textbf{Equation 3} \quad - & 18d + 18c & = & 198 \\ \hline & 14d & = & 42 \end{array}$$

$$\frac{14d}{\mathbf{14}} = \frac{42}{\mathbf{14}}$$

$$d = 3$$

Now go back to **Equation 1**, and work it out using your new information.

Use d = 3 in **Equation 1**:

$$3 + c = 11$$
$$3 + c \mathbf{- 3} = 11 \mathbf{- 3}$$
$$c = 8$$

Check the figures in **Equation 2**:

$$32d + 18c = 240$$
$$32 \times 3 + 18 \times 8 = 240$$
$$96 + 144 = 240$$

This is true.

Answer: **Natalia bought 3 wooden desks and 8 wooden chairs**. She already had her mother's old kitchen table to be used for meetings.

She had all the furniture sprayed black, the floorboards sanded and varnished. As there wasn't much light, the walls were painted white with metallic silver blinds at the windows. The effect? Economical chic.

Exercises

Hint: Remember to read the whole paragraph, then highlight or underline only the information you need for this maths problem – because this is how maths problems tend to occur in Real Life.

20) Tania, now a publicist at her agency, is taking Willow out for a business lunch at Swaddles, a smart but not-too-expensive restaurant. (Tania is hoping that Willow will engage her firm to raise the profile of Willow's art gallery, so Tania doesn't want her PR firm to look stingy or too extravagant). They agree to meet again the following week, with Tania's financial director, Juliet, to discuss a draft proposal together.

When Tania gives the receipts for the two business lunches to the accounts department, she realises that the bills are not itemised. However, Tania remembers what was bought on each occasion.

Business lunch 1: two set menus, **2** glasses house wine, totalled **£65.00**.
Business lunch 2: three set menus, **6** glasses house wine, cost **£111.00**.
Calculate cost of set menu and a glass of house wine.

After another formal meeting, Tania's proposal was accepted and Willow's sales nearly doubled that year, so again they lunched at Swaddles to celebrate.

21) TV garden presenter, Anthea Bennett employs two gardeners, Bill and Tony, to look after her garden. Bill is a more experienced gardener and so is paid more than Tony.

Anthea has forgotten exactly how much each gardener is paid per hour, but when her accountant asks, she remembers that when the two gardeners work together she pays them **£27** per hour.

Last week, Bill did **5** hours' work for Anthea while Tony worked **7** hours. Anthea Bennett paid them a total of **£159**. How much does she pay each gardener for an hour's work?

And you thought those TV gardeners did all the weeding themselves.

Quadratic Equations

Quadratic Equations sent many a shiver down many a student's spine. They might concern quite complicated calculations about **area** or **velocity**. Unless you are an experienced mathematician, you are unlikely to need them in Real Life, so they are not included in this Course.

In case anyone ever asks you, a **quadratic equation** is an equation that contains a term which is **squared**, e.g. x^2. But the equation never contains an unknown quantity raised to a higher power, such as x^3.
It might also contain a term with a plain x and /or a number alone.

Example
$y^2 + 3y - 4 = 0$

With a bit of extra maths, it is possible to work out that in this example, the solutions are $y = 1$ or $y = -4$.
Instead of Y, put either of these numbers in the equation and the equations will work out correctly.

Friendship.

Answers to Part 28

1) Write two different ways of placing the terms on the left hand side of the following equation.

$6 + 5p - 2q = a$

Any of the following answers will do:

Answers:

$5p + 6 - 2q = a$

or $5p - 2q + 6 = a$

or $6 - 2q + 5p = a$

or $- 2q + 6 + 5p = a$

or $- 2q + 5p + 6 = a$

2) $11a - 5b = y$

Answer: $- 5b + 11a = y$

3) Use the examples in a, b, c and d, to do the same calculations when $w = 7$.

a) $2(w + 7) = 2(7 + 7)$

$= 2(14)$

$= \mathbf{28}$

b) $4w + 6 = 4(7) + 6$

$= 28 + 6$

$= \mathbf{34}$

c) $\dfrac{5w + 3}{2} = \dfrac{5(7) + 3}{2}$

$= \dfrac{35 + 3}{2}$

$= \dfrac{38}{2}$

$= \mathbf{19}$

d) $w^2 = 7^2$

$= \mathbf{49}$

4) Use the examples in a, b, c and d, to do the same calculations when **w = 12**.

a) $2(w + 7) = 2(12 + 7)$

$= 2(19)$

$= \mathbf{38}$

b) $4w + 6 = 4(12) + 6$

$= 48 + 6$

$= \mathbf{54}$

c) $\frac{5w + 3}{2} = \frac{5(12) + 3}{2}$

$= \frac{60 + 3}{2}$

$= \frac{63}{2}$

$= \mathbf{31.5}$

d) $w^2 = 12^2$

$= \mathbf{144}$

5) **d + 6 = 11**

As **d** has **6** added to it, subtract **6** from both sides:

$d + 6 \mathbf{- 6} = 11 \mathbf{- 6}$

$d = 5$

Answer: **d = 5**

6) **f – 3 = 5**

As **f** has **3** subtracted from it, add **3** to both sides:

$f - 3 \mathbf{+ 3} = 5 \mathbf{+ 3}$

$f = 8$

Answer: **f = 8**

7) **10 = g – 8**

As **g** has **8** subtracted from it, add **8** to both sides:

$10 \mathbf{+ 8} = g - 8 \mathbf{+ 8}$

$18 = g$

Answer: **g = 18**

8) $4 = h + 10 - 3$

First simplify the equation to $4 = h + 7$.

As h has 7 added to it, subtract 7 from both sides:

$$4 - 7 = h + 7 - 7$$
$$-3 = h$$

Answer: $h = -3$

9) $6y = 42$

As y is multiplied by 6, divide by 6:

$$\frac{6y}{6} = \frac{42}{6}$$
$$y = 7$$

Answer: $y = 7$

10) $\frac{k}{7} = 5$

As k is divided by 7, multiply by 7:

$$\frac{k}{7} \times 7 = 5 \times 7$$
$$k = 35$$

Answer: $k = 35$

11) $10 = 5m$

As m is multiplied by 5, divide by 5:

$$\frac{10}{5} = \frac{5m}{5}$$
$$2 = m$$

Answer: $m = 2$

12) $13 = \frac{n}{2}$

As n is divided by 2, multiply by 2:

$$13 \times 2 = \frac{n}{2} \times 2$$
$$26 = n$$

Answer: $n = 26$

13) Solve the equation: **7b + 8 = 29**.

Do you divide or subtract first? Solve the equation both ways.

It's best to do **subtraction first**.

$$7b + 8 \mathbf{- 8} = 29 \mathbf{- 8}$$

$$\frac{7b}{\mathbf{7}} = \frac{21}{\mathbf{7}}$$

$$b = 3$$

Answer: **b = 3**

If you divide first:

$$\frac{7b + 8}{7} = \frac{29}{7}$$

$$b + \frac{8}{7} = \frac{29}{7}$$

$$b + \frac{8}{7} - \frac{8}{7} = \frac{29}{7} - \frac{8}{7}$$

$$b = \frac{21}{7} = 3$$

Answer: **b = 3**

14) Solve the equation: $6 = \frac{a}{2} - 4$.

Do you multiply or add first? Try both ways.

It's best to **add first**.

$$6 \mathbf{+ 4} = \frac{a}{2} - 4 \mathbf{+ 4}$$

$$10 = \frac{a}{2}$$

$$\mathbf{2 \times} 10 = \frac{a}{2} \mathbf{\times 2}$$

Answer: **a = 20**

If you multiply first, both terms on the right-hand side must be multiplied, show this by putting the terms in brackets.

$$6 \mathbf{\times 2} = (\frac{a}{2} - 4) \mathbf{\times 2}$$

$$12 = \frac{a}{2} \mathbf{\times 2} - 4 \mathbf{\times 2}$$

$$12 = a - 8$$

$$12 \mathbf{+ 8} = a - 8 \mathbf{+ 8}$$

Answer: **a = 20**

15) Find the value of **q** in the following equation:

$$7 + \frac{q}{3} = 25$$

First take away **7** from both sides.

$$7 + \frac{q}{3} \mathbf{- 7} = 25 \mathbf{- 7}$$

$$\frac{q}{3} = 18$$

q is being divided by **3**, so multiply both sides by **3**.

$$\frac{q}{3} \mathbf{\times 3} = 18 \mathbf{\times 3}$$

Answer: **q = 54**

16) Solve the following equation to find **w**.

$$\frac{5w + 3}{2} = 9$$

The whole left-hand side of the equation is being divided by **2**, so multiply both sides by **2**.

$$\frac{\mathbf{2}\,(5w + 3)}{2} = 9 \mathbf{\times 2}$$

$$5w + 3 = 18$$

Now subtract **3** from both sides:

$$5w + 3 \mathbf{- 3} = 18 \mathbf{- 3}$$

$$5w = 15$$

Divide both sides by **5**:

$$\frac{5w}{\mathbf{5}} = \frac{15}{\mathbf{5}}$$

Answer: **w = 3**

Colour and pattern in nature.
Photo by Carole Latimer.

17) Take the equation $7b + 8 = 29$ from question 13.
If you double all the numbers in the entire equation, will the value of b be the same?

$7b + 8 = 29$ multiplied by two becomes:

$$14b + 16 = 58$$

$$14b + 16 - \mathbf{16} = 58 - \mathbf{16}$$

$$\frac{14b}{\mathbf{14}} = \frac{42}{\mathbf{14}}$$

$$b = 3$$

Answer: The answer is YES. **b** has the same value that it has in question 13, **b = 3**. The answer is not doubled, **b** is not **6**, because by doubling the **entire** equation, you have obeyed **Algebra Rule 4, which is: whatever you do to one side of the equation you must also do to the other.**

18) Najma runs a small accountancy partnership. She asks her bank for a business loan of **£3,000** to buy four identical computers and a new printer.
The printer costs **£300**.
Use algebra to work out the maximum Najma can spend on each computer.

If cost of the computer = t, then four computers = $4t$

Total cost = Printer + four computers

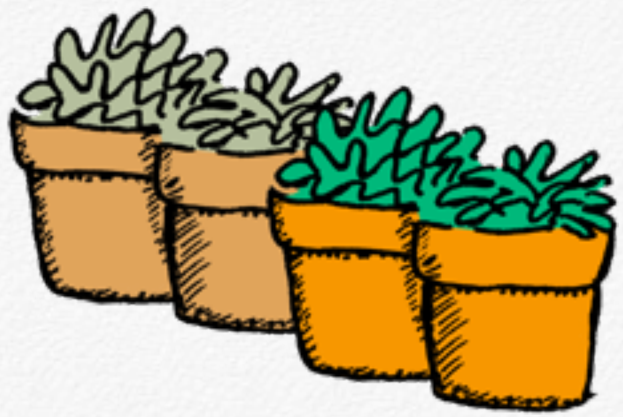
$$3000 = 300 + 4t$$

$$3000 - \mathbf{300} = 300 + 4t - \mathbf{300}$$

$$\frac{2700}{\mathbf{4}} = \frac{4t}{\mathbf{4}}$$

$$675 = t$$

Answer: **£675 is the maximum price Najma can spend on each computer to stay within budget.**



19) Anna imports hand-embroidered lingerie from Portugal then sells it at her boutique in Edinburgh. She makes **£8** more profit on a bra than on a pair of knickers.
On Valentine's Day, Anna made **£480** profit on her bras and knickers. She sold **5** pairs of knickers and **8** bras.

a) If the profit for one pair of knickers **= k**, what would the profit for a bra be in terms of **k**?

Profit on bra = profit on a pair of knickers plus **£8**, so **k + 8**.

Answer: **Profit for one bra = k + 8.**

b) What is the profit for **5** knickers in **k**?

Profit for one pair of knickers = **k**, so **5** knickers = **5k.**

Answer: **The profit on five pairs of knickers = 5k.**

c) What is the profit for **8** bras in **k**?

Profit for one bra = **k + 8**, so profit for **8** bras = **8(k + 8)**.

Simplified this is **8k + 64**.

Answer: **The profit on eight bras is 8k + 64**.

d) What is the total profit for **5** knickers and **8** bras in **k**?

Profit of five knickers plus profit of eight bras =

$5k + 8k + 64 =$

$13k + 64$

Answer: **Total profit for the knickers and bras = 13k + 64**.

e) If the total profit is **£480**, work out the value of **k**.

So... $13k + 64 = 480$

$13k + 64 - \mathbf{64} = 480 - \mathbf{64}$

$$\frac{13k}{\mathbf{13}} = \frac{416}{\mathbf{13}}$$

$$k = 32$$

Answer: **k = 32**

f) What is Anna's profit on a matching knicker-and-bra set?

Profit on a pair of knickers = k = £32.

Profit on a bra $= k + 8$

$= 32 + 8$

$= £40$

Answer: **Anna's profit on a knicker-and-bra set is £72**.

20) Tania, now a publicist at her agency, is taking Willow out for a business lunch at Swaddles, a smart but not-too-expensive restaurant. (Tania is hoping that Willow will engage her firm to raise the profile of Willow's art gallery, so Tania doesn't want her PR firm to look stingy or too extravagant). They agree to meet again the following week, with Tania's financial director, Juliet, to discuss a draft proposal together.

When Tania gives the receipts for the two business lunches to the accounts department, she realises that the bills are not itemised.

However, Tania remembers what was bought each time.
Business lunch 1: two set menus, **2** glasses house wine, totalled **£65.00**

Business lunch 2: three set menus, **6** glasses wine, cost **£111.00**.
Calculate cost of set menu and a glass of house wine.

First, simplify the information to a minimum.
Let **m** represent the cost of a set menu and **w** the cost of a glass of house wine.

Meal 1
2 set menus and **2** glasses of wine = **£65.00**
(in algebra shorthand: $2m + 2w = 65$)

Meal 2
3 set menus and **6** glasses of wine = **£111.00**
(in algebra shorthand: $3m + 6w = 111$)

You aim to knock out either **m** or **w**. You need to make either the number of **m** the same in both equations or the number of **w** the same in both equations. Here, multiply Meal 1 by **3** to make **6w** in both equations:

Meal 1
$2m + 2w = 65\ (x\ 3)$ becomes $6m + 6w = 195$.
Call this **Meal 3.**

Now **6w** can be eliminated by subtracting Meal 2 from Meal 3. Always subtract the smaller equation from the bigger one:

Meal 3		$6m + 6w = 195$
Meal 2	–	$3m + 6w = 111$
		$3m = 84$

So three set menus cost **£84**.

$$\frac{3m}{\mathbf{3}} = \frac{84}{\mathbf{3}}$$

$$m = 28$$

So one set meal cost **£28**.

Use **m = 28** in Meal 1 and solve it to find the cost of one glass of wine.

Meal 1

$$2m + 2w = 65$$

$$2 \times 28 + 2w = 65$$

$$56 + 2w = 65$$

$$56 + 2w \mathbf{- 56} = 65 \mathbf{- 56}$$

$$\frac{2w}{\mathbf{2}} = \frac{9}{\mathbf{2}}$$

$$w = 4\frac{1}{2}$$

So one glass of house wine costs **£4.50**.

Check your figures in Meal 2:

Meal 2

$$3m + 6w = 111$$

$$3 \times 28 + 6 \times 4.50 = 111$$

$$84 + 27 = 111$$

This is true, so the figures are correct.

Answer: **Each set menu cost £28 and each glass of house wine cost £4.50.**

21) TV garden presenter, Anthea Bennett, employs two gardeners, Bill and Tony, to help look after her garden. Bill is a more experienced gardener and so is paid more than Tony. Anthea has forgotten exactly how much each gardener is paid per hour, but when her accountant asks, she remembers that when the two gardeners work together she pays them **£27** per hour. Last week, Bill did **5** hours work for Anthea while Tony worked **7** hours. Anthea Bennett paid them a total of **£159**.
How much does she pay each gardener for an hour's work?

First, simplify the information.
Call Bill's hourly rate **b** and Tony's hourly rate **p**.

The two gardeners together = **£27** per hour:
$b + p = 27$ **(Equation 1)**

Bill worked **5** hours, so earned **5 x** Bill's hourly rate (**b**), so **5b**.
Tony worked **7** hours, so would earn **7 x p**.
When Bill worked **5** hours and Tony **7** hours, it totalled **159**:
$5b + 7p = 159$ **(Equation 2)**

You aim to knock out either **b** or **p**. Multiply equation **1** by **5** to make **5b**, then there will be **5b** in both equations:

Equation 1 $b + p = 27$ $(\times 5)$ becomes:
$5b + 5p = 135$ **(Equation 3)**

Both equation 3 and equation 2 contain **5b**. Subtracting the smaller equation from the bigger one, to eliminate **5b**:

$$\begin{array}{lcr} \textbf{Equation 2} & & 5b + 7p = 159 \\ \textbf{Equation 3} & - & 5b + 5p = 135 \\ \hline & & 2p = 24 \end{array}$$

$$\frac{2p}{2} = \frac{24}{2}$$

$$p = 12$$

Now, use $p = 12$ in **Equation 1:**

$$\begin{aligned} \textbf{Equation 1} \quad b + p &= 27 \\ b + 12 &= 27 \\ b + 12 - \mathbf{12} &= 27 - \mathbf{12} \\ b &= 15 \end{aligned}$$

Check in **Equation 2**:

$$5b + 7p = 159$$
$$5 \times 15 + 7 \times 12 = 159$$
$$75 + 84 = 159$$

This is true, so the figures are correct.

Pattern in a wild iris.

Answer: **Anthea pays Bill £15 per hour and Tony £12 per hour**.

YOUR BRAIN WORKOUT

Question 1 of 5.

If 4a = 28, what does 4 equal?

- **A.** a = 9
- **B.** a = 7
- **C.** a = 8
- **D.** a = 4

YOUR BRAIN WORKOUT

Question 2 of 5.

If b + 7 = 11, what does b equal?

- **A.** b = 18
- **B.** b = 3
- **C.** b = 4
- **D.** b = 16

YOUR BRAIN WORKOUT

Question 3 of 5.

If 8 = f – 3, what does f equal?

- **A.** f = 11
- **B.** f = 5
- **C.** f = 7
- **D.** f = 9

YOUR BRAIN WORKOUT

Question 4 of 5.

If $\frac{k}{3} = 12$, what does k equal?

- **A.** k = 4
- **B.** k = 15
- **C.** k = 9
- **D.** k = 36

YOUR BRAIN WORKOUT

Question 5 of 5.

If $2m - 3 = 15$, what does m equal?

A. $m = 9$

B. $m = 6$

C. $m = 10$

D. $m = 16$

YOUR BRAIN WORKOUT

Answers

Q1. a = 7

Q2. b = 4

Q3. f = 11

Q4. k = 36

Q5. m = 9

PART 29
GEOMETRY

Quick Quiz

Question 1 of 4.

How is **two hundred and five thousand and thirty** written in numbers?

- **A.** 215,130
- **B.** 25,30
- **C.** 205,030
- **D.** 205,130

Quick Quiz

Question 2 of 4.

Which of the following sums is true?

- **A.** $27 \div 3 = 9$
- **B.** $9 \div 3 = 27$
- **C.** $3 \div 9 = 27$
- **D.** $3 \div 27 = 9$

Quick Quiz

Question 3 of 4.

20% of £587.69 is approximately?

- **A.** £50
- **B.** £60
- **C.** £120
- **D.** £12

Quick Quiz

Question 4 of 4.

What is the probability of getting heads when tossing a fair coin?

- **A.** 0.4
- **B.** 0.1
- **C.** 0.9
- **D.** 0.5

Quick Quiz

Answers

Q1. 205,030

Q2. 27 ÷ 3 = 9

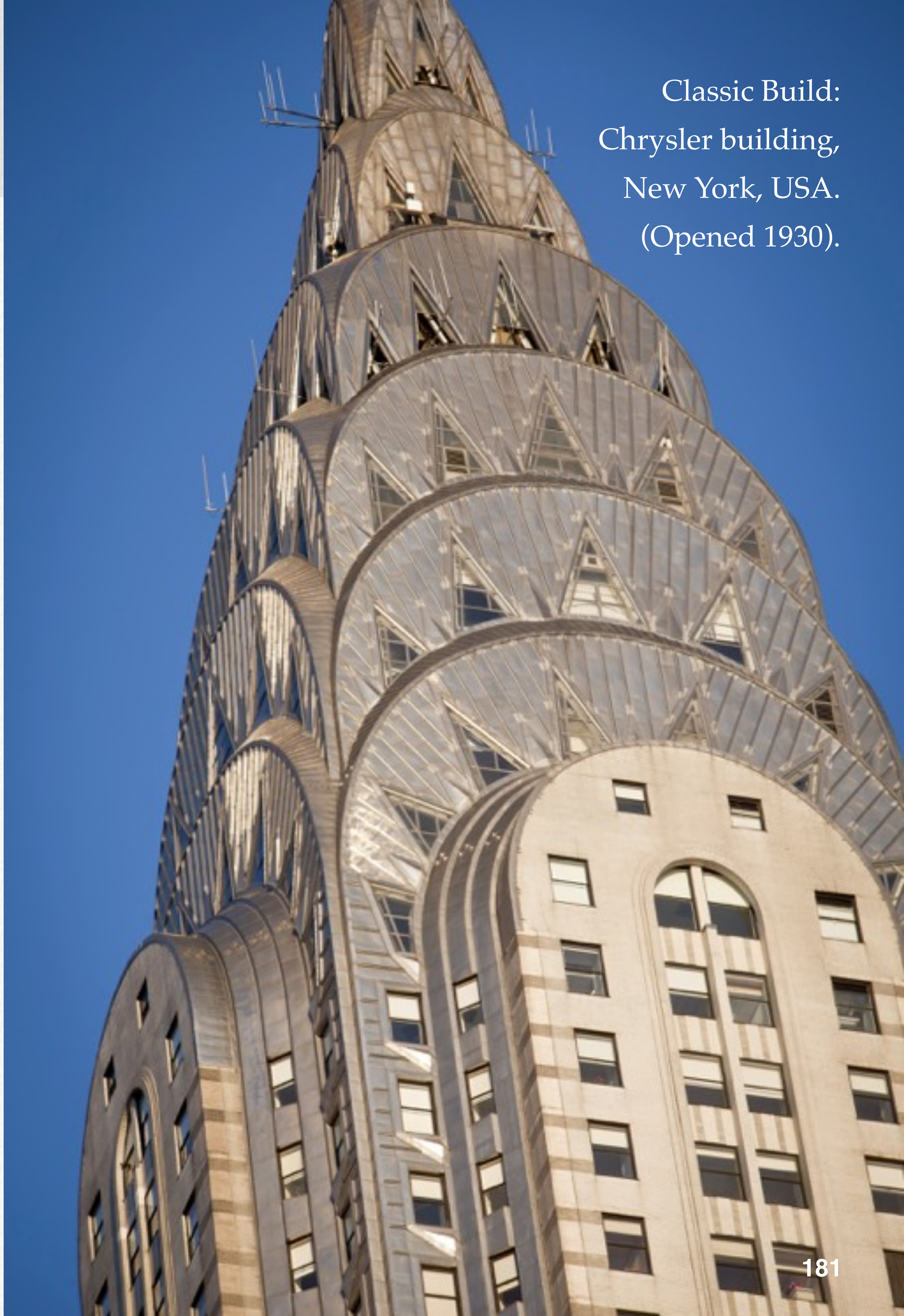
Q3. £120

Q4. 0.5

The Rock Bottom Basics

You'll be surprised how often Real Life money situations involve geometry: when you're planning the furniture layout in your first flat; when you're building a kitchen extension or (when you're famous) ordering a swimming pool. But, in Real Life – unless you have an A level in maths – you are unlikely to be involved in **geometric** calculations. However, you need to understand **angles** and know the different **shapes**.

On journeys, look at the buildings you pass, to spot the circles and semi-circles, squares and rectangles, cubes and cylinders, spheres and spirals. You are surrounded by geometry.



Classic Build:
Chrysler building,
New York, USA.
(Opened 1930).

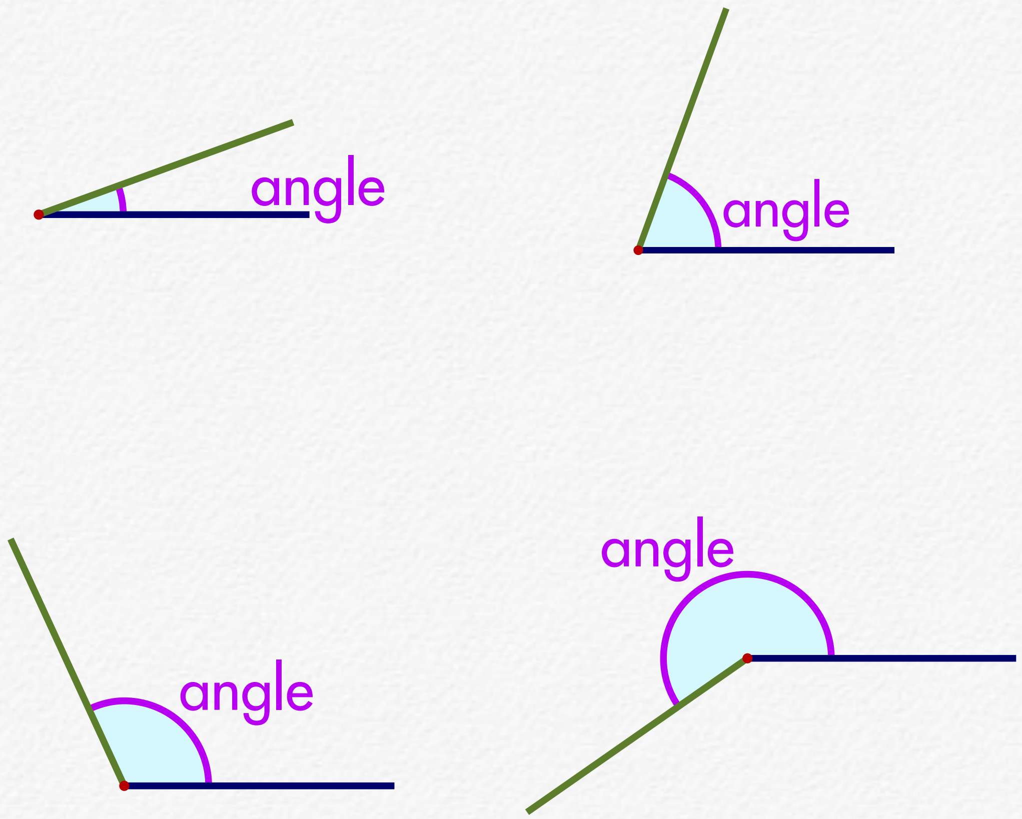
Classic Build: The Colosseum, Rome, Italy (1st Century).

Angles

- Use a dot as a starting point:

- Draw two straight lines from the starting point (in two different directions).

The space between the lines is called an **angle**.



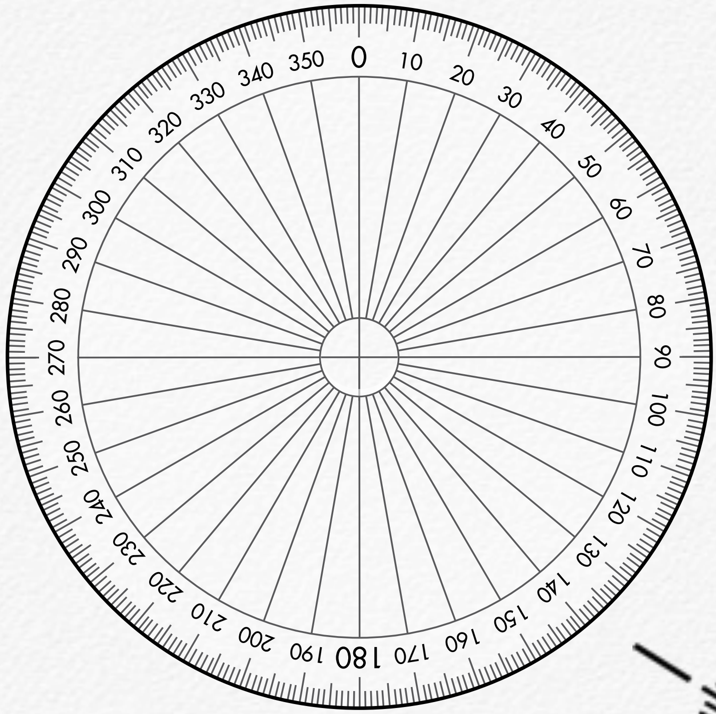
A little curve is drawn between the two lines to mark the angle you are interested in.

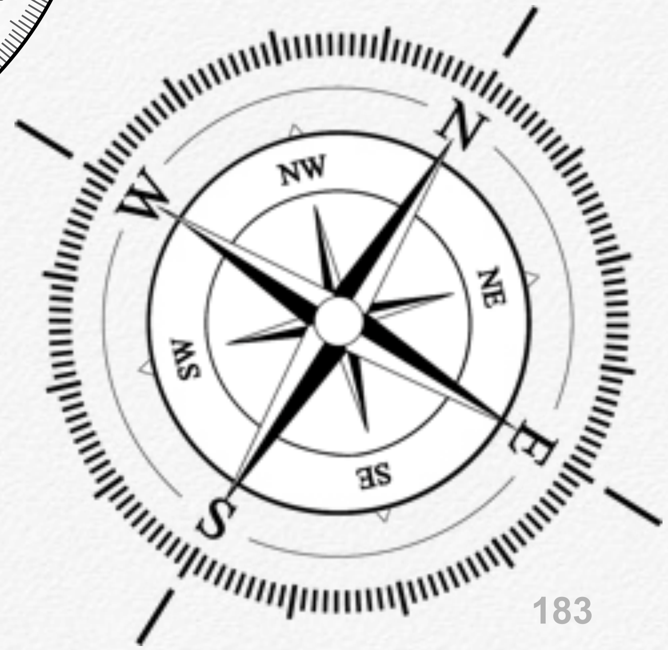
- A **circle** can be divided into 360 **equal** tiny angles; each is called a **degree**.

The shorthand for degrees is a tiny circle hanging in the air: °.

5° = 5 degrees. 83° = 83 degrees.

360° = 360 degrees, which is a complete turn of a compass.

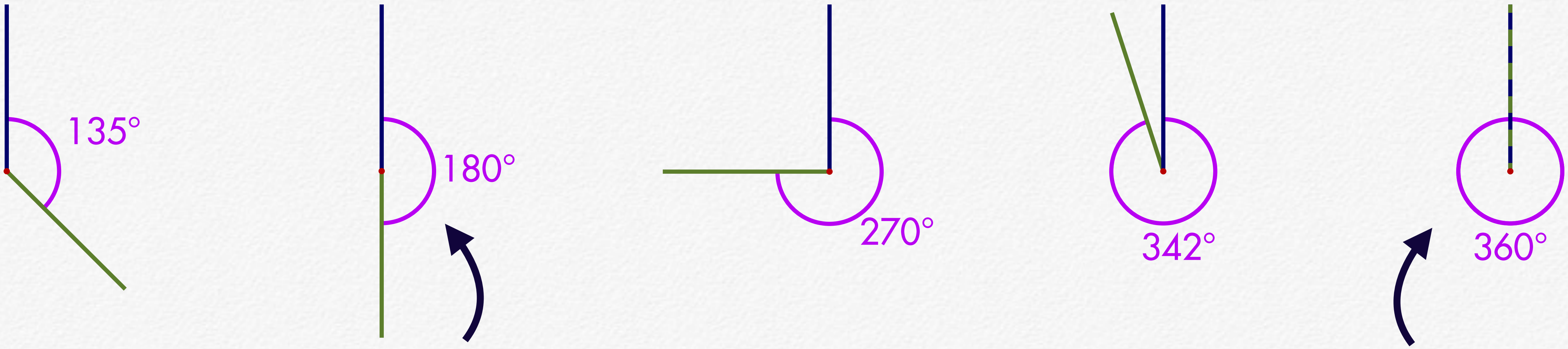




- **Degrees** are used to measure an **angle**.

One degree is a very small angle, ten degrees is larger, and so on…

A **right angle** is an angle that measures **90°**. Instead of a little curve, you draw a little square to show it is **90°**

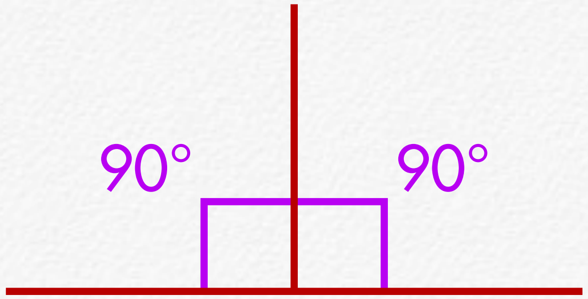


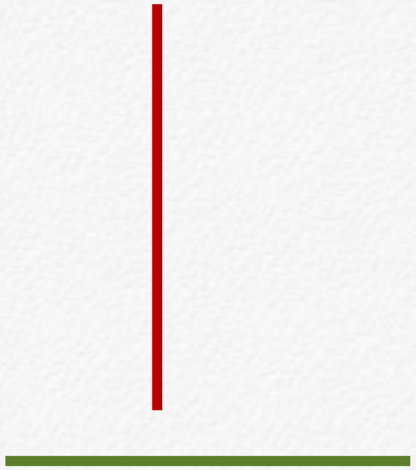
An angle of **180°** will give you a **straight line**.

If your two lines meet, then the angle is a full turn, **360°**

Perpendicular Lines

Perpendicular lines are lines which are at right angles to each other.





Parallel Lines

Parallel lines are two lines – which keep the same distance apart – that never meet: like railway lines.

Geometry was essential
to design this ceiling.
King's College Chapel, Cambridge.
England (15th Century).

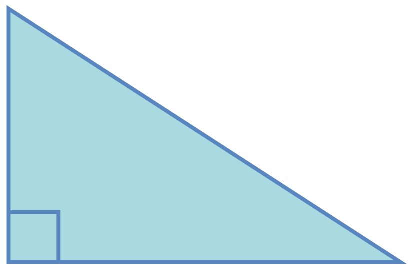
Triangles

Triangles have **3** sides:

Scalene Triangle

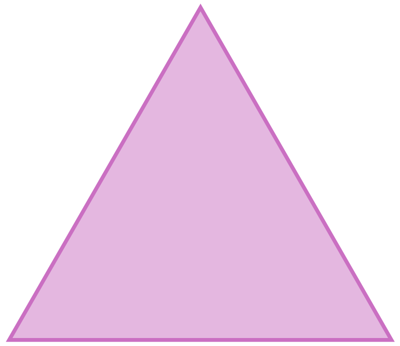
no sides are equal:
no angles are equal.
Check it yourself.

Right-Angled Triangle



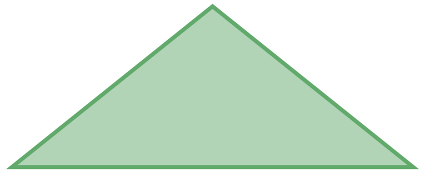
any triangle with one $90°$ angle.

Equilateral Triangle



all sides & all angles are equal.

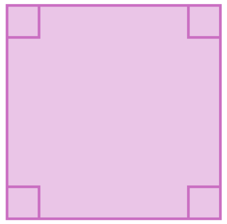
Isosceles Triangle



two equal sides & two equal angles.

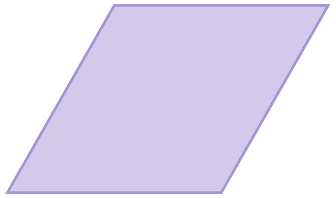
Quadrilaterals

Square



All four sides are equal:
all angles are $90°$.

Rhombus



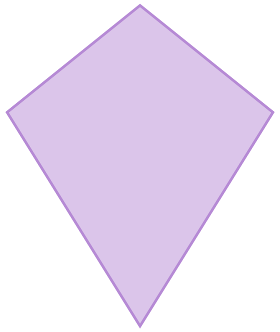
a wonky square
Someone leaned against it. All sides are equal in length. No $90°$ angles.

Rectangle

a square stretched
Sideways. Opposite sides are parallel and equal in length.
All angles are $90°$.

Parallelogram

a wonky rectangle
Opposite sides are parallel and equal in length. No $90°$ angles.



Kite

The top two adjoining sides are equal in length and so are the bottom two.

Trapezium

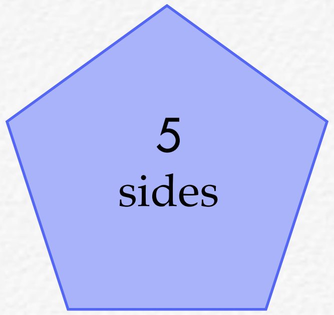
Has only one pair of parallel lines.
Think of it as a skirt shape.

or

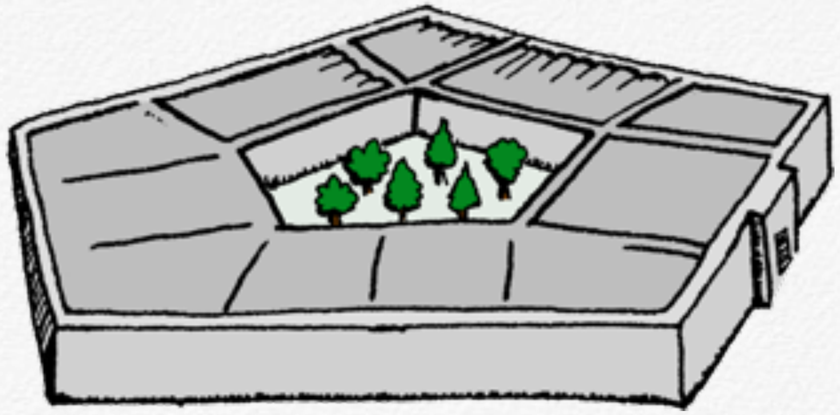
Polygons

Polygons have many sides; all of the following examples are **regular polygons** (every side equal in length).

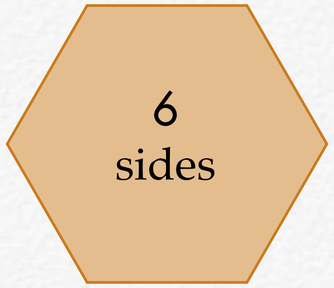
Pentagon



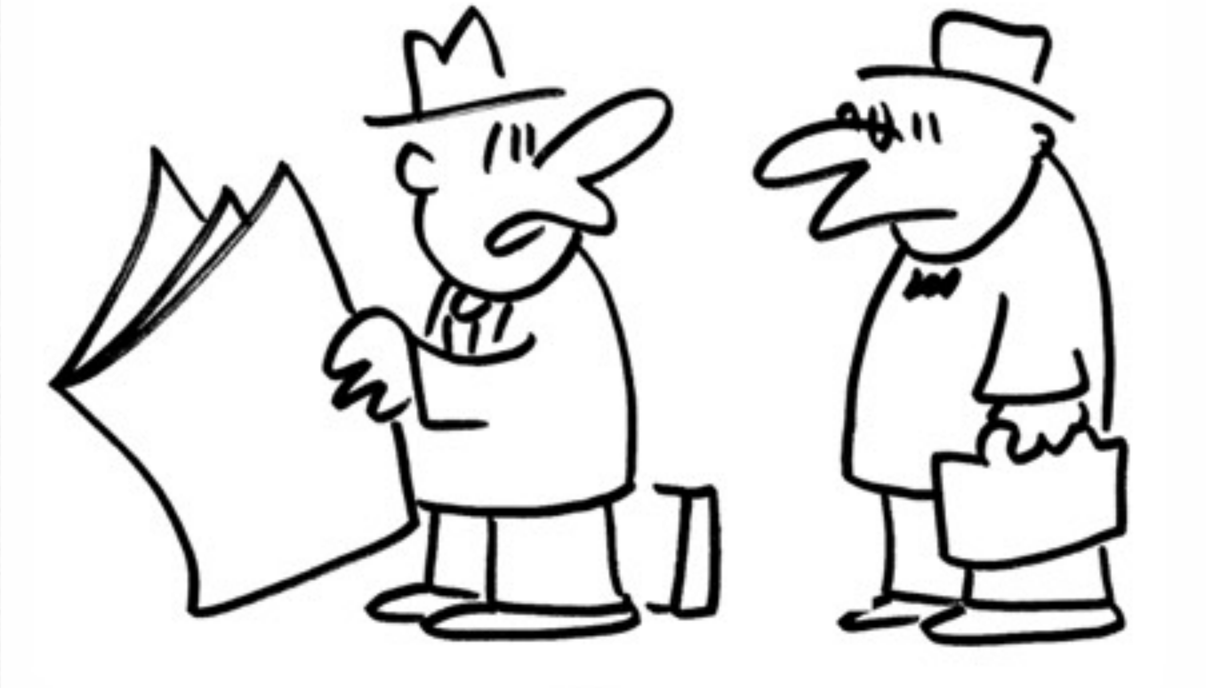
The shape of the US Department of Defence HQ building; that's why it's called the Pentagon.



Hexagon

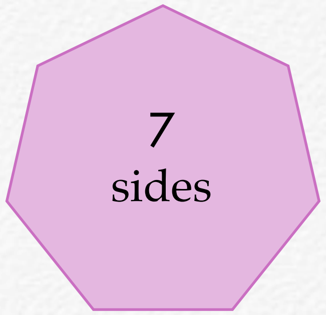


A honeycomb shape.



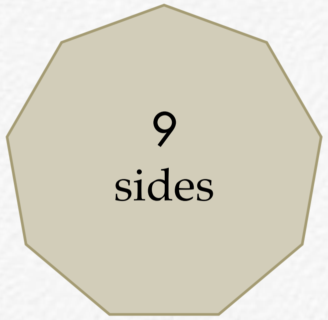
"I'm all for increased military spending, but now they want to build a *Hexagon!*"

Heptagon



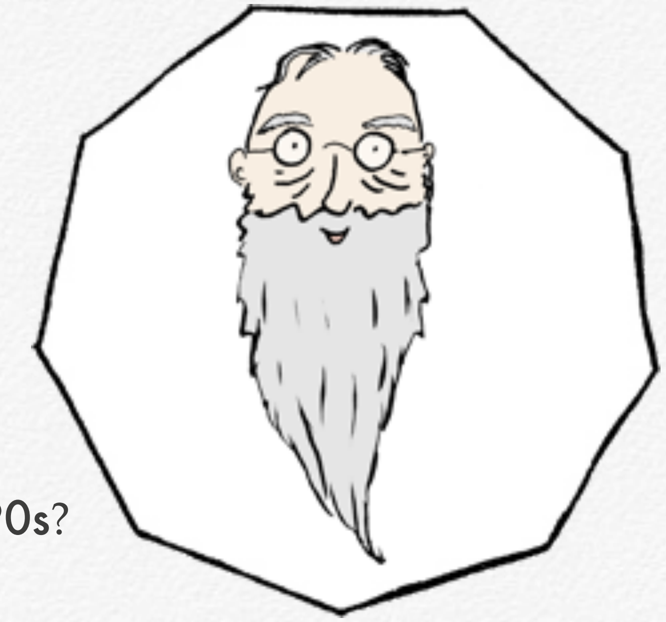
The shape of a 50p coin.

Nonagon

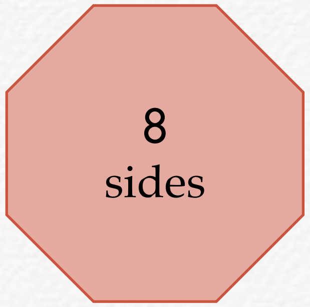


Is your grandfather in his 90s?

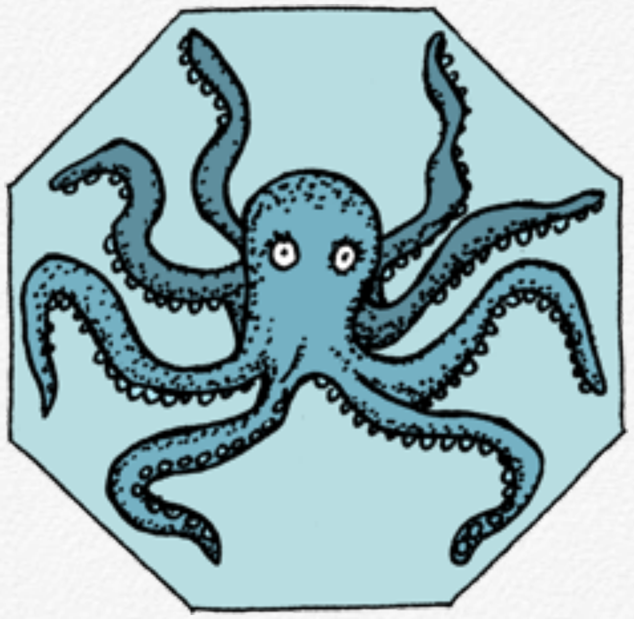
If so, he is a nonagenarian.



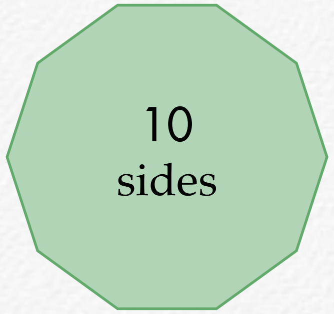
Octagon



An octopus has 8 arms.



Decagon



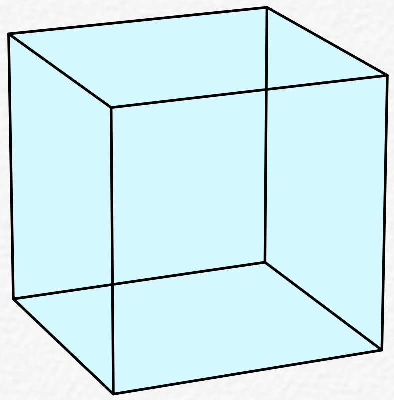
The decimal system
is based on the number 10.

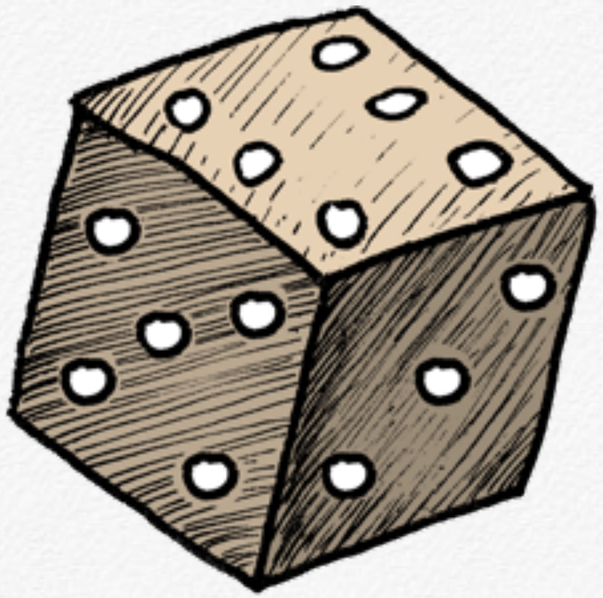
Classic Build:
Moorish arches in
The Alhambra Palace,
Granada, Spain
(13th Century).

3D Shapes

Cube

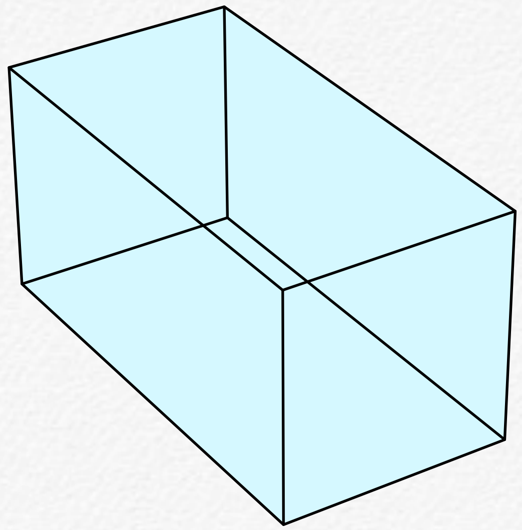
All edges have the same measurements

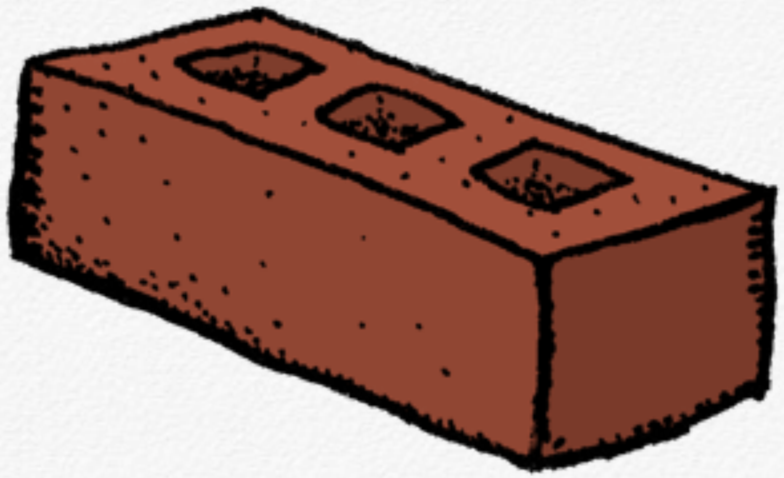




Cuboid

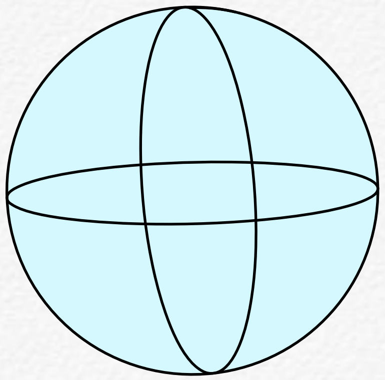
The shape of a brick

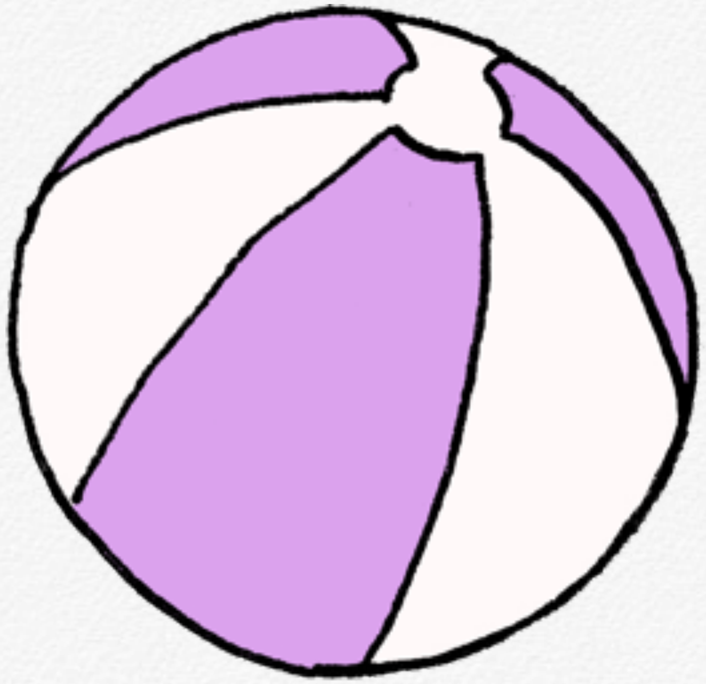




Sphere

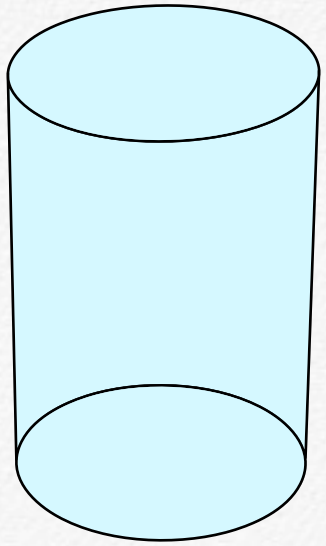
The shape of a ball





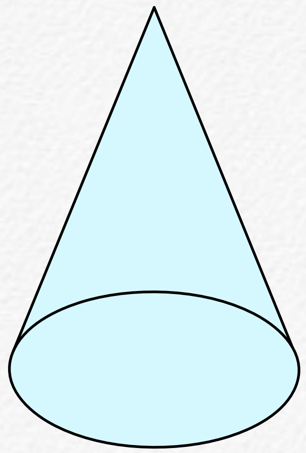
Cylinder

A drain pipe



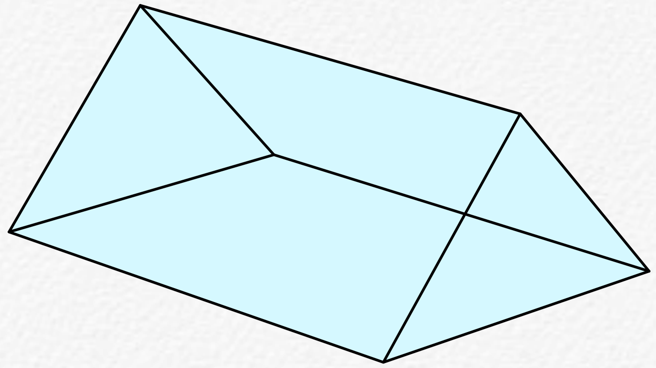
Cone

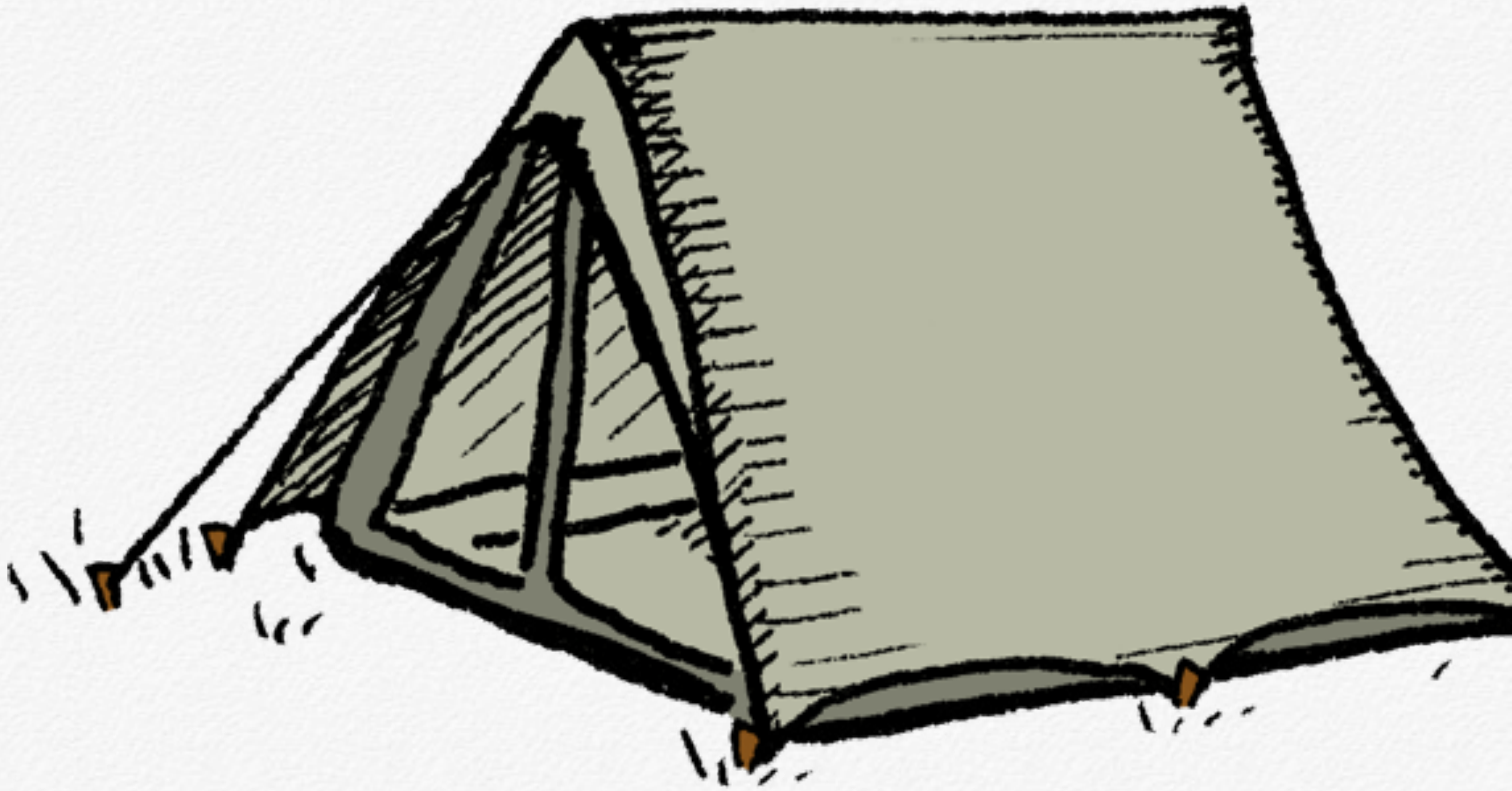
Think ice-cream cone



Triangular Prism

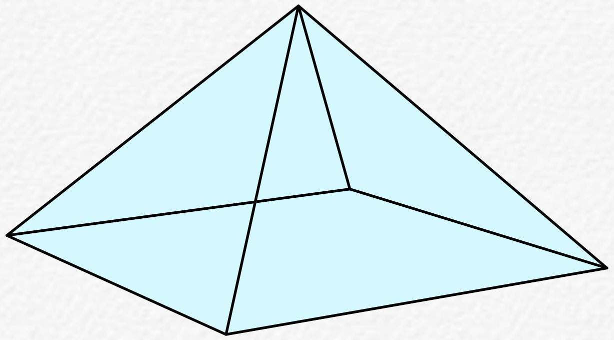
An old fashioned tent

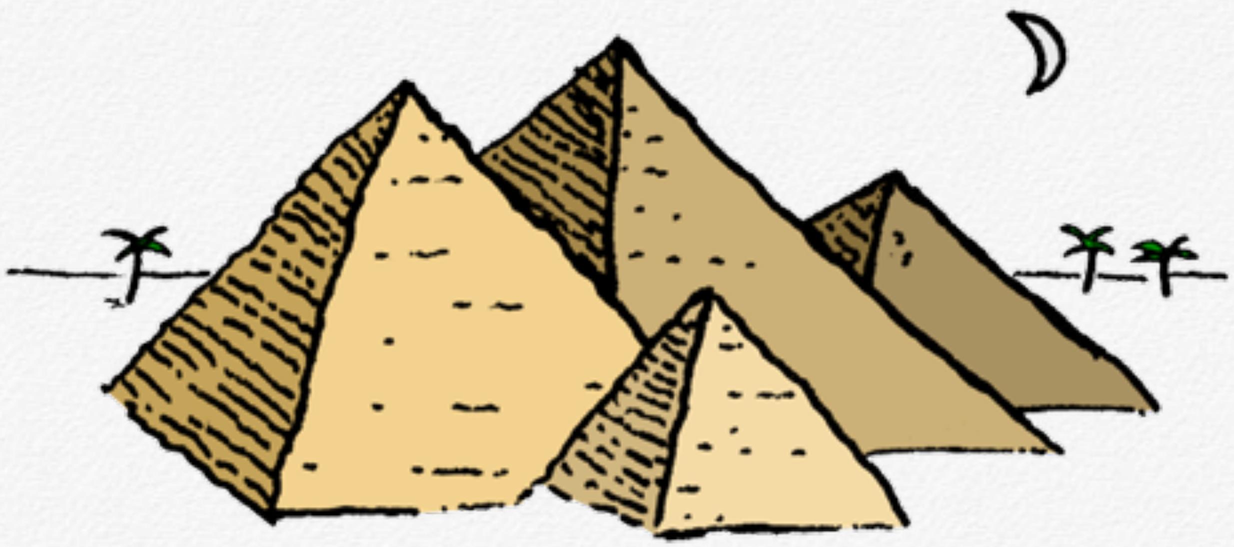




Square-based Pyramid

Like the Egyptian ones

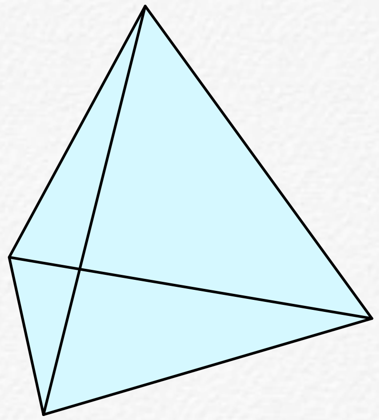


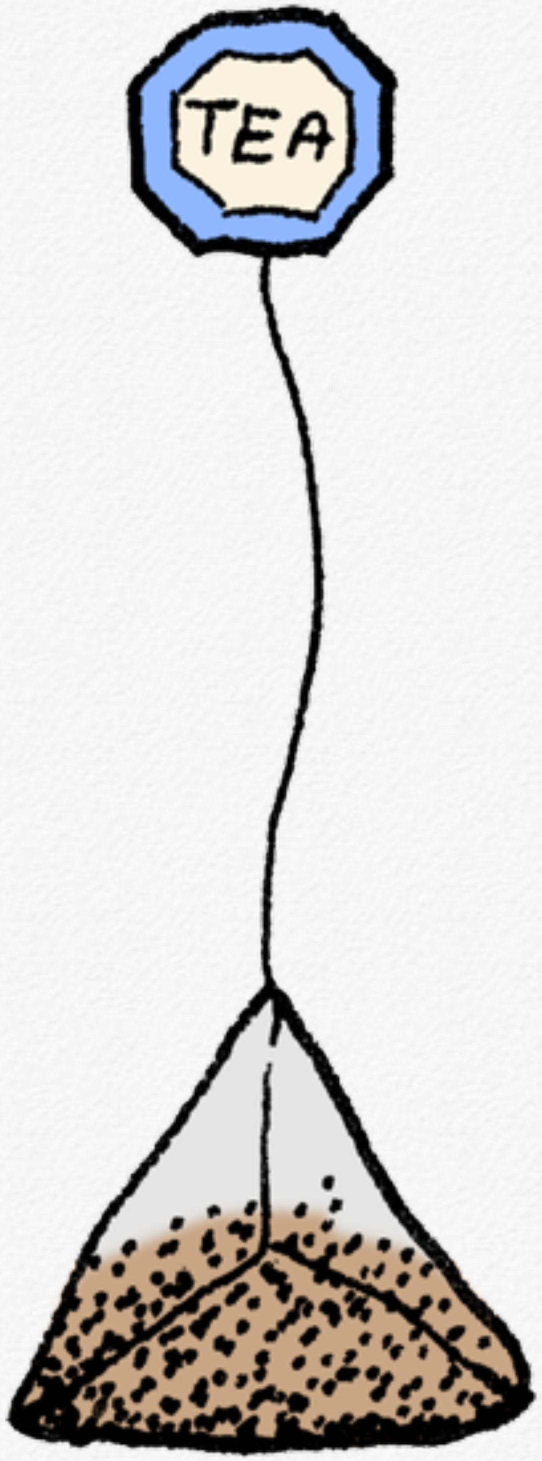


Tetrahedron

(Triangular-based pyramid)

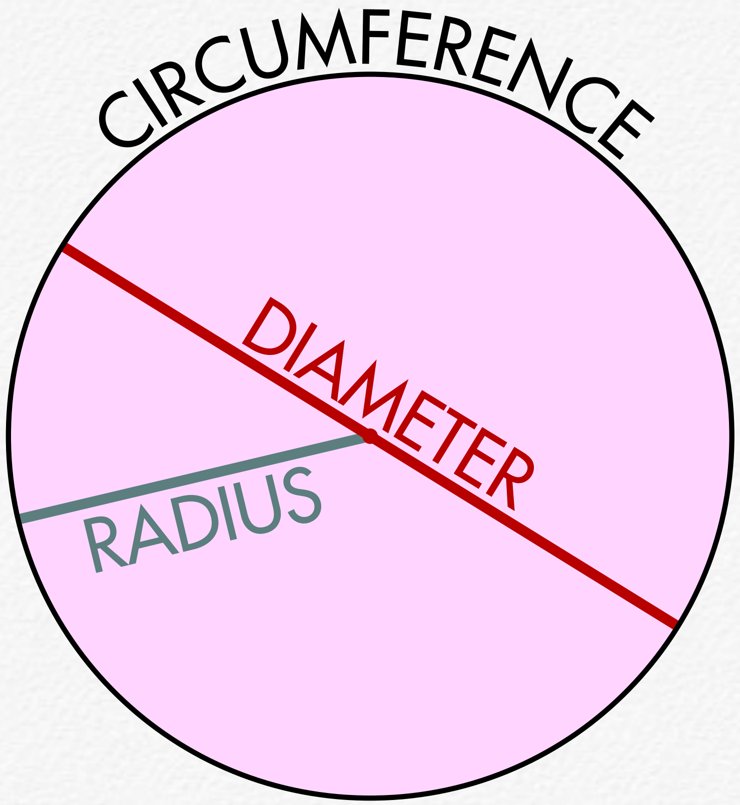
Like the pyramid tea bag

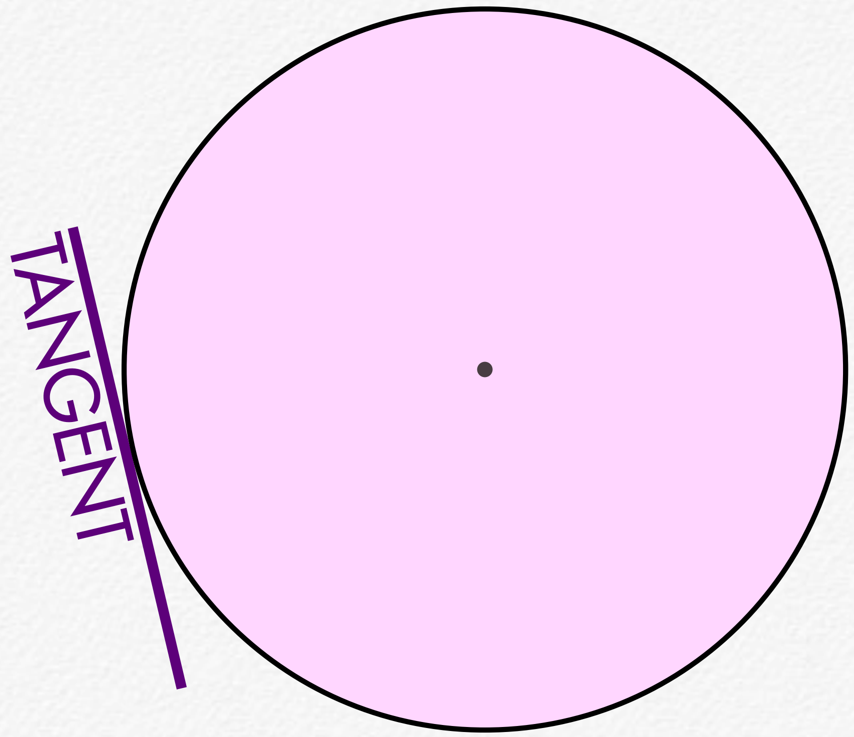




Classic Build:
Milan Cathedral,
Milan, Italy
(15th Century).

Parts of a Circle





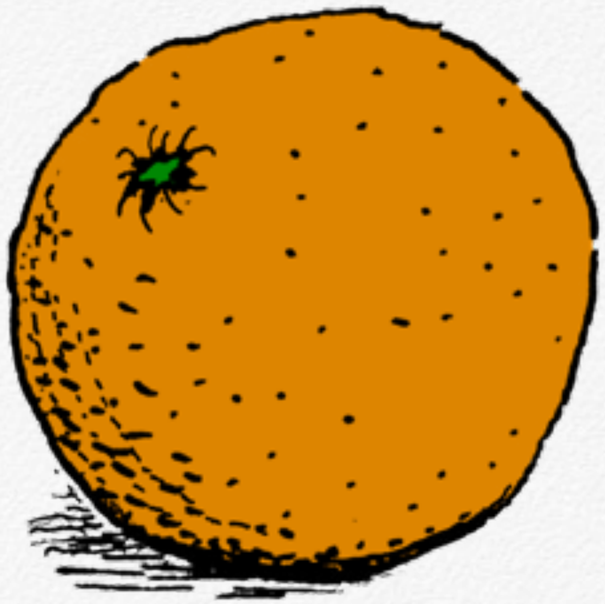
The **tangent** is a line that touches a curve at only one point.

The **radius** is a line that goes from the centre to the edge of the circle.

The **diameter** is twice the length of the **radius**.

The **circumference** is the distance all the way around the edge of the circle.

Incidentally, the distance around ANY shape is called the **perimeter**.



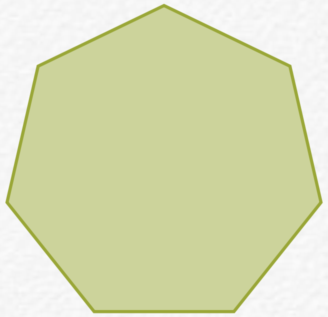
Exercises

1) Test yourself by naming the following shapes:

a)

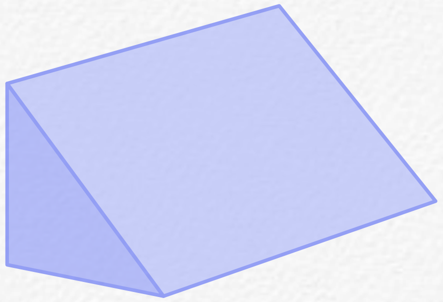
b)

c)

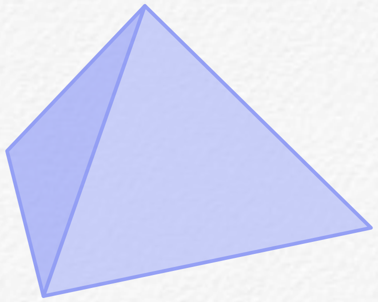


d)

e)



f)



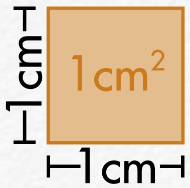
Islamic geometric pattern at The Alhambra Palace, Granada, Spain (11th Century).

Area

Area is a measure of flat, two-dimensional space. This could be a measurement of the space on a piece of paper or fabric; the size of a wall to be painted; the size of a floor to be carpeted or the size of a plot of land to apply for planning permission.

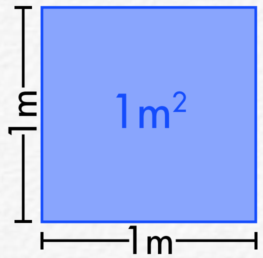
In the **metric system**, area is measured in **square centimetres** (cm^2), **square metres** (m^2) and, for very large areas, **square kilometres** (km^2), if you are measuring an entire country for a geographical survey.

A **square centimetre** is the area of a square that measures **1 cm** by **1 cm**:



For example, if the display on your mobile phone is **4 cm x 3 cm** it has an area of **12 cm²**.
That means that **12** of those yellow square centimetres would fit on the surface of your mobile phone display.

A **square metre** is the area of a square that measures **1 m** by **1 m**:



If your bedroom measures **3m** long by **3m** wide, it would have an area of **9m²**. This would be **9** separate square metres of carpet to fit onto the floor space of your bedroom.

Any unit of length becomes **a unit of area** by being **squared**. In the **imperial system of measurements** there are **square inches** (**sq.in**), **square feet** (**sq.ft**) and **square miles** (**sq.mile**).

Hectares and **acres** are two special units of area, which are used mainly by geographers, ecologists, farmers and estate agents. Acres (ac.) belong to the **imperial system**. Hectares (ha.) belong to the **metric system**.

A hectare is **10,000 m²**. That's approximately the size of Trafalgar Square in London.

An acre is a bit less than half the size of a hectare, approximately **4,000 m²**, about the size of half a professional football pitch.

Buying Property

It's important to understand what area means, because estate agents increasingly use area to describe the properties they are selling. This is already standard practice in Europe, where the area, expressed in square metres, of the floor space of an apartment or house is always specified.

In Britain, estate agents tend to concentrate on telling you the **number** of rooms but it's important to know how **big** these rooms are, so you need to know the **area** of the floor space in the property, in square feet or square metres. Office space sold in Britain is nearly always listed in square feet.

Farm land is bought and sold according to its area in acres or hectares.

To get a better idea of the size of area measurements, use the following crib:

1 m^2 = 10.8 sq.ft. A car parking space = 12 m^2 = 130 sq.ft. A small studio flat size = 35 m^2 = 380 sq.ft.
1 hectare = 10,000 m^2 = 2.5 acres (approx. Trafalgar Square) 4,000 m^2 = 1 acre (a bit more than half a professional football pitch)
1 square mile = 640 acres = 259 hectares

So a hectare is a bit bigger than a football pitch, an acre is about half that size.

Ideal home? USA (21st Century).

Calculating Area

Area of Squares and Rectangles

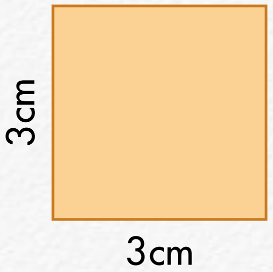
It's easy to calculate the area of rectangles and squares. You simply multiply the **length** of the rectangle (or square) by its **width**.

Area of a rectangle = length x width

Before paper and pencil were invented, for geometric drawings, ancient mathematicians used sticks on sand or earth.

First Example

What is the area of a square measuring **3cm** by **3cm**?



First, write down the formula. Then insert the numbers:

Area of a rectangle = **length** x **width**

= 3 x 3

Answer = 9 cm^2

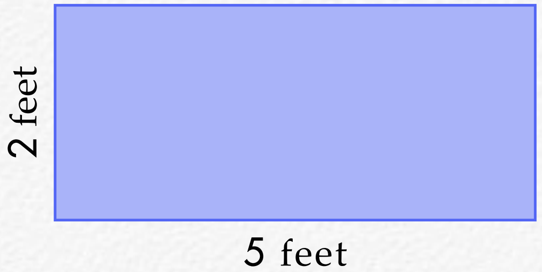
Note

A square is a rectangle with sides of equal lengths.

Second Example

Six year old Lydia's bed measures **2** feet by **5** feet.

What area does Lydia's bed occupy?

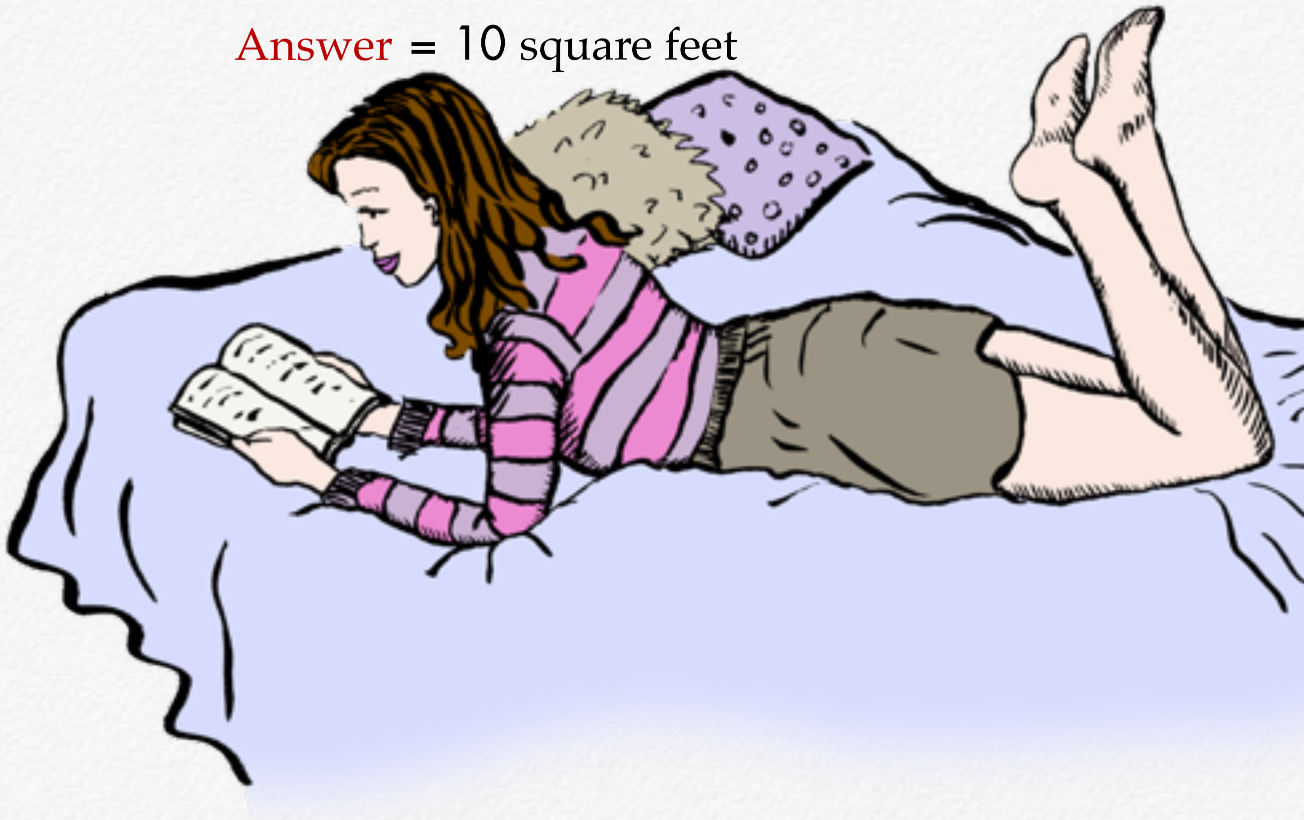


First, write down the formula. Then insert the numbers:

Area of a rectangle = **length** x **width**

= 5 x 2

Answer = 10 square feet



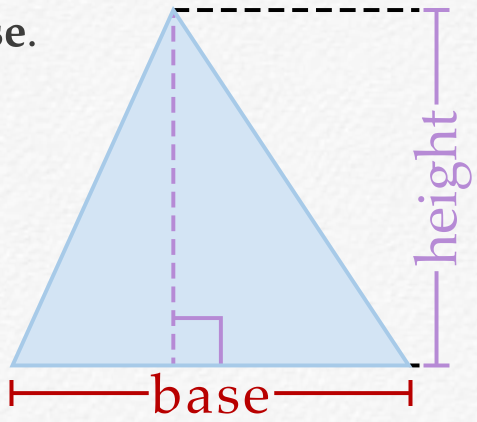
Handmade Islamic tiles used to form an irregular, repeating pattern, based on geometric shapes. Topkapi Palace, Istanbul, Turkey.

Area of a Triangle

The area of a triangle is calculated by the following formula:

Area of a triangle = (base x height) ÷ 2

Choose any side of a triangle as **base**.
The **height** is a bit more tricky.
Imagine a dotted line drawn **straight down** from top of triangle to the base… that is the **height**.

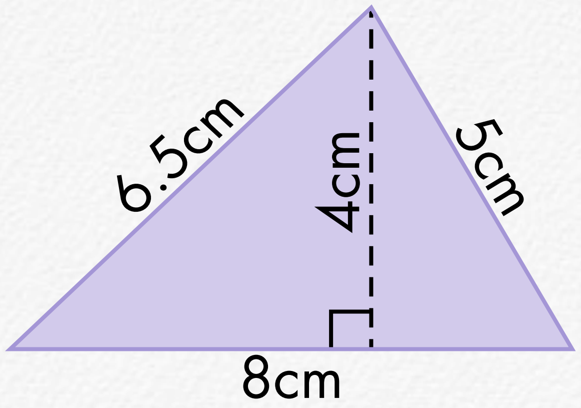


The **height** is **NOT** necessarily the length of one of the sides of the triangle (see diagram).

The **height** and the base of a triangle always form a right angle, so the **height** and the base lines are **perpendicular** to each other. (See dotted lines in diagram).

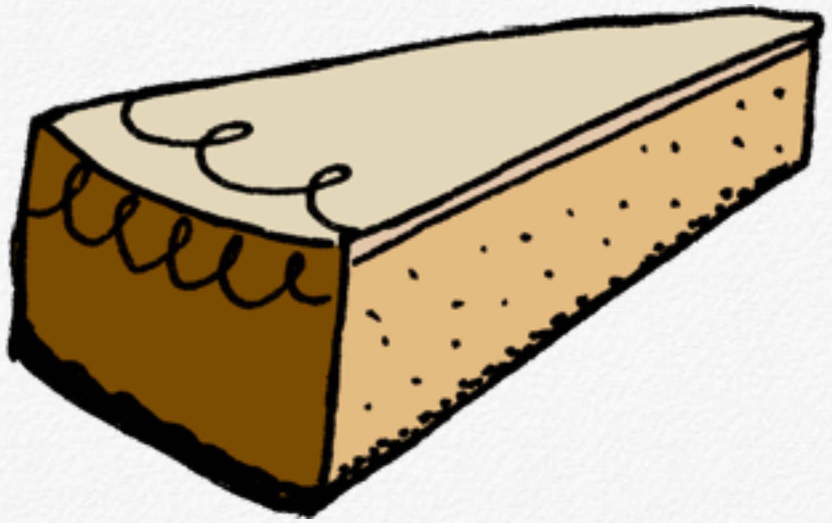
First Example

Calculate the area of the triangle in the following diagram.



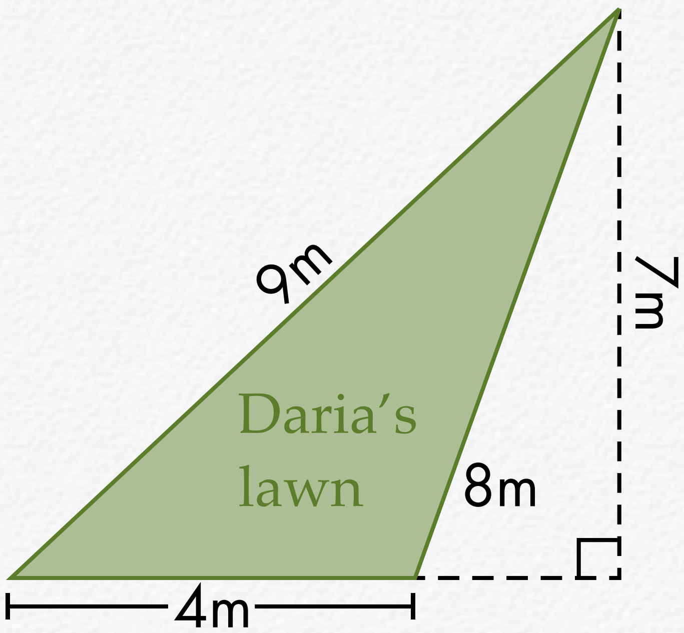
First, write down the formula:

Area of a triangle = (base x height) ÷ 2
= (8 x 4) ÷ 2
= 32 ÷ 2
Answer = 16 cm^2



Second Example

Daria has redesigned her garden. In order to know how much grass seed to buy for her new lawn, she needs to know the area of the patch that will become lawn.
What is the area of Daria's lawn?



First, write down the formula:

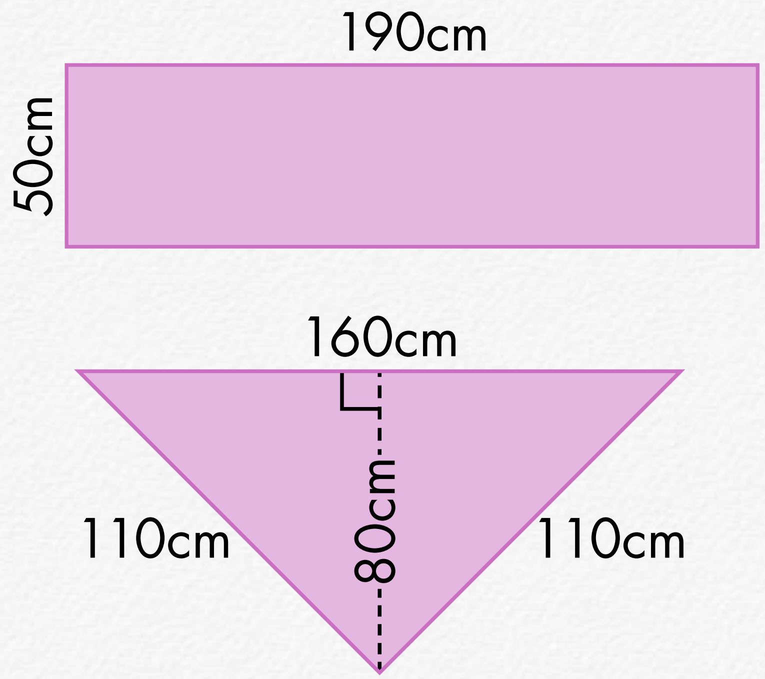
$$\textbf{Area of a triangle} = (\textbf{base} \times \text{height}) \div 2$$
$$= (\ 4 \quad \times \quad 7\ ) \div 2$$
$$= 28 \div 2$$
$$\text{Answer} = 14\ m^2$$

Exercises

2) Rima is painting her new flat. She decided to paint one wall in her living-room in a deep shade of 'raspberry diva'. The instructions on the tin tell Rima that each tin of 'raspberry diva' paint will cover **40 m²**.

Rima's living-room wall measures **5 m** long and **3 m** high. Will one tin of paint be enough to paint two coats of 'raspberry diva' on Rima's living-room wall?

3) Fashion designer Maria needs to minimise the amount of silk she will need for her new line of shawls, which are individually woven. Maria's two designs are drawn below. Which shape shawl will use the least amount of silk?



Classic Build:
Sydney Opera House,
Sydney, Australia.

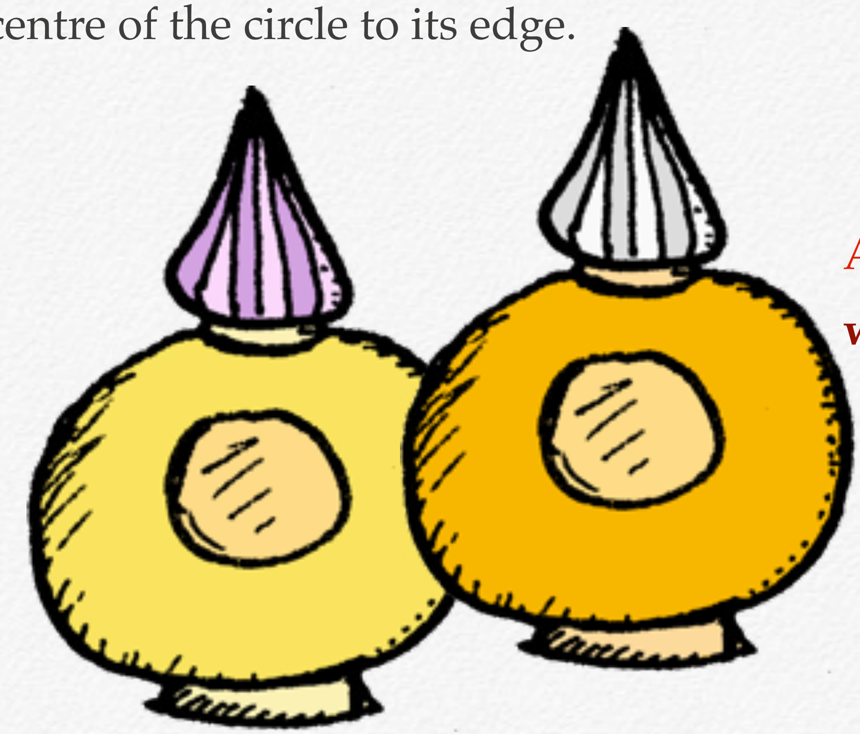
Area of a Circle

To calculate the area of a circle, you use the following formula:

$$\text{Area of a circle} = \pi r^2$$
$$\text{Area of a circle} = \pi \times r \times r$$

π is a Greek letter that is short-hand for the number used in circle calculations. π is written 'pi' but pronounced 'pie' as in apple pie. **The π number is a bit more than 3**.
To be more precise, π is 3.1415926535897932…

Remember that **r stands for the radius of the circle**, which is the distance from the centre of the circle to its edge.



Example
You want a skinny dip pool built in your small garden.
What will the area of the pool be?

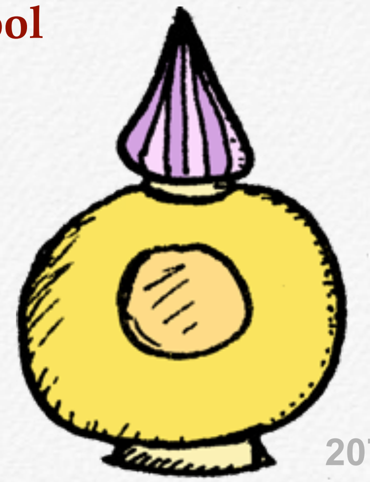
All you need know to work out the area of any circle is the radius.

You decide that your pool will be **4 m** across (the diameter), so the **radius** (half the diameter) **will be 2 m**.
Use $\pi = 3$ in this calculation.

To work out the area of your skinny dip pool, the calculation is: πr^2.

$$\begin{aligned} \text{Area} &= \pi \times r \times r \\ &= 3 \times 2 \times 2 \\ &= 3 \times 4 \\ &= 12\ \text{m}^2 \end{aligned}$$

Answer: **The approximate area of the pool will be 12 m²**.



To calculate the area of the pool more accurately, use $\pi = 3.142$

The calculation will be πr^2.

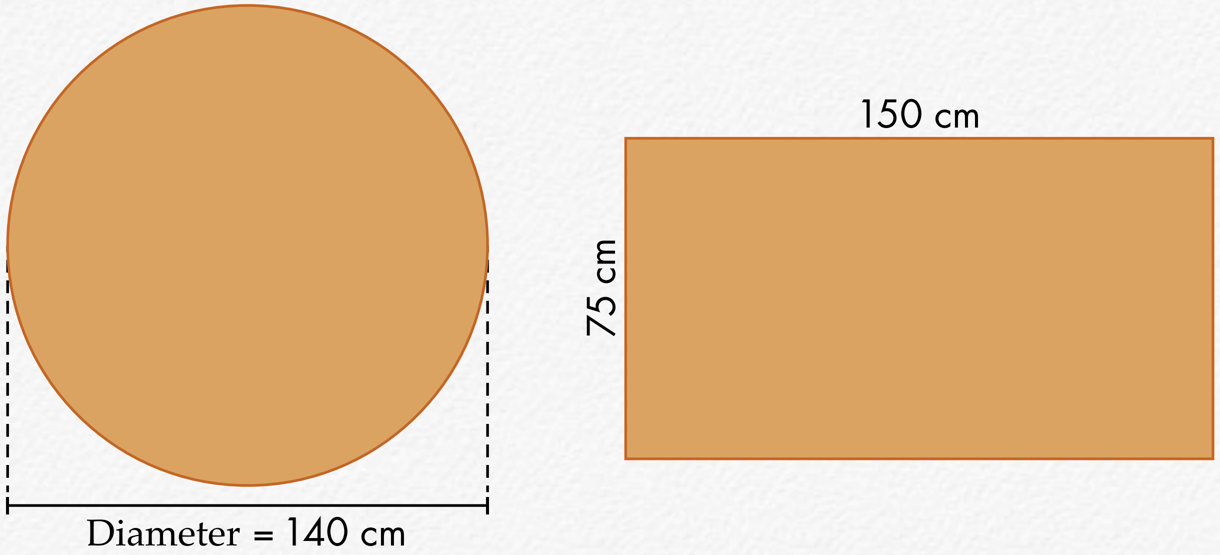
$$\text{Area} = 3.142 \times 2 \times 2$$
$$= 3.142 \times 4$$
$$= 12.568 \text{ m}^2$$

You can see that the approximate value is only a little less than the accurate value, so in Real Life, using $\pi = 3$ may be good enough for rough calculations of small area.

Skinny dip pool.

Exercise

4) Janice wants to buy a kitchen table that will seat **6** people and that is not too big for her small kitchen, so she needs to choose the table with the smallest area. The measurements of the two tables are shown below. Should Janice choose the rectangular or circular table?



Useful tip: For each person round a table, allow a width of **75 cm** or $2\frac{1}{2}$ ft.

Will they need a mortgage deposit?

Area of a Compound Shape

Some shapes are composed of two or more shapes.
To find the entire area of one of these compound shapes, split the shape up into smaller rectangles, triangles and semi-circles.

Note

A semi-circle is exactly half of a circle.

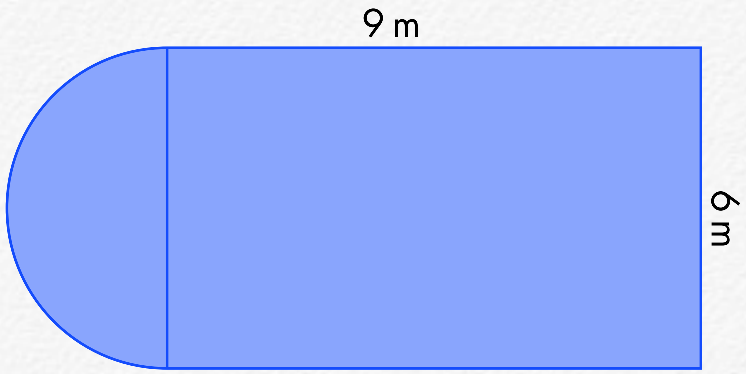
__Example__

A diagram of Princess Caroline's indoor swimming pool is shown below. What is the area of Princess Caroline's swimming pool?

Diagram A

By adding an extra line to diagram A, you will discover that the pool is actually made of a rectangle and a semi-circle. See below.

Diagram B



The area of the pool = the area of the rectangle plus the area of the semi-circle.

Area of a rectangle = length x width

= 9 x 6

= 54 m^2

The area of a semi-circle is half the area of a circle.

In diagram B you can see that the diameter of the semi-circle is **6 m**, so the radius of the semi-circle is **3 m**.

Area of a circle = $\pi \times r \times r$

= 3 x 3 x 3

= 27 m^2

Area of semi-circle = half the area of the circle

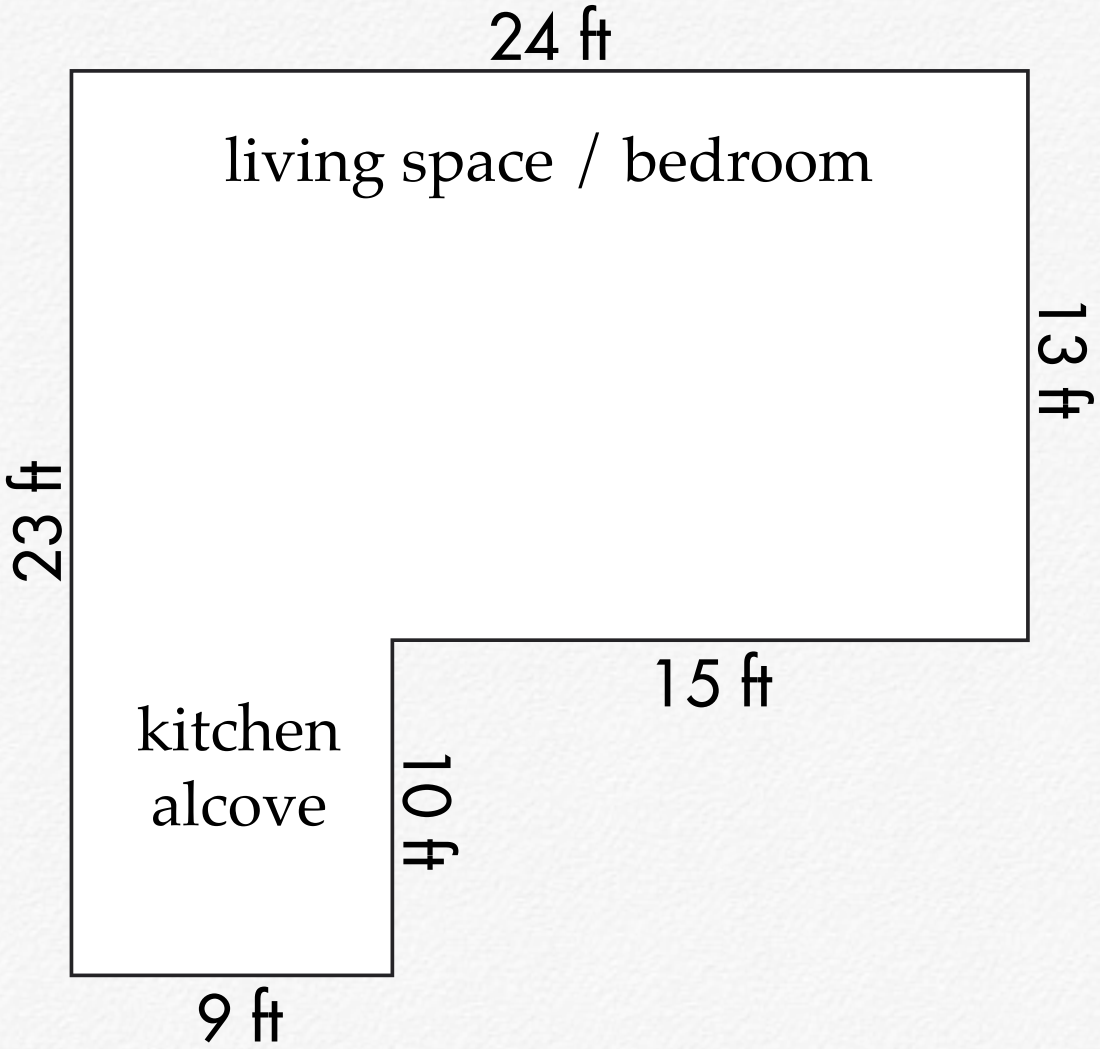
= 27 ÷ 2 = 13.5 m^2

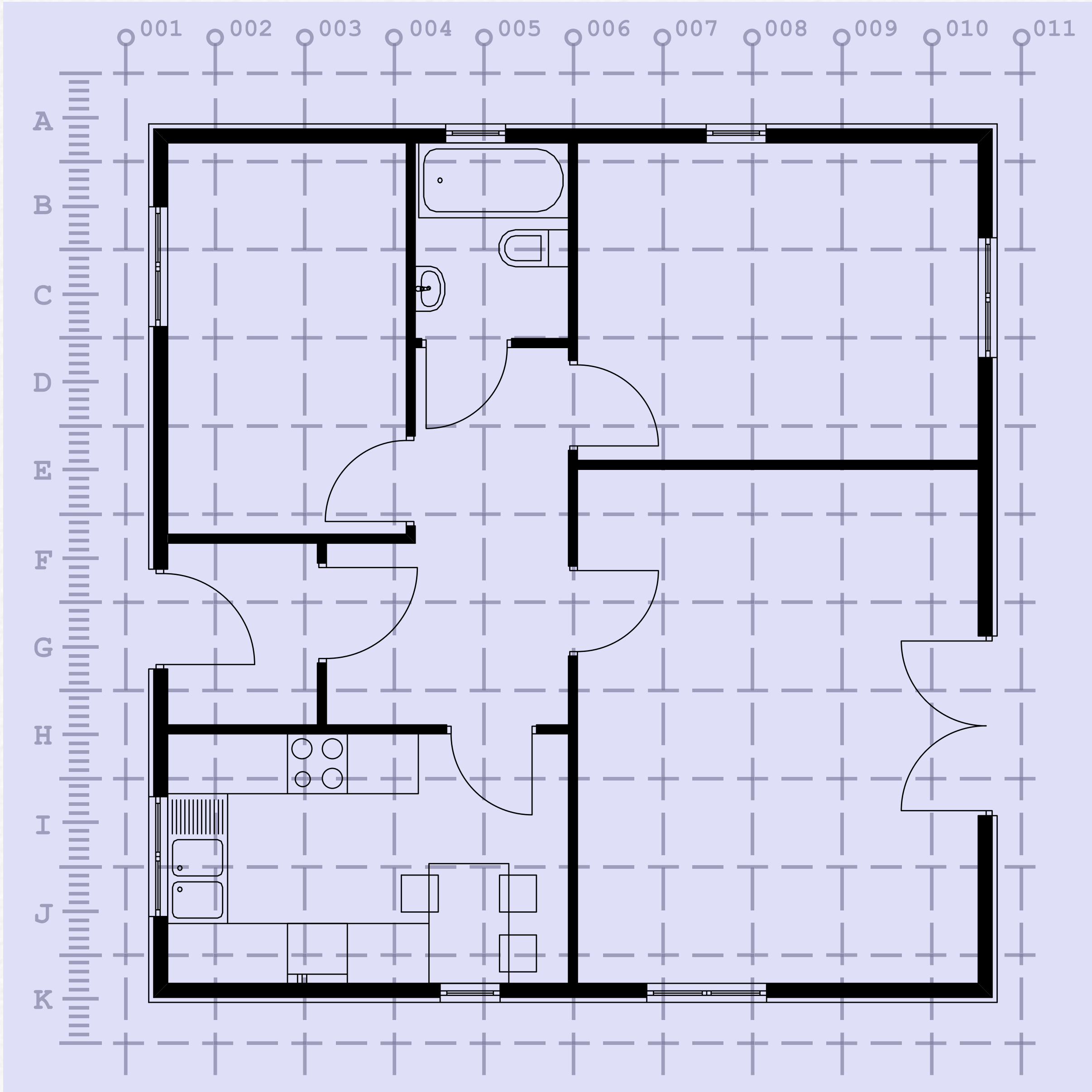
Total area of pool = 54 m^2 + 13.5 m^2 = 67.5 m^2

Answer: **The area of Princess Caroline's swimming pool is 67.5 m^2.**

Exercise

5) Rita wants to sell her studio flat, and her local estate agent wants to know the area. Rita measured her flat and drew the plan shown below. What is the area of Rita's studio flat?





Typical floor plan: ground floor of a house.

Classic Build: Skyscraper city with the United Nations complex, New York, USA (opened 1952).

Area of Irregular Shapes

Many areas can't be cut neatly up into squares, triangles and circles, so use a grid and count the squares, to get a good estimate of the area.

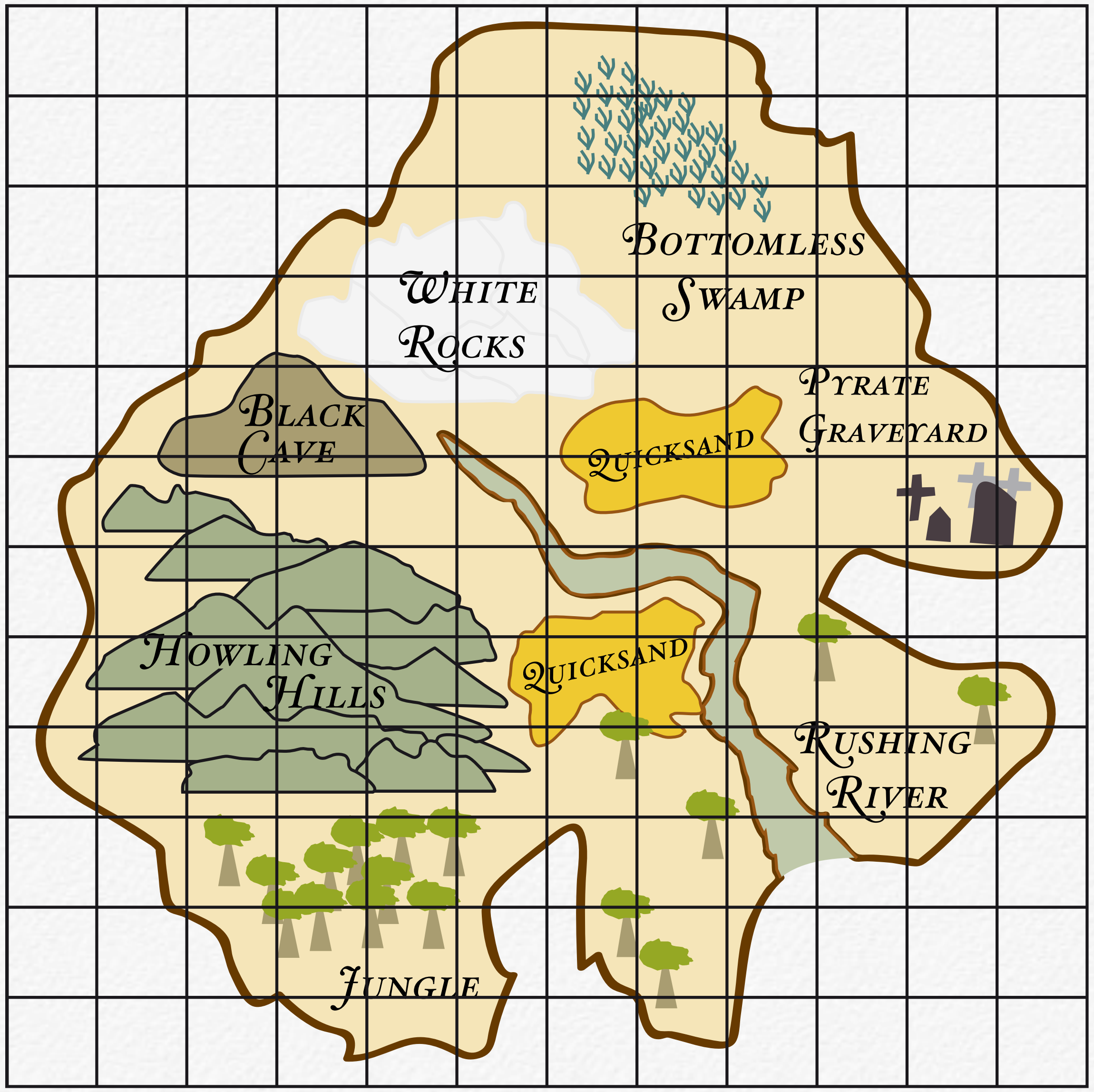
Example

Find the area of the Treasure Island below.

The area can be estimated by putting a grid over the map of the island. Use the map scale to decide the grid scale: here, each grid square is an acre.

Scale: 1 grid square = 1 acre.

Remember an acre is the approximate size of half a professional football pitch.



Here are the counting rules:

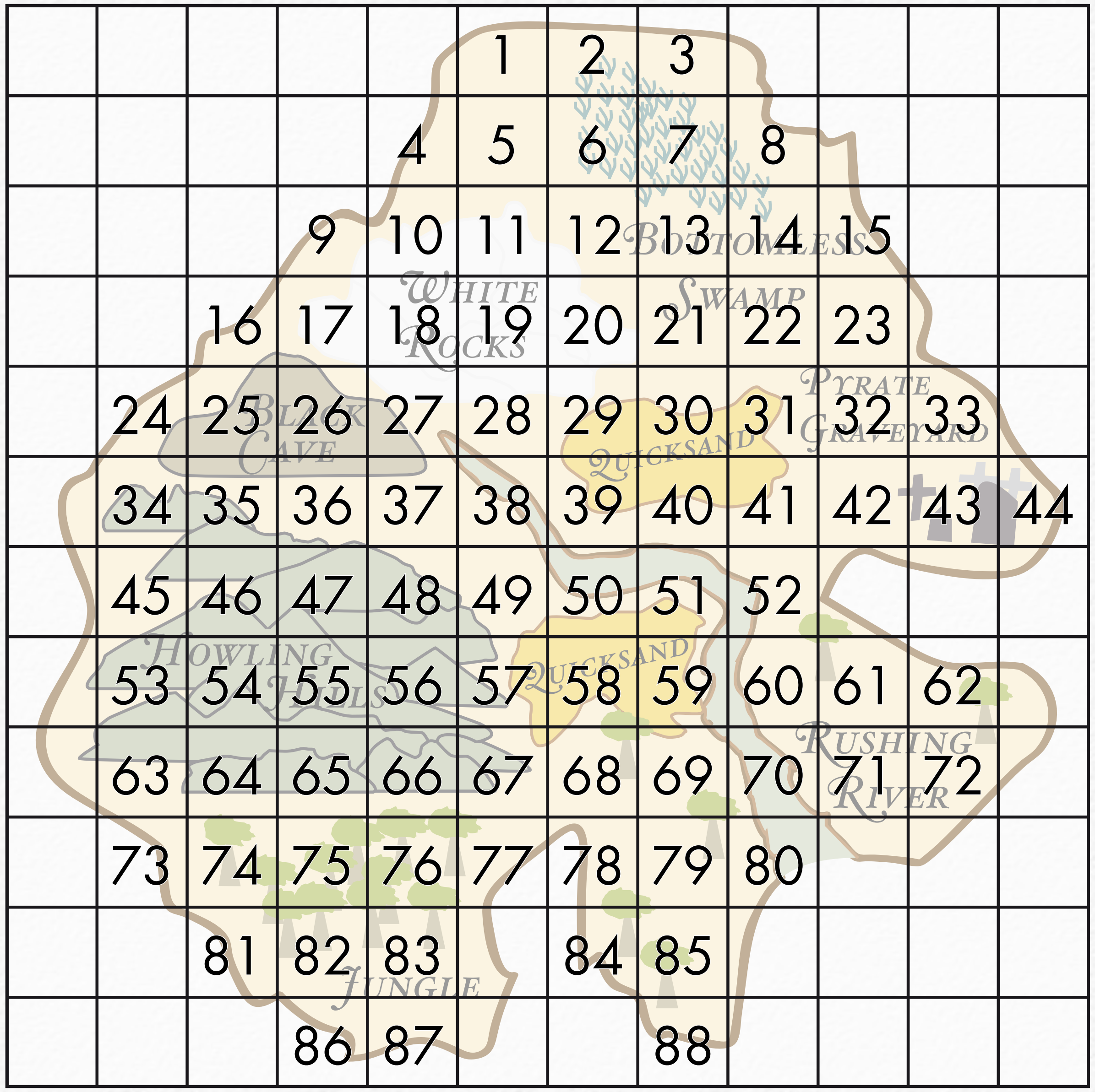
Each square completely covered by land is one acre.

At the edge of the island:

- count any square that is half (or more than half) covered by land as one acre.
- don't count any square that is **less** than half covered by land.

A good engineer's working habit is always to work from left to right: so count each line of these squares, from left to right.

The theory is that the bits of land that are **not** counted make up for the bits of sea that **are** counted.



Answer: **The area of Treasure Island is approximately 88 acres.**

Useful tip: A grid is also useful to systematically search for the buried treasure with a metal detector.

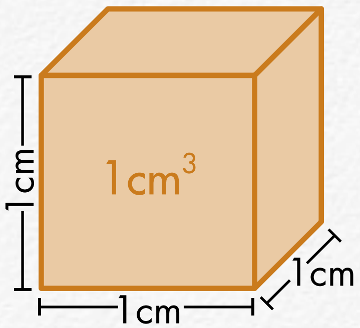
Volume

Volume is used to measure the space inside something: a cup, bottle, hot water tank, petrol tank or battleship store. Volume can also measure the space that a solid object occupies.

In the **metric system**, volume is measured in **cubic centimetres** (cm^3), **cubic metres** (m^3) or in **litres**.

One cubic centimetre = 1 millilitre.

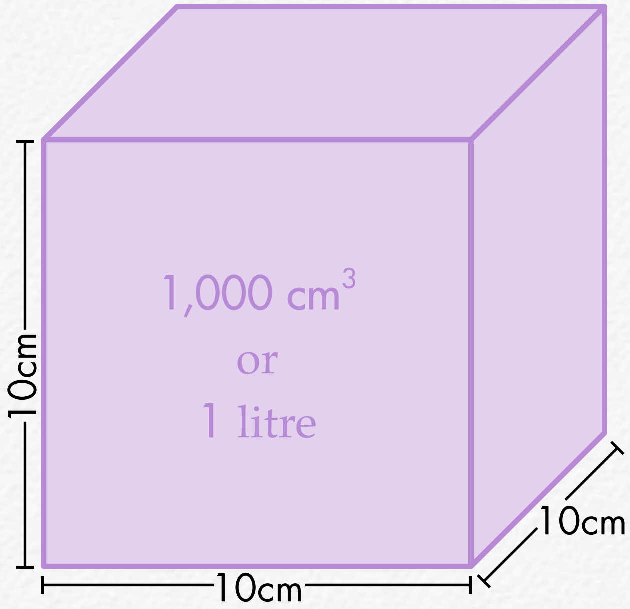
$1\ cm^3 = 1\ ml$



These diagrams are not to scale.

$1{,}000\ cm^3$ = 1,000 millilitres = 1 litre

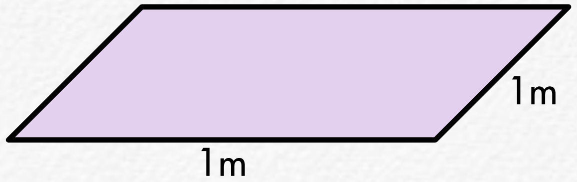
The cube in this diagram measures **10 cm long** by **10 cm wide** by **10 cm high** and its volume is **1 litre**.



A **measurement of length** can be used as a **measurement of volume** by being **cubed**.

Example

1 metre x 1 metre gives you an area of **1 m^2**

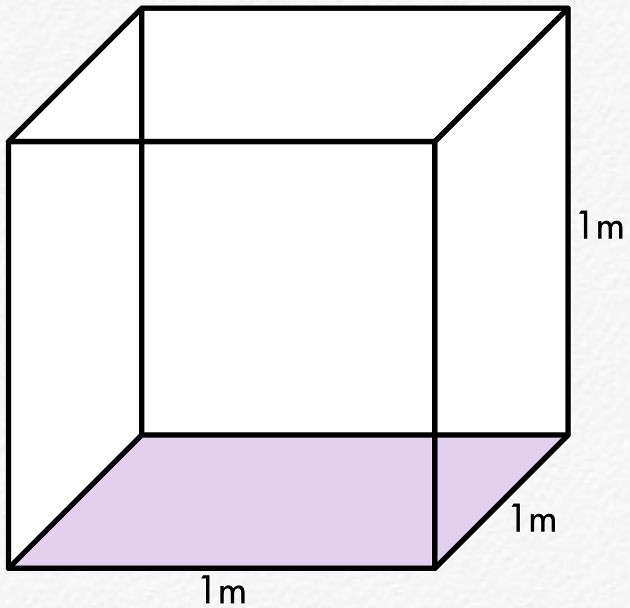


$1\ m \times 1\ m = 1\ m^2$

If you multiply **1 m^2** by one further metre, it becomes a **measurement of volume** .

$1\ m \times 1\ m = 1\ m^2$

$1\ m^2 \times 1\ m = 1\ m^3$



In the **imperial system of measurements** you will seldom come across cubic inches (**cu.in**) and cubic feet (**cu.ft**). Imperial volume measurements are usually fluid ounces, pints, quarts and gallons (look back to STEP 3).

Basic solid shapes
are used to form buildings.

Volume has become increasingly important as people shop on the internet. They need to understand the measurement in a description, in order not to be mistaken or cheated. My grandson Sam was carrying a small briefcase. “Where did you get that?” I asked him. He grinned, “It’s the suitcase you ordered for me on the internet.”

Example

Is a 10 litre suitcase big enough to pack all you need for your holiday?

To visualise a 10 litre suitcase, think of a one litre carton of milk.

Answer: **You will only be able to pack 10 cartons of milk in that tiny 10 litre suitcase.**

Note: Each of the 6 sides of a cube is a square shape.

A **cuboid** has either four rectangular sides or six rectangular sides – like the suitcase on the next page.

How can you calculate the volume of your own suitcase? Use the following formula for the volume of a cuboid.

Volume of a cuboid = length x width x height

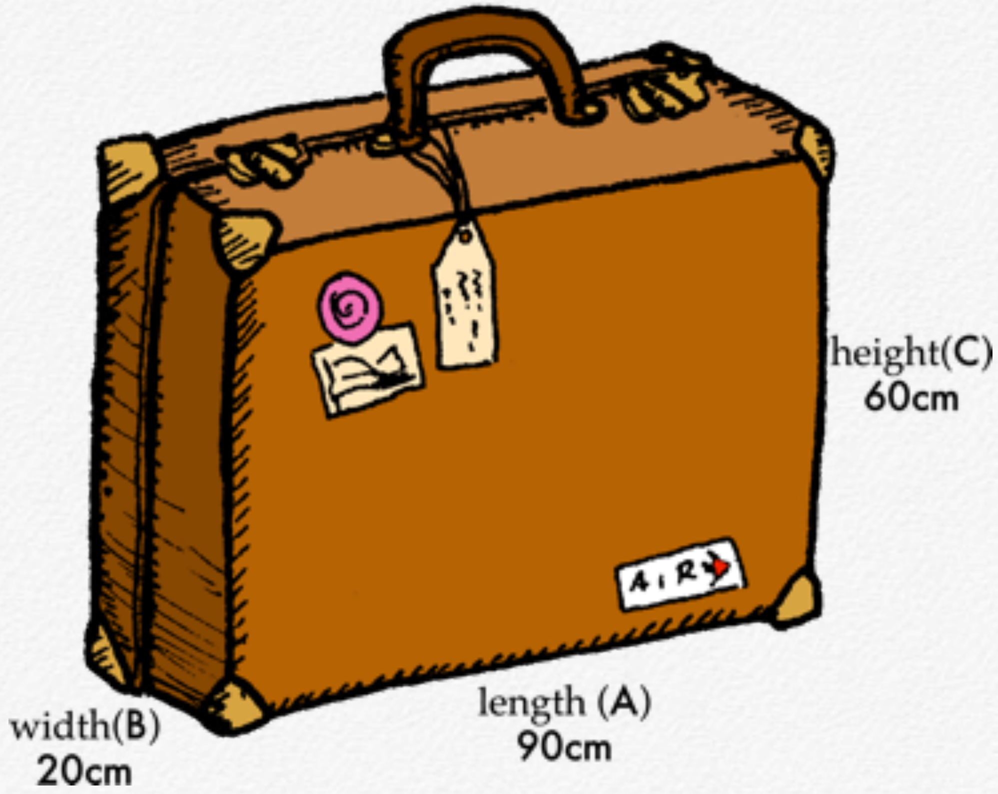
A memory aid is: it’s a **L**ong **W**ay **H**ome

Property developers need geometry.

Sarah Beeny, Property developer, TV presenter, mother of four.

Example

Calculate the volume of the suitcase to the right, in litres.



In centimetres, measure the length (A), the width (B) and the height (C).

Volume of a cuboid = length x width x height

$= A \times B \times C$

$= 90 \times 20 \times 60$

$= 108{,}000\ cm^2$

The volume of the suitcase is **108,000 cm^3**,

but you need to **convert cm^3 to litres**; use the chart method for this.

You know that **1** litre is equal to **1000 cm^3**. Insert the amount you want to convert in the correct column, for its unit.

	litres	cm^3	
1 litre = 1000 cm^3	1	**1000**	Multiply the diagonal numbers
108,000 cm^3 = ?	?	108,000	Divide by the remaining number

Remember the rule? **Multiply the diagonal numbers, then divide by the remaining number**.

The calculation is:

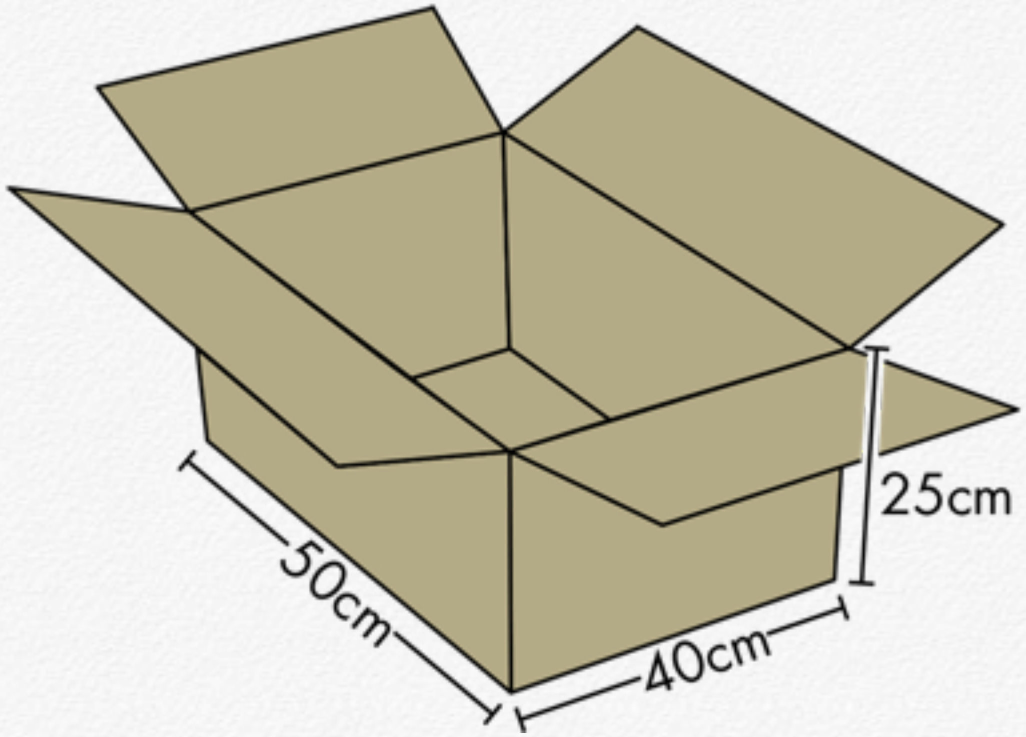
$1 \times 108{,}000 = 108{,}000$

$108{,}000 \div 1000 = \mathbf{108}$

Answer: **The suitcase has a volume of 108 litres.**

Exercise

6) What is the volume, in litres, of the cardboard box shown below?



Military commanders use geometry.
Racheli Levantal takes a fitness test.
She is Israeli platoon commander, Karakal Battalion.

Answers to Part 29

1) Name the following shapes:

a) 

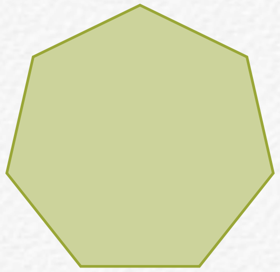
This shape has two pairs of parallel lines which are not **all** equal in length. The pairs of parallel lines aren't perpendicular, so it is a parallelogram.

Answer: **Parallelogram**

b) 

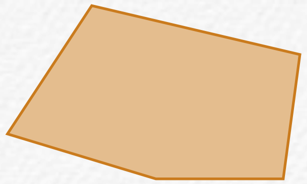
This shape has one pair of parallel lines which are not equal in length, so it is a trapezium.

Answer: **Trapezium**

c) 

This shape has seven sides, so it is a heptagon. As all the sides are equal length, it is sometimes called a regular heptagon.

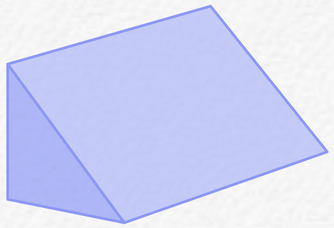
Answer: **(Regular) heptagon**

d) 

This shape has five sides, so it is a pentagon.
It is sometimes called an irregular pentagon because the sides are not all equal.

Answer: **(Irregular) pentagon**

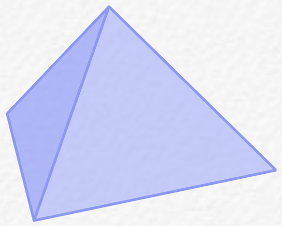
e)



This shape has triangular ends,
which are the same size and shape at both ends.
If you cut a triangular prism anywhere along its length,
the chopped-off bit will have the same triangular ends.

Answer: **Triangular prism**

f)



This shape is a pyramid, with a triangular base.
It is a **triangular based pyramid**, or a **tetrahedron**.

Answer: **Triangular based pyramid or tetrahedron**

Classic Build: The Golden Gate Bridge,
San Francisco, USA (opened 1937).

2) Rima is painting her new flat. She decided to paint one wall in her living-room in a deep shade of 'raspberry diva'.

The instructions on the tin tell Rima that each tin of 'raspberry diva' paint will cover $40\ m^2$.

Rima's living-room wall measures $5\ m$ long and $3\ m$ high. Will one tin of paint be enough to paint two coats of 'raspberry diva' on Rima's living-room wall?

It always helps to visualise the problem, so first draw a diagram of Rima's living-room wall:

Then, calculate the area of the wall. Write out the formula:

Area of rectangle = length x width

Next, insert the numbers: $= 5 \times 3$

Area of living-room wall $= 15\ m^2$

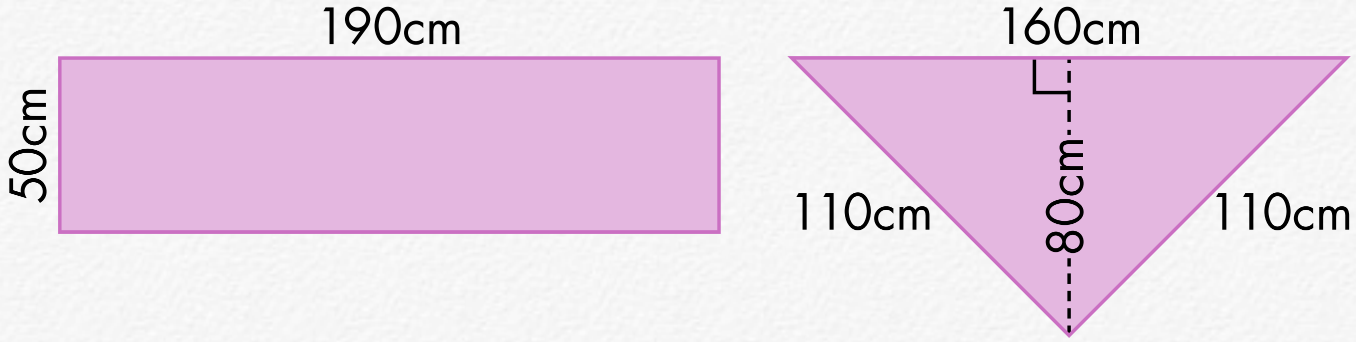
Two coats of paint are required,

so double the area: $2 \times 15\ m^2 = 30\ m^2$

Answer: **The area to be painted is $30\ m^2$, so one tin of 'raspberry diva' paint will be sufficient to paint two coats on Rima's living-room wall, because one tin contains sufficient paint for $40\ m^2$.** (If Rima puts the lid firmly back on the tin, the paint will keep for later touch-ups.)

Classic Build: Interior of City Hall, London, England (opened 2002).

3) Fashion designer Maria needs to minimise the amount of silk she will need for her new line of shawls, which are individually woven.
Maria's two designs are drawn below.
Which shape shawl will use the least amount of silk?



Traditional Russian Shawl.

Calculate the area of each shawl separately:

Area of the rectangular shawl.

First, write down the formula: → **Area of a rectangle = length x width**

Then insert the numbers: → = 190 x 50

Area of the rectangular shawl = 9500 cm^2

Area of the triangular shawl.

First, write down the formula: → **Area of a triangle = (base x height) ÷ 2**

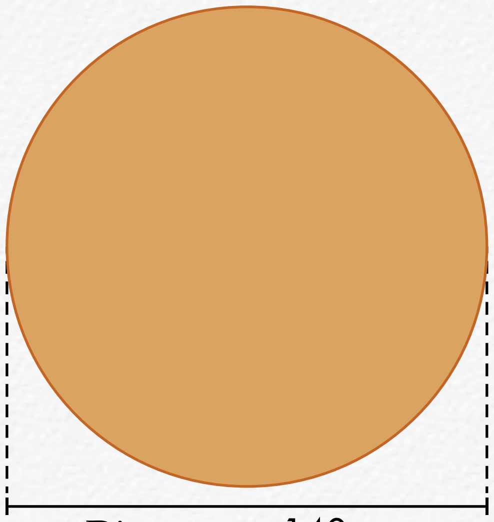
Then insert the numbers: → = (160 x 80) ÷ 2

Use BIDMAS: brackets first: = 12800 ÷ 2

Area of the triangular shawl = 6400 cm^2

Answer: **The triangular shawl will use the least amount of silk.**

4) Janice wants to buy a kitchen table that will seat **6** people and that is not too big for her kitchen, so she needs to choose the table with the smallest area. The measurements of the two tables are shown below.
Should Janice choose the rectangular or circular table?



Calculate the area of each table separately:

Area of a circular table.

The only measurement you need to know is the radius. The diameter of the table is 140 cm. The radius is half the diameter, which is 70 cm.

First write down the formula: **Area of a circle** = π x r x r

Then insert the numbers: = 3 x 70 x 70

= 3 x 4900

Area of a circular table = 14,700 cm^2

Area of a rectangular table.

First write down the formula: **Area of a rectangle = length** x **width**

Then insert the numbers: = 150 x 75

Area of a rectangular table = 11,250 cm^2

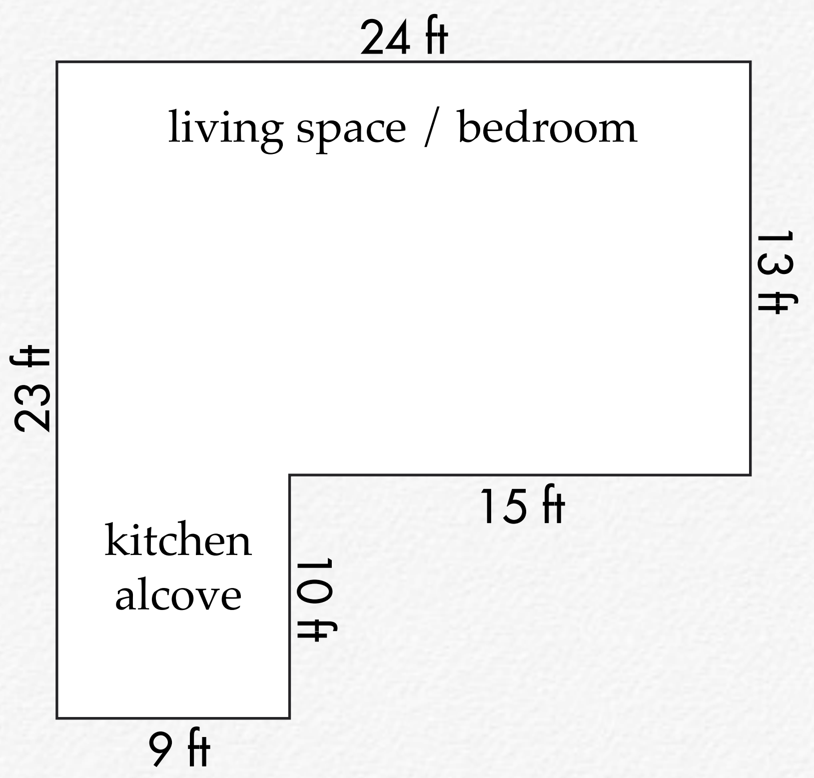
Answer: **Janice should choose the rectangular table as it takes up least room**.

5) Rita wants to sell her studio flat.

Her local estate agent wants to know the area.

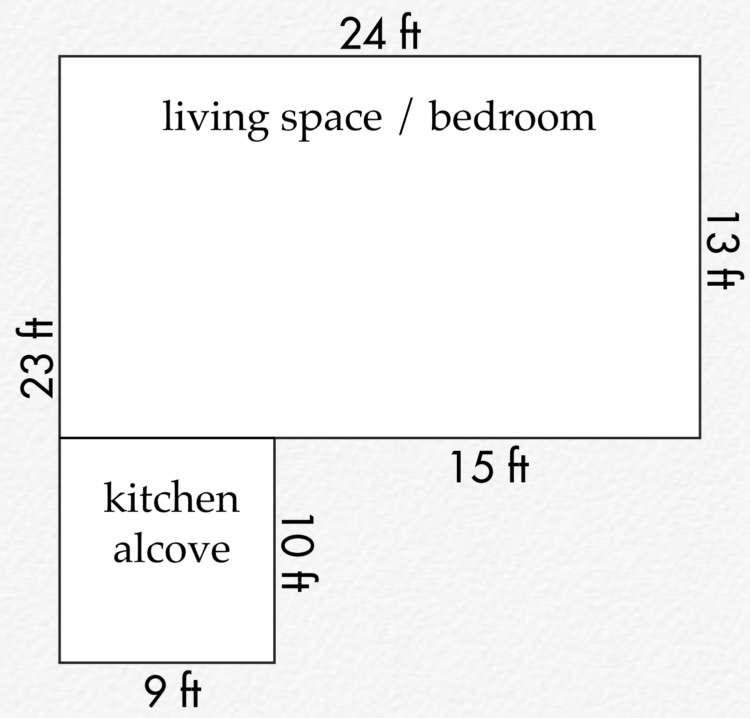
Rita measured her flat and drew the plan shown below.

What is the area of Rita's studio flat?



To calculate the area, add an extra line

to change this L-shape into two rectangles, as shown.



Now calculate the area of each rectangle separately.

Area of a living space / bedroom = **length** x **width**

Be careful to choose the correct length & width: = 24 x 13

= 312 sq.ft

Area of a kitchen alcove = **length** x **width**

= 9 x 10

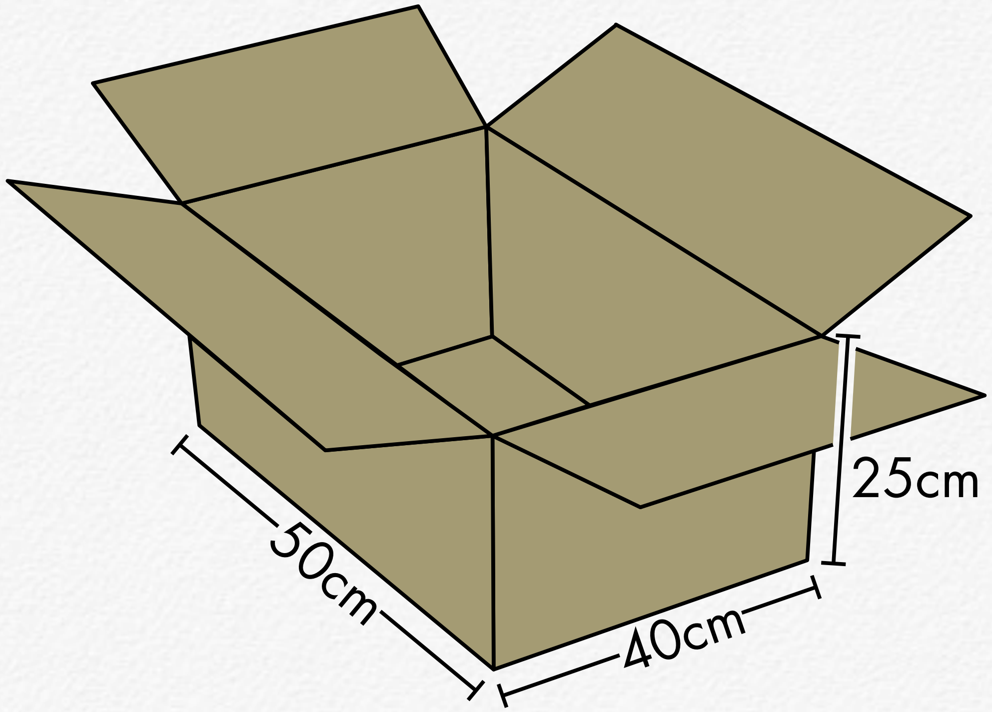
= 90 sq.ft

To calculate the total area, **add** the two areas together.

312 sq.ft + 90 sq.ft = 402 sq.ft

Answer: **The area of Rita's studio flat is 402 sq.ft.**

6) What is the volume, in litres, of the cardboard box shown to the right?



Volume of a cuboid = length x width x height

= 50 x 40 x 25

= 50,000 cm^3 (cubic centimetres)

The volume of the box is **50,000 cm^3**, but you now need to **convert cm^3 to litres**; use the chart method for this.

You know that **1** litre is equal to **1000 cm^3**. Insert the amount that you want to convert in the correct column for its unit.

	litres	cm^3	
1 litre = 1000 cm^3	1	**1000**	Multiply the diagonal numbers
50,000 cm^3 = ?	?	50,000	Divide by the remaining number

The calculation is:

1 x 50,000 = 50,000

50,000 ÷ 1000 = **50**

Answer: **The volume of the cardboard box is 50 litres**.

Classic Build:
Tower Bridge
(opened 1894).
City Hall Building
(opened 2002).
London, England.

YOUR BRAIN WORKOUT

Question 1 of 7.
Which of the following triangles has three equal sides?

- **A.** Scalene
- **B.** Equilateral
- **C.** Isosceles
- **D.** Right angled

YOUR BRAIN WORKOUT

Question 2 of 7.

How many right angles are there in a rectangle?

- **A.** 1
- **B.** 2
- **C.** 3
- **D.** 4

YOUR BRAIN WORKOUT

Question 3 of 7.

Which of the following shapes has seven sides?

- **A.** Heptagon
- **B.** Pentagon
- **C.** Octagon
- **D.** Hexagon

YOUR BRAIN WORKOUT

Question 4 of 7.

The 3D shape of a ball is called

- **A.** Circle
- **B.** Cylinder
- **C.** Cone
- **D.** Sphere

YOUR BRAIN WORKOUT

Question 5 of 7.

A typical shoe box is a...

- **A.** Cube
- **B.** Cuboid
- **C.** Tetrahedron
- **D.** Triangular prism

YOUR BRAIN WORKOUT

Question 6 of 7.

A triangle with two sides the same length is called

- **A.** an equilateral triangle
- **B.** a right angled triangle
- **C.** an Isosceles triangle
- **D.** a scalene triangle

YOUR BRAIN WORKOUT

Question 7 of 7.

How many sides does a nonagon have?

- **A.** 6
- **B.** 9
- **C.** 7
- **D.** 8

YOUR BRAIN WORKOUT

Answers

Q1. Equilateral

Q2. 4

Q3. Heptagon

Q4. Sphere

Q5. Cuboid

Q6. An Isosceles triangle

Q7. 9

YOUR BRAIN WORKOUT

Question 1 of 3.

What is the area of
a room measuring
3 metres by 4 metres?

YOUR BRAIN WORKOUT

Question 2 of 3.

What is the area of
a garden measuring
12 metres by 8 metres?

YOUR BRAIN WORKOUT

Question 3 of 3.

What is the area of
a rug measuring
2 metres by 1.5 metres?

YOUR BRAIN WORKOUT

Answers

Q1. $12\ m^2$

Q2. $96\ m^2$

Q3. $3\ m^2$

PART 30
WATCHING YOUR INTERESTS

A party can cost more than you calculated,
so always add an extra 20% to your planned budget.

Quick Quiz

Question 1 of 4.

What is the answer to :

5 – 2 x 4 + 3 =

- **A.** 15
- **B.** 21
- **C.** 0
- **D.** 1

Quick Quiz

Question 2 of 4.

Which of the following fractions is equal to a third?

- **A.** $\frac{6}{12}$
- **B.** $\frac{4}{8}$
- **C.** $\frac{3}{9}$
- **D.** $\frac{3}{6}$

Quick Quiz

Question 3 of 4.

What is the answer to :

$3a - 2 - a + 5a + 5 =$

- **A.** $9a + 7$
- **B.** $9a + 3$
- **C.** $8a + 3$
- **D.** $7a + 3$

Quick Quiz

Question 4 of 4.

What is the correct calculation to work out how to share 30 cherries between 10 children?

- **A.** $30 \div 10$
- **B.** $10 + 30$
- **C.** $30 - 10$
- **D.** 10×30

Quick Quiz

Answers

Q1. 0

Q2. $\frac{3}{9}$

Q3. $7a + 3$

Q4. $30 \div 10$

Interest

This section of MONEY STUFF will save you the most money… and saving money is as important as earning it. If you save your money in a piggy bank, it just stays there until you want to spend it; but you can use that money to **make more money** if you take it out of the piggy bank and save it in a real bank, which will pay you a small amount every year.

This small amount is called **interest**. Your interest is always calculated as a percentage of the money deposited in your deposit account, which is called the 'capital sum', or '**capital**'. (More later.)

Interest rates vary from bank to bank and from day to day, depending on world market conditions: in 2012, bank interest varied from 0.5% in Canada or 2.3% in France or 3.6% in Australia. Higher interest rates were available on money deposited, as follows: 11.6% in Kenya, 16.4% in Turkey, 22.3% in Belarus. Different banks pay different interest rates in the same country. So if you have money to deposit, always shop around, try different banks.

Note

The bank pays you **5%** interest, then they lend your money to someone else at **7%** interest and keep the difference, which is **2%**. That's basically how High Street banks pay their business costs and make a profit.

Watch out for credit card interest!

Deposit Account

If you put your money in a **current account**, you can withdraw whatever you please whenever you please, but the interest rate will be either zero or very low. It's better to save money in a **deposit account**, which you can start at any time. This may be an account in which your money is deposited only for a stipulated period, during which time you can't withdraw a penny without paying a penalty. The agreed period (called a **fixed term**) may be **1** month, **3** months, **6** months, a year, two years, or even longer. You may earn higher interest in a **long-term account**.

Bank transfers. It takes a bank a few minutes to transfer a sum of money from your account A to your account B. It takes the same few minutes to transfer money from Manchester in Britain to Perth in Australia. Watch out for banks taking **2** or **3** days – or longer – to transfer big sums from account A to account B, because the bank is embezzling **2** or **3** days interest on your money. If you are making a big transfer, make sure this doesn't happen to you. Speak to a real person in the bank and then confirm your verbal instructions by email. Make it clear that you do not expect this money transfer to take longer than one working day and get this agreed, **in writing and dated**. Email is fine, because messages are automatically dated.

Too many credit cards can be dangerous.

How to Calculate Interest

Interest rates are written as an annual percentage: for example **4.8% pa** (per annum) means that your money will earn **4.8%** of the amount in your savings account for every year that you leave it on deposit.

If you put **£100** on deposit on 1st January, by the end of the year you will have: **£100** $\times \frac{4.8}{100}$ = **£4.80** interest.

So you will then have **£104.80** in your account.

Numbers are needed
to measure cooking quantities.

To calculate how much your interest will be, get in the habit of using the basic chart method:

Example of 1st year's interest calculation

Would-be pop-singer, Tamara Pink, decides to save **£800** of her earnings in a Deposit Account with interest at **6.6%**.

If Tamara doesn't withdraw any of her money in the next **12** months, she will gain **6.6%** interest.

How much interest will Tamara's money earn in a year?

First, decide which number represents **100%** of Tamara's savings. The **whole amount** of money deposited in the bank is **always 100%**, = **£800**, in Tamara's case.

Next, draw the chart with headings for **£** and **%**, then fill it in.

Insert the equivalent amounts that you know:
£800 = 100% of cash deposit.
Remember this important point.

Interest rate = 6.6% →

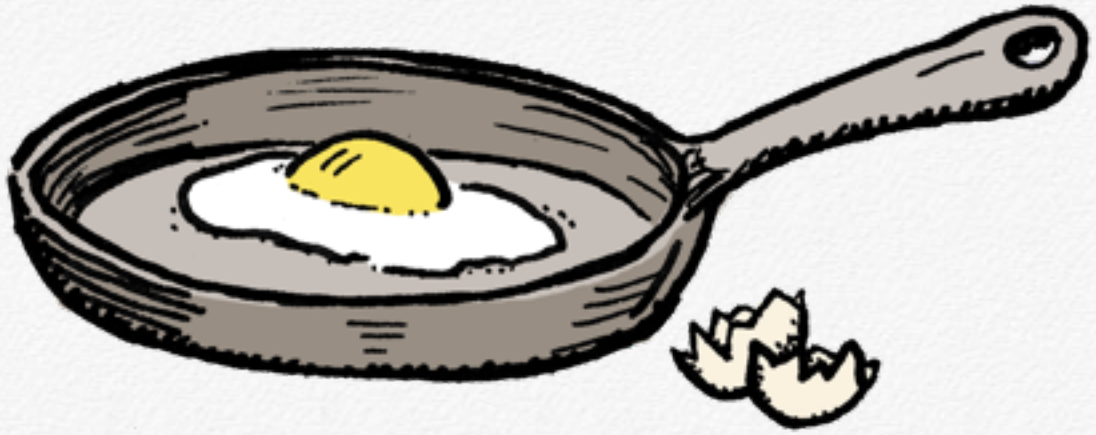
£	%
800	100
?	6.6

Remember the chart rule: **Multiply the diagonal numbers, then divide by the remaining number**.

£	%
800	**100**
?	6.6

Multiply the diagonal numbers

Divide by the remaining number



The calculation is: 800 x 6.6 ÷ 100 = 52.8

So, the total amount of money Tamara has in her bank account is:

£800 (her original deposit) **+ £52.80** (one year's interest on **£800**) **= £852.80**

Answer: **In 12 months, Tamara Pink's savings will earn £52.80 interest on her £800 bank deposit**. The bank will automatically add **£52.80** to her bank balance (unless she gives different instructions to her bank). So at the beginning of the second year, Tamara Pink will have **£852.80** in her bank account.

Looking back, that last calculation can be worked out in just one step rather than two. You start by adding **100%** (representing all Tamara's money) and **6.6%** (the interest rate). **100 + 6.6 = 106.6%**. Then in only one step, the basic chart method will give you the **total amount** in Tamara's bank account at the end of the year. (Note: Before adding those two numbers, I removed both the percentage symbols.)

	£	%
Remember £800 = 100% of cash deposited	800	**100**
	?	106.6

As always, **multiply the diagonal numbers, then divide by the remaining number**.

So the calculation is: 800 x 106.6 ÷ 100 = 852.80

Answer: **In 12 months, Tamara's £800 will have grown to £852.80**.

Career choice?
You'll need numbers.

Compound Interest

Einstein supposedly said that **compound interest** was the greatest invention of mankind.

The basic principle is a simple concept. So here's how to calculate compound interest.

For the second year that Tamara leaves her **initial deposit** (the original capital sum of £800) in her deposit account, her money will earn interest on her initial capital, and will also earn a little bit of interest on the first year's interest: Tamara's money earned £52.80 in the first year on deposit.

So, from the end of the first year, **Tamara will earn interest on her interest of £52.80, which increases her capital sum**. 6.6% interest on £52.80 is £3.48.
Not much you might think… but wait and see.

Rabbits multiply on a similar principle (see later).

Example of 2nd year's interest calculation

At the start of the second year, Tamara Pink has additional money in her deposit account, due to the addition of the first year's interest. At the end of the second year, Tamara's money will have earned **6.6%** of her increased deposit of **£852.80**.

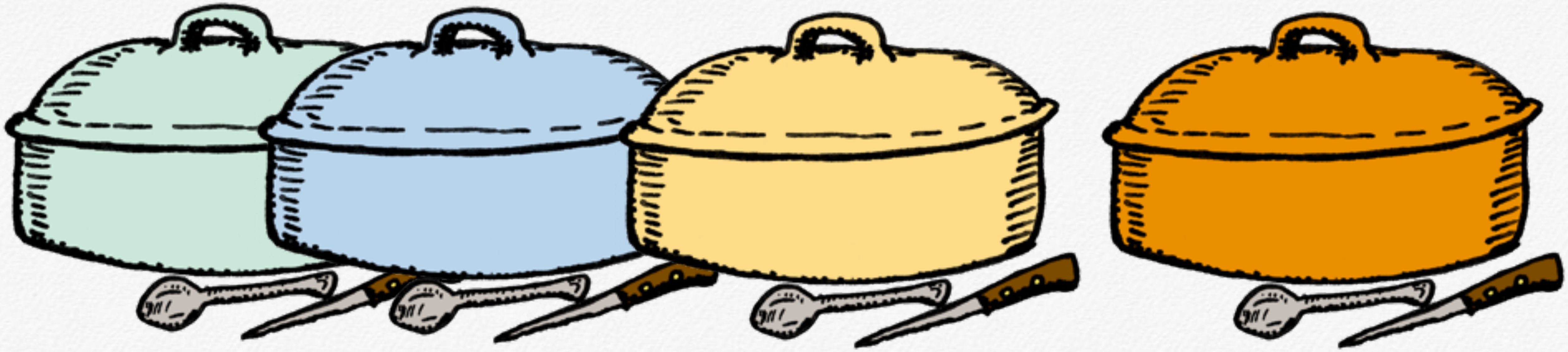
Use the chart method to calculate how much money Tamara will have by the end of the second year.

The calculation is:

	£	%	
£852.80 = 100% of Tamara's savings.	852.80	**100**	Multiply the diagonal numbers
100% plus interest rate = 106.6% →	?	106.6	Divide by the remaining number

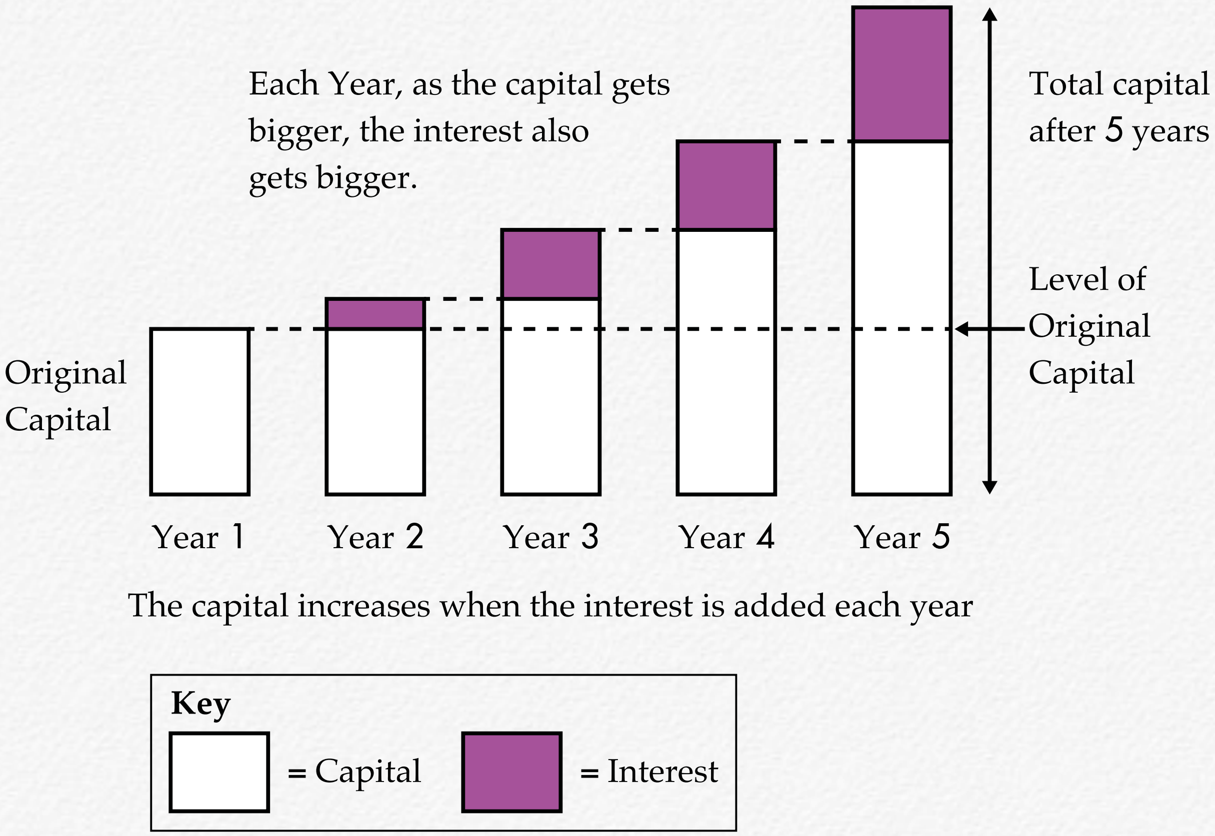
852.80 x 106.6 ÷ 100 = 909.08

Answer: **At the end of the second year, Tamara's savings will have increased to £909.08**.



Each year the amount of interest grows bigger because the interest is calculated on Tamara's original capital **plus** the accumulated interest that is piling up, year after year.

Diagram that Demonstrates the Theory of Compound Interest



For each successive year that Tamara leaves her money in that deposit account, the capital will increase, so the interest will **also** increase every year.

You need to clarify whether any interest that you are being paid is simple interest or compound interest, because it makes a huge difference in the long run. This clarification is even more important if **you** are paying interest on a debt, such as a mortgage, business bank loan or a credit card debt.

The reason that bewildered people find themselves with enormous credit card debt is because THEY ARE BEING CHARGED COMPOUND INTEREST, not simple interest; but because they don't understand the difference, they haven't done the calculations to check the maths.

Makeup artist **concentrates** on model's face.

Look at the chart below for the difference between compound interest and simple interest.

Simple interest is calculated when interest is earned only on the original capital.

Compound interest is calculated when the interest is earned on the original capital, plus on the accumulated interest.

Simple interest calculations:

For **£800** deposited at **6.6%** annual interest for **4** years,

Starting bank balance	Year 1		£800.00
Interest	6.60%	+	£52.80
End balance is starting balance + interest	Year 1		£852.80
Starting bank balance	Year 2		£852.80
Interest	6.60%	+	£52.80
End balance is starting balance + interest	Year 2		£905.60
Starting bank balance	Year 3		£905.60
Interest	6.60%	+	£52.80
End balance is starting balance + interest	Year 3		£958.40
Starting bank balance	Year 4		£958.40
Interest	6.60%	+	£52.80
End balance is starting balance + interest	Year 4		£1,011.20

the total money in account at the end of 4^{th} year, (which includes 4 years' interest) of £211.20 = £1,011.20 total.

If Tamara's savings account earns **simple interest**, Tamara's money earns only **£52.80** every year, so by the end of the fourth year, the total interest earned is:
£52.80 x 4 = £211.20.

If Tamara's savings account earns **compound interest**, by the end of the fourth year, the total interest earned will be **£233.04**. Not much? Wait and see...

Compound interest calculations:

for **£800** deposited at **6.6%** annual interest for **4** years.

Starting bank balance	Year 1		£800.00
Interest	6.60%	+	£52.80
End balance is starting balance + interest	Year 1		£852.80
Starting bank balance	Year 2		£852.80
Interest	6.60%	+	£56.28
End balance is starting balance + interest	Year 2		£909.08
Starting bank balance	Year 3		£909.08
Interest	6.60%	+	£60.00
End balance is starting balance + interest	Year 3		£969.08
Starting bank balance	Year 4		£969.08
Interest	6.60%	+	£63.96
End balance is starting balance + interest	Year 4		£1,033.04

Tamara's balance at the end of year **1** is her starting balance of year **2**, and so forth.

Notice that the interest rate percentage STAYS AT 6.6% OVER THE 4 YEAR PERIOD.
So, the amount of interest earned ...increases every year.

In four years, compound interest has earned **£22** extra on Tamara's saving; this may not seem much, but **the interest increases every year**.
(This is an example of **exponential growth**, see later).
The difference between the money earned with **simple interest** and the money earned with **compound interest** will get bigger AND BIGGER.

Remember that Tamara started with **£800** in her deposit account. After **20** years, Tamara would have earned **£1,056** in simple interest but with compound interest, Tamara would have earned **£2,072.33**: this is a difference of **£1,016.33**… which is more than Tamara's original deposit of **£800**.

In that example, Tamara was putting her **£800** savings in the bank and compound interest worked in Tamara's favour.

But supposing Tamara had arranged a **compound interest mortgage** for **£150,000**. The amount she would have to repay would be far larger than if she had arranged a **simple interest mortgage**. This is because compound interest would be in the bank's favour and so it would cost Tamara far, far more.

She’s almost at the end.

So are you.

Compound Interest Formula

Here's **the theory behind the formula for calculating compound interest**. Skip this bit if you don't want to know the theory and go straight beyond it to **the compound interest formula**.

From the preceding examples of Tamara's investment of **£800**, the calculation to find the amount of money in Tamara's account at the **end of the first year** is:

1^{st} year calculation

$$800 \times 106.6 \div 100 = 852.80$$

Alternatively, this can be written as $800 \times \frac{106.6}{100} = 852.80$

The calculation at the **end of the second year** is:

2^{nd} year calculation

$$852.80 \times 106.6 \div 100 = 909.08$$

Alternatively, this can be written as $852.80 \times \frac{106.6}{100} = 909.08$

Notice the fraction $\left(\frac{106.6}{100}\right)$ is in both the 1^{st} year calculation and the 2^{nd} year calculation.

Multiply the amount of money in the bank account at the start of the year by this fraction to calculate the amount of money in the account at the end of the year.

The amount of **852.80** was the result of the 1^{st} year calculation, so replace **852.80** with $\left(800 \times \frac{106.6}{100}\right)$ in the 2^{nd} year calculation.

$$\left(800 \times \frac{106.6}{100}\right) \times \frac{106.6}{100} = 909.08$$

This calculation can be rewritten in a shorter way, using the powerful power of two.

$$800 \times \left(\frac{106.6}{100}\right)^2 = 909.08$$

If Tamara keeps her money in the account for a **third year**, the equation is as follows:

$$800 \times \left(\frac{106.6}{100}\right)^3 = 969.08$$

So after **3** years, Tamara's nest egg will total **£969.08**.

...Meanwhile...

The last calculations explain **how to arrive at the compound interest formula**.
Fill in the formula, by replacing numbers with words in the last calculation.

800 = the original amount of money invested at the beginning of the 1st year

= **starting amount**

106.6 = the percentage interest rate of 6.6% plus 100% (all Tamara's money)

= 100 + **percentage interest rate per annum.**

The **100** below the line changes the percentage to a fraction, for the purpose of the calculation.

The power of **3** represents the number of years of investment = **number of years**.

The compound interest formula is:

$$\text{Total savings} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{to the power of the number of years}}$$

<u>1st Example of Compound Interest</u>

How much money will be in Tamara's account at the end of **5** years?

At sea, nothing is cheap.

First, write out the formula:

$$\text{Total savings} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{number of years}}$$

Next, list the information you will insert into the formula, as follows:

The **starting amount** = £800 (the amount Tamara originally invested)

The **percentage interest rate pa** = 6.6%

No. of years = 5 years

Now, insert those quantities into the formula, replacing the words with numbers.

Next, do the calculation. Remember to do the calculations in the bracket **first**.

$$\text{Total savings} = 800 \times \left(\frac{100 + 6.6}{100}\right)^5$$

$$= 800 \times \left(\frac{106.6}{100}\right)^5$$

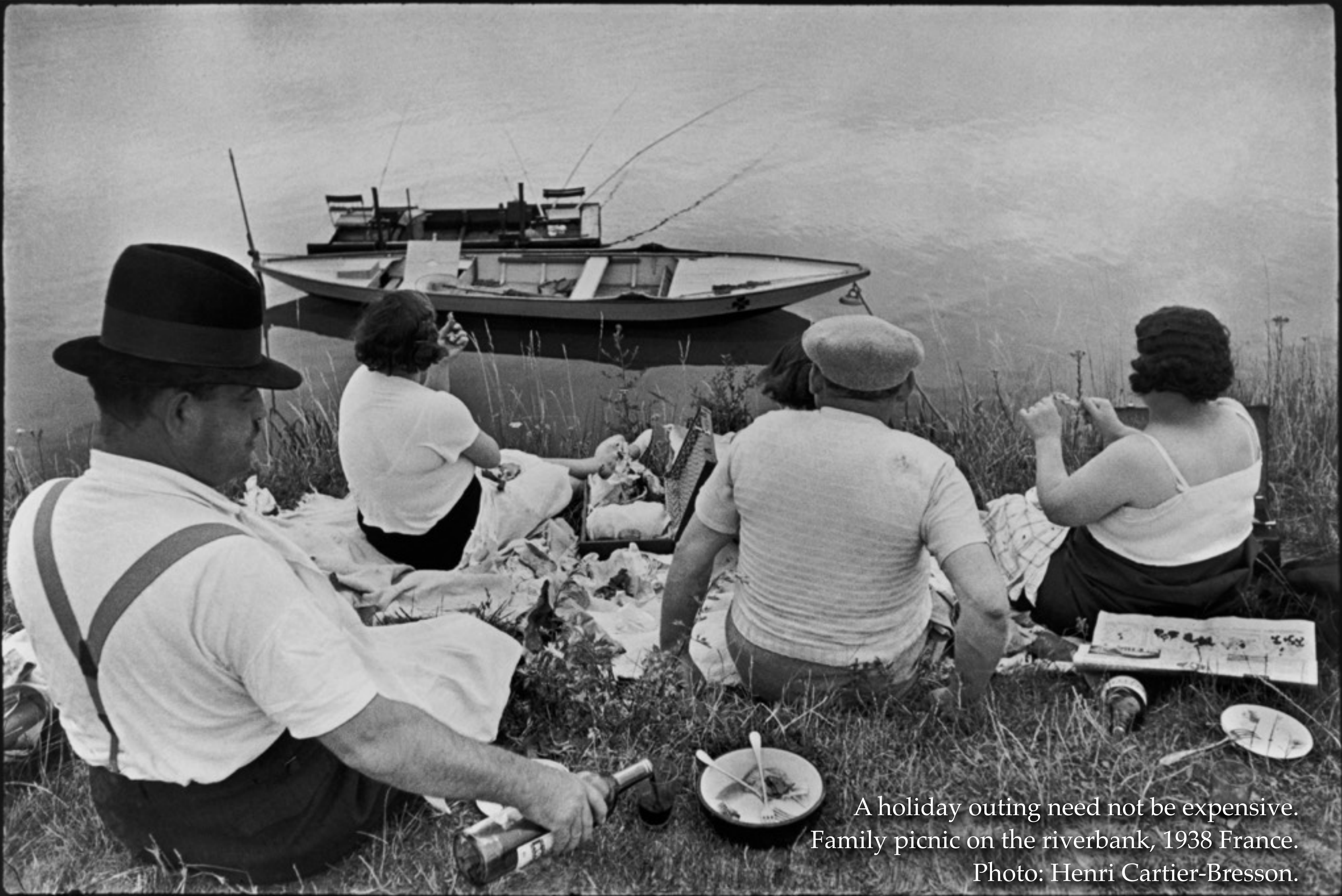
$$= 800 \times (1.066)^5$$

On a **normal calculator**, calculate 1.066 to the power of 5 by tapping in:

800 x 1.066 x 1.066 x 1.066 x 1.066 x 1.066 (Note that 1.066 will be tapped in five times).

It's vital to understand that this is **not** the same as 1.066 x 5, which would give the **wrong answer** of 5.33, whereas the correct answer is 1.377 (rounded to 3 decimal places).

Check this for yourself on your calculator.



A holiday outing need not be expensive.
Family picnic on the riverbank, 1938 France.
Photo: Henri Cartier-Bresson.

The calculation is quicker with a **scientific calculator** or alternatively on your computer calculator, after you select view / scientific.

To work out **1.066** to the power of **5**, tap in **1.066**, then press the $\boxed{x\hat{}y}$ or $\boxed{xy}$ or $\boxed{x^{\blacksquare}}$ or $\boxed{\wedge}$ button, then $\boxed{5}$ and $\boxed{=}$.
The answer should be **1.377** (to **3** d.p.).

Hot tip: you can also type '**1.066** to the power of **5**' into Google and it will give you the answer on the Google calculator.

Getting back to the preceding calculation, you now know:

$$800 \times (1.066)^5 = 800 \times 1.377$$
$$= 1101.22$$

Answer: **After 5 years in her savings account with 6.6% interest pa,**
Tamara's £800 will have grown to £1101.22.

Note: be aware that when you put your money on deposit in a bank, you are lending it to the bank – **so the money no longer belongs to you.**

In Control.

2nd Example of Compound Interest:

How much will Tamara's investment of **£800** be if she invests it for a total of **12** years in the **6.6%** saving account?

First, write out the formula:

$$\text{Total savings} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{no. of years}}$$

Next, list the information to insert in the formula, which is of course the same information that you used in the 1st example, the only difference being the number of years that the money is on deposit.

The **starting amount** = £800
The **percentage interest rate pa** = 6.6%
No. of years = 12 years

Insert those quantities into the formula, replacing the words with numbers.
Next, the calculation (remember to do the calculations in the bracket **first**):

$$\text{Total savings} = 800 \times \left(\frac{100 + 6.6}{100}\right)^{12}$$

$$= 800 \times (1.066)^{12}$$

$$= 800 \times 2.153 \text{ (rounded to 3 decimal places)}$$

$$= 1722.57$$

Answer: **After 12 years in the savings account with 6.6% interest pa,**
Tamara's nest egg of £800 will have more than doubled, to £1,722.57.

To double your money over **12** years sounds good, but remember that income tax may need to be paid on the interest earned, and… because of inflation… in **12** years' time, Tamara's money has doubled – but so has the value of everything else. So Tamara might then still only be able to buy the same sofa that she can purchase now for **£800**. (See later for inflation.)

Temptation.

Interest on All Debts, Including Credit Cards

(Triffid-like Growth Rates)

Compound Interest works **for you** if you are saving money and **rolling over** (not taking out) the interest at the end of each year.

Compound interest works **against you**. If you borrow money from a bank without discussion, you will pay compound interest rates – and the interest rates charged on debts are far higher than the interest earned on savings accounts.

Credit card debt is a trap for many people. A credit card company operates like a bank and is often owned by a bank, and is run like a bank, even if it is a fashion store card.

Once you have been given a credit card by your bank, it's easy to spend… even when you don't have money in the bank. With your credit card, you can buy something now (and pay for it later) with money you have borrowed - that's the same as a small bank loan.

From the moment your credit card is used for a purchase – say a white jacket costing **£150** – you owe **£150** to the credit card company.

Unless you pay the money in full back to the credit card company **EVERY MONTH AND ON TIME**, you will be charged a **high interest rate and perhaps penalty charges**. If your debt of **£150** is left unpaid, the interest to be paid on your debt is added to your debt, which grows bigger and bigger very quickly. The bigger your debt grows, the harder it is to pay it back.

Typically, if you don't pay your minimum repayment amount within two weeks of receiving your bill, the credit card company will charge you a late fee of say **£12** plus an interest rate of about **0.05%**.

Don't be fooled by a credit card interest rate of **0.05%**: that may sound low, but it's **per day**, not per year. This works out at about **20%** per year. Some credit card companies charge up to **29%** a year! (see following calculations).

Example of Credit Card Debt

Chris ignores the beige envelope bills that flutter through her letterbox; she hides these frightening bills in a drawer, together with official letters, often from the bank or credit card company. Out of sight, out of mind, thinks Chris.

The interest rate on Chris's credit card is **0.07%** per day, which doesn't seem much money to her. Chris doesn't realise a) how much she spends, using her credit card and b) that **0.07%** is a **compound** interest rate.

If a debt of **£1,000** is left unpaid on Chris's credit card for a year, how much will the debt be after **12** months – without including any charges?

The compound interest formula for a deposit account is **exactly the same for calculating debt**. First, write the formula.

Because the interest rate given is **per day** instead of per year, **use days instead of years** for this calculation.

$$\text{Total debt} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate per day}}{100}\right)^{\text{no. of days}}$$

Next, list the information you will insert into the formula:

The **starting amount** = £1,000

The **percentage interest rate per day** = 0.07%

Number of days = 365 days

Few people need more than two credit cards.

Insert those quantities in the formula, replacing the words. Next, do the calculation. (Remember BIDMAS. Do the calculations in the bracket **first**):

$$\text{Total debt} = 1000 \times \left(\frac{100 + 0.07}{100}\right)^{365}$$

Because the bracketed fraction contains a plus sign, **you must not cancel out the zeros**.

$= 1000 \times (1.0007)^{365}$

$= 1000 \times 1.29099$

$= £1,290.99$

Answer: **After 1 year, Chris' credit card debt has increased from £1,000 to £1,290.99**.

Chris cut her credit card problem with a pair of scissors: she chopped up all her credit cards. From then on, she only used her bank debit card and was careful not to go overdrawn.

Remember not to mix days, months or years in one calculation: **always use the time period used for the interest rate**. Chris's interest rate is calculated at **0.07% per day**, so **days** must be used for all calculations.

If **penalty charges are included**, the debt will continue to increase, because each month - lets say - an extra **£12** will be added onto the credit card bill. This extra amount of **£12** will **also** accumulate interest and… Chris may be staring incredulously at a total debt of **£1,450** for borrowing **£1,000** for a year.

Because the **£12** penalty charge is added each month, the monthly compound interest rate has to be calculated for each month:

You need maths to be an architect or a designer of any sort.

Example

Example of a credit card interest calculation on a bill of **£1,000** charged at **0.07%** per **day**

= an annual percentage rate (APR) of approx. **29%**.

If the minimum monthly payment is not paid, then the penalty is **£12**.

	Credit card bill when penalty charges are not included.	When penalty charges ARE INCLUDED... plus interest charged on the penalty.	**Total bill**
Jan	1,021.93	12.00	1,033.93
Feb	1,042.15	24.26	1,066.41
March	1,065.00	36.79	1,101.79
April	1,087.59	49.57	1,137.14
May	1,111.44	62.66	1,174.10
June	1,135.02	75.99	1,211.01
July	1,159.91	89.66	1,249.57
Aug	1,185.35	103.63	1,288.89
Sept	1,210.50	117.83	1,328.33
Oct	1,237.05	132.41	1,369.46
Nov	1,263.29	147.22	1,410.51
Dec	1,290.99	162.45	1,453.44

The credit card company is legally obliged to tell you what their **annual percentage rate** (APR) is.
Always look at the APR as a guide – but remember that it is only a rough guide.

Because different credit card companies calculate their rates in different ways, it is impossible to check their calculation without knowing exactly how these are calculated.

So the examples given in MONEY STUFF are **simplified** examples, to show you how compound interest can be calculated: they are **only a guide** to how your credit card bills are calculated.

To suit themselves (naturally) credit card companies set a very low minimum repayment each month in order
a) To encourage you to use the card.
b) Because if you only pay the minimum, you could pay them money for years, if not for ever.

Jemima, who uses her credit card carefully, spends about **£500** a year and always pays the minimum monthly repayment. Nevertheless, because she is only paying the tiny monthly repayment amount, **most of Jemima's minimum monthly payment is paying off interest alone**.

IF YOU CAN, PAY ALL YOUR CARDS OFF MONTHLY.

You can use a scientific calculator for these exercises.

Exercises

1) Sally has deposited **£500** into a post office savings account on behalf of her new granddaughter, Iris. The interest rate is **6% pa**.

Use the formula for compound interest, to calculate how much the **£500** in the savings account will be when Iris receives the account book on her 18^{th} birthday.

2) Melanie paid for an impulse trip to Marrakesh on her credit card. Unfortunately, it was a trip she couldn't really afford.

Melanie's bill is **£1,200**. Melanie decides to pay **£50** to the credit card company each month. The credit card charges interest at **0.045%** per day. (Assume there are **30** days in all months in this question.)

a) If Melanie pays **£50** in the first month, how much interest will she be charged on the remaining debt for the first month?

b) What percentage of Melanie's second month **£50** payment is used to pay the interest on her debt?

c) How many months of **£50** payments need to be made by Melanie before her debt is less than **£1,000**?

"We want you to believe that we are a trustworthy, multinational business that welcomes women."

Answers to Part 30

1) Sally has deposited **£500** into a post office savings account on behalf of her new granddaughter, Iris. The interest rate is **6% pa**. Use the formula for compound interest to calculate how much the **£500** in the savings account will be when Iris receives the account book on her **18th** birthday.

First, write out the formula:

$$\text{Total savings} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{no. of years}}$$

Next, list the information you will insert into the formula:

The **starting amount** = £500

The **percentage interest rate pa** = 6%

No. of years = 18 years

Now, insert those quantities into the formula, replacing the words, and do the calculation
(remember to do the calculations in the bracket **first**):

$$\begin{aligned}\text{Total savings} &= 500 \times \left(\frac{100 + 6}{100}\right)^{18}\\ &= 500 \times (1.06)^{18}\\ &= 500 \times 2.854\\ &= 1427.17\end{aligned}$$

Answer: **By the time baby Iris is 18, the money in her post office savings account will be £1,427.17.**

Sally looks after Iris when her teenage mum, Michelle, is on tour. Aged fifty, Sally finds it tough to look after a baby as well as running her b&b.

Then 'The Sound of Music' played in Manchester, where Michelle's dresser, Imogen, was about to retire!

Eventually, Imogen moved to Brighton to help Sally run her b&b and together, they obtained a mortgage to buy the house next door, to expand the b&b business.

2) Melanie paid for an impulse trip to Marrakesh on her credit card. Unfortunately, it was a trip she couldn't really afford. Melanie's bill is **£1,200**. Melanie decides to pay **£50** to the credit card company each month. The credit card charges interest at **0.045%** per day. (Assume there are **30** days in all months in this question.)

a) If Melanie pays **£50** in the first month, how much interest will she be charged on the remaining debt for the first month?

Melanie paid **£50**, so her debt of **£1200** is **£50** less. **£1200 – £50 = £1150**

The formula:

$$\text{Total debt} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{no. of years}}$$

The **starting** amount = **£1,150**

The **percentage interest rate** = **0.045%** per day

Use **no. of days** instead of no. of years since the interest is per day = **30** days

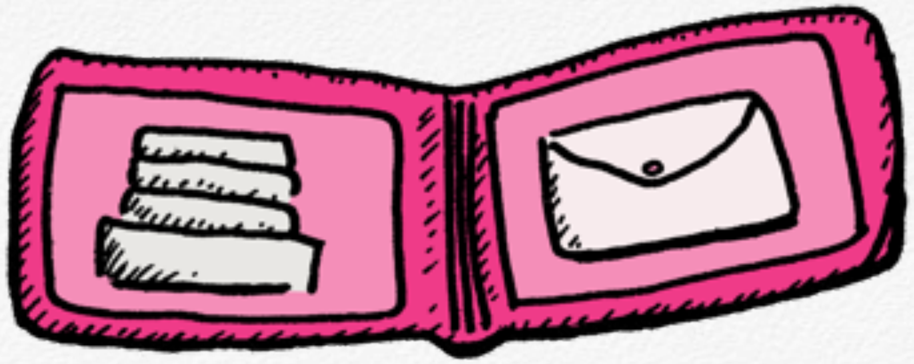
Insert the quantities into the formula:

$$\begin{aligned}\text{Total debt} &= 1150 \times \left(\frac{100 + 0.045}{100}\right)^{30} \\ &= 1150 \times (1.00045)^{30} \\ &= 1150 \times 1.0135 \\ &= 1165.63\end{aligned}$$

Melanie's total debt is now **£1,165.63.**

£1,165.63 – £1150 = £15.63 interest.

Answer: **The interest on £1,150 for one month on Melanie's credit card was £15.63**, for a holiday that Melanie could hardly remember because she was either drinking or hung-over. After being fired from her job at the RSPCA, because she permanently stank of alcohol, Melanie dragged herself to her local substance abuse team and is now drying out in rehab.



b) What percentage of Melanie's second month **£50** payment will be used to pay the interest on her debt?

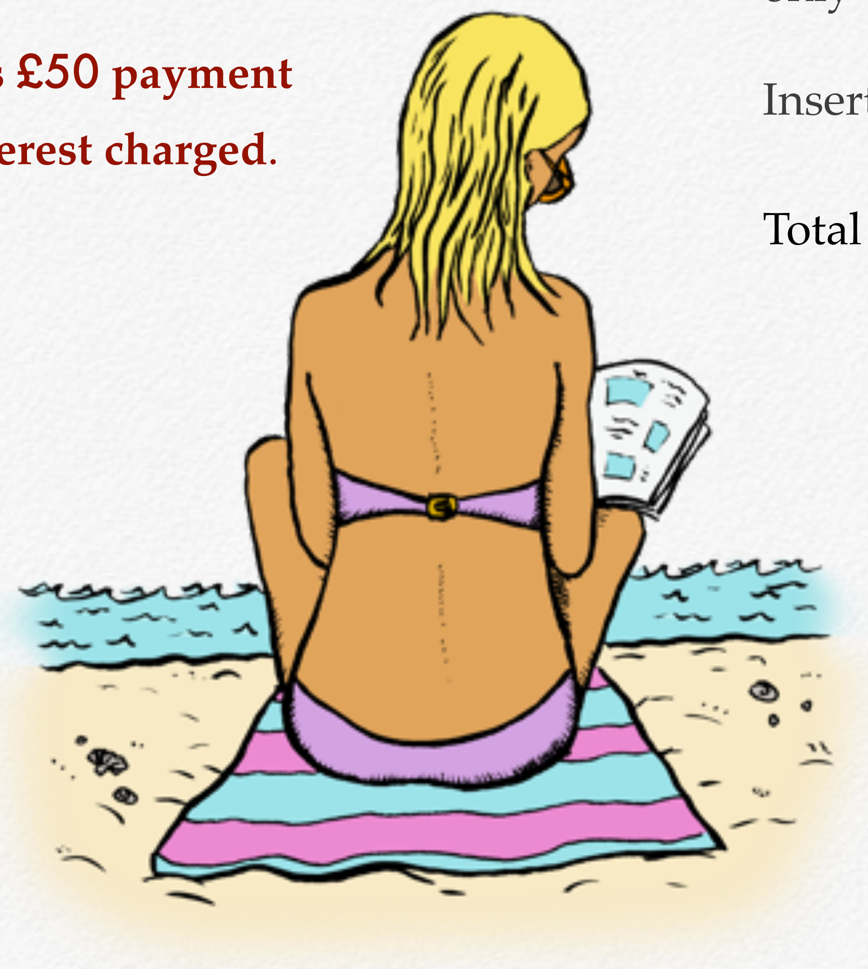
Use the chart method as this is a question of simple percentages and not compound interest.

	£	%
£50 payment = 100%	**50**	100
£15.63 interest = ?%	15.63	?

Multiply the diagonals and divide by the other number.

100 x 15.63 ÷ 50 = 31.3%

Answer: **31.3% of Melanie's £50 payment will be spent paying the interest charged.**



c) How many months of **£50** payments need to be made by Melanie before her debt is less than **£1,000**?

2^{nd} Month

Melanie paid another **£50**, so her debt of **£1165.63** is **£50** less. **£1165.63 – £50 = £1115.63**

The formula:

$$\text{Total debt} = \text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate per day}}{100}\right)^{\text{no. of days}}$$

All the quantities are the same as part a), only the **starting amount** is different = **£1115.63**

Insert the quantities into the formula:

$$\text{Total debt} = 1115.63 \times \left(\frac{100 + 0.045}{100}\right)^{30}$$

$$= 1115.63 \times (1.00045)^{30}$$

$$= 1115.63 \times 1.0136 \text{ (to 4 d.p.)}$$

$$= 1130.80 \text{ (at end of } 2^{nd} \text{ month)}$$

3rd Month

£1130.80 – £50 = £1080.80

Change the starting amount in the previous formula (skip to the last stage of the calculation)

= 1080.80 x 1.0136

= 1095.50 (at end of 3rd month)

4th Month

£1095.50 – £50 = £1045.50

Change the starting amount in the previous formula (skip to the last stage of the calculation)

= 1045.50 x 1.0136

= 1059.72 (at end of 4th month)

5th Month

£1059.72 – £50 = £1009.72

Change the starting amount in the previous formula (skip to the last stage of the calculation)

= 1009.72 x 1.0136

= 1023.45 (at end of 5th month)

6th Month

£1023.45 – £50 = £973.45

Change the starting amount in the previous formula (skip to the last stage of the calculation)

= 973.45 x 1.0136

= 986.69 (at end of 6th month)

Answer: **It took Melanie 6 months to reduce her debt to less than £1,000**.

She paid a total of **£300** in those **6** months which only reduced her overall debt by **£213.31**.

YOUR BRAIN WORKOUT

Question 1 of 7.

What is the interest on £100 invested for a year at 3.5% p.a.?

- **A.** £35
- **B.** £3.50
- **C.** 35p
- **D.** 3.5p

YOUR BRAIN WORKOUT

Question 2 of 7.

What is the interest on **£200** invested for a year at **3.5%** p.a.?

- **A.** £35
- **B.** £70
- **C.** £7
- **D.** 70p

YOUR BRAIN WORKOUT

Question 3 of 7.

What is the interest on £1,000 invested for a year at 3.5% p.a.?

- **A.** £35
- **B.** £3.50
- **C.** 35p
- **D.** 3.5p

YOUR BRAIN WORKOUT

Question 4 of 7.

What is the interest on £3,000 invested for a year at 3.5% p.a.?

- **A.** £35
- **B.** £70
- **C.** £105
- **D.** £130

YOUR BRAIN WORKOUT

Question 5 of 7.

What is the interest you would have to pay on a credit card bill of **£100** that you didn't pay off for a month? The credit card charges **5%** per month.

- **A.** £5
- **B.** £50
- **C.** 50p
- **D.** 5p

YOUR BRAIN WORKOUT

Question 6 of 7.

What is the interest you would have to pay on the credit card debt of **£100** for the second month? The credit card charges you **5%**, per month, and the amount you owe has grown to **£105**.

- **A.** £10.50
- **B.** £5.25
- **C.** £52.50
- **D.** 52.5p

YOUR BRAIN WORKOUT

Question 7 of 7.

After 11 months, the credit card debt has grown to £171.03. What will be the interest on the twelfth month?

- **A.** £17.10
- **B.** £1.71
- **C.** £8.55
- **D.** 85p

YOUR BRAIN WORKOUT

Answers

Q1. £3.50

Q2. £7

Q3. £35

Q4. £105

Q5. £5

Q6. £5.25

Q7. £8.55

YOUR BRAIN WORKOUT

Question 1 of 4.

Use your knowledge of compound interest to **guess** the following answers.

How much will a **£60** dress, bought on a credit card, cost if the bill isn't paid for a year? The interest rate is **3%** per month.

- **A.** £60
- **B.** £65
- **C.** £75
- **D.** £85

YOUR BRAIN WORKOUT

Question 2 of 4.

Use your knowledge of compound interest to **guess** the following answers.

As a rough percentage, how much more expensive does the dress become if the credit card bill is left unpaid for a year?

- **A.** 3%
- **B.** 10%
- **C.** 25%
- **D.** 40%

YOUR BRAIN WORKOUT

Question 3 of 4.

Use your knowledge of compound interest to **guess** the following answers.

Roughly how much much will a **£10,000** car cost if bought on hire purchase, where the buyer pays in monthly instalments for **5** years? The interest rate is **10%** p.a.

- **A.** £10,000
- **B.** £10,100
- **C.** £11,000
- **D.** £12,500

YOUR BRAIN WORKOUT

Question 4 of 4.

Use your knowledge of compound interest to **guess** the following answers.

Roughly how much interest is paid on a mortgage of **£200,000** at an interest rate of **4%** p.a., paid monthly over a **20**-year payment period?

- **A.** £8,000
- **B.** £9,000
- **C.** £90,000
- **D.** £900

YOUR BRAIN WORKOUT

Answers

Q1. £85

Q2. 40%

Q3. £12,500

Q4 £90,000

PART 31
HOW TO MAKE MONEY

Keep alert.

Quick Quiz

Question 1 of 4.

Which is a correct sequence from the 75 times table?

- **A.** 75, 150, 220, 300...
- **B.** 75, 105, 225, 330...
- **C.** 75, 150, 225, 300...
- **D.** 75, 150, 225, 305...

Quick Quiz

Question 2 of 4.

Write 18 as a product of its prime factors.

- **A.** 3 x 6
- **B.** 2 x 9
- **C.** 4 x 4.5
- **D.** 2 x 3 x 3

Quick Quiz

Question 3 of 4.

Which is **not** a square number?

- **A.** 12
- **B.** 9
- **C.** 16
- **D.** 4

Quick Quiz

Question 4 of 4.

How many centimetres in **2** metres?

- **A.** 1,000
- **B.** 2,000
- **C.** 200
- **D.** 100

Quick Quiz

Answers

Q1. 75, 150, 225, 300...

Q2. 2 x 3 x 3

Q3. 12

Q4. 200

Exponential Growth

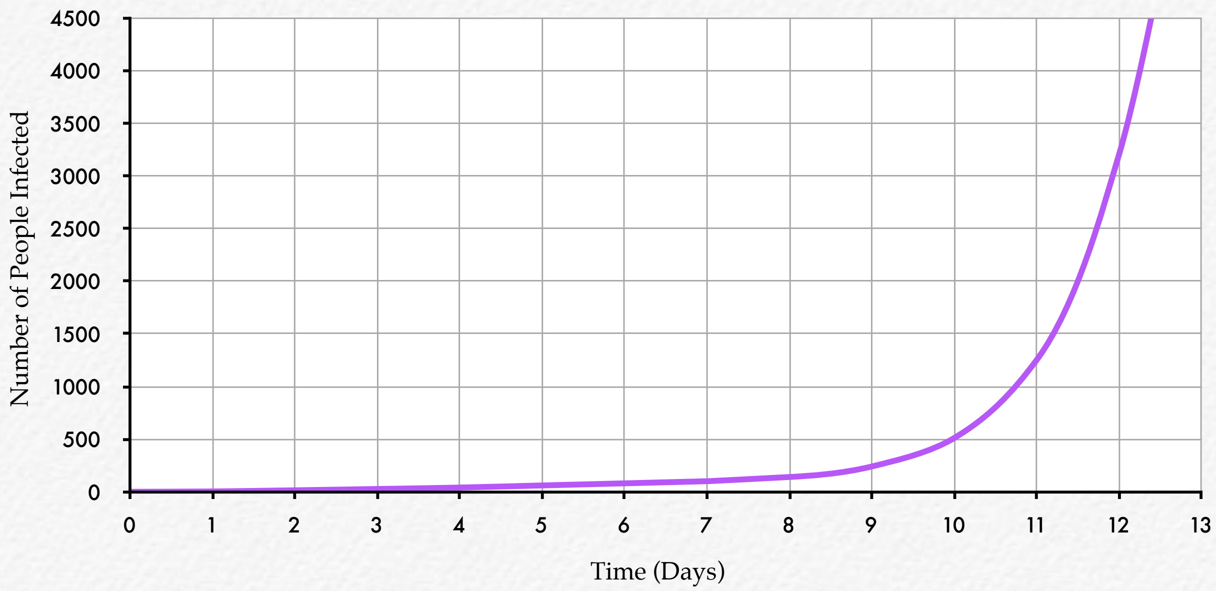
Have you ever had a chain letter asking you to send a five pound note to somebody and assuring you that, shortly afterwards, you could expect to have a thousand fivers sent to you?

But those fivers never arrive. The supposed growth of your fiver into a thousand fivers is an example of **exponential growth** (see later).

In everyday understanding, 'exponential' is simply an adjective that describes a particular sort of growth. It is a **growth rate which starts slowly and escalates surprisingly quickly**; for example, think of the virus that caused the Black Death. The Black Death, a ghastly plague which originated in Central Asia, reached Britain in 1348.
It crept across the country at the rate of half a mile a day. Approximately half the British population perished.

Put simply, an exponential growth describes a number which is multiplied by itself again and again and again. A measles or influenza epidemic starts with one person, the virus quickly spreads to two people who each spread it to another two people, at the same rate… so the infection quickly **multiplies** – possibly beyond control. The following example is particularly relevant to everyday life and emphasises the need for an infected person to avoid crowded areas.

Fig 1. Graph showing the number of people infected with flu over a 12 day period



The curve of this graph shows **the exponential growth of a flu infection**. Within the first **ten days**, the flu virus spreads to 500 people; within the next **two days**, the flu virus spreads to 4,000 people. However, an influenza epidemic never increases according to such a neat progression, because not everyone will pass it on to the same number of people.

You already know the two main components of **exponential growth**: the **powers** and **compound interest**.

Example of a simple exponential growth:

Using powers.

3^1 = **3**

$3^2 = 3 \times 3$ = **9**

$3^3 = 3 \times 3 \times 3$ = **27**

$3^4 = 3 \times 3 \times 3 \times 3$ = **81**

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$ = **243**

The number **3** has quickly grown to **243** in just four jumps.

Using Compound Interest.

Check, "Diagram that Demonstrates the Theory of Compound Interest". Check the pink blocks. You can see that the amount of interest that's added to the money already in Tamara's bank account **grows bigger BY A BIGGER AND BIGGER AMOUNT every year**. This is a neat demonstration of exponential growth.

Watch Out!

Steady growth is **not** exponential growth.

With **steady growth**, the number increases steadily, **by the same amount** every time: every week, month, year, etc...

Examples of steady growth:

a) 3, 5, 7, 9, 11… Here, the number **2** is added every time.

3 (+ 2) 5 (+ 2) 7 (+ 2) 9 (+ 2) 11… The number has not grown much bigger in **4** jumps.

b) 4, 7, 10, 13, 16… Here, the number **3** is added every time.

4 (+ 3) 7 (+ 3) 10 (+ 3) 13 (+ 3) 16… The number has not grown much bigger in **4** jumps.

c) 5, 9, 13, 17, 21… Here, the number **4** is added every time.

5 (+ 4) 9 (+ 4) 13 (+ 4) 17 (+ 4) 21… The number has not grown much bigger in **4** jumps.

Fashion design earns billions, worldwide.

How to Bankrupt Your Parents

Once a smart child has learned the secret of **exponential growth**, she could – in theory – bankrupt her parents by asking for a change in pocket money.

Example

Sunita's parents run a prosperous corner grocery store and punish Sunita if her maths marks aren't good, "For you will never be rich if your maths is not good", they scold.

On her 14th birthday, Sunita asks her parents for a change in her pocket money. She suggests that instead of getting her usual **£5** per week, she get **1p** in the first week, **2p** in the second week, 4p in the third week, **8p** in the fourth week and so on, **doubling the amount each week**. If her parents agree to the new scheme, will Sunita be given more or less pocket money in the next six months?

In 26 weeks (6 months), Sunita's pocket money will increase in the following pattern:

1p, 2p, 4p, 8p, 16p…… Sunita is cunningly using exponential growth: each week, her pocket money increases by a **bigger** amount.

Each week, the previous week's pocket money is multiplied by 2:

1st Progression

Week 1 = 1p = 1p

Week 2 = 1p x 2 = 2p

Week 3 = 1p x 2 x 2 = 4p

Week 4 = 1p x 2 x 2 x 2 = 8p

Week 5 = 1p x 2 x 2 x 2 x 2 = 16p

These calculations can be written more easily if you use **powers** (also referred to as **index notation**).

Example:

$2^2 = 2 \times 2$, and $2^4 = 2 \times 2 \times 2 \times 2 = 16$, and so on.

Remember what a power is? The little number hanging in the air to the right of a big number, which indicates how many times that big number is to be multiplied by itself.

2^{nd} Progression

Week 1 = 1p	= 1p
Week 2 = 1p x 2	= 2^1p
Week 3 = 1p x 2 x 2	= 2^2p
Week 4 = 1p x 2 x 2 x 2	= 2^3p
Week 5 = 1p x 2 x 2 x 2 x 2	= 2^4p

You can now see that, in each successive week Sunita's pocket money is building up by **2 to the power of an increasingly large number**.

In order to calculate Sunita's pocket money progression for 26 weeks, her parents would need to continue the progression calculation from week **5**, to week **6**, to week **7**, ... and so on to week **26**, which would involve twenty five **x2**s, = 2^{25}.

As there are **26** weeks in the progression, why is the little power number **25** not **26**?
Because Sunita gets **1p** for the first week, so the progression doesn't start until the second week, when it doubles from **1p** to **2p**.

Here's the progression written only using powers:

3^{rd} Progression

Week 1	=	2^0p*
Week 2	=	2^1p
Week 3	=	2^2p
Week 4	=	2^3p
Week 5	=	2^4p
⋮		
Week 25	=	2^{24}p
Week 26	=	2^{25}p

*At this point you need to know a new fact.
Any non-zero number to the power of zero = **1**.

So $2^0 = 1$, $4^0 = 1$, $25^0 = 1$, $942^0 = 1$.
At this stage, please accept that this is so.

Sunita's Pocket Money Growing Exponentially

Week	Amount (£)
1	0.01
2	0.02
3	0.04
4	0.08
5	0.16
6	0.32
7	0.64
8	1.28
9	2.56
10	5.12
11	10.24
12	20.48
13	40.96
14	81.92
15	163.84
16	327.68
17	655.36
18	1,310.72
19	2,621.44
20	5,242.88
21	10,485.76
22	20,971.52
23	41,943.04
24	83,886.08
25	167,722.16
26	335,544.32
27	671,088.64
28	1,342,177.28
29	2,684,354.56
30	5,368,709.12

If Sunita had bargained for another 4 weeks – making 30 weeks in all – you work out what she would have claimed from her parents.

At the end of the **26** weeks, Sunita's pocket money will have increased to 2^{25}p.

So how much money would Sunita receive in week **26**?

Use the $\boxed{x^y}$. button on your scientific calculator, the $\boxed{x\hat{\ }y}$. button on your computer, scientific calculator, or Google the answer.

Answer: 2^{25} = **33,554,432p**. To change the pence to pounds, divide the answer by **100** = **£335,544.32**, undoubtedly more than Sunita's parents had expected. "What you ask for is not pocket money but a dowry fit for a princess", fumed her father, at the twelfth week. "But Dad, I'm just being good at maths which is what you've always wanted", said Sunita. "True, true", said Sunita's dad. Eventually, he doubled her pocket money to an allowance of **£10** a week, because now Sunita was fourteen, she had outsmarted her parents using maths, and was no longer a child.

Number Patterns

Wallpaper patterns and fabric patterns are designs that use the repetition of shapes, such as the polka dot pattern. In maths, a pattern is formed by the repetition of a mathematical action. The three-times table is a pattern, formed by repeatedly adding 3. The four-times table is a pattern formed by repeatedly adding 4. Sunita's pocket money progression was a pattern of exponential growth formed by repeatedly multiplying by 2 each week $= 2^0, 2^1, 2^2, 2^3, 2^4, 2^5 \ldots$

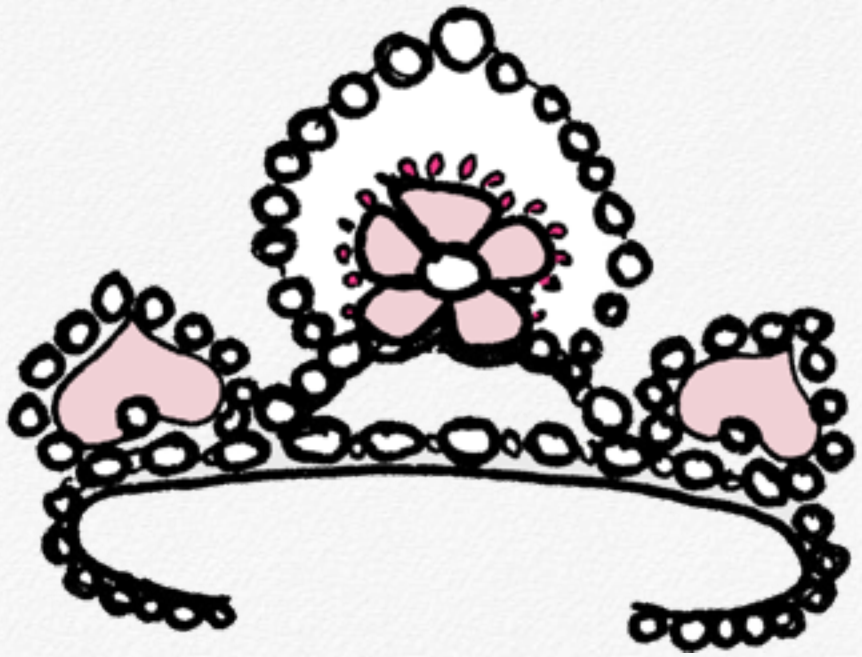
Sunita goes shopping on week 16

Good design adds value.
Designer: Tom Dixon Studio.
Photographer: Tom Mannion.

The Rothschilds' Secret

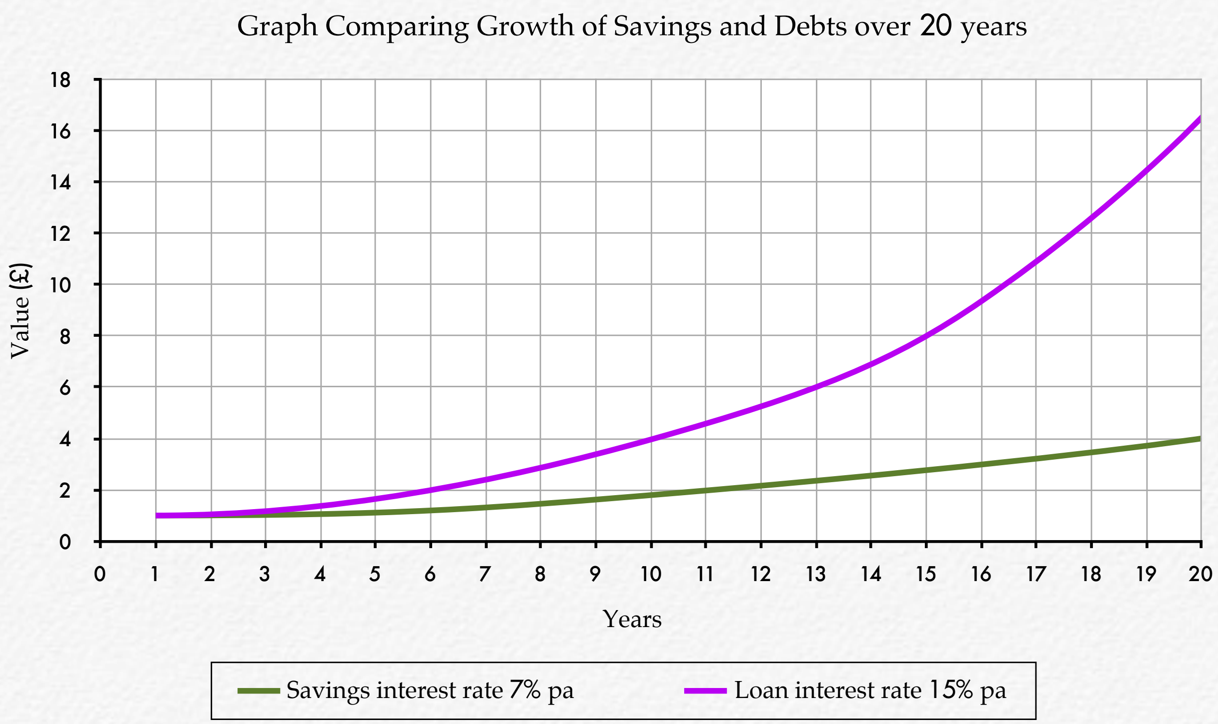
The founder of the famously rich Rothschild dynasty was a banker, who started the family fortune by using exponential growth to calculate the interest on big sums of money that he loaned to kings and governments to pay for their wars.

Any number that has an increasing **power** (or index number), **grows exponentially**. The rate of exponential growth depends on the size of that number. 1 does not change when it is multiplied by itself, for example $1^{99} = 1$. But any number greater than 1 will grow exponentially. Big numbers grow much faster than small ones. The following graph shows this:

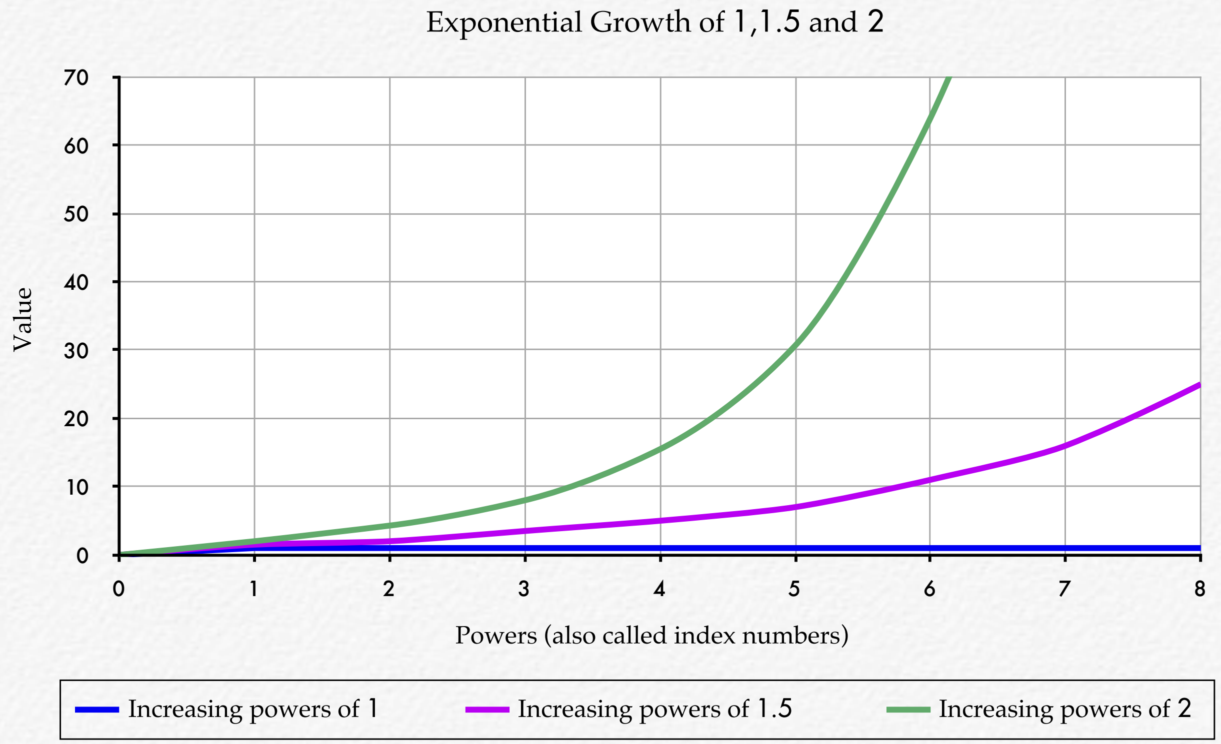


Any figure that grows exponentially will also have the same shaped graph; a curve which starts off almost horizontal and ends up almost vertical. The speed with which the curve becomes almost vertical depends on the starting number.

Powers are used in the formula for compound interest, so compound interest also grows **exponentially**, whether it's on your savings (good) or interest claimed by your bank on your unpaid debts (oh dear). The following graph compares a savings account **high** interest rate of 7% pa, with a loan account **low** interest rate of 15% pa.



Football has grown exponentially from a local interclub game to a maths-essential global industry.



If you look back at the **compound interest formula**, the size of the number being multiplied by **powers** depends on the interest rate, so the curve for the debt is steeper than curve for the savings.

That is the basis of banking. The banker pays low interest on the money that savers deposit with the bank and charges much higher interest on loans made to the bank's customers. So those loans are, theoretically, funded from the money deposited by the bank's savers.

Steer clear of pyramid schemes.

The Pyramid Money-Making Scheme...

...and why it won't work for you.

An example of **extreme exponential growth** is a pyramid money-making scheme:

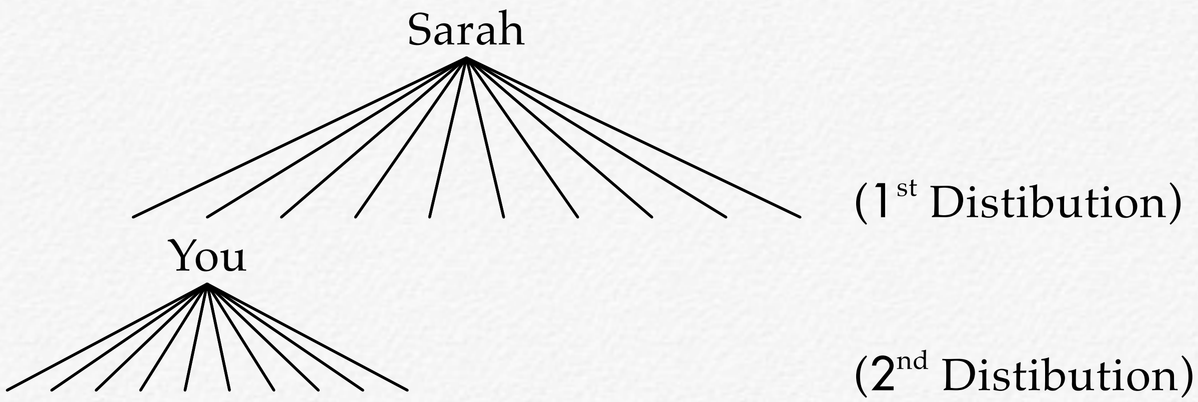
A **typical pyramid scheme** is a chain letter. Someone you know sends you a letter which contains a list of five names. You are asked to pay **£5** to the person at the top of the list and send the letter on to 10 more people – but this time adding **your** name to the bottom of the list, and removing the name of the top person on the list.

The idea sounds good. You can easily think of 10 friends who can afford **£5** and they would also benefit! So you invest **£5** which you send to the person at the top of the list. If the chain remains unbroken, when your name reaches the top of the list, you will receive 100,000 fivers – that is **£500,000**. Of course, some people will break the chain but you'll still be rich if you get half the amount.

Pyramid schemes don't work like that. Why not? Because the total number of people needed to participate is virtually impossible to achieve:

Example

Your friend, Sarah, starts the pyramid, with you in the first distribution of letters.



The theory. Sarah writes the letter and sends it to 10 people; you are one of those 10 people. Each of Sarah's ten people sends the letter to 10 more people; so in the second distribution already 100 letters have been sent – too many to draw on the diagram.

If each of the 100 people that receive the letter in the 2nd distribution sends it to another 10, then 1,000 people will receive the letter in the 3rd distribution. … and so it will increase… with the number of recipients being multiplied by 10 in each distribution.

The catch. Normally, you don't know where you stand in the distribution line of a pyramid scheme. Did the person at the top of your list start it? Or was the pyramid started by someone whose name has already been taken off the list, a few distributions ago?

All you know is that your name is at the bottom of the list and it will take 5 distributions, all growing exponentially, before your name is at the top of the list, by which time 100,000 people should be sending you £5. If they do, that's your mortgage paid off.

But they won't.

Why won't you get the money?

When you receive the list there are 5 names already on it. In theory, many other people are receiving the same letter with the same top name.

But perhaps Sarah didn't start the list. Perhaps Sarah sends out the 5th distribution. That means 100,000 people are receiving a similar letter to yours (with the same top name).

Here's the Proof. A person sends the letter to 10 people (1^{st} distribution). Each of those 10 people send it to 10 more people = 100 people (2^{nd} distribution).

Those 100 people send it to 10 more people = 1,000 people (3^{rd} distribution) Similarly, there are 10,000 people in the 4^{th} distribution, one of whom is Rachel.

When Rachel sends her letter to you, she is one of 100,000 people in the 5^{th} distribution. The 5^{th} distribution is 10 x 10 x 10 x 10 x 10 which is 10^5 = 100,000 people.

What is supposed to happen after the letter reaches you?

Suppose that, like you, every one of those 100,000 people in the 5^{th} distribution, sends the letter to 10 more people (6^{th} distribution), who each send it on to 10 more people..... By the time you are – theoretically – receiving your fivers in the post – **which won't be until the 10^{th} distribution** – there will be **10^{10} = ten billion people** who will have received the chain letter in the 10^{th} round – plus the people who received it in the 9 earlier rounds!!!

Unfortunately there are only 6.5 billion people in the world, which is why you won't get that tantalising cash.

Microbanking has grown exponentially. Very small loans enable poor women in poor countries to start a business – and earn their own income.

A successful pyramid scheme depends on worldwide obedience – and not even Hitler was mad enough to expect that. That is why chain-letters are usually accompanied by a threat such as, if you break this chain then a year's bad luck will follow you; a few people are sufficiently gullible to believe this. This is why the chain letter scheme is illegal. A Ponzi financial scheme is illegal and so are many other fraudulent schemes based on the theory of extreme potential growth, which is why I believe that this information is essential to Real Life.

Exponential growth doesn't just apply to money. **Population growth** of people or animals, **epidemics** or **internet gossip** can also spread exponentially. Other factors will eventually **limit** the growth.

Competition for food or an increase in the number of predators or fatal illness will limit populations - human, animal or bacteria. Isolation or the lack of uninfected victims in an area can limit the spread of disease.

And just as the pyramid scheme will die out through lack of participants, the gossip about your encounter with the local hero will eventually end… because there is a limited number of people who are interested.

Subject of gossip?

The Chain of Causality

This is a sequence of events that can **sometimes** be linked to growth.

'A stitch in time saves nine', is a simple example: if you don't stitch up your hem, then the thread will unravel until you have nine times the amount to mend.

Another example is in a proverbial verse about the English Battle of Bosworth Field of 1485 which was lost by King Richard III, thrown when his horse lost a shoe and stumbled. The King shouted, "A horse, a horse, my kingdom for a horse!" Richard knew that he could not rally his troops on foot, and was more vulnerable to an enemy attack on the ground: he was killed.

Here is that ancient verse, "For want of a nail, the horseshoe was lost. For want of a shoe, the horse fell and was lost. For want of a horse, the king was lost. For want of a leader, the battle was lost. For want of a victory, the kingdom was lost…all for want of a horseshoe nail."

Example of Possible Population Growth or Decline

Statistics from Russia show that because of high mortality rates (due to excessive smoking, drinking and violent crime) and low birth rates, the working-age population in Russia is projected to shrink by nearly **20%** in just 25 years. Russia's working-age population is currently about **100** million. If the decline continues at the same rate, what will be the working-age population of Russia in **100** years?

Use the compound interest formula for exponential growth calculations.

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{number of years}} = \text{Total amount}$$

The starting amount is **100 million** (the current working-age population of Russia). The percentage interest rate in this example is a percentage **decrease of 20%**, so subtract **20** rather than add **20**.

The decrease by **20%** occurs over a **25-year period**. The 'number of years' should read for this calculation 'the number of periods'. In **100** years there are **four 25**-year-periods.

So with the numbers in place the calculation will read:

$$100{,}000{,}000 \times \left(\frac{100 - 20}{100}\right)^4 = \text{Population in 100 years' time.}$$

$$100{,}000{,}000 \times \left(\frac{80}{100}\right)^4 =$$

$$100{,}000{,}000 \times (0.8)^4 =$$

$$100{,}000{,}000 \times 0.4096 = 40{,}960{,}000$$

Answer: **In 100 years time, the working-age population of Russia is projected to fall from 100 million to only 41 million.**

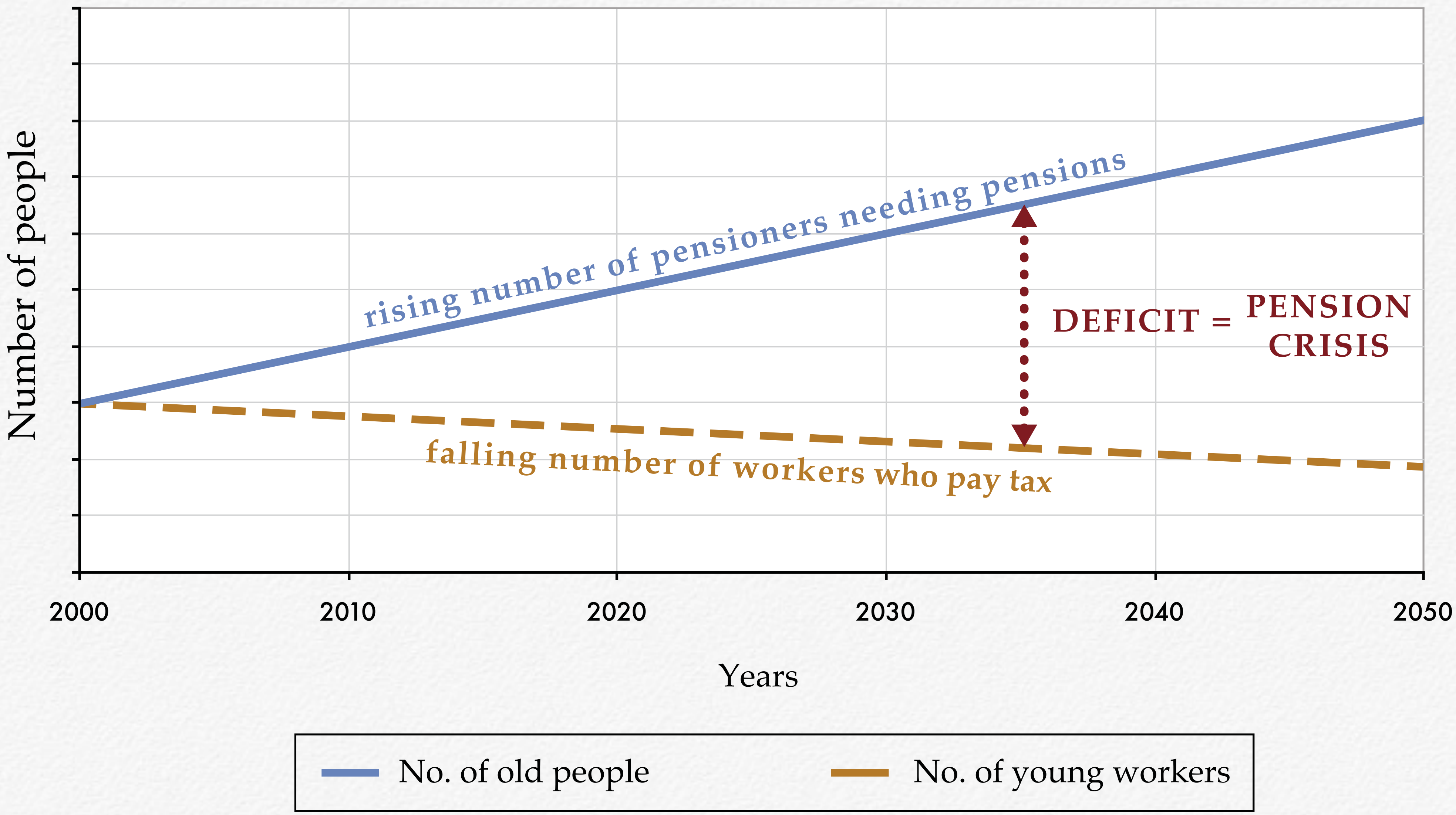
Projection example warns of a possible pension crisis in the fictitious country of MALMANAJA

Pensions are paid to old people by the government with money that the government raises by taxing the wages of workers.

However, the young people of Malmanaja are deciding to have fewer children – and the old people now live longer because of the Malmanaja national health scheme.

Soon there will not be enough income tax every year to pay all the pensions every year.

The projection below was commissioned by the Government of Malmanaja. It shows that they need to raise more money, or lower the amount that each old person receives in pension.



YOUR BRAIN WORKOUT

Question 1 of 5.

By what mathematical operation does the number pattern 7, 11, 15, 19,....increase?

- **A.** + 2
- **B.** + 4
- **C.** + 7
- **D.** x 4

YOUR BRAIN WORKOUT

Question 2 of 5.

By what mathematical operation does the number pattern 1, 2, 4, 8, 16,....increase?

- **A.** + 2
- **B.** + 4
- **C.** x 2
- **D.** x 4

YOUR BRAIN WORKOUT

Question 3 of 5.

By what mathematical operation does the number pattern 4, 7, 10, 13,....increase?

- **A.** + 3
- **B.** + 2
- **C.** x 2
- **D.** x 4

YOUR BRAIN WORKOUT

Question 4 of 5.

By what mathematical operation does the number pattern 4, 2, 0, -2,....increase?

- **A.** + 2
- **B.** x −2
- **C.** x 2
- **D.** − 2

YOUR BRAIN WORKOUT

Question 5 of 5.

By what mathematical operation does the number pattern 1, 3, 9, 27,....increase?

- **A.** + 3
- **B.** +3 then + 6, then + 9...
- **C.** + 2
- **D.** x 3

YOUR BRAIN WORKOUT

Answers

Q1. + 4

Q2. x 2

Q3. + 3

Q4. - 2

Q5. x 3

YOUR BRAIN WORKOUT

Do the following number sequences have steady or exponential growth?

Question 1 of 4.

2, 6, 10, 14, ...

YOUR BRAIN WORKOUT

Do the following number sequences have steady or exponential growth?

Question 2 of 4.

2, 10, 18, 26, ...

YOUR BRAIN WORKOUT

Do the following number sequences have steady or exponential growth?

Question 3 of 4.

2, 6, 18, 54, ...

YOUR BRAIN WORKOUT

Do the following number sequences have steady or exponential growth?

Question 4 of 4.

2, 4, 8, 16, ...

YOUR BRAIN WORKOUT

Answers

Q1. Steady Growth (+4)

Q2. Steady Growth (+8)

Q3. Exponential growth (x3)

Q4. Exponential growth (x2)

PART 32
INFLATION

Quick Quiz

Question 1 of 4.

What is 500 x 20?

- A. 1,000
- B. 10,000
- C. 100,000
- D. 1,000,000

Quick Quiz

Question 2 of 4.

Which of the following is **not** equal to three quarters?

- **A.** 0.75
- **B.** 0.75%
- **C.** 75%
- **D.** $\frac{6}{8}$

Quick Quiz

Question 3 of 4.

If $4n + 3 = 19$, what is the value of n?

- A. 4
- B. 3
- C. 5.5
- D. 2

Quick Quiz

Question 4 of 4.

What is the cost of a **£500** sofa with **30%** off?

- A. £470
- B. £450
- C. £350
- D. £300

Quick Quiz

Answers

Q1. 10,000

Q2. 0.75%

Q3. 4

Q4. £350

Inflation

Inflation is when prices increase.
Deflation is when prices decrease.

When there are more buyers than goods, prices go up. When there are only 20 Christmas turkeys left at the farm but there are 100 late shoppers outside the farmer's back door, he increases the price of each turkey. So prices rise, which is **inflation**.

When there are more goods than there are buyers, prices fall. If there are 100 punnets of strawberries in the supermarket and it's near closing time, but only twenty people want to buy strawberries, the supermarket decreases the price of the surplus strawberries, to get rid of them before they go bad and have to be dumped.
So prices go down, which is **deflation**.

1929–1932 was a **depression period** in Britain, when many workers could not get jobs, so had no pay, so could not buy goods. The only way that producers could sell their goods was by reducing their normal prices. So prices went down which meant that the producers of goods didn't make enough money, so more workers had to be fired… and so on: it was a miserable time.

Annual inflation is calculated by the Government as the cost of the contents of an imaginary shopping basket of goods and services (referred to as the **Inflation Basket**), that ranges from potatoes and milk to travel and electricity.

First Example

Between 1950 and 1983, inflation in the country of Malmanaja increased by an average of 7.28% per year.

If the Inflation Basket is priced £100 in 1950, how much would it cost 25 years later in 1975?

Use the **compound interest formula** for **inflation** calculations.

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage interest rate pa}}{100}\right)^{\text{number. of years}} = \text{Total amount}$$

£100 is the starting amount, 7.28% is the inflation per year and 25 is the number of years, so the calculation in the brackets needs to be multiplied by itself 25 times (i.e. to the power of 25).

$$\text{Starting amount in 1950} \times \left(\frac{100 + \text{percentage inflation of 7.28}}{100}\right)^{\text{no. of years is 25}} = \text{Basket price in 1975}$$

$$100 \times \left(\frac{100 + 7.28}{100}\right)^{25} = £579$$

Answer: **If a gran in Malmanaja purchased £100 of goods in 1950, when she was twenty and newly married, by 1975, when she was 45, she would have had to pay £579 for the same goods.**

Exercises

1) How much would a gran in Malmanaja have to pay for the same **£100** worth of goods, bought in **1950**, if she visited the supermarket thirty three years later in **1983**?

2) Assuming that Malmanaja inflation from **1983** to **2000** continued at an average rate of **7.28%** a year, how much would a gran – now seventy – have to pay in the year **2000** for goods that cost her **£100** in **1950**?

Does a bride need to know her numbers?

Yes, to balance her budget.

Inflation Rates

The **inflation rate** varies in different countries. Here are the **1950–1983** average figures for 6 countries:

	UK	USA	France	Germany	Italy	Japan
Average Inflation Rate 1950 - 1983 (%)	7.28	4.39	6.82	3.30	7.72	5.95

In which country would you have got the best value for your money between **1950** and **1983**?

Obviously, Germany, which had the lowest inflation figure.

Example

Use the compound interest formula to calculate the inflation increase between **1950–1983** in each national Inflation Basket of **£100** in the following countries. Give the answer rounded to the nearest pound.

	UK	USA	France	Germany	Italy	Japan
Inflation 1950 - 1983 (Average Per year, %)	7.28	4.39	6.82	3.30	7.72	5.95
Equations for each	$100 \times \left(\frac{107.28}{100}\right)^{33}$	$100 \times \left(\frac{104.39}{100}\right)^{33}$	$100 \times \left(\frac{106.82}{100}\right)^{33}$	$100 \times \left(\frac{103.30}{100}\right)^{33}$	$100 \times \left(\frac{107.72}{100}\right)^{33}$	$100 \times \left(\frac{105.95}{100}\right)^{33}$
Answers	= £1,017	= £413	= £882	= £292	= £1,164	= £673

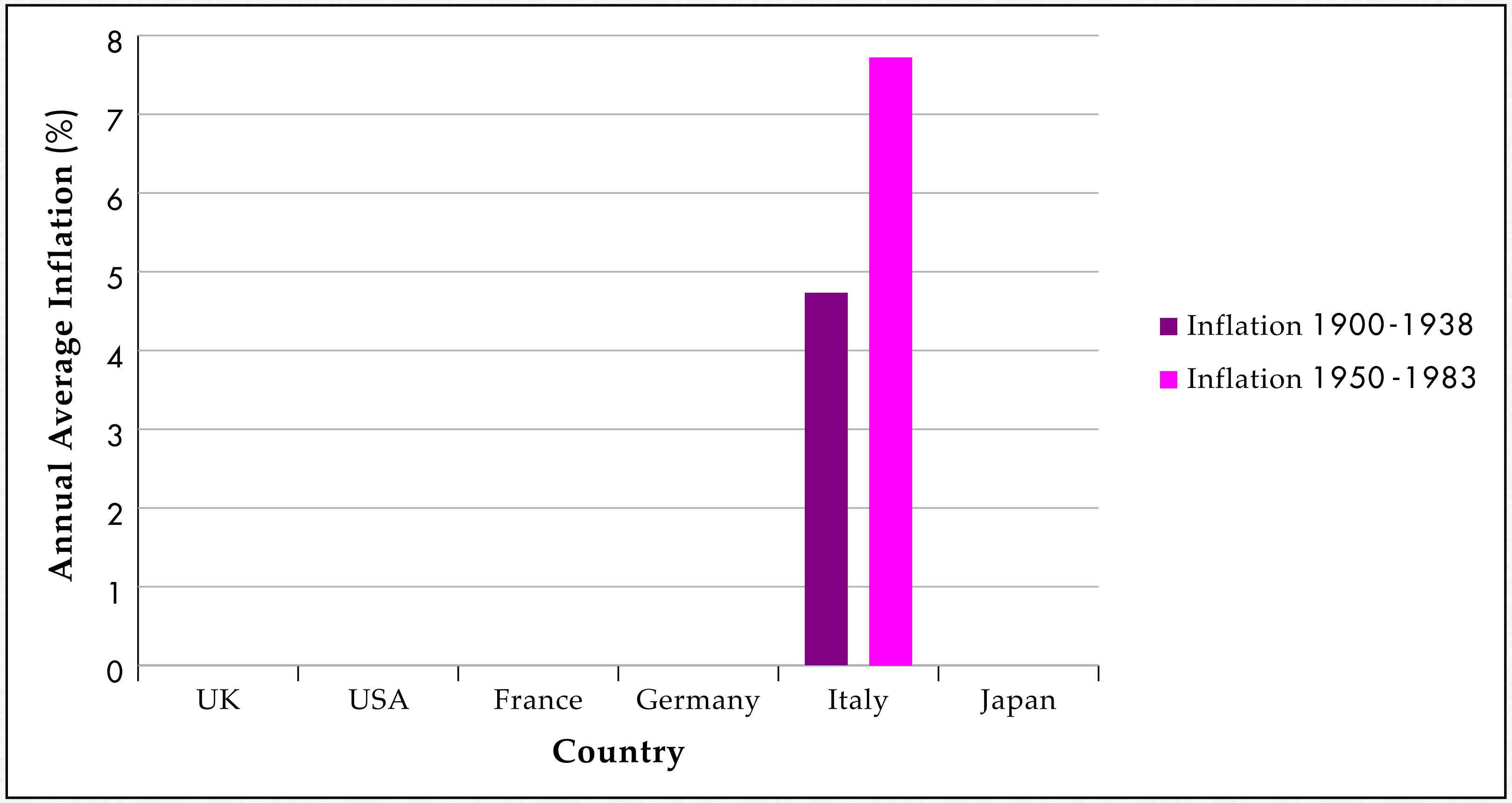
Exercises

3) Draw the following bar chart yourself, preferably on squared paper. To compare further the inflation rates in each of the countries previously listed, plot the inflation rates given in the table that follows onto the compound bar chart beneath it. Italy has already been done for you. (For a reminder of how to plot a bar chart, see STEP 3.)

INFLATION TABLE

	UK	USA	France	Germany	Italy	Japan
Inflation 1900-1938 (Average Per Year, %)	1.47	1.38	5.38	1.31	4.73	(not availible)
Inflation 1950-1983 (Average Per Year, %)	7.28	4.39	6.82	3.30	7.72	5.95

BAR CHART



You can now see that inflation given in a bar chart form is easier for a reader to absorb at a glance.

4) a) Between 1900 and 1938, in which of those countries did prices rise i) the least and ii) most?

b) Between 1950 and 1983, in which of those countries did prices rise i) the least and ii) most?

5) Which of those countries showed the greatest difference between the inflation rate 1900–1938 and the inflation rate for 1950–1983?

Whatever the inflation rate, the sea is still free.

Official Inflation and Real Inflation

Governments cheat. For example, when the price of oil shot up in **1975**, the British Government simply decided to leave oil out of the Inflation Basket, in order to keep the inflation figure as low as possible… possibly so that people would vote for them again at the next election. The **1975** official British inflation figure was a horrifying **24.2%**, whereas the oil price increased by **70%** that year! The British government normally aims to keep inflation lower than **2%**.

Does this mean that you can't trust official inflation figures? Yes, some would say. And if **all** the relevant facts were factored in, would the government inflation figure be higher? Yes, some would say. "Few people believe the Office of National Statistics' figures anymore," said respected financial correspondent Jeremy Warner in The Daily Telegraph, November 2012.

Example

If the official average inflation rate is **4.5%** per annum, how many years will it take for your **£1** coin to halve its value?

For your **£1** coin to halve its value, the value of **£1** needs to have inflated to **£2**. So you need to calculate the number of years it would take for **£1** to inflate to **£2**, using the compound interest formula:

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage inflation}}{100}\right)^{\text{no. of years}} = \text{Value at end of period}$$

£1 is the starting amount, **4.5%** is the inflation percentage per year, and **£2** is the value at the end of the period.

As you need to calculate the number of years, refer to this figure as **n**.

$$1 \times \left(\frac{100 + 4.5}{100}\right)^{n} = 2$$

This calculation simplifies to: $(1.045)^{n} = 2$

Remember: any number multiplied by 1 will not change. So 100 x 1 = 100 and 1 x 100 = 100.

Brush up your maths.

Example continues on the next page

The simplest way to find the value of **n** is by **trial and improvement**:

Choose any number as a first guess for **n**, say 10.

Next, using a scientific calculator find the value of $(1.045)^n$ using **n** = 10.

If the answer is too small (less than 2), try a bigger value for **n**.

If the answer is too big (more than 2), try a smaller value for **n**.

With **n** = 10:	$(1.045)^{10} = 1.55$	**n** = 10 is too small.
Try **n** = 20:	$(1.045)^{20} = 2.41$	**n** = 20 is too big.
Try **n** = 15:	$(1.045)^{15} = 1.94$	**n** = 15 is just a little bit too small.
Try **n** = 16:	$(1.045)^{16} = 2.02$	**n** = 16 is just too big.

So the answer for **n** is between 15 and 16, but closer to 16.

Answer: **It will take 16 years for a £1 coin to halve its value at 4.5% inflation**. So £100 in cash will also halve its value.

So a chocolate bar that cost £1 in 2000, in 16 years time – 2016 – will cost £2.

Not all objects rise in price at the same rate: for instance, the average British house may rise higher or sink lower than the British inflation rate.

Exercises

6) If your real inflation rate is 10% per annum, how many years will it take for your saved £1 coin to halve its value and so buy half as much stuff?

Your Personal Inflation Index

Pick your own yearly price index of something you use a lot and notice how the price rises or falls every year. My friend Lorraine's private inflation index is the Hellman's mayonnaise index; my private inflation index is the Cooper's Oxford marmalade index.

Check that **the amount you get for your money doesn't decrease**, which would distort the inflation comparison.

Check that the size of bottles, jars, Crunchy bars or Mars bars don't get smaller.

FUEL COSTS RISE

Cheapest seaside holiday.

Your personal inflation indicators.

Answers to Part 32

1) How much would your gran have to pay for the same **£100** worth of goods, bought in **1950**, if she visited the supermarket in **1983** when she was **53** years old?

Use the compound interest formula:

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage inflation}}{100}\right)^{\text{no. of years}} = \text{Basket price at end of period}$$

£100 is the starting amount, **7.28%** is the average inflation per year and **33** is the number of years:

$$100 \times \left(\frac{100 + 7.28}{100}\right)^{33} =$$

$$100 \times \left(\frac{107.28}{100}\right)^{33} =$$

$$100 \times (1.0728)^{33} =$$

$$100 \times 10.1653 = 1016.53$$

Answer: **Gran's £100 shopping basket in 1950 would cost £1,017 in 1983**.

So at the end of **33** years, your gran needed over **10** times more money to buy those same goods! Inflation is tough for pensioners – or anyone – on a fixed income.

2) Assuming that inflation from **1983** to **2000** continued at an average rate of **7.28%** a year, how much would your gran – now seventy – have to pay in **2000** for goods that cost her **£100** in **1950**?

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage inflation}}{100}\right)^{\text{no. of years}} = \text{Basket price at end of period}$$

Use the compound interest formula:

$$100 \times \left(\frac{100 + 7.28}{100}\right)^{50} =$$

$$100 \times \left(\frac{107.28}{100}\right)^{50} =$$

$$100 \times (1.0728)^{50} =$$

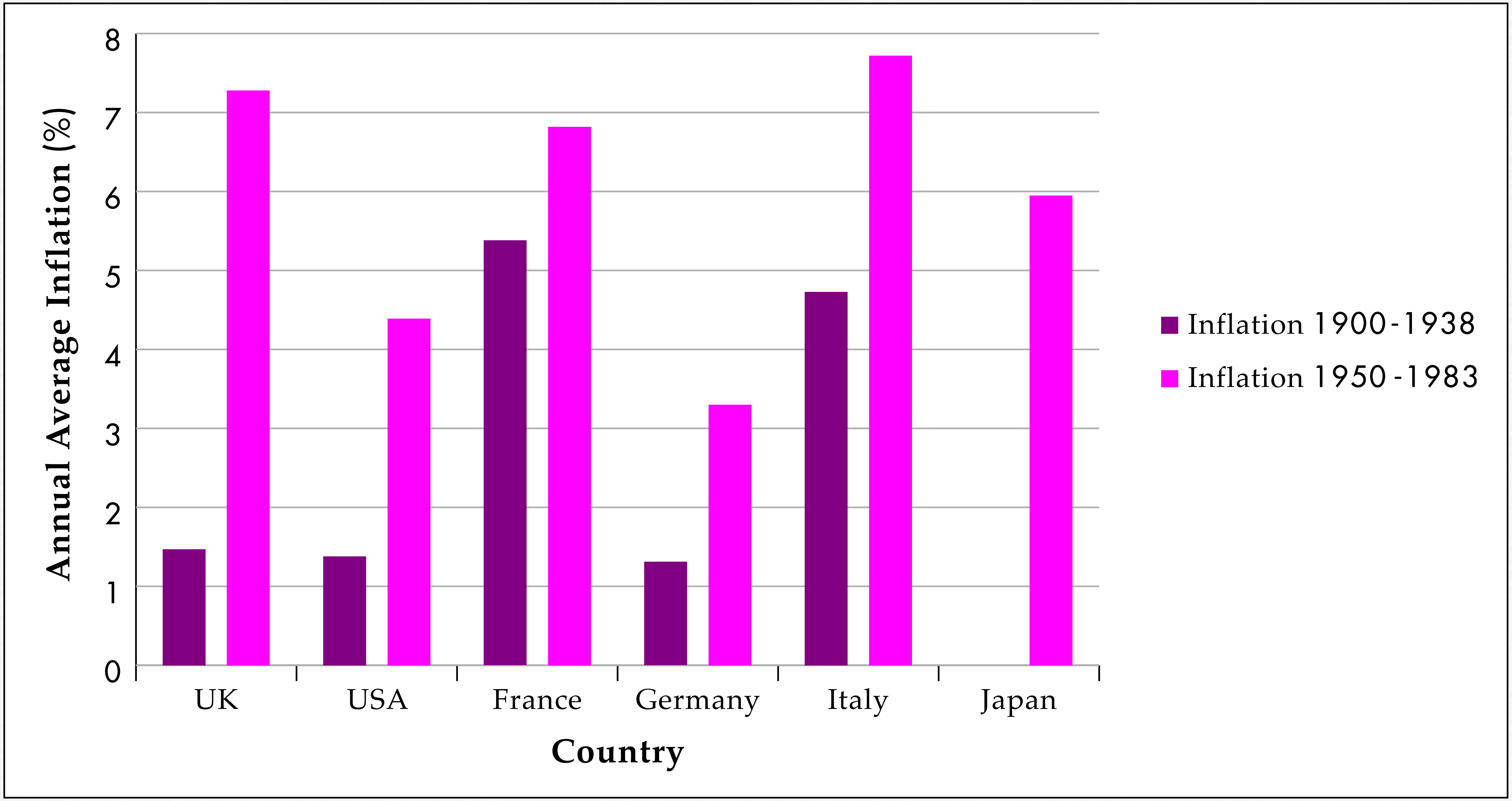
$$100 \times 33.5690 = 3356.90$$

£100 is the starting amount, **7.28%** is the inflation per year and **50** is the number of years:

Answer: Gran's **£100** shopping basket in **1950** would cost **£3,357** in the year **2000**.

You need maths to follow a recipe.

3) Your bar chart should look like this:



4)

a) Between 1900 and 1938, in which country did prices rise i) the least and ii) most?

Answer: i) **Prices rose the least in Germany** ii) **Prices rose the most in France**.

b) Between 1950 and 1983, in which country did prices rise i) the least and ii) most?

Answer: i) **Prices rose the least in Germany** ii) **Prices rose the most in Italy**.

5) Which country showed the greatest difference between the inflation rate 1900–1938 and the inflation rate for 1950–1983?

Answer: **The UK showed the greatest difference between the inflation rates in the periods 1900–1938 and 1950–1983**.

Anything is possible.

6) If your real inflation rate is 10% per annum, how many years will it take for your saved £1 coin to halve its value?

For your **£1** coin to halve its value, the value of **£1** needs to have inflated to **£2**. You need to find the number of years it will take for **£1** to inflate to **£2**, using the compound interest formula:

$$\text{Starting amount} \times \left(\frac{100 + \text{percentage inflation}}{100} \right)^{\text{no. of years}} = \text{Value at end of period}$$

Here **£1** is the starting amount, 10% is the inflation per year and **£2** is the value at the end of the period. You need to find the number of years, so call this **n**.

$$1 \times \left(\frac{100 + 10}{100} \right)^{n} = 2$$

This calculation simplifies to: $(1.10)^{n} = 2$

Find the value of **n** by **trial and improvement**:

With **n** = 10:	$(1.10)^{10} = 2.95$	**n** = 10 is too big.
Try **n** = 5:	$(1.10)^{5} = 1.61$	**n** = 5 is too small.
Try **n** = 7:	$(1.10)^{7} = 1.95$	**n** = 7 is just too small.
Try **n** = 8:	$(1.10)^{8} = 2.14$	**n** = 8 is just too big.

So the answer for n is between 7 and 8, but closer to 7.

Answer: **It will take just over 7 years for the £1 coin to halve its value at 10% inflation**.

YOUR BRAIN WORKOUT

Question 1 of 9.

The price of a pint of milk has risen from 34p in 1999 to 45p in 2009. What is the percentage increase in the price of milk over this 10-year period?

- **A.** 11%
- **B.** 32%
- **C.** 6%
- **D.** 50%

YOUR BRAIN WORKOUT

Question 2 of 9.

The price of an 800g loaf of sliced white bread has risen from 51p in 1999 to £1.26 in 2009. What is the percentage increase in the price of white bread over this 10-year period?

- **A.** 50%
- **B.** 75%
- **C.** 112%
- **D.** 147%

YOUR BRAIN WORKOUT

Question 3 of 9.

The price of 1 kg of sugar has risen from 61p in 1999 to 88p in 2009. What is the percentage increase in the price of sugar over this 10-year period?

- **A.** 20%
- **B.** 27%
- **C.** 44%
- **D.** 60%

YOUR BRAIN WORKOUT

Question 4 of 9.

The average price of a bottle of wine has risen from £3.55 in 1999 to £4.18 in 2009. What is the percentage increase in the price wine over this 10-year period?

- **A.** 6%
- **B.** 18%
- **C.** 63%
- **D.** 75%

Question 5 of 9.

The price of a packet of 20 king-sized cigarettes has risen from £3.37 in 1999 to £5.39 in 2009. What is the percentage increase in the price of cigarettes over this 10-year period?

- A. 22%
- B. 202%
- C. 40%
- D. 60%

YOUR BRAIN WORKOUT

Question 6 of 9.

The average price of a hamburger has risen from £1.90 in 1999 to £1.99 in 2009. What is the percentage increase in the average price of a hamburger over this 10-year period?

- **A.** 1%
- **B.** 4.7%
- **C.** 9%
- **D.** 90%

YOUR BRAIN WORKOUT

Question 7 of 9.

The average cost of a house has risen from £73,305 in 1999 to £163,969 in 2009. What is the percentage increase in house prices over this 10-year period?

- **A.** 124%
- **B.** 83%
- **C.** 40%
- **D.** 10%

YOUR BRAIN WORKOUT

Question 8 of 9.

The price of a pint of draught lager has risen from £1.93 in 1999 to £2.79 in 2009. What is the percentage increase in the price of draught lager over this 10-year period?

- **A.** 86%
- **B.** 45%
- **C.** 9%
- **D.** 90%

YOUR BRAIN WORKOUT

Question 9 of 9.

The average salary has risen from £18,396 in 1999 to £20,900 in 2009. What is the percentage increase in the average salary over this 10-year period?

- **A.** 200%
- **B.** 251%
- **C.** 25%
- **D.** 14%

YOUR BRAIN WORKOUT

Answers

Q1. 32%

Q2. 147%

Q3. 44%

Q4. 18%

Q5. 60%

Q6. 4.7%

Q7. 124%

Q8. 45%

Q9. 14%

SOME OF THE SECRETS OF THE UNIVERSE

Sunrise Himalayas.

All that we know of the Universe was discovered by using maths.

Creativity

I left school thinking, "Hooray! No more maths," because I was a below-average maths student. Since then, I've realised that from the day you leave school – you need maths. I had to remember what little maths I know and, bit by bit, teach myself the rest.

Maths trains your brain for Real Life, it doesn't only teach you how to add and subtract. **Maths sharpens the mind**. Maths makes you understand the reasons for thinking in a **logical** way, working in an **orderly** fashion. You learn to be **neat**, and **efficient with your time**. Maths shows you how to **persevere** and **focus** only on what is important in a problem, and not be distracted by the rest.

In ancient Greece, mathematicians were seen as very wise people, so learning maths was thought to be a way in which a human mind can become closer to the infinitely clever mind of the Creator.

In particular, geometry was considered a discipline that develops your soul, as well as enabling you to cut out a toga pattern or build a temple.

That may be why we still talk of being made to feel "**uplifted**" or "**on a higher plane**" by a piece of music, or of being "**carried away**" by an ancient place of worship, or a beautiful view.

When this happens, you can feel in the peaceful presence of a timeless quality of perfection which makes you catch your breath, and feel as if you had been lifted off the ground by a few inches. You get a tingly feeling at the back of your neck.

Everyone is creative. The satisfaction, the joy of it are the same feelings whether you have made something yourself, or whether you appreciate something made by somebody else.

When you persevere with something that you don't understand in maths, there comes a time when you suddenly stumble out of the depressing, grey clouds of Not-Understanding into that joyful, exhilarating moment that makes you lift your head and beam, "I GOT IT!"

You're delighted by **your cleverness** in understanding it. You appreciate **the cleverness of the idea** you've been working on, the neatness, the elegance of it.

You, too, feel the pleasure of understanding what – in ancient times – was considered a secret, known only to wizards and priests.

Two Big Secrets

Everyone enjoys beauty in some form, whether it is a huge piece of sculpture like **the Angel of the North**, or a pair of shoes. (Shoes **are** sculpture, with a practical purpose.)

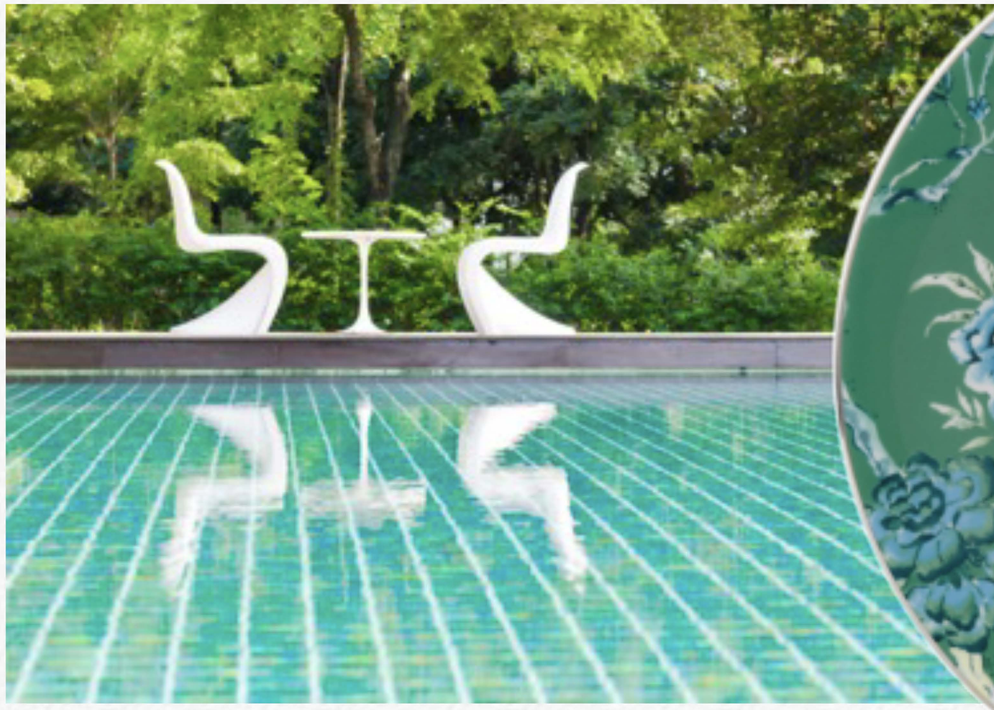
Angel of the North, England. Sculptor Antony Gormley.

Needlework picture, Embroiderer unknown. Silk on linen.

Everything man-made has been designed:
someone thought about it before it was made
– although you might doubt that, sometimes.

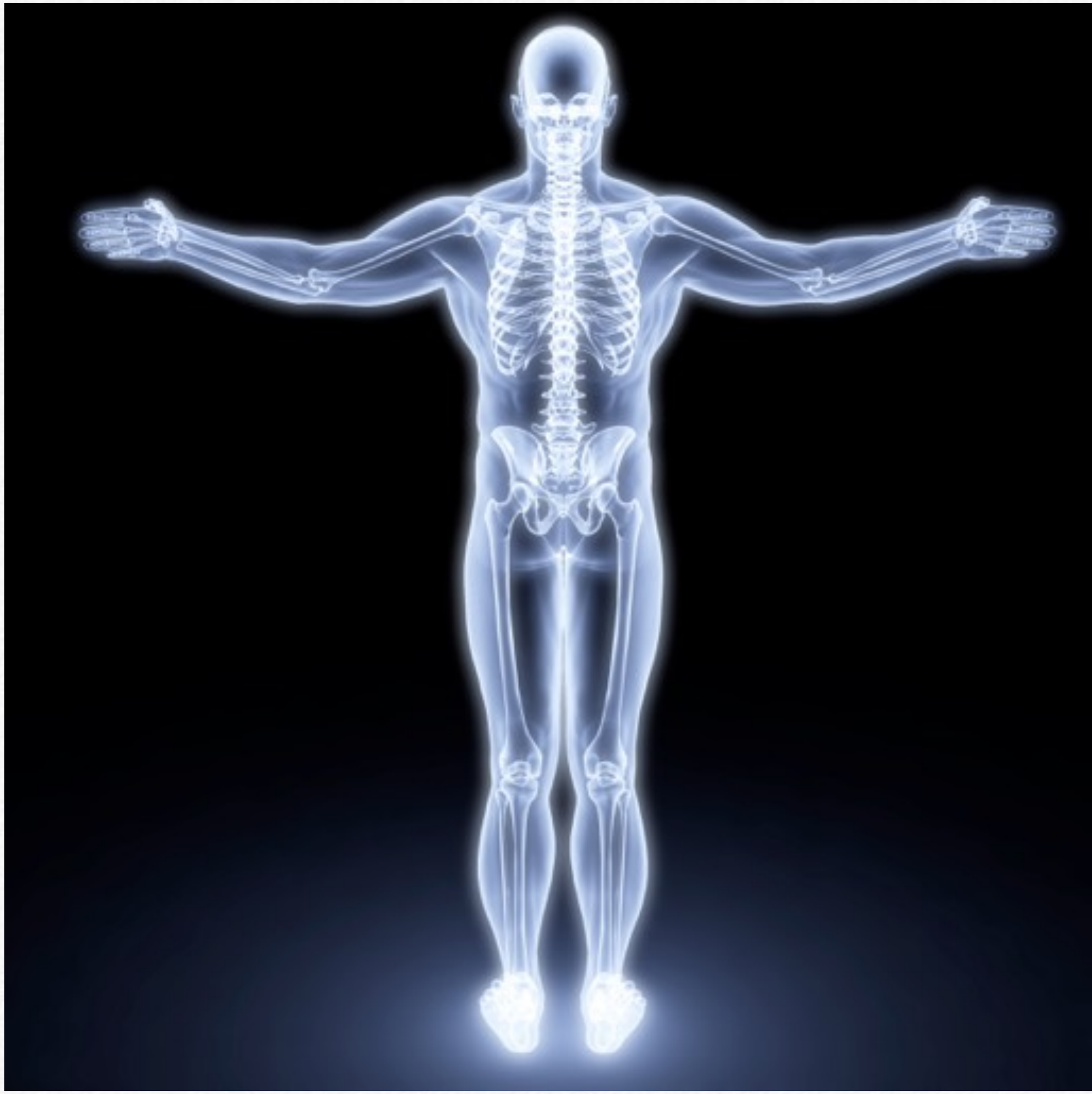
Not many people deliberately set out to create something dull or ugly, but then not many people know what guarantees success, whether you are composing a song, designing a pair of shoes or building a concert stadium.

Concorde	Philippe Starck Chairs	Wedgwood plate
Japanese Temple	Racing Motorcycle	



There are two clever methods that help anyone to achieve creative success.

An ancient Greek mathematician produced a formula – **The Golden Ratio** – that would guarantee beautiful proportion in anything – a table, a bedroom or an open-air theatre.



Much later, a young, Italian mathematician discovered a series of numbers that would also produce harmonious proportion – **The Fibonacci Sequence.**

Today, all over the planet, designers, architects and artists still use the **Golden Ratio** and the **Fibonacci Sequence**. So do scientists, hoping to discover more secrets of the Universe.

These two methods are arguably some of the formulas that Nature uses to create living things. They can be seen in the construction of flowers, plants, trees, shells, sea creatures, birds, beetles, animals – and you.

Number Patterns and The Fibonacci Sequence

PLEASE READ SLOWLY

A sequence is a series of numbers, connected by a **rule**. **Example:** The answers to the two times table is a sequence that you can easily spot: 2, 4, 6, 8, 10, 12.

The rule is the unit **2**. You add **2** to each number to get the next number

2 + 2 = 4   4 + 2 = 6   6 + 2 = 8   8 + 2 = 10   10 + 2 = 12

Because **2** is an **even number**, that is a **sequence of EVEN numbers**.

Here is another sequence:
3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

The **rule** is the unit **3**. You add a **3** to each number to get the next number.
3 + 3 = 6   6 + 3 = 9   9 + 3 = 12   12 + 3 = 15

Can you work out the next five sums?

In the two previous sequences, there is **steady growth**: the difference between one term and the next is constant. Such sequences are called **Arithmetic Sequences**.
All the familiar multiplication tables, where you **add the same number** each time, give Arithmetic Sequences.

In **Geometric Sequences**, the difference between one term and the next term changes. For example if each term is **multiplied by 2**, you get the following sequence:

2, 4, 8, 16, 32, 64, 128...

Here the difference between the terms gets bigger as the sequence goes along.
This is an example of **exponential growth**
(see previous Part 30, WATCHING YOUR INTERESTS).

Fibonacci pattern of the centre of a daisy.

Note: In Italy, Fibonacci is pronounced FIB-ON-ACH-EE.

Long ago in Italy, a young mathematician called Fibonacci discovered a simple but special sequence. **His rule was, in order to reach the next number in a progression, you add the two previous numbers.**

The Fibonacci Sequence always starts with 0, 1.

The rule: in order to get the next number, add the sum of the two previous numbers, so 0 + 1 = 1.

The progression is now 0, 1, 1.

The rule: in order to get the next number, add the sum of the two previous numbers, which are 1 + 1 = 2.

The progression is now 0, 1, 1, 2.

What is the next number in the progression?

The rule: add the two previous numbers, which are 1 + 2 = 3.

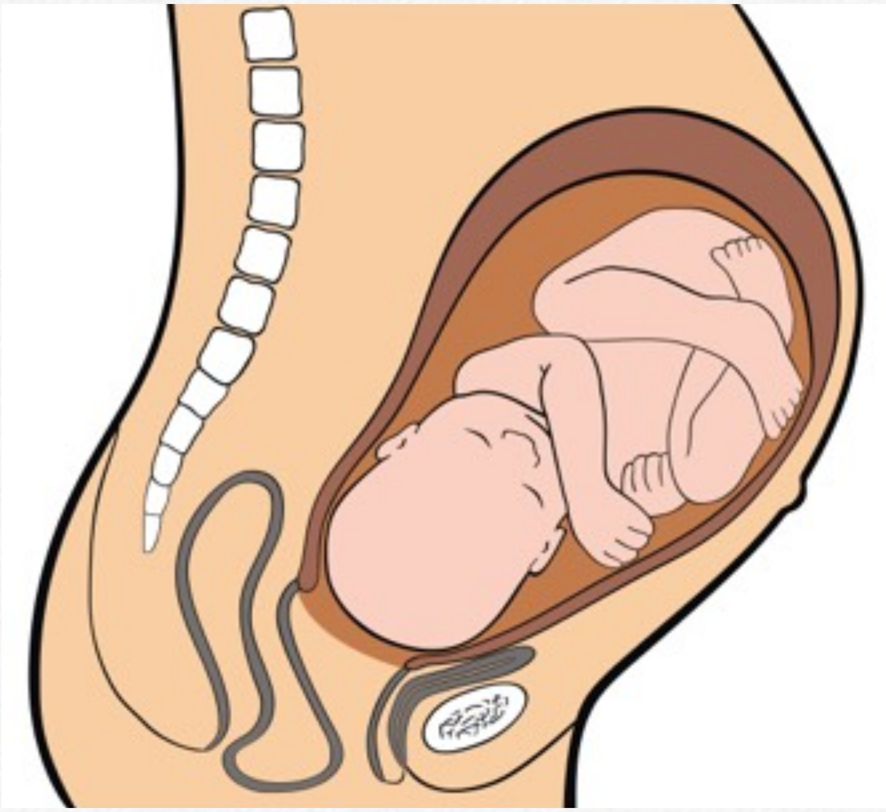
The progression is now 0, 1, 1, 2, 3.

What is the next number in the progression?

The rule: add the two previous numbers, which are 2 + 3 = 5.

Here is the start of the Fibonacci sequence, which continues into eternity.

0	1	1	2	3	5	8	13	21
		↑	↑	↑	↑	↑	↑	↑
		[0 + 1]	[1 + 1]	[1 + 2]	[2 + 3]	[3 + 5]	[5 + 8]	[8 + 13]



Packaging.

Now, switch your attention to packing a suitcase. If you just throw your clothes into it, perhaps the lid won't shut. You need to take everything out and fold each item carefully.

Nature is a brilliant packaging designer. A human baby is efficiently packed in its skin; a baby bends and folds up efficiently in its mother's womb. To use minimum space, Nature uses minimum packaging space and minimum materials for something to grow with minimum energy.

You can also see that Nature knows the best way to use space if you notice how efficiently she packs the seeds of a sunflower, the petals of a buttercup or the arrangement of leaves on a stem. As petals and leaves slowly unfurl in the sun and expand to their full size, the growth pattern is part of the Fibonacci sequence.

Slice a banana (across the middle) and you see a central pattern of 3 seeds.

Cut an apple in half (through the middle) and you stare at a central pattern that is the shape of a 5-pointed star, which contains five dark seeds.

You can see number patterns from the **Fibonacci Sequence** in the designs of fish, shells, insects, butterflies and other life forms studied by biologists, chemists, physicists and other scientists around the planet.

You have one thumb, two bones in that thumb, three bones in each finger. You have five items on each hand – four fingers plus a thumb – and eight fingers on both hands. 1, 2, 3, 5, 8 is part of the Fibonacci sequence.

Exercises

1. Without using a calculator, add the next six numbers to the Fibonacci sequence:

0 1 1 2 3 5 8 13 21 – – – – –

2. Are the following numbers in the Fibonacci series?

a) 17 d) 142

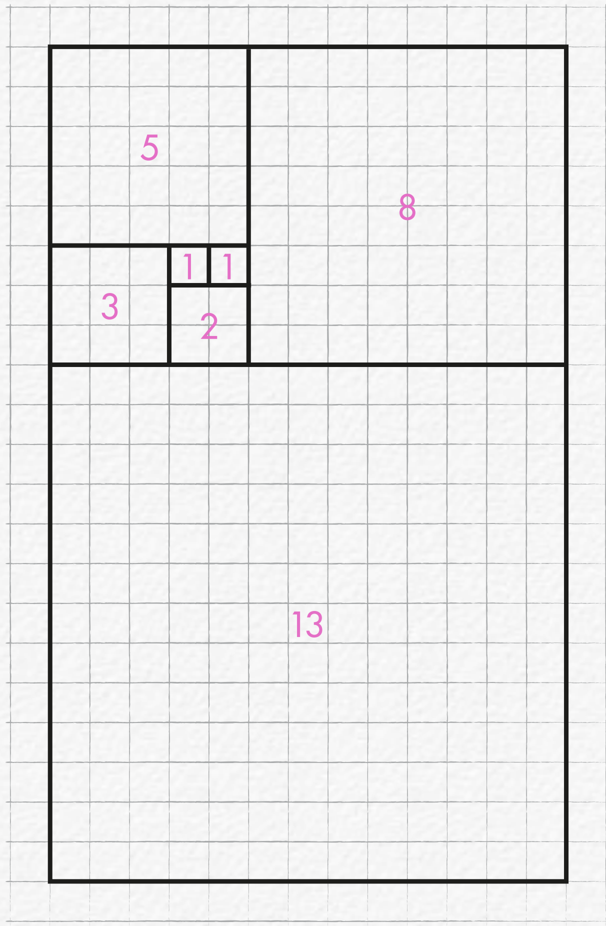
b) 39 e) 377

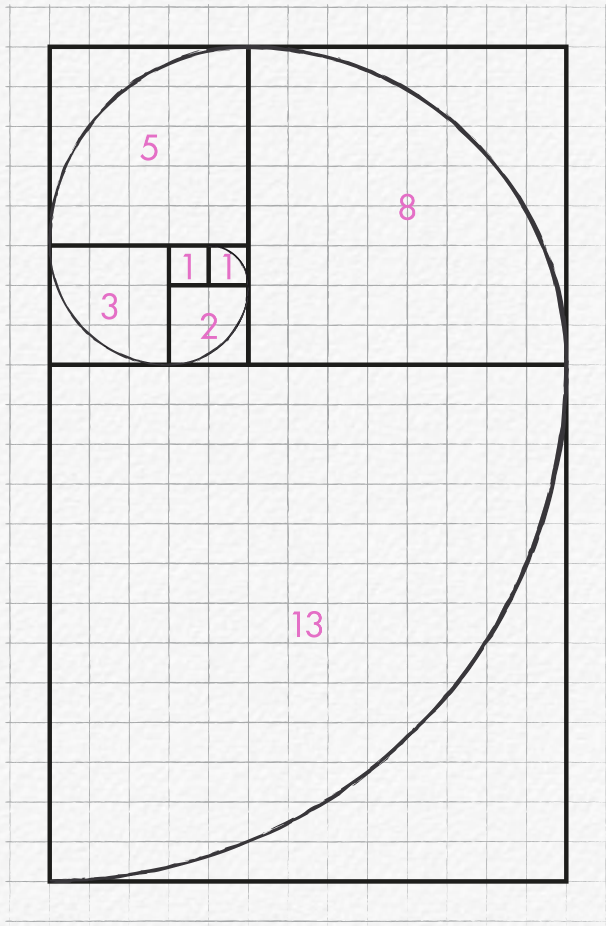
c) 89

More good packaging.

If squares are drawn with side lengths in the Fibonacci Sequence, they pack together well, as you can see in these grids.

If curves are drawn with a compass, to connect opposite corners of each square, the Fibonacci Spiral appears. This spiral can be found over and over again in nature.





Tornado sky.

River whirlpool.

Spiral galaxy - Milky Way.

Nautilus shell.
Artwork shows interior construction.

Answers to Exercises

1. Without using a calculator, add the next six numbers to the Fibonacci sequence:

0 1 1 2 3 5 8 13 21 – – – – –

Add the two final numbers in the sequence above:

13 + 21 = 34

So **34** is the next number in the sequence.

Then add the new two final numbers:

21 + 34 = 55

So **55** is the next number in the sequence.

Repeat by adding the new two final numbers:

34 + 55 = 89

So **89** is the next number in the sequence.

Repeat by adding the new two final numbers:

55 + 89 = 144

So **144** is the next number in the sequence.

Repeat by adding the new two final numbers:

89 + 144 = 233

So **233** is the next number in the sequence.

Repeat by adding the new two final numbers:

144 + 233 = 377

So **377** is the next number in the sequence.

Answer:
0, 1, 1, 2, 3, 5, 8, 13, 21, **34, 55, 89, 144, 233, 377**…

2. Are the following numbers in the Fibonacci series?

a) 17 Answer: **No**

b) 39 Answer: **No**

c) 89 Answer: **Yes**

d) 142 Answer: **No**

e) 377 Answer: **Yes**

The Guggenheim Museum
of Modern Art, Bilbao, Spain.
Architect: Frank Gehry.

The Golden Ratio

A **formula** – like a cookery **recipe** – is a **method** which you follow to get a good result. One of the most useful formulae in design is called **The Golden Ratio**. This method was invented thousands of years ago, in ancient Greece, and has been used ever since to produce good-looking buildings, machines, and paintings.

You need to understand three words:
ratio, proportion and harmony.

A **RATIO** compares one thing to another that it resembles, in number, size or some other way.
(If you don't remember, look back to STEP 2, Part 15, **Ratios**.)

Example 2:4 4:8 8:16 16:32

These four comparisons all cancel down to 1:2, which is the **common ratio** of all four of those groups of figures.

Take two apartment buildings. If the second building is twice as **high** as the first building, the **height** ratio is 1:2.

If the second building is twice as **wide** as the first building, the **width** ratio is 1:2.

If the depth of the second building is twice as **deep** as the first building, the **depth** ratio is 1:2.

Note: ratios are always separated by two dots **:**

The **PROPORTION** is a different sort of relationship between two similar things. When something is described as, "**in proportion**", whether it refers to a human being or a building, it means that – to the eye – **it looks right**, it is a pleasure to look at. It isn't too high or too short, it isn't too narrow or too wide. It is well-balanced.
It is NOT **out of proportion.**

A man with legs that look much too short for his body could be described as "out of **proportion**", and so might a woman with arms that reach to her ankles. Apparently it is not easy for ballet schools to find pupils who are perfectly proportioned, because so few human beings are perfect in this way.

HARMONY is what you get when a group of things fits well together. A **harmonious group** of colours pleases your eye, the colours don't make you wince. A **harmonious group** of people might be The Mozart Club or a Punk group. When choosing a small group of people that will need to live and work closely together – a submarine crew or an exploring expedition – the organizers try to assemble a **harmonious group** that is unlikely to quarrel all the way to the North Pole.

Architects use mathematics to plan buildings that are **in proportion**. Such buildings are geometrically planned, often using **the Golden Ratio** to achieve **proportions** that **harmonise**.

From ancient times, when planning cathedrals, temples and other sacred places of worship, the **architect's** aim has been to create a building that encourages solemnity, wonder and awe in those who enter it, so that they may feel in a suitably reverential state to communicate with their god. The best of these buildings are said to have soul.

HARMONY.

Harmony, rhythm and melody in music can also be based upon ratios, and the Golden Ratio has supposedly been used by **composers** of work ranging from early church music to modern chart-toppers.

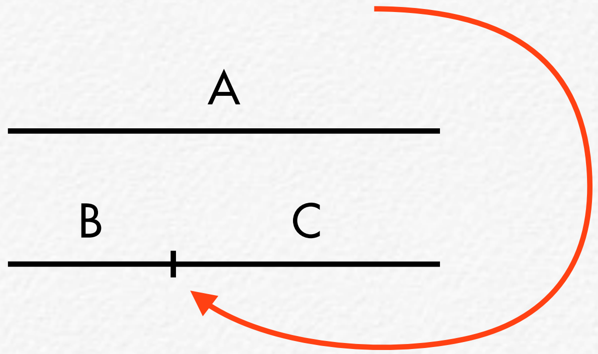
To make sure of a good balance, **artists** have planned their pictures with the use of the Golden Ratio. Artist Albrecht Durer (1471-1528) said, "Geometry is the right foundation of all painting."

Modern designers still use the Golden Ratio to design items that range from a table or an IT device to a spacecraft.

So what is the Golden Ratio?

It's a straight line that is cut – in one place only – into a small line and a larger line.
(Originally, a piece of rope was cut in two.)

Here's the straight line



Here is the small line B and the larger line C

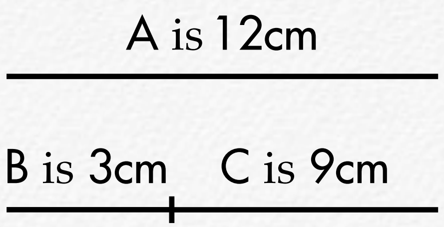
Here's the **formula**.

The ratio of line **B** to line **C** is the same as the ratio of line **C** to line **A.**

In other words, the ratio **B:C** = the ratio **C:A**

Using the Goldilocks method, try to guess what the ratio is.

First guess: Let's try a ratio of **1:3**



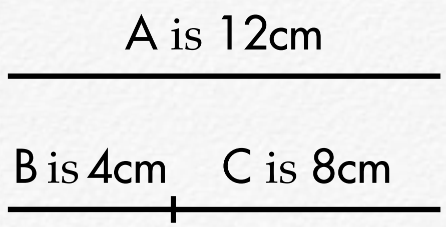
B to **C** would be **3:9**, which cancels down to **1:3**

C to **A** would be **9:12**, which cancels down to **3:4** or **1:1.3**

C to **A** is **1:1.3** which is not the same as ...
B to **C** which is **1:3**
so here the ratio **A:C** does not equal the ratio **B:C**.

The Golden Ratio may be between **1:1.3** and **1:3**.
Perhaps around **1:2**?

Second guess: Let's try a ratio of **1:2**



B to **C** would be **4:8**, which cancels down to **1:2**

C to **A** would be **8:12**, which cancels down to **2:3** or **1:1.5**

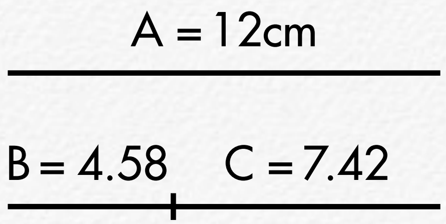
C to **A** is **1:1.5**. This is not the same as **B** to **C** which is **1:2** so here the ratio **A:C** does not equal the ratio **B:C**.

Here the Golden Ratio may be between **1:1.5** and **1:2**

In fact, the Golden Ratio is 1:1.62 (to 2 decimal places).

Which means that :

A = 12cm



B = 4.58cm

C = 7.42cm

B to **C** would be **4.58:7.42**, which simplifies to **1:1.62**

C to **A** would be **7.42:12**, which simplifies to **1:1.62**

So here the ratio **A:C** is equal to the ratio **B:C**.

Those measurements are in the Golden Ratio.

Note: **The Golden Ratio** to six decimal places is **1** to **1.618034**.

In fact, the Golden Ratio is an infinite decimal: it continues to infinity.

The Greeks used a symbol to represent The Golden Ratio.

It's a letter from the Greek alphabet, which is written like this **Φ** and written in the Western alphabet as phi, to represent ONLY the number **1.62** (to **2**dp).

Φ = 1.62 (to 2dp)

Note: phi is pronounced "fy", to rhyme with 'horrify'.

Exercises

3. Try this with your ruler. Rule accurately.

Draw line **A = 1.6cm**.

Underneath it, draw line **B = 0.6cm** and line **C = 1cm**.

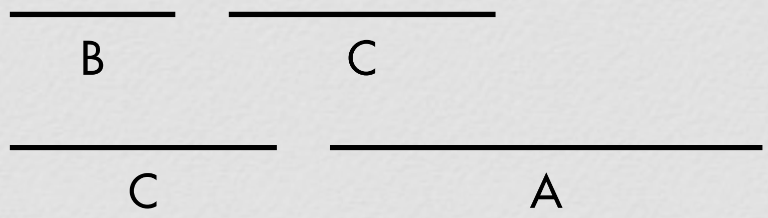
4. As that was perhaps too fiddly to be exact, try it again with your ruler, only, ten times bigger.

Make **A = 16.2cm**, **B = 6.2cm**, **C = 10cm**.

Here's the ancient Greek definition of the golden ratio.

The **small** is to the **large**, as the **large** is to the **whole**.

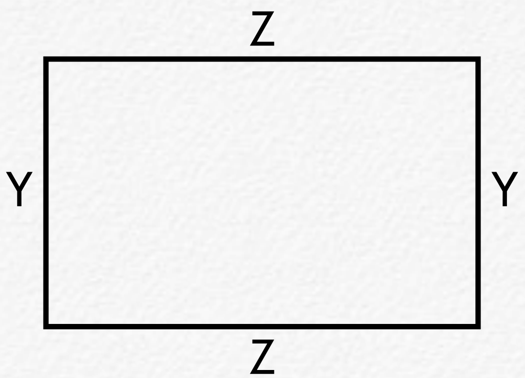
So on your drawings, the small (line **B**) is to the large (line **C**) as the large (line **C**) is to the whole (line **A**).



The ratio **B : C** is the same as the ratio **C : A**.

The Golden Rectangle

The Golden Rectangle is a rectangle with the dimensions in the Golden Ratio. In the following rectangle, the ratio is Y : Z which is 1 : 1.62 (to 2 decimal places).



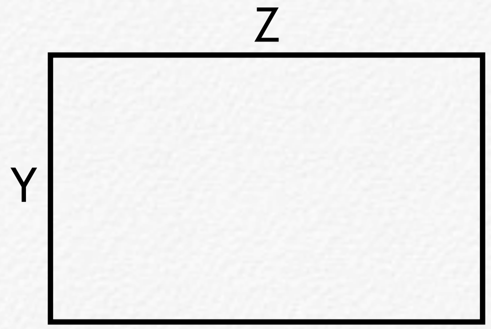
Here, Z = 3.24cm and Y = 2cm.

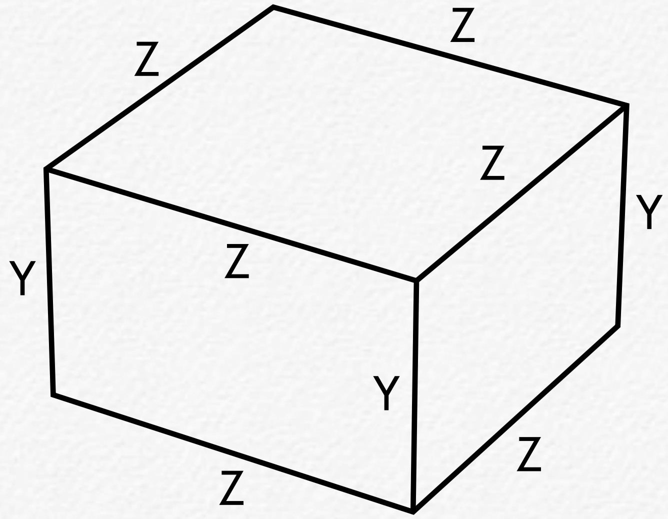
The **lengths** of Z and Y are different to the previous example, but the ratios remain the same, at 1.62 : 1

Note: The Golden Ratio can also be called The Golden Section.

Here is the start of my temple design, using the Golden Ratio for the width, depth and height of the building.

Z = width and depth Y = height





Temple of Hephaestus, Athens, Greece.

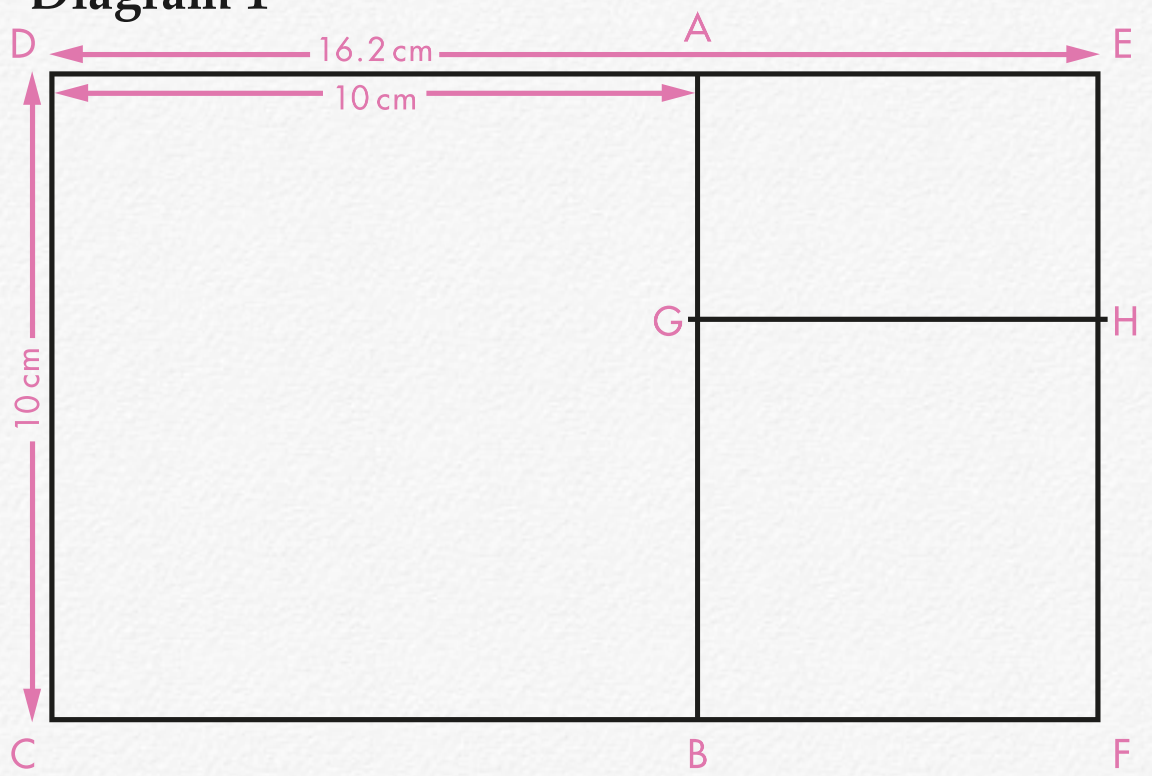
The Golden Spiral

If you were writing instructions to your Martian friend, telling her how to make a cup of tea, it would appear very complicated to the Martian – until she had done it once.

Similarly, after doing the following exercise once, a second time may take you only a few minutes.

Geometry has its own, easy-to-pick-up shorthand, but I'm not using it to write this.

Diagram 1



Exercise 5 (You will need a ruler and a compass)

How to Draw a Golden Spiral.

You will need: a big sheet of paper (preferably squared paper), a compass fitted with a pencil, another pencil and a ruler. Maybe an eraser would be prudent.

Stage 1

Look at diagram 1 on the left and copy it as follows.

- Draw a rectangle **10cm** high **x 16.2cm** wide. This rectangle is in the **Golden Ratio** of **1 : 1.62.**
- You are going to divide this rectangle into a square and a smaller rectangle, as follows.
- Along the top line of the rectangle, working from the left, measure **10cm** and label this point **A**.
- From **A** draw a **10cm** vertical line downwards and label the bottom point **B**, to form a square on your left. Label the other corners of the square **C** (bottom left) and **D** (top left).
- Now look at the vertical line on the far right of your diagram. Label the top point **E** and the bottom point **F**.

You now have a square on the left (A, B, C, D) and a smaller rectangle on the right (A, E, F, B).

Stage 2

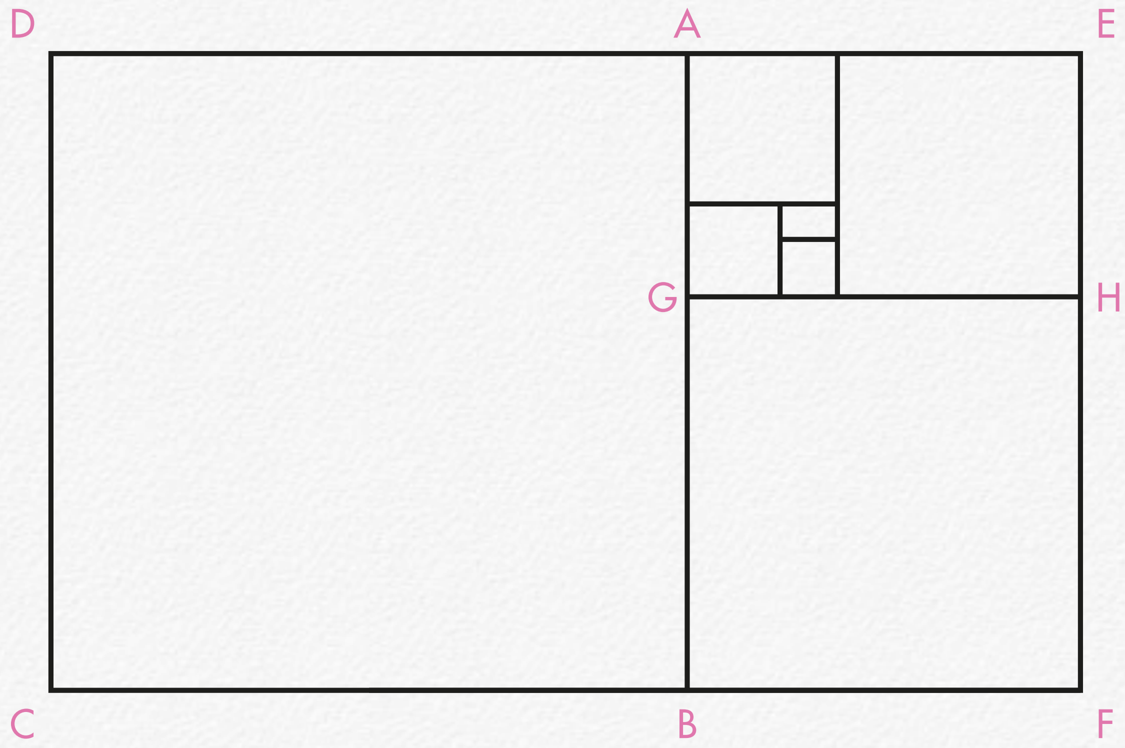
- Turn your page clockwise by **90°**, so that the line **EF** is at the bottom of the page.
- Measure the left hand vertical line, **B** to **F**.
- Mark the same length on the top horizontal line, from **B** towards **A** and label this point **G**.
- From **G**, draw a vertical line downwards, to meet line **EF**, label at point **H**.

FROM NOW ON, ALWAYS WORK IN THE SMALLER RECTANGLE.

Example: in Diagram 2, the rectangle **AEFB** is smaller than the rectangle **DEFC**.

Shell of a Nautilus pompillus.

Diagram 2

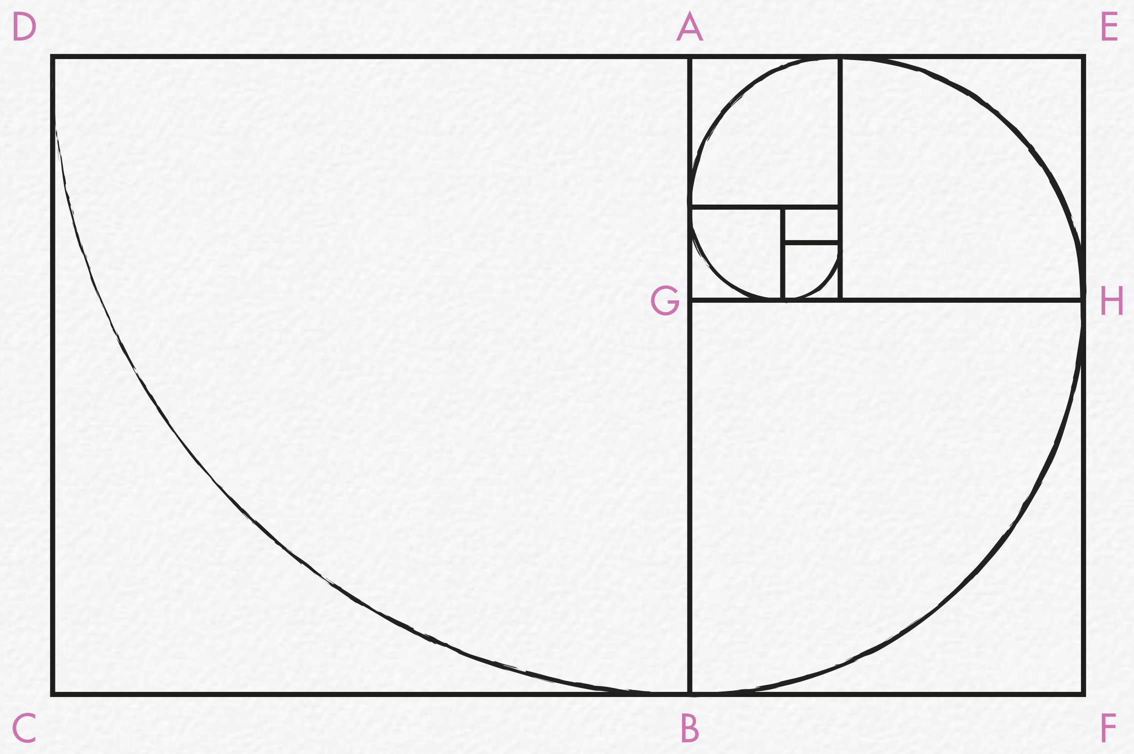


Stage 3

Repeat stage 2 without using the reference letters ABCDEFGH, as follows:

Turn your page clockwise by 90°. Measure the left side of the smaller rectangle. Mark the same length on the top line of the rectangle. From that point, draw a vertical line downwards to form a square on the left and a smaller rectangle on the right.

Diagram 3



Stage 4

Continue to repeat stage 3 until the smaller rectangle becomes too small to divide. At every stage, the rectangles get smaller. Do this six times in all.

At each step, turn the page 90° clockwise, or you risk getting in a muddle. NEVER TURN YOUR PAGE ANTICLOCKWISE. There should always be a square on your left, and a rectangle on your right.

Stage 5

Position the page as you started, with **DAE** on the top line. In the square **ABCD**, put your compass point at **A**. Pull open your compass, so the pencil reaches **D**. Draw a quarter circle from **D** to **B**.

FROM NOW ON, WORK ONLY IN THE SQUARES, which get smaller at every stage.

Turn your page clockwise by **90°**, so a smaller square now faces you. Put your compass point at the top right hand side **G**, and continue the curve by drawing a quarter circle from the top corner **B** to the bottom corner **H**.

Stage 6

Repeat stage 5, without the letters. Turn the page **90°** at the start of each stage. Adjust your compass to the length of each new, smaller square.

Always position your compass point on the top, right corner of the square.

Always continue the curve from the top, left corner of the square.

When the curve reaches your final square...

Congratulations, you have drawn a Golden Spiral.

Do you notice any similarity to the Fibonacci Spiral? The Fibonacci Spiral is **almost** the same as the Golden Spiral.

In the Fibonacci Sequence, if you work out the ratio between any two consecutive numbers, for the smaller numbers, it will give a ratio close to the Golden Ratio. The further you go along the Sequence, the nearer you get to the exact ratio of 1: φ. For example, the ratio of 21 to 34 is 1.62 (to 2 decimal places) and so on.

YOUR LITTLE GREEK CRIB

Proportions

Φ is 1.6 (to one d.p.) that's **a bit over one and a half**.

The formula for The Golden Ratio is 1:1.6

Circumference of a circle

π is 3.1 (to one d.p.) that's **a bit over three**.

The formula for area of a circle is πr^2

The marvel of cabbage.

The Golden Spiral is one of Nature's growth structures.

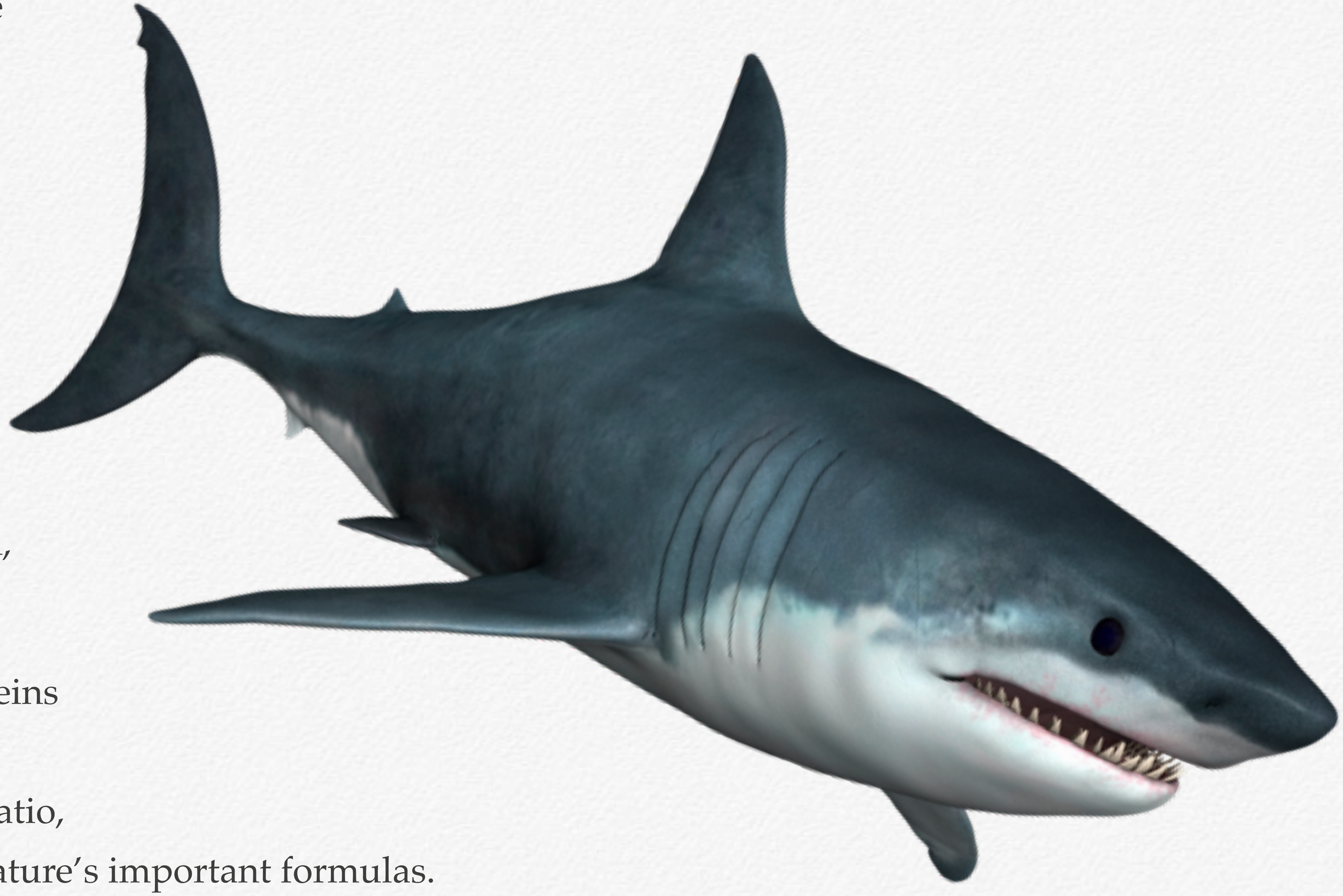
The growth factor is **φ** or **1.62**… The organism grows by a ratio of **φ:1** at every growth stage. The Nautilus seashell (see previous pages) is the best example of this; starting from the tiny spiral, it grows larger and larger. Always expanding in the same ratio, the shell grows bigger without changing shape or proportion.

Note: the Golden Spiral is an example of a logarithmic spiral.

Golden Ratio spirals fill space efficiently, which is why scientists find them over and over again in nature: in the arrangement of petals and seeds in a flower, leaves on a branch, branches on a stem.

In animals, you can see it in the branching of veins and nerves, and the proportions of skeletons.
Not everything in nature follows the Golden Ratio, but it occurs enough to show that it is one of nature's important formulas.

"Whenever we are pleased with our (design) results, we find that the proportions of our cars correspond to the Golden Section...If that's not the case, then it just doesn't look right." "Making Aston Martin", by Dr. Ulrich Bez. © 2013, pub TeNeuer.



Elegant, efficient design.

How to Make a Star

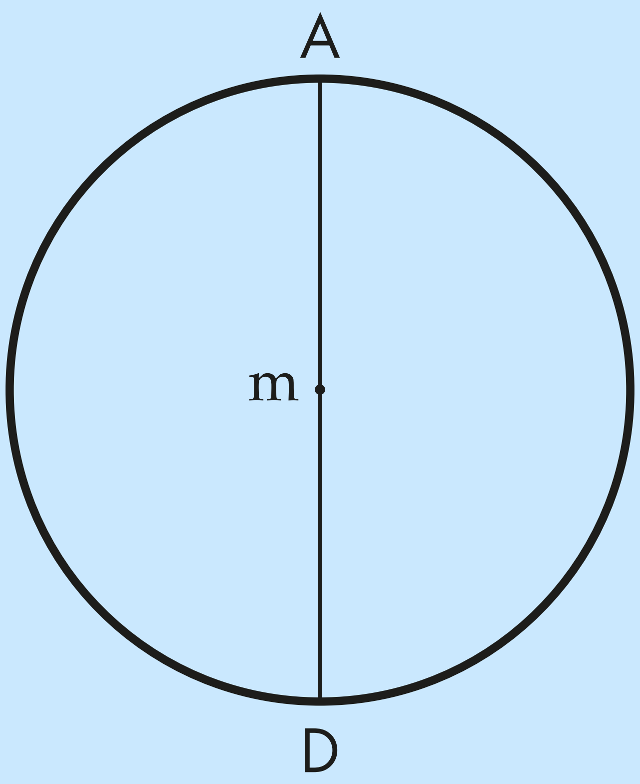
Just for fun - my fun - here's how to draw a six-pointed star. This star is frequently used in Islamic art, and when you can draw this, plus a circle, a rectangle and a triangle, you can make other shapes and plan almost any geometric, flat pattern design for wall covering, fabric and so on.

You need a sheet of paper, a pencil, a compass, a ruler and a protractor.

Step 1.
Open your compass to measure **8cm** between each arm, then draw a circle.

With your ruler and pencil, draw a **diameter** line that runs vertically through the centre of the circle.

Label **A** and **D**, the two points where the diameter touches the edge of the circle (called the **circumference**). **A** needs to be at the top of the circle.



Step 2.
Place your protractor on line **AD**, with the curved side of the protractor to your right and - of course - the centre point of the protractor on the centre of the circle, at m.

Measure an angle of **60°** from **A** to the right. Mark the point with a dot at the edge of the protractor.

Remove the protractor from the paper. With a ruler, connect the dot and the centre of the circle with a straight line. Extend both ends of this line to the circumference of the circle and label the right-hand end **B** and the other end **E**.

Place your protractor on the line **BE**, with the curvy side of the protractor nearest you. Measure another angle of **60°**, and mark with a dot, as before.

With a ruler, join the dot to the centre of the circle and extend both ends of this line to the circumference, **C** (on the right) and **F** (on the left).

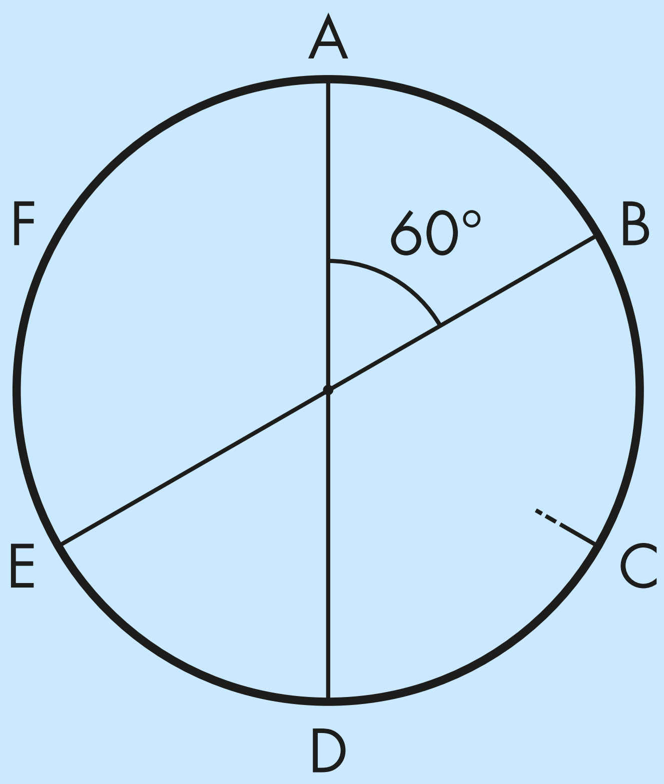
You should now have a circle with the points **A, B, C, D, E** and **F** marked in that order on the circumference, at equal distance apart.

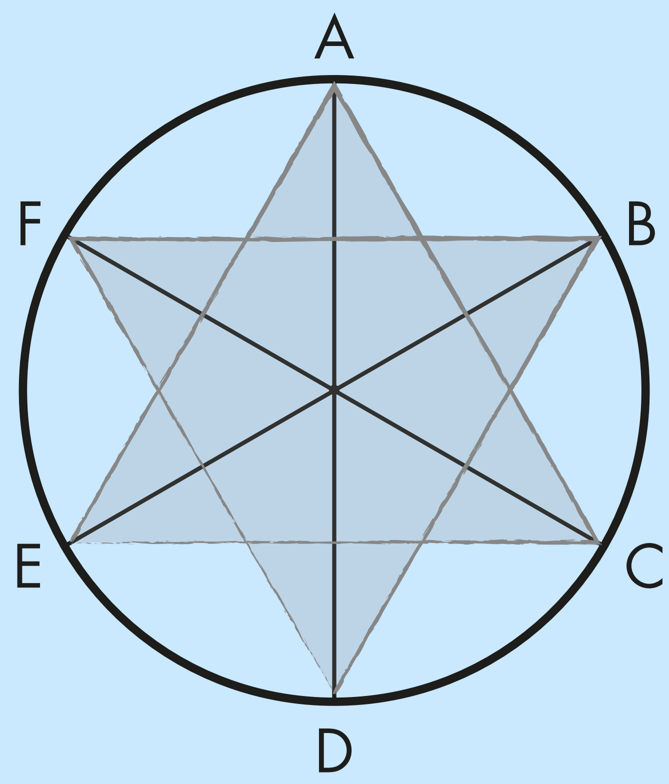
Step 3.
With your ruler, draw the following lines:

Join **A** to **C, C** to **E, E** to **A**. You have a triangle.

Join **B** to **D, D** to **F, F** to **B**. You have a second triangle. In fact, you have a lot more triangles. Count them.

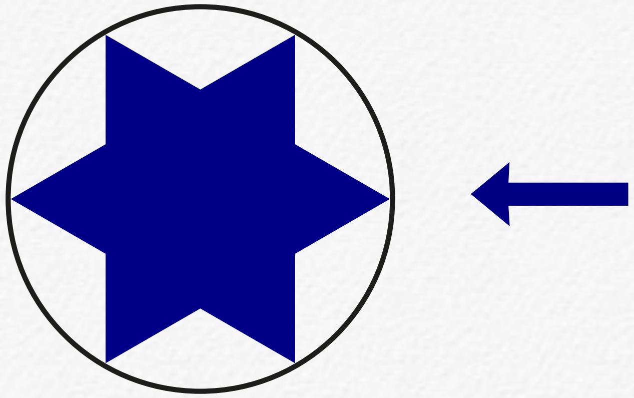
With your pencil, shade the inside of triangle **ACE**, then shade triangle **BDF**, and you have a star. Colour it gold. I award it to you.





The Sultan Ahmed Mosque, Turkey.

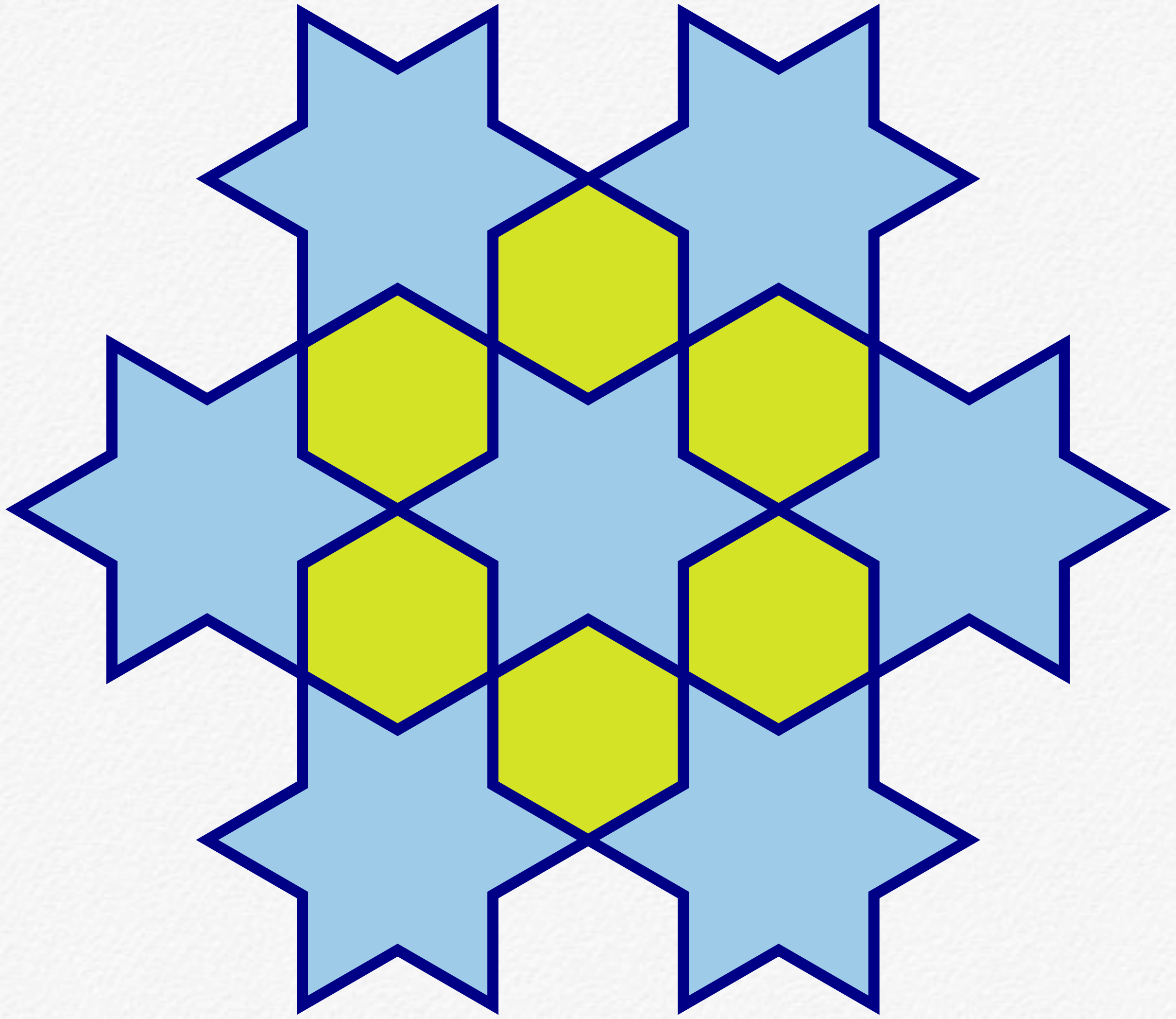
Note: Draw, then cut out, a star on cardboard. Use it as an outline guide to draw, then paint dozens of stars – imagine the Milky Way on your walls or bedroom ceiling.

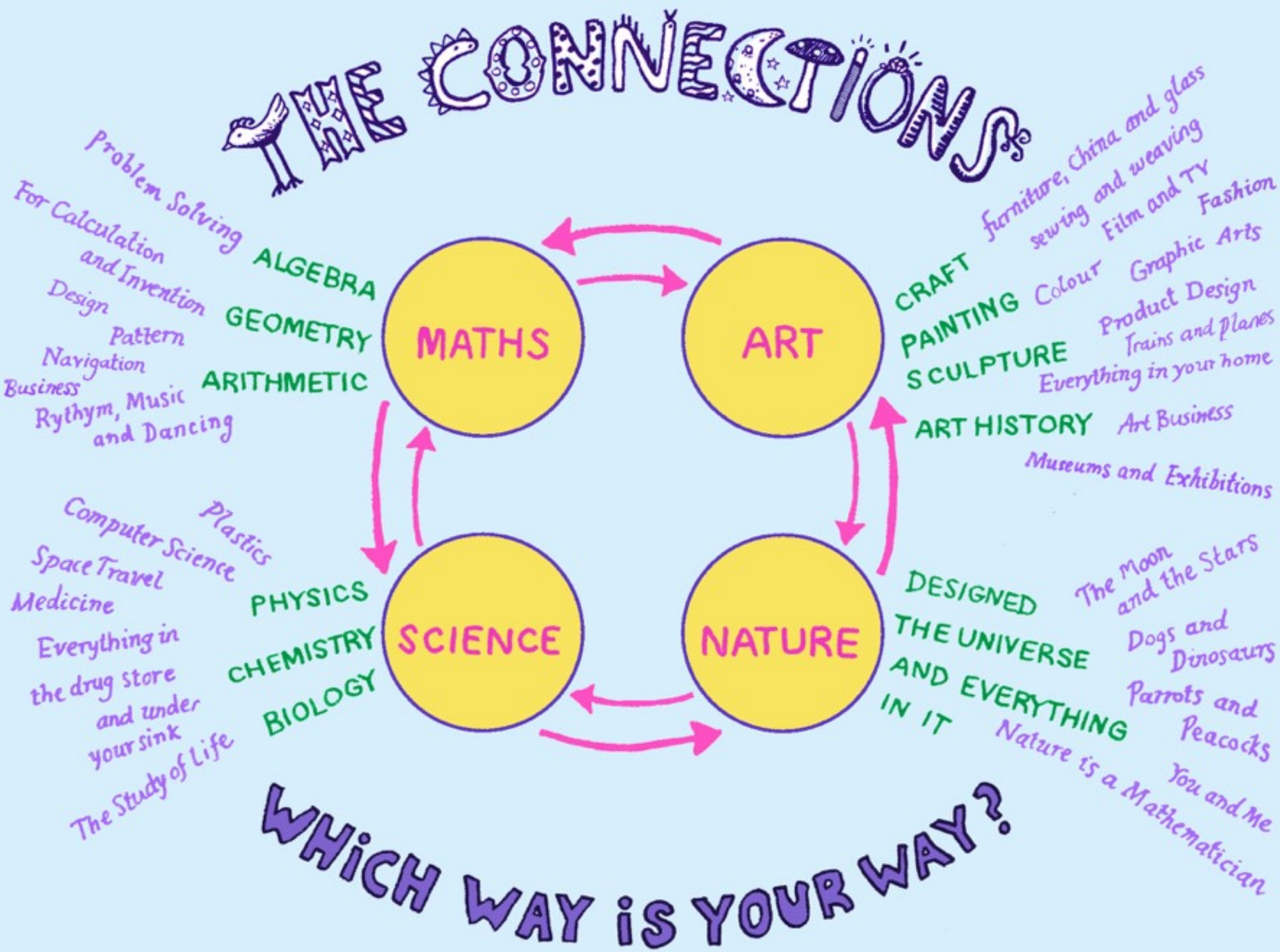


This six-point star is called a **hexagram.**

Flight to fame? Hopeful 1940s starlets.

Create a star pattern:




THE CONNECTIONS
Problem Solving
For Calculation
and Invention
Design
Pattern
Navigation
Business
Rythym, Music
and Dancing
ALGEBRA
GEOMETRY
ARITHMETIC
MATHS
ART
CRAFT
PAINTING
SCULPTURE
ART HISTORY
Furniture, China and glass
sewing and weaving
Film and TV
Fashion
Colour
Graphic Arts
Product Design
Trains and planes
Everything in your home
Art Business
Museums and Exhibitions
Computer Science
Plastics
Space Travel
Medicine
Everything in
the drug store
and under
your sink
The Study of Life
PHYSICS
CHEMISTRY
BIOLOGY
SCIENCE
NATURE
DESIGNED
THE UNIVERSE
AND EVERYTHING
IN IT
The Moon
and the Stars
Dogs and
Dinosaurs
Parrots and
Peacocks
You and Me
Nature is a Mathematician
WHICH WAY IS YOUR WAY?

The Connections

That was a very brief explanation of the connections between: maths + art & design; maths + nature; maths + science.

You need maths for many JOBS, and ALL CAREERS.

Musicians need arithmetic. **Artists** use arithmetic and geometry, whether painting on paper or working with some IT product which could not have been invented without maths.

An Art & Design craft specialisation can lead to a design career: **sewing** leads to **fashion**, **carpentry** leads to **furniture**, **pottery** can lead to **china** and **glassware design**.

Designers, **engineers** and **architects** need arithmetic, algebra and geometry. **Astronomers** use maths to get information from stars that are light-years away, **chemists** use maths to write their formulae.

All over the world, mathematics is the language of science.

The basic goal of **physicists** is to discover all the laws that govern the Universe, from "How do you make sand?" to "What makes volcanoes erupt?" Like most people, a physicist sees beauty in a rainbow but can also delight in discovering – with maths – how a rainbow is made.

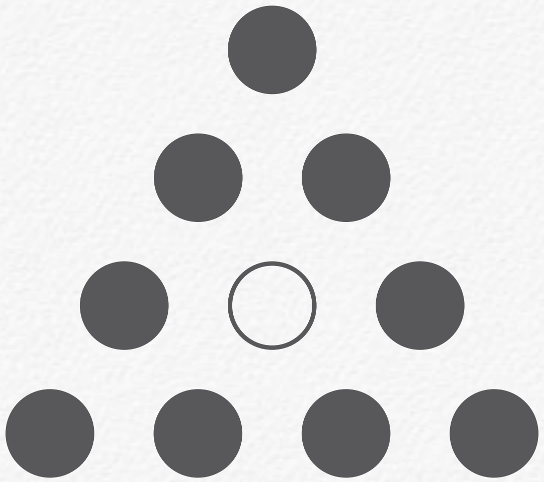
Nature uses maths to design
EVERYTHING THAT EXISTS,
from a wave, or a mollusk on the ocean floor,
to a baby chick and the stars in the Universe.

TO UNDERSTAND THE MYSTERIES OF THE UNIVERSE – YOU NEED MATHS.

Slice a red cabbage to see the pattern.

How Nature Designs

The ten **Decad** numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 can provide all the other numbers and formulas found throughout the Universe.



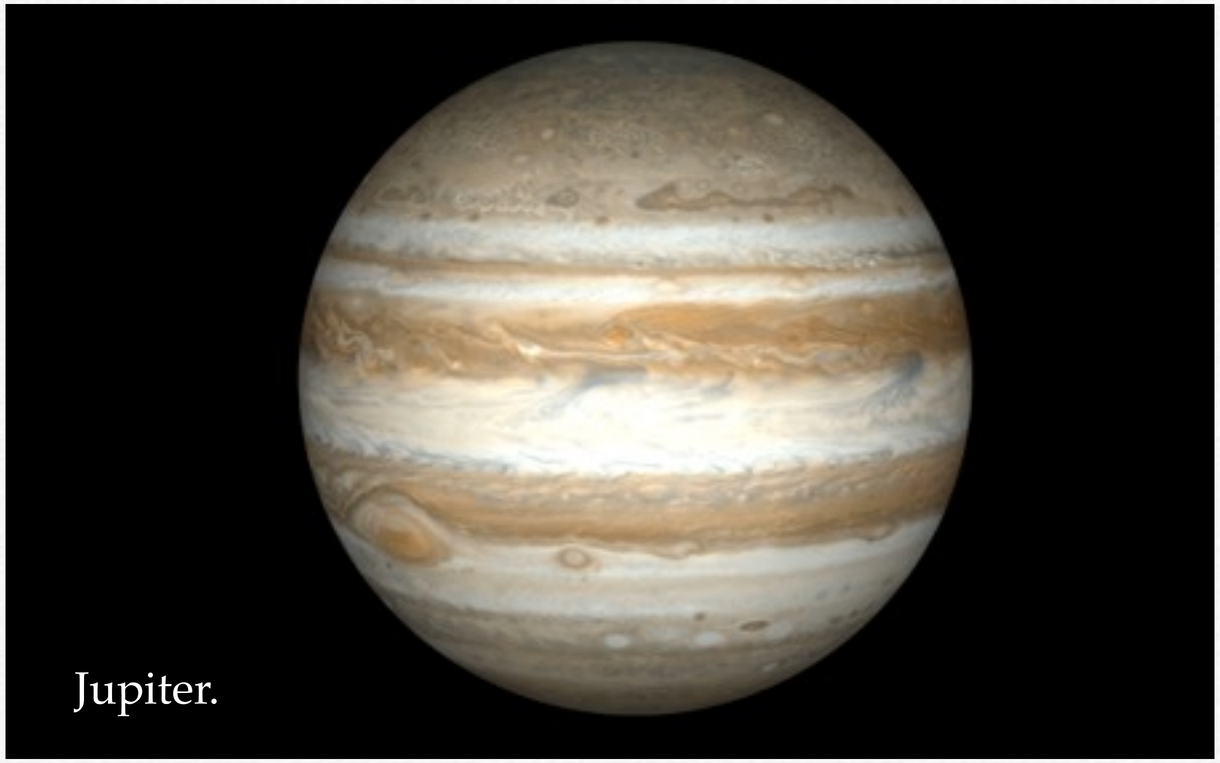
THE DECAD

Those ten numbers plus a few simple geometric shapes – line, square, triangle, circle – are the basics used by Nature to produce everything in the natural Universe.

deka = Greek for ten. **Decade** = English for a ten year period.

Nature uses **a few maths formulae**, and always refines the design she constructs until she achieves the most efficient system, which uses the least energy, whatever the scale, from snowflake to snowstorm.

Satellite photos show that there is only one weather pattern and it is global, not local. A local weather pattern may seem chaotic, but the big picture is harmonious.



Jupiter.

"The branching shapes of trees to attract the maximum sunlight are one of the universal patterns – like the structure of lungs, roots, coral and mathematical solutions." From "A Short Book about Drawing" by Andrew Marr. © 2013, pub Quadrille Publishing.

A famous seventeenth century scientist called Galileo, said that Nature is a book written in the language of mathematics and the more you start to look for that language, the more you see it everywhere.

Just look at a group of trees and you see the complexity of Nature; but beneath it lies the simplicity of a few mathematical laws. From leopard’s spots and leaves on trees to sea waves and the Galaxy, “In Nature, there are patterns on every scale,” says Professor Brian Cox in his BBC TV series “Human Universe: Why Are We Here?”

In ancient times, Indian, Persian and Greek mathematicians studied naturally occurring rhythms – such as the repetitious four seasons – and other behavior patterns, for clues that might lead to a better understanding of the Universe.

They’re still working on it.

- Nobel prize-winner, physicist Professor Richard Feynman, described to a friend his feelings about the beauty of the world.

  "It's (like) the feeling one has in religion, that has to do with a god that controls everything in the whole universe… when you think about how things that appear so different and behave so differently are all run, 'behind the scenes' by the same organization, (with) the same physical laws."

  "It's an appreciation of the mathematical beauty of Nature, of how she works inside… a feeling of how dramatic and wonderful it is. It's a feeling of awe – of scientific awe – this feeling about the glories of the universe."

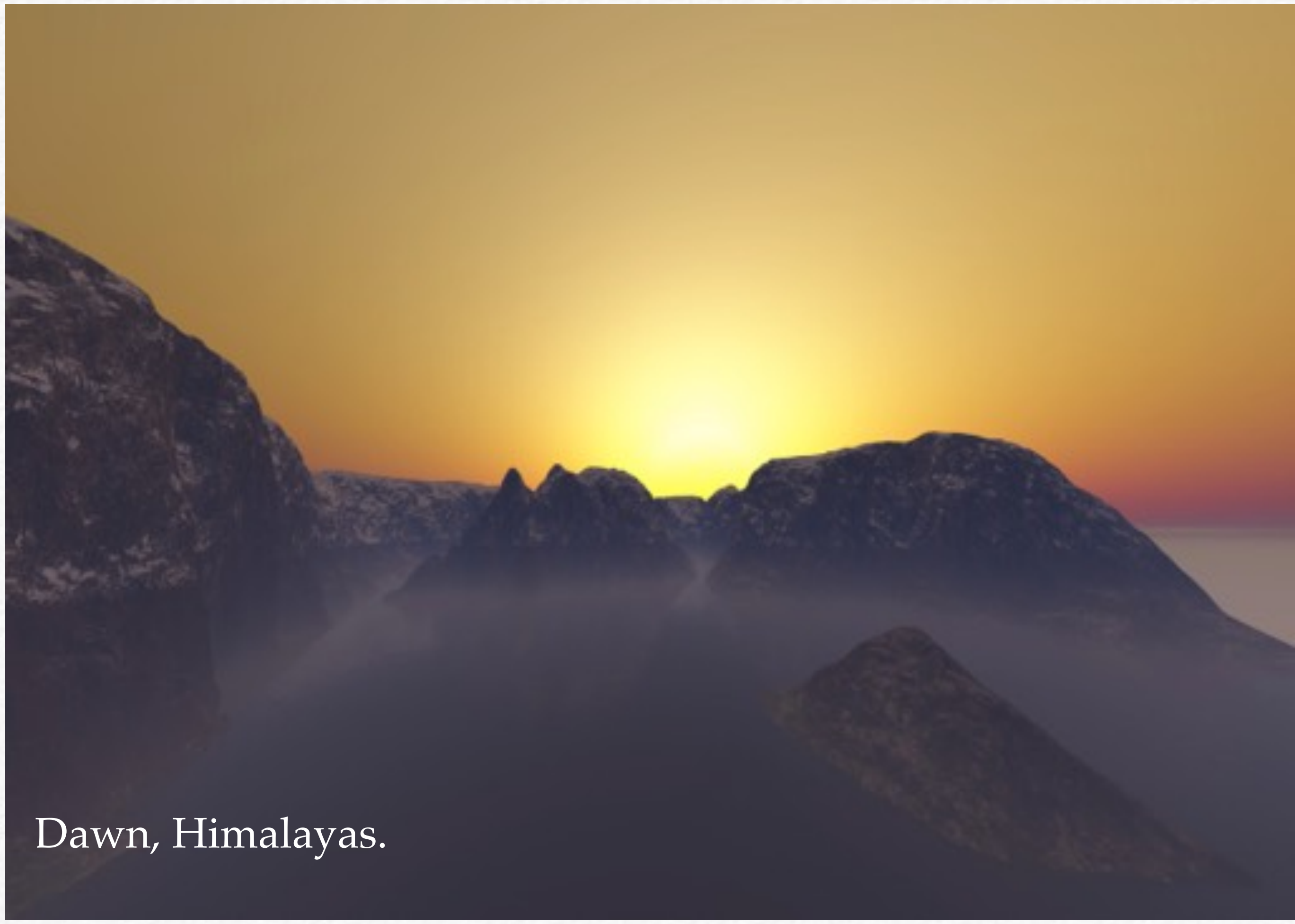
From "Surely you're joking, Mr. Feynman!"
by Richard P Feynman.
© 1865, pub W.W.Norton & Company Inc.

Note: When a quotation includes dots…like that…it means that the quoter is cutting out stuff.

When a quotation includes words in brackets in an odd place (like this) it means that the author has cut out stuff and has instead inserted "linker" words – the ones in brackets.

- "To learn to resee the world in terms of its (mathematical) patterns requires a shift within us. But once this shift occurs, a light turns on and the world brightens."

From "A Beginner's Guide to Constructing the Universe"
by Michael S. Schneider,
© 1994, pub Harper Collins.


Dawn, Himalayas.


The Himalayas.

Nearly there!

YOUR FUTURE

You may have a clear plan for your life or you may have a vague plan for your life...

But life is like an obstacle race in the dark - you never know what's coming next.

Whatever it is, you are now better equipped to deal with it, because you have...

FINISHED THE MONEY STUFF COURSE.

So give yourself a pat on the back. Give yourself a wrap-party, as they do on a film set after the last take. Celebrate your terrific achievement.

YOUR FUTURE

Keep up with your MONEY STUFF practice.

If you forget a bit, find an hour – when you're not tired – and read the whole part again. Good practice - check every bill you get.

Keep the MONEY STUFF Course because you'll need it as a reference all your life.

Use your numbers skill to manage your money – and you will undoubtedly have more money, now you've reached...

THE END

...but always remember...

You might be a celebrity...

You might not......

But now you know your money stuff...

You are prepared for stardom

And certainly you can be...

THE STAR OF YOUR OWN LIFE !!!

Money Stuff

4 STEP MATHS PLAN

I judge myself competent in the following:

PROBABILITY

Calculating basic probability for one, two or more events.
Compiling work-based Risk Assessments.

ALGEBRA

Simplifying and solving algebraic equations. Using formulae.
Using algebra to solve real life problems, including those with simultaneous equations.

GEOMETRY

Basic geometric terms. 2D and 3D shape names.
Calculating areas of rectangles, triangles, circles, compound and irregular shapes.
Calculating volumes of cubes and cuboids.

BANK INTEREST, GROWTH & INFLATION

Calculating simple and compound interest.
How compound interest relates to exponential growth and inflation.

Signed ....

Date ....

STEP 4

Money Stuff

THE END

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Mathematics consultant
ELIZABETH FAGERLUND
Illustrator SASHA SPYROU

Editor
LINDSAY NICHOLSON

Final production
Technical drawings
DAVID MOSELEY

Thank you

Ten years ago I decided to produce a book about maths and money that would make it easier for women not to feel apprehensive about maths, make their own financial decisions and understand business. I was given a shove by my dear friend, American writer, Cyra McFadden and then an extra shove by the legendary British journalist (and astrophysicist) Lindsay Nicholson.

Above all else, I would like to thank my mathematics tutor, Elizabeth Fagerlund. Elizabeth prepared the maths working draft and also did the groundwork for the exercises which I then worked on. The original idea was that Elizabeth should teach me and that I would then write down what I had learned. In fact, Elizabeth **often** objected – for mathematical reasons – to what I had written, and I **always** objected to her proposed alterations – for communication reasons. We argued happily for four years, and every day I woke up thinking, "Oh good, it's maths today!"

One day Elizabeth overheard me tell someone that she was thirty-two. No, she said, "I was thirty-two when we started, I'm thirty seven now.". That was five years ago, and since then, Elizabeth has acquired a husband and three children– and we're still happily discussing maths, something my schoolgirl self would never have believed possible.

Why not? Because when I was twelve, I dreaded maths so much that during a maths lesson that I hadn't prepared, I told the maths teacher that I had a stomach pain. Immediately, I was sent to the school nurse. When asked the exact location of the pain, left or right, I played it safe, "In the middle." The nurse took me home in a taxi. My white-faced mother called the doctor. Too terrified to tell the truth, I ended up in hospital for observation.

After two months in hospital, they prudently took out my appendix, as insurance, and I missed the tennis season. I didn't know that my mother's only brother, aged eight, had died after an emergency appendix operation, which explained my mother's white face.

Having caused so much trouble, I would have been astonished to hear that I would one day enjoy doing maths – perhaps because I loved working with Elizabeth, who also introduced me to other maths teachers who worked on MONEY STUFF.

The maths text was tested separately by classroom maths teacher, Shirley Castle, who checked the maths both before and after the ebook version (twice). After that, all text was checked by mathematical tutors Luke Scott and Matthew Shaw and I am very grateful to them. I'm also grateful to Alfred Munkenbeck who kindly checked "Some of the Secrets of the Universe", and for the advice of Roger Green, Head of Maths at St Paul's Girls' School; his sons, James and Charles, kindly tested STEP 1 and STEP 2.

I'm also grateful to anthropologist Dr Samantha Callan – now Director of the Centre for Social Justice – for early research work which ranged from the historical social conditioning of young girls, to what seventy-year-olds wish they had been taught by their mother.

Apple's iBooks Author software was used for text, drawings, photographs and other illustrations. Illustrator Christopher Brown found the ideal illustrator, Sasha Spyrou who produced all the drawings and was a pleasure to work with – mostly by email, as Sasha and her two daughters live in Liverpool. Georgina Harding kindly helped me for two weekends with photo research.

For the first year of electronic input I worked with Lukas Bott in the studio of Sebastian Conran Associates, who provided all technical support, supervised by Studio Manager Helena Chadd. After Lukas emigrated to Thailand, I did the layouts at home and they were put on screen at the studio by designer David Moseley, who also did all the technical drawings. The first ebook version was criticised by graphics consultant Maya Wilson. I am also grateful for the generous advice and encouragement given to me by Louisa Fitch, CE of website design firm, Silent Deer.

Creative consultant Elke Hanspach of InkValley was responsible for the MONEY STUFF brand which includes the logo, the marketing strategy, the visual direction of the website; working with Elke is like jumping on a moving train, which always arrives on time.

My thanks also to Adam Hoyle, Creative Director of Do Tank Studios, which was responsible for my two websites moneystuff.com and shirleyconran.com and also the free

animated quiz, “Have You Got What it Takes to be a Millionaire?” which is at Apple iStore. Adam was also my technical consultant on the final work.

My brilliant publicity team included the legendary Jacqui Graham (traditional media), Liberty 842 (social media) and Think Jam (internet).

After the text of MONEY STUFF was finished – and **before any visuals were inserted** – a series of trials was arranged. The first was held over six weeks at Langley Park School for Girls in Kent (1,600 pupils) and arranged by business coach Diane Carrington – now Chair of Governors at Langley Park – where 13 and 14 year old students did a course from MONEY STUFF, designed by Lauren Davie. This was followed by a separate trial in the 6th Form, where students were preparing to leave school.

Head Teacher, Jan Sage, Head of Maths, Lesley Hine and maths teacher, Jenny Brown were very supportive and open to new ideas. Five years later, I revisited Langley Park, and met again some of the original MONEY STUFF graduates, who by that time had left school and were using their MONEY STUFF knowledge in the Real World. This was a real and unexpected pleasure for me.

Part of the Langley Park trial was filmed; the 8-minute DVD was produced by Najma Kazi after it had been filmed by students from the Ealing School of Media, which kindly allowed the use of its production facilities. Thanks are due to Godslove Mensah, (director) Stuart Fletcher, Julian Wright, Rikki Udagawa, Dominique Yeung.

During the Langley Park trials, I found reinforcement of my theory that – among many women – feelings about maths can range from apprehension to fright. So, with co-author, psychologist Helen Whitten, a self-confidence course was written, to weave in the text. For advising me on the determination plan, that was similarly added to the latter part of MONEY STUFF, I would like to thank football coach Thaddeus Cox for his help and the books to back up his advice.

The pilot held for 33 students in the Art & Design School of Loughborough University was organised by Professor Carol Robinson and her team, and the adjudicator was Dr David Green. I’m very grateful to them and to the Vice Chancellor, Professor Shirley Pearce, for allowing the pilot and for being so supportive.

In Essex, Jane Colby, CE of The Tymes Trust for Young ME Sufferers, kindly arranged for two long-term, home-bound girls, Shannen Dabson and Chloe Halstead, to do MONEY STUFF, which both finished triumphantly. These tests were deliberately not timed and they took place with the consent of and supervision by parents.

In London, Jane Denton and two other mothers, Jane McKenzie and Corinne Abbott, were considering sharing the cost of their children's maths master, to teach them once a week, so that they could help with homework and follow the financial news during the bank crisis of 2008. Instead, they met every week to eat cake and do MONEY STUFF. Elizabeth Fagerlund supervised this test.

Jane McKenzie was a secondary school supply teacher for everything except maths. At the beginning of a term, she was told she had to teach maths **that morning**, because a returning teacher's plane had been grounded after a volcano eruption in Iceland. Jane grabbed MONEY STUFF and successfully taught fractions. I was as thrilled as she was.

Among the other people, aged 13 to 70, who tested MONEY STUFF on their own, was Jane Ware (who also did some testing), Susan Anderson, and also, my former colleague at the Daily Mail, Janet Fitch. Janet has helped me from the beginning – and not only with editing, but with quiet support and encouragement.

Donald (Bones) Morris and I worked enjoyably together on text adaptation for the male version of MONEY STUFF.

My thanks also to Lauren Davie; as well as designing two of the courses, Lauren kindly kept track of all the IT and paper work behind MONEY STUFF; this Archive is carefully preserved in my garage.

For ten years, tolerant friends have supported me, in particular the generous Adrienne Katz, Director of Youth Works Consulting Ltd; Professor Cary Cooper CBE; Dr Carl Hylton of Black Men's Forum; Andrew Holdenby, CE of Reform Britain; Joel Cadbury, CE of Longshot; Dr Terence Brown, whose passion is the history of maths; Jacqui Graham, Publicity Director of Macmillan Publishing and Peter York, CE of Social Research Unit.

Peter introduced me to Caroline Shott, Founder and CEO of The Learning Skills Foundation, who for several years has steadily supported me and advised me.

I would also like to thank Anne Sibbald of Janklow & Nesbit, New York, for her advice, and literary agent Claire Conrad for her considerable work in London; I am indebted to David Solomon, CE of MD Management Corporation, Monaco, who kindly checked the budgets & bookkeeping section.

I am also indebted to designer Georgina Godley, for shrewd visual advice. It was Georgina who originally told me about the connections, then sent me ten books about it, wham!

Five years into the project, Zenna Atkins – then Chair of OFSTED – introduced me to Jerry Jarvis, the dynamic MD of Edexcel, who has since been a constant source of encouragement and support.

One day, Jerry said he wanted me to meet a friend for coffee; the friend was Keith Pledger, author and Chair of Examiners for Edexcel, a man who is a fountain of ideas in human form. Keith leaned forward and said, "The joy of your book is – it's so simple. I'm going to write an introduction. It will cost you nothing." If HM The Queen had popped in to prepare my breakfast tray, I could not have been more surprised or delighted by this generosity.

Gently, Keith nudged me into including the revision areas and the Achievement Certificates, and when I needed cheering up, after illness, he sent me a photograph of his bluebell wood.

I could not believe my luck when my daughter-in-law, Gertrude Thoma, offered to do the final edit. This meant weeks of work and meticulous attention to detail, so I am truly grateful for her contribution.

My gratitude beyond thanks goes to my M.E. mentor, Alex Howard of the Optimum Health Clinic and to Professor Peter White and his associates at the CFS Clinic, St Bartholomew's Hospital, London. Thanks also to osteopath Richard Budd of Re-Think Body who helped with the Refresher exercises.

At this point in the Gratitude List, authors thank their wonderful PA who has been their right arm, struggled through snow and ice to get to the bus stop in order to turn up for work, etc. My first thought was that this sort of thing encourages other people to pinch your PA. Should I perhaps say that MONEY STUFF was written despite my PA's awful typing, bad timekeeping and endless duvet days?

In fact, Rosemary Tross is everything that anyone could wish for in a PA – my left leg, second brain, etc – and only she knows how much I value her help, her consideration and her big contribution to MONEY STUFF.

Finally, I would like to thank my son, Sebastian Conran, who for ten years continually encouraged this project, criticised this project and pointed the way ahead for it. It was Sebastian who gave me my first iPad for my birthday, and Sebastian who had me working on Apple's iBooks Author software, within a week of its launch. What more could a mother want?

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