BY INTERNATIONAL AUTHOR OF SUPERWOMAN **DAME SHIRLEY CONRAN**

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THE DO-IT-YOURSELF
6 STEP MATHS PLAN
FOR REAL LIFE

THIS IS STEP 2

Money Stuff

MATHEMATICS WILL GIVE YOU GIRL-POWER

HOW TO NAVIGATE MONEY STUFF IS AT THE START OF STEP 1.

MONEY STUFF is a new way to learn, and each Part is built on the previous Part so - even if you know the basics - PLEASE START AT THE BEGINNING, with STEP 1. Some older testers started in the middle, got in a muddle and had to go back to the start. Don't risk it.

HOW TO GET THINGS DONE

World synchronized skating championships, Finland.

How to Get Things Done

When you are learning to ride a bike, you can easily give yourself an incentive, because you know that the result will be wonderful – because you see other people happily riding a bicycle.

When you are practising maths, you cannot see the end result, so it's not so easy to give yourself an incentive.

Remember that when you know your numbers, you will have more money, and an easier, better time – and you will use your increased maths skill for the whole of your life.

Remember, if you want to be richer, you need maths; if you don't want to be poorer – you also need maths.

MONEY STUFF contains all the maths you should need in Real Life. What it **doesn't contain** is an INCENTIVE, because that can only be provided by you.

So to get things done, work out your motivator – your own incentive – the thought of which gets you **into action mode**.

Here's how a good incentive works. My grandson, Sam, aged 6 could not count to ten. His anxious parents arranged for different tests: nothing wrong. I didn't know why Sam couldn't count: maybe he was lazy, maybe he liked to see his parents fuss over him; maybe he just didn't see the point.

So, one day on the kitchen table, I built several stacks of gold coins, £100 in all. I told Sam that if he could count to 100 by the following Saturday, he could keep the gold coins (incentive). He would only be allowed one try. If he made ONE mistake, he would lose all the gold coins (another incentive).

With two incentives to motivate him to work, on the following Saturday, Sam counted perfectly to 100. I had proved that he could learn something by heart – if he would focus on it.

Before going any further, **you** may need to choose your own really good incentive; your incentive needs to be something you will really enjoy: a walk in the park, a film, a cup of hot chocolate, a pair of slinky shoes.

When you achieve something you've never done before, you realise **you can do more than you think you can**: and that's a fantastic feeling, an added bonus.

One of the best things you can achieve in life is self-confidence. Anyone who thinks she is **totally** self-confident is **over-confident**, an arrogant know-all. On the other hand, if you have no self-confidence, you will magnify your worries and minimise your chance of dealing with any worry stuff.

What sort of worry stuff? Being short of money can feel a major worry, but so can having spots, feeling too fat or the perennial not-being-pretty-enough.

There's also the worry of doing something new for the first time, or meeting new people. Will they like me, will they accept me? Will I do something embarrassing? Will they take me seriously? Will I disappoint them?

Then there's the really personal worry: Will I fall for someone who won't fall for me? Have I just done that? How can I patch up the row with my beloved? Should I pretend I don't care? What if he does the same? We'll never meet again!

In this Course, you will find plenty of self-confidence tips, which will help you to achieve more than you think you can, in maths and in Real Life.

PS. Sam's mother grabbed most of Sam's prize coins to start a saving account for him, but she left plenty for him to spend at the toyshop.

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OUR MOTTO

Life is too short

to be short of money

!!! Watch out for prices !!!

(Another warning)

The cost of living has been zig-zagging upwards for hundreds of years. In the sixteenth century, Queen Elizabeth I worried about the increasing costs of feeding and equipping her army and navy. Today, you can still expect prices to rise unsteadily in the unforeseeable future.

What causes prices to rise? Many reasons, including bad weather, which increases farmers' food prices. So workers need higher wages, which means that the cost of the goods they make will increase. If the prices of bricks, cement and steel increase then so will the cost of housing and rents.

Sometimes the price rises are so small you don't notice them – but you will certainly notice if your home energy bill shoots up in a few months and mum starts switching off the lights and heating.

These rising prices are called **inflation**.

When I started to write this maths course, the prices I used in the exercises were the same as the prices in the shops – but by the time I had finished **Step 1**, the shop prices had risen - so the exercise prices were out-of-date. That is why the prices in **MONEY STUFF** are not current prices; they are historically correct prices, paid by your grandmother and mother in the early **21st** century.

In maths, as in life, people have different ways to writing numbers. For example, you can write a fraction as either ½ with a diagonal line, as we do, or as $\frac{1}{2}$ which you may also see. Whichever you use, the meaning is the same. Likewise, some people write **1,000** or **1,000,000** as we do, with commas to break up the digits, others prefer just to leave a space, like this **1 000** or **1 000 000**. The choice is yours – that's the joy of maths!

Shop prices will alter throughout your life.

But the maths you need to shop will never alter.

Dame Shirley Conran

CONTENTS

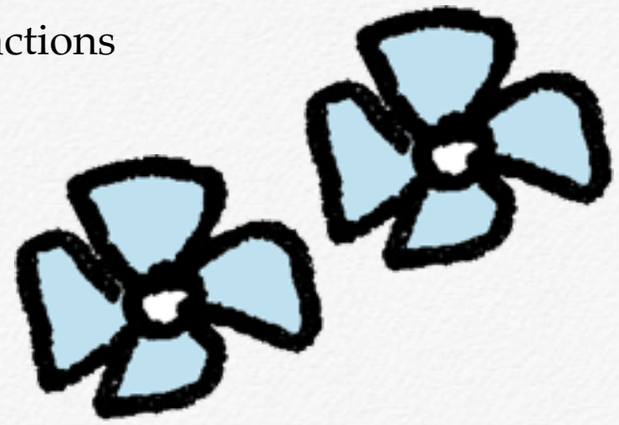
Fashion front row.

Chinese competitors warm up before women's three metre Springboard Synchro Final, Diving World Series 2011.

Contents

PART 12: DECIMALS

PART 13: PERCENTAGES



PART 14: QUICK SWITCH

PART 15: RATIOS

Chinese competitors in actual competition.

Relationships

(Fractions, Decimals and Percentages)

Remember, you need not learn by heart any of what follows – you only need to understand it. You can refer back to bits of MONEY STUFF when you need them.

Unfortunately, many schools teach fractions, decimals and percentages so that a student understands only the basic concepts of each system. A student is **not** always taught the underlying principles or the importance of relationships between the three subjects.

It is important to stress that **fractions**, **decimals and percentages all relate to each other as part of one whole concept – which is essential for Real Life**.

Relationships.

In Real Life, **all of them** are used everywhere: in shops, on labels, in magazines, in business. You can't get away from them – and you won't want to because the three systems are so useful in different situations.

Fractions Example

Star sale item: a pair of gold lamé pants with $\frac{1}{4}$ off the price.

So the gold hot-pants cost only $\frac{3}{4}$ of the original price.

If the original price was £4, the sale price is £3.

Percentages Example

'One per cent' means one out of 100 (more later), so if 70% of magazine readers took part in the win-a-sports-car competition, the editor immediately knows that the competition was a success because 70 out of every 100 readers tried to win the sports car.

But only 7% of readers entered the win-a-summer-holiday-in-Iceland competition. The editor immediately realised that, compared to previous competitions, this competition was a failure because only 7 out of every 100 readers entered it.

Decimals Example

The decimal system is where a whole thing is cut into 10 equal parts (tenths) or 100 equal parts (hundredths) and so on. Whole things are ALWAYS separated from part things by a decimal point.

To repeat: **fractions**, **decimals** **and** **percentages** are numbers used when you want to represent a part of a whole.

Fraction example

$3\frac{1}{8}$ means you have a bit more than 3 whole things.

Percentage example

99% means you don't quite have a whole 100%.

Decimal example

12.4 means you have a bit more than 12.

Later, you'll see that fractions, decimals and percentages are interchangeable.

If you say, "I'll buy half a kilo of apples," in one short sentence you have used two measuring methods - **half** and **kilo**.

You don't say, "I'll buy nought point five of a kilo of apples."

PART II
FRACTIONS

Quick calculations.

Quick Quiz

Q1.

How is **nine million and ten** written in numbers?

- A. 9,000,010
- B. 9,010,010
- C. 9,010,000
- D. 9,000,000,010

Quick Quiz

Q2.

Which of the following is a prime number?

- **A.** 21
- **B.** 23
- **C.** 25
- **D.** 27

Quick Quiz

Q3.

What is the answer to?

8 + 8 ÷ 2 – 5 x 2 = ?

- **A.** 2
- **B.** –2
- **C.** 6
- **D.** 14

Quick Quiz

Q4.

What is **72,827** rounded to the nearest thousand?

- **A.** 70,000
- **B.** 80,000
- **C.** 72,000
- **D.** 73,000

Quick Quiz

Answers

Q1. 9,000,010

Q2. 23

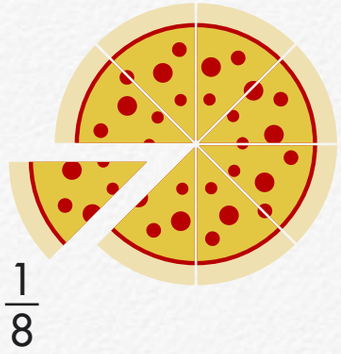
Q3. 2

Q4. 73,000

Fractions

Pizzas are often used in maths examples because it's easy to imagine a pizza cut into pieces: I don't know what maths teachers would do without Italian cooking.

To check that you know what a **fraction** is, think of a pizza. If there are eight people at supper, then one pizza is divided into eight equal parts and everyone gets one slice – one eighth of the pizza.



$\frac{1}{8}$

This is written as $\frac{1}{8}$ of a pizza.

If you eat two slices, you've had $\frac{2}{8}$ of the pizza.

Fractions also apply to groups of things. If you eat 7 chocolates from a box of 10, you have eaten $\frac{7}{10}$ of the chocolates. Bang goes your diet.

As you have just seen, fractions are written as two numbers, one on top of the other with a **dividing line** between. The bottom number (**the denominator**) tells you how many equal parts a whole thing has been divided into, and the top number (**the numerator**) tells you how many of those parts you have.

$\frac{1}{8}$ ← the numerator

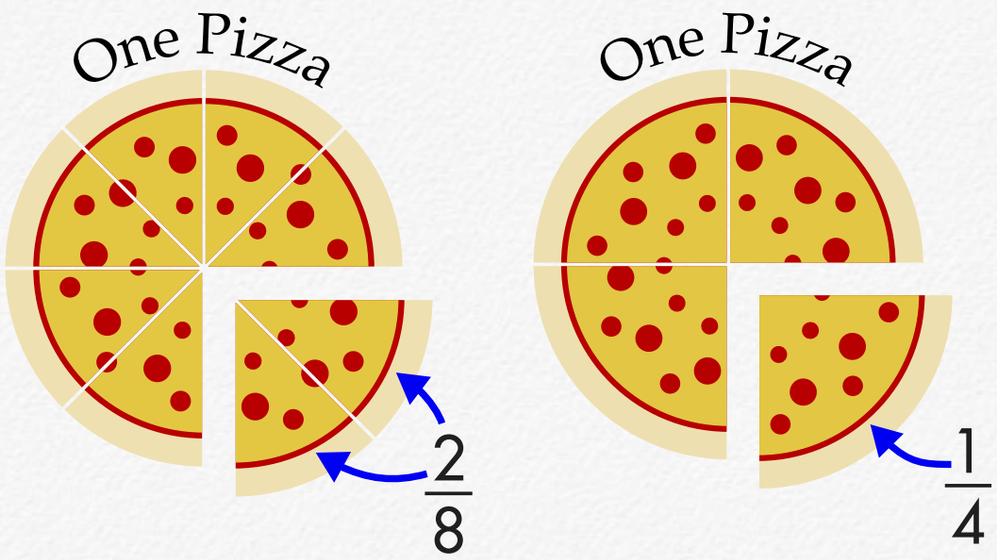
$\frac{1}{8}$ ← the denominator

Some fractions that look different on paper are really the same thing. These are called **equivalent fractions**.

Example 1

If you ate $\frac{2}{8}$ of a pizza that you divided into **8** slices, you have really eaten a quarter ($\frac{1}{4}$) of the pizza.

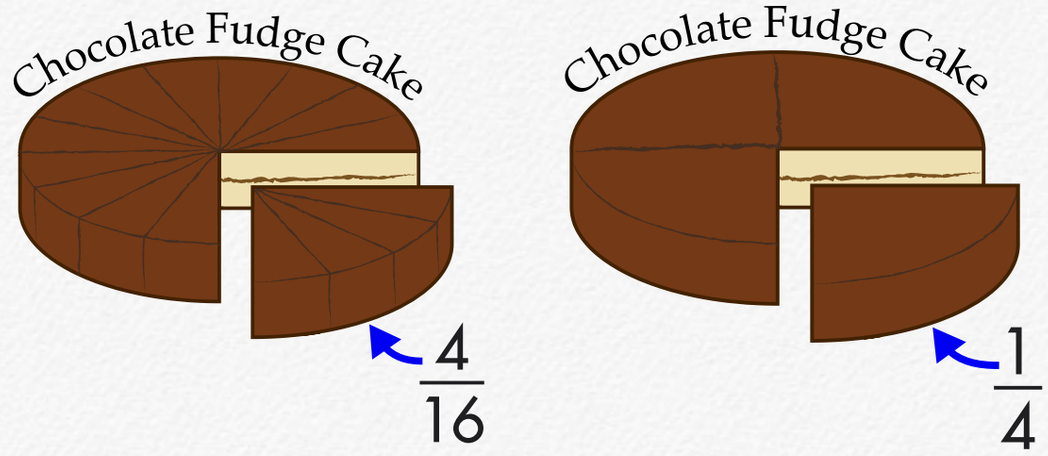
So $\frac{2}{8} = \frac{1}{4}$.



Example 2

Divide a chocolate fudge cake into **16** slices. **4** slices are eaten at tea time.

a) What fraction of the cake has been eaten?
b) What fraction of the cake remains?



Answers:

a) $\frac{4}{16}$ of the cake was eaten = $\frac{1}{4}$ eaten.

b) $\frac{12}{16}$ of the cake was left = $\frac{3}{4}$ remains.

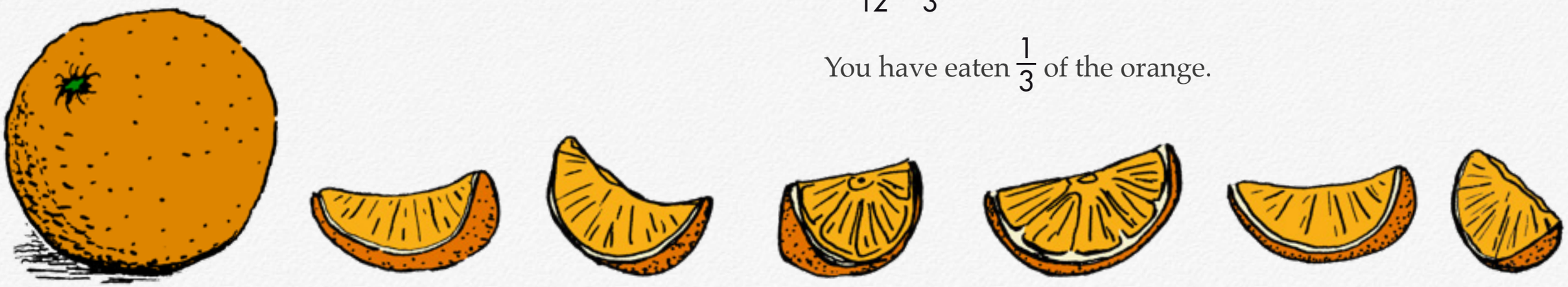
The Equivalent Fraction Rule

You can make an equivalent fraction mathematically, without having to draw slices of pizza or cake. You alter, by multiplication or division, both the number on top and the number underneath the dividing line in the same way. The resulting fraction, although it has different numbers, is an equivalent fraction.

EQUIVALENT FRACTION RULE

Whatever you do to the top, do the same to the bottom

(only to multiply or divide)



First Example

From the previous chocolate fudge cake example we can see that $\frac{4}{16} = \frac{1}{4}$.

$\frac{4}{16}$ becomes $\frac{1}{4}$ if you **divide** both the top and bottom by **4**.

$$\frac{4}{16} \overset{\div 4}{=} \frac{1}{4}$$

Second Example

If you eat **4** segments of an orange out of a total of **12**, then you have eaten $\frac{4}{12}$ of the orange.

Divide both the top and the bottom by **4** and you get the equivalent fraction which is $\frac{1}{3}$.

So $\frac{4}{12} = \frac{1}{3}$.

You have eaten $\frac{1}{3}$ of the orange.

Third Example

The circle below is divided into **5** equal parts of $\frac{1}{5}$ each.

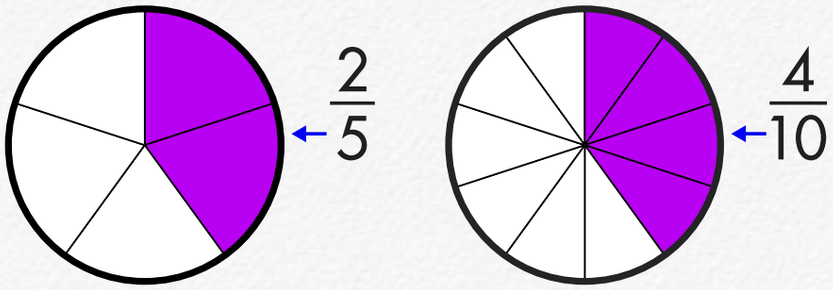
What is the equivalent fraction of $\frac{2}{5}$ in tenths?

It's easier if you write it like this: $\frac{2}{5} = \left(\frac{?}{10}\right)$

Next, ask yourself how to change the denominator **5** into 10. The answer is, multiply by **2**.

Using the equivalent fraction rule, you also need to **multiply** the top by **2**.

Answer: $\frac{2}{5} \overset{\times 2}{=} \frac{4}{10}$

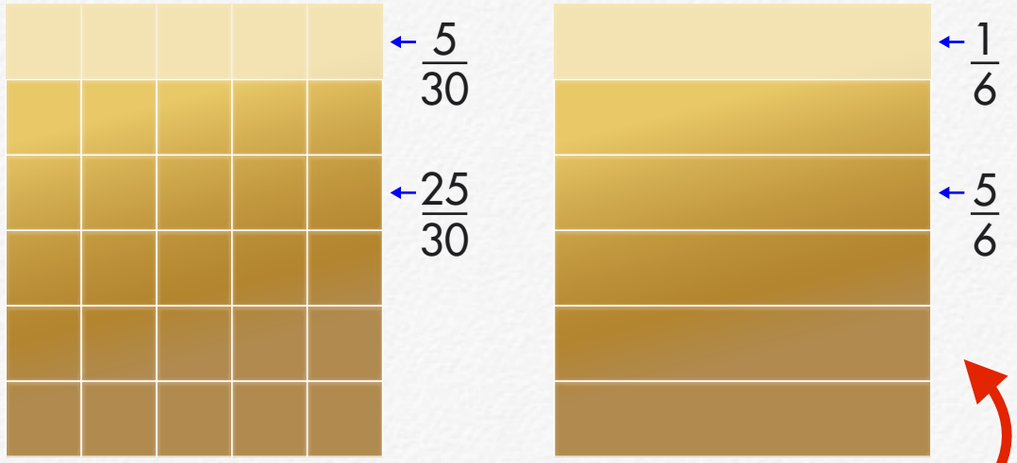


Fourth Example

Belinda has made a tray of toffee for the village fête. She cuts it up into **30** squares and gives **25** squares to sell at the fête, but keeps **5** squares for herself. What fraction of the toffee has Belinda given to the fête?

Answer: $\frac{25}{30} \overset{\div 5}{=} \frac{5}{6}$

Belinda gave $\frac{5}{6}$ of the toffee to the fête.



If Belinda cuts the toffee into **6** bars, not squares, and keeps one bar for herself, she would still keep $\frac{1}{6}$ of the toffee for herself, and give $\frac{5}{6}$ of the toffee to the fête.

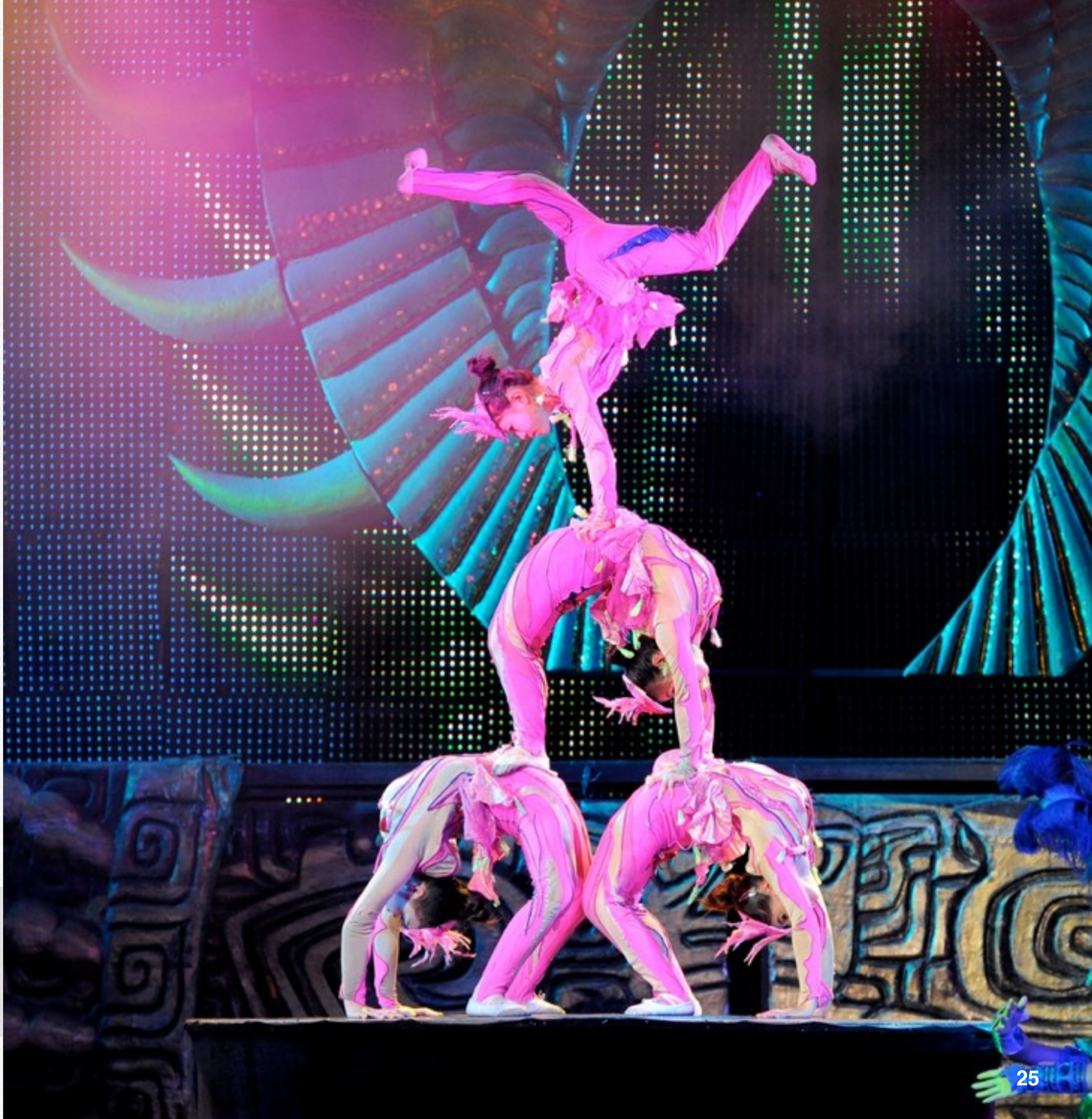
Exercises

Use the equivalent fraction rule to fill in the following blanks:

1) $\frac{1}{4} = \frac{?}{8}$

2) $\frac{3}{9} = \frac{?}{3}$

3) $\frac{12}{16} = \frac{?}{4}$



In balance.
Chinese acrobats
in Beijing theatre.

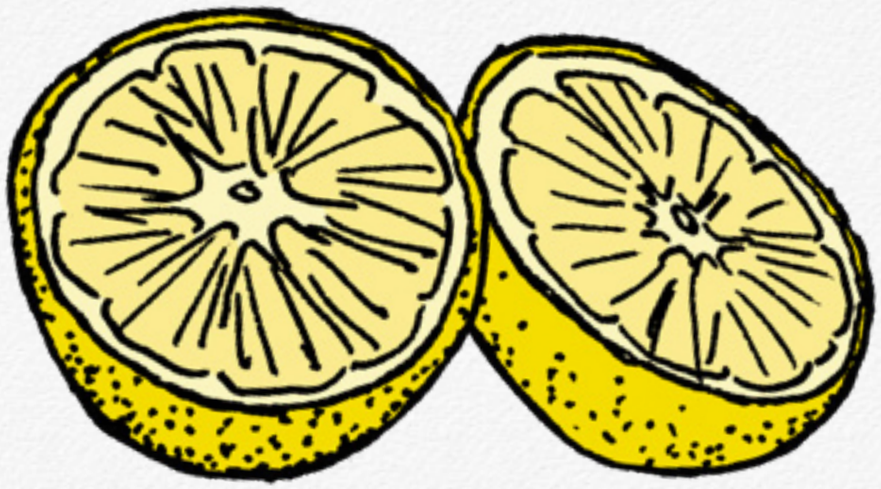
How to Simplify a Fraction

Writing a fraction in its simplest form will make it easier to understand.

For instance, $\frac{1}{4}$ is easier to visualise than $\frac{4}{16}$

The previous exercises **2** and **3** are both examples of simplifying a fraction. So you've just done it!

To simplify a fraction, you always **divide** both the top and bottom number by the same number. Keep simplifying the fraction until you cannot find a number that divides both the top and bottom number. When this happens, you know your fraction is in its simplest form.



First Example

Simplify $\frac{16}{32}$ to its simplest form.

Divide both the top and bottom numbers by 2 until you can't go any further: $\frac{16}{32} \overset{\div 2}{=} \frac{8}{16} \overset{\div 2}{=} \frac{4}{8} \overset{\div 2}{=} \frac{2}{4} \overset{\div 2}{=} \frac{1}{2}$

So the answer is: $\frac{16}{32} = \frac{1}{2}$

Simplifying a fraction is sometimes called '**reducing a fraction**', or '**cancelling down**'.

Second Example

Cancel down $\frac{9}{15}$ to its simplest form.

Divide both the top and the bottom by **3**, since **3** goes into both numbers: $\frac{9}{15} \overset{\div 3}{=} \frac{3}{5}$

You can't divide any further...

So the answer is: $\frac{9}{15} = \frac{3}{5}$

Third Example

Reduce $\frac{45}{60}$ to its simplest form.

Since **5** goes into both **45** and **60**, divide by **5**:

$$\frac{45}{60} \overset{\div 5}{=} \frac{9}{12}$$

Both **9** and **12** can be divided by **3**, so continue:

$$\frac{9}{12} \overset{\div 3}{=} \frac{3}{4}$$

You can't divide any further...

So the answer is: $\frac{45}{60} = \frac{3}{4}$

Exercises

4) Simplify the following fractions to their simplest forms:

a) $\frac{4}{20}$ b) $\frac{7}{28}$ c) $\frac{11}{77}$

d) $\frac{12}{18}$ e) $\frac{35}{63}$ f) $\frac{48}{88}$

Self-Confidence Tip 4

SWITCH YOUR MOOD

A girl with nothing to do might feel depressed, with low energy; but if her boyfriend calls unexpectedly… she's all smiles and energy.

Because you can switch your mood instantly, there's no need to be a victim of your feelings.

When you don't understand something, it's easy to get depressed and switch OFF. You may not realise that it's just as easy to switch ON. Someone who switches on 'aggression' or 'defiance' or 'sullen gloom' before breakfast is deciding to have a nasty day whatever happens. Someone who switches on 'cheerful' is likely to have a good day.

Practice being optimistic. When you're cleaning your teeth, say aloud, "I feel terrific!" (This works for me, but pick your own booster phrase).

Particularly when you try something new – ATTITUDE is all-important.

How to Compare Fractions

Fractions are easy to compare if they have the same denominator.

It's easy to see that $\frac{6}{7}$ is bigger than $\frac{3}{7}$, but not so easy to see if $\frac{3}{8}$ is bigger than $\frac{2}{5}$

To make it easier, we change the fractions, using the **equivalent fraction rule**, so that they have the same common denominator.

Getting it right.
International tennis star,
Venus Williams whacks it back in Melbourne, Australia.

First Example

Natalia, who loves to cook, produced a small blueberry pie for supper. Conrad ate $\frac{2}{5}$ of the pie and Natalia ate $\frac{3}{7}$ of the pie. Who ate more pie, Natalia or her brother, Conrad?

Which fraction is bigger, $\frac{2}{5}$ or $\frac{3}{7}$?

These fractions need a common denominator.

An easy way to find a common denominator is to multiply the two denominators together.

Rewrite it like this:

$$\frac{2}{5} \times \frac{3}{7} = \frac{\quad}{35} \quad \frac{\quad}{35}$$

Then extend the multiply sign upwards and multiply the numbers on the ends of each line as shown by the purple arrows:

$$\frac{2}{5} \times \frac{3}{7} = \frac{(2\times7)}{35} \quad \frac{(5\times3)}{35} = \frac{14}{35} \quad \frac{15}{35}$$

In effect, you have used the **equivalent fraction rule.** You multiplied both top and bottom of the left fraction by **7**.

You multiplied both the top and the bottom of the right fraction by **5**.

You can now see that Conrad ate $\frac{14}{35}$ and Natalia ate $\frac{15}{35}$

Answer: **Natalia ate more pie**.

Getting the balance right.

Second Example

Who has done more of their chemistry revision? Gemma who has revised $\frac{2}{3}$ of the course, or Lily who has revised $\frac{5}{8}$ of the course?

First, find the common denominator:

$$\frac{2}{3} \times \frac{5}{8} = \frac{\quad}{24} \quad \frac{\quad}{24}$$

Then extend the multiply sign upwards. Multiply the numbers on the ends of each line as shown below:

$$\frac{2}{3} \times \frac{5}{8} = \frac{(2\times8)}{24} \quad \frac{(3\times5)}{24} = \frac{16}{24} \quad \frac{15}{24}$$

You can now see that Gemma has revised $\frac{16}{24}$ of the course, and Lily has revised $\frac{15}{24}$

Answer: **Gemma has done more chemistry revision**.

Exercises

5) Which fraction is bigger, $\frac{2}{9}$ or $\frac{2}{10}$?

6) Which fraction is bigger, $\frac{3}{11}$ or $\frac{1}{4}$?

Getting the balance right.

How to Add Fractions

Fractions can only be added if the **denominators are the same**.

You simply add the top numbers and keep the denominator the same.

Example for adding fractions

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ This cancels down to $= \frac{1}{2}$

$\frac{5}{103} + \frac{23}{103} = \frac{28}{103}$

But, you have a problem if the denominators are not the same:

$\frac{1}{3} + \frac{4}{7} = ?$

You need to find a **common denominator**.

Do this in the same way as when you compared fractions.

Rewrite the calculation like this:

$\frac{1}{3} \times \frac{4}{7} = \frac{\quad}{21} + \frac{\quad}{21}$

Now extend the multiply sign upwards to multiply the numbers, as before.

$\frac{1}{3} \times \frac{4}{7} = \frac{(1 \times 7)}{21} + \frac{(3 \times 4)}{21} = \frac{7}{21} + \frac{12}{21} = \frac{7+12}{21}$

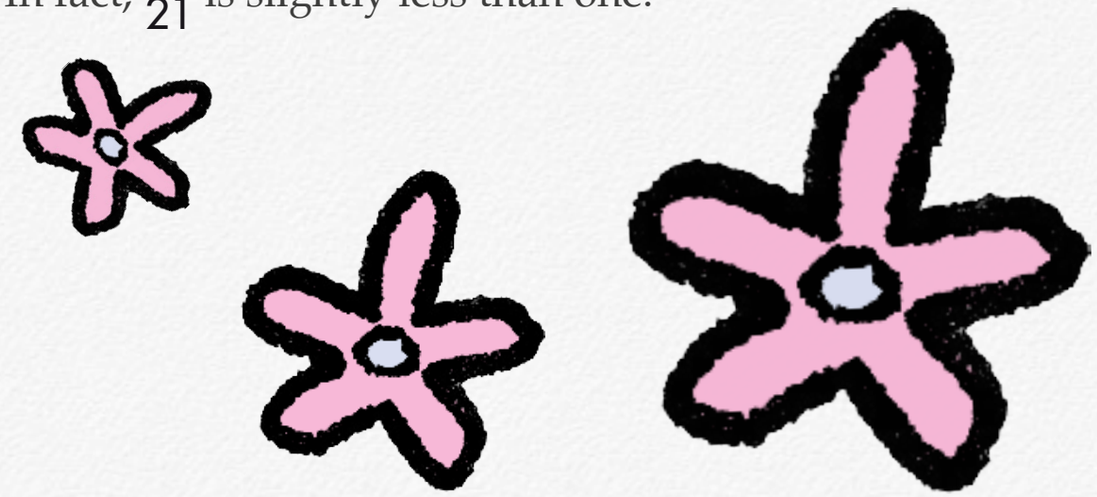
$= \frac{19}{21}$

Mental Check

Always do a common sense, mental check on your answer.

In the preceding example, $\frac{1}{3}$ is a bit less than half a thing (pizza, block of toffee, whatever), and $\frac{4}{7}$ is a bit more than half a thing, so the answer to $\frac{1}{3} + \frac{4}{7}$ should be, 'about one'.

In fact, $\frac{19}{21}$ is slightly less than one.



Exercises

7) Add $\frac{1}{4}$ and $\frac{2}{5}$ together.

8) What is $\frac{1}{6} + \frac{3}{7}$?

9) Lily shared $\frac{1}{2}$ of her whole packet of wine gums with her friends after school. She then ate $\frac{1}{3}$ of the whole packet of wine gums while waiting for her bus.
What fraction of Lily's packet of wine gums had been eaten by the time she got on the bus?

Quick calculation.
Hungarian NBI League women's volleyball game.

How to Subtract Fractions

Subtraction of fractions also relies on having a **common denominator**. If the denominators are not the same, still use the preceding method, with the central sign as a minus, not a plus:

Example for subtracting fractions

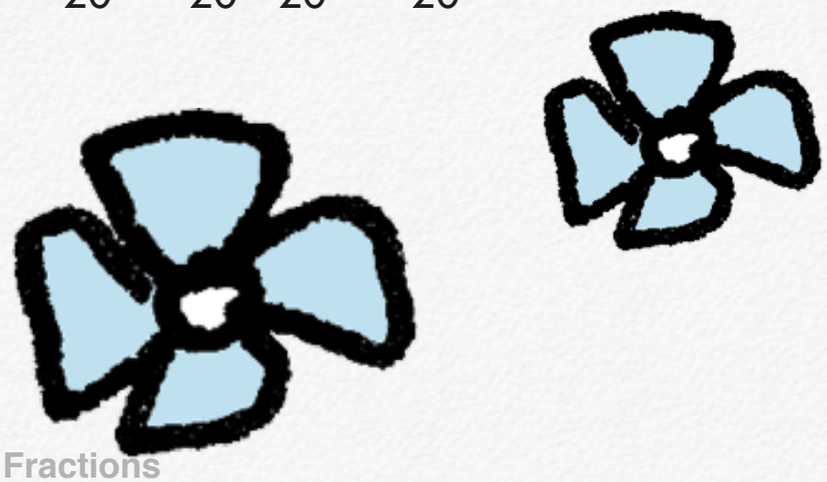
$\frac{3}{4} - \frac{2}{5} = ?$

First adjust the calculation so that the denominators are the same. **Remember to subtract, not add at the end**:

$\frac{3}{4} \times \frac{2}{5} = \frac{}{20} \quad \frac{}{20}$

$\frac{3}{4} \times \frac{2}{5} = \frac{(3\times5)}{20} - \frac{(4\times2)}{20} = \frac{15}{20} - \frac{8}{20} = \frac{15-8}{20}$

$= \frac{7}{20}$



How to subtract a fraction from a whole thing

A whole thing contains **ALL** the separate fractions in it.

If you cut your pizza into **8** pieces, each piece is $\frac{1}{8}$ of the pizza. The **whole** pizza is $\frac{8}{8}$

If you decided to cut your pizza into **12** pieces, then each piece is $\frac{1}{12}$ of the pizza. The whole pizza is $\frac{12}{12}$

Example of subtracting a fraction from a whole thing:

Natalia, hungry after a football match, boasts that she ate $\frac{11}{20}$ of a pizza. How much of the pizza was left for her brother, Conrad?

Natalia's pizza was cut into twentieths, so the whole pizza $= \frac{20}{20}$ So subtract $\frac{11}{20}$ from $\frac{20}{20}$

$\frac{20}{20} - \frac{11}{20} = \frac{9}{20}$

Exercises

10) What is $\frac{4}{5} - \frac{1}{2}$?

11) Lily made an enormous strawberry meringue gateau for a family picnic in the park. $\frac{5}{8}$ was eaten at the picnic and $\frac{1}{3}$ of the original gateau was eaten after they returned home.

a) How much of the gateau was eaten altogether?

b) How much of the gateau was left over? (Hint: look back at the Example of subtracting a fraction from a whole thing.)

12) Ruth runs a travel agency and only has time to cook at the weekends. One Saturday, Ruth's delicious raspberry tart was shared out between her two children and her mother. Ruth's mother took a quarter of the tart, leaving $\frac{3}{4}$ for Ruth's children. At tea time, Ruth's children ate $\frac{2}{5}$ of the original tart. How much of the tart was left over for Ruth?

13) In the art college canteen, Jane and Karen shared a banoffee pie. Jane ate $\frac{3}{5}$ and Karen ate $\frac{1}{4}$

How much more pie did Jane eat than Karen?

14) Rachel's dad is a schoolmaster and runs the local scouts group. For a scouts' supper party buffet, Rachel made three big ham and cheese quiches. Afterwards, Rachel collected the leftovers. $\frac{1}{4}$ of one quiche was left and $\frac{2}{5}$ of the other two quiches were left. How much ham and cheese quiche was left over for Rachel's pig?

Rhythmic gymnastic competition in Romania.

Self-Confidence Tip 5

TALK POSITIVE

REMEMBER to use confident language. Carefully choose the words that you use to describe your maths work, as those words will make a difference to how you feel.

Avoid, "I **have** to do it", or "This will be **boring**".

Keep telling yourself, "If I **focus carefully** on this, then I will understand it and then I will get many benefits". Tell yourself, "When I complete this bit, I will feel a **sense of achievement**".

Feeling shy
Does not describe
The agony of sticking out,
The fear of ridicule,
Pointed fingers, howls of laughter,
“Did you hear what that idiot just said?
And her outfit !!!”

“Just be yourself,”
Is what you’re told.
This smug advice
Sends me into a silent rage.
I want to scream, to howl,
That’s exactly what I DON’T want to be.

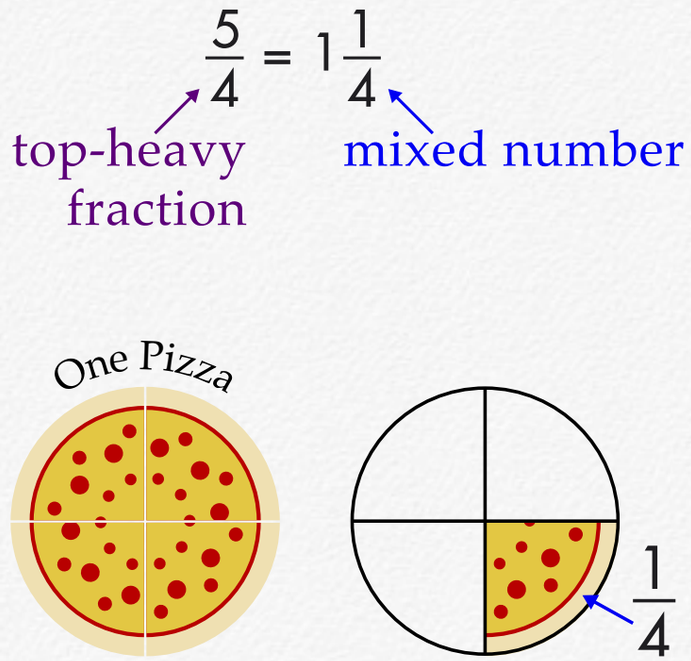
My mother
Spends a fortune
Trying to look different
And stick out.

Isn’t life ironic?

Vulgar Fractions and Mixed Numbers

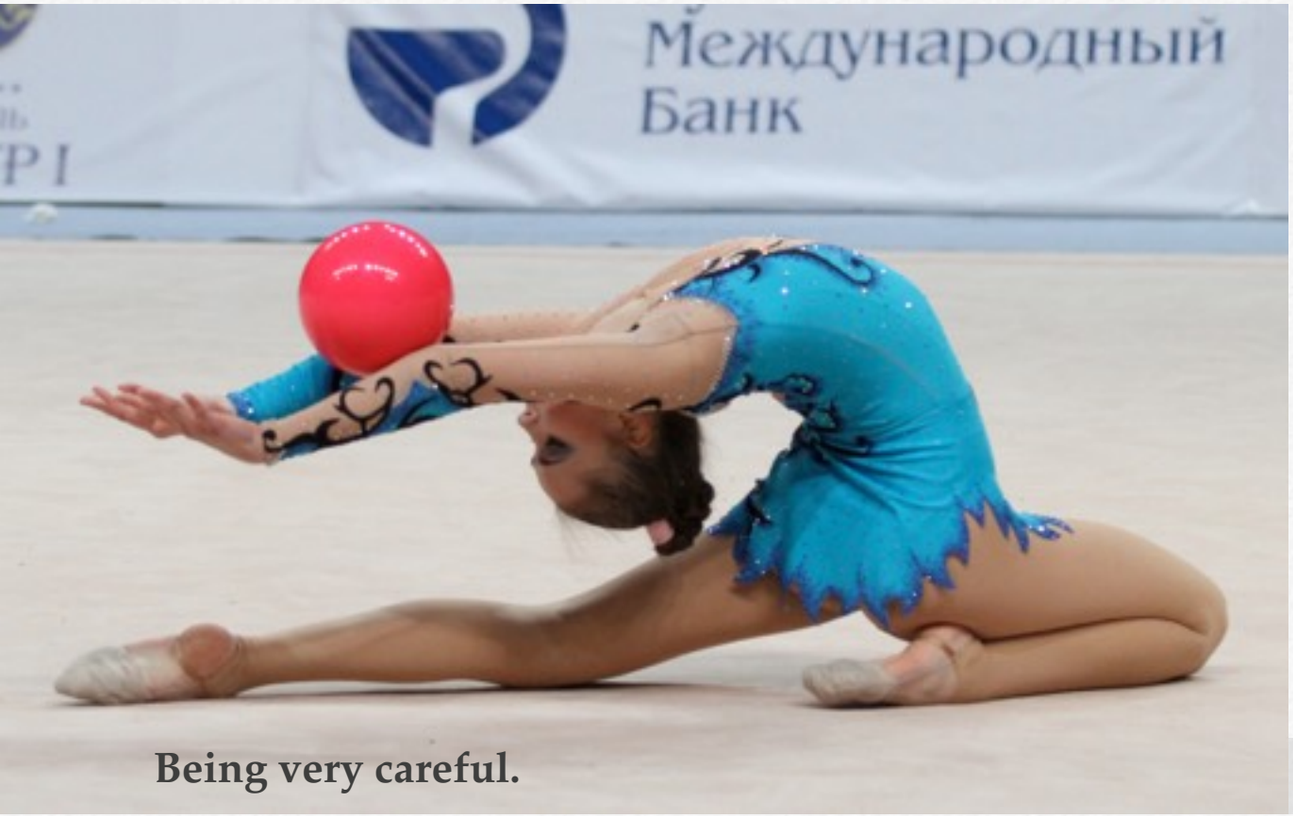
When the top number is bigger than the bottom number, we call it a **vulgar fraction** or **top-heavy fraction**.

All top-heavy fractions can be transformed into a **mixed number**, which is a whole number with a fraction attached.



Example

In other words, $\frac{5}{4}$ is equal to one whole thing (pizza, cake, whatever) and one extra quarter.



Being very careful.

Rhythmic gymnastics championship, Moscow, Russia.

How to Change Top-Heavy Fractions into Mixed Numbers

It may be easier to think in terms of mixed numbers than top-heavy fractions.

$1\frac{1}{4}$ is more easily visualised than $\frac{5}{4}$ so it's useful to know how to change top-heavy fractions into mixed numbers.

First, you need to know that the dividing line in the middle of the fraction actually means '**divide**'.

First Example

$\frac{6}{3}$ means $6 \div 3$ $\quad\quad$ $\frac{6}{3} = 2$

$\frac{12}{4}$ means $12 \div 4$ $\quad$ $\frac{12}{4} = 3$

So, in order to change a top-heavy fraction into a mixed number, you do this division.

Look at the introduction, $\frac{5}{4}$ means **5 ÷ 4**. Do the division.

You will find that **4** goes into **5** once, leaving a **remainder** of **1**. Now simply stick the remainder on top of the dividing number, which is **4** in this calculation.

So $\frac{5}{4}$ is the same as $1\frac{1}{4}$

What's the secret of a winner?
Practice.

Second Example

Change $\frac{28}{5}$ to a mixed number.

$\frac{28}{5} = 28 \div 5 = 5$ with a remainder of 3

The remainder

$= 5\frac{3}{5}$

the whole number

the number you divided by (the denominator)

Third Example

What is $\frac{20}{6}$ as a mixed number?

$\frac{20}{6} = 20 \div 6 = 3$ with a remainder of 2

$= 3\frac{2}{6}$

or, after simplifying the fraction

$= 3\frac{1}{3}$

Careful calculation needed.

How to Change Mixed Numbers into Top-Heavy Fractions

In order to **multiply** or **divide** fractions (more later), it is necessary to change mixed numbers into top-heavy fractions.

First Example

Change $4\frac{2}{3}$ into a top-heavy fraction.

First take the whole number, **4**. Multiply **4** by the denominator, which is **3**. This equals **12**.

Now add **12** to the numerator, which is **2**.

12 + 2 = 14

This final figure stands over the original denominator, which is **3**.

The bottom number (denominator) always stays the same.

Answer: $4\frac{2}{3} = \frac{14}{3}$

Second Example

What is $7\frac{3}{4}$ as a top-heavy fraction?

Work out the new numerator:

7 x 4 = 28

28 + 3 = 31

The denominator stays the same.

Answer: $7\frac{3}{4} = \frac{31}{4}$

Racers need maths.

Women's road cycling race, Olympic Games, Britain.

Exercises

15) What is $\frac{7}{2}$ as a mixed number?

16) What is $5\frac{1}{6}$ as a top-heavy fraction?

17) How much wine is left over from a supper party if one bottle is left $\frac{2}{3}$ full, and another is left $\frac{3}{4}$ full? Give your answer as a mixed number.

18) Rachel cuts apple pies into quarters, to share out amongst the scout group. She has $3\frac{3}{4}$ pies to cut up.

How many quarters of pie will Rachel have altogether?

Careful calculation needed.
Climber is focused on her next move.

Self Confidence Tip 6

HANDLING CHANGE

Think of the changes you have successfully handled in the last year.

Perhaps a new mobile phone, new software, a new entertainment system…Your brain processed all that information – simply because you wanted to learn it.

Remember how apprehensive you felt when you first started school and knew nothing? Remember that when you were quite young, you learned to read and write, two terrific achievements.

If you are eager to learn new things, because your brain has infinite potential…you can do so.

What you need to do is – train your brain. If you follow these instructions carefully, and then put in the time, there is no reason why you can't succeed at maths.

If, previously, you had a problem with maths, that may have been because the wrong teaching approach was used, one that didn't work with your brain. You are now using a new approach… so you can feel confident about your ability with numbers.

How to Multiply Fractions

This is the easiest form of fraction arithmetic. You simply multiply the two top numbers together, then multiply the two bottom numbers together.

Examples

1) $\frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$

2) $\frac{5}{6} \times \frac{3}{10} = \frac{15}{60}$

Here the answer can be simplified: $\frac{15}{60} = \frac{3}{12} = \frac{1}{4}$

3) Natalia eats $\frac{1}{4}$ of a square apple crumble and leaves her brother, Conrad, $\frac{3}{4}$ of this pudding. Conrad gobbles $\frac{2}{3}$ of what he is given. How much of the original apple crumble did Conrad eat?

Conrad ate $\frac{2}{3}$ **of** $\frac{3}{4}$ of the crumble.

Note

'Of' in maths means multiply.

So the calculation for Conrad's apple crumble is

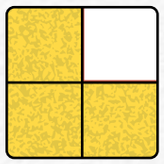
$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

$$\frac{6}{12} = \frac{3}{6} = \frac{1}{2}$$

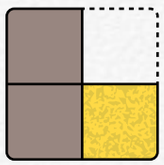
Answer: **Conrad ate $\frac{1}{2}$ of the apple crumble.**

Conrad's greed can be proven as follows:

$\frac{3}{4}$ of the crumble



$\frac{2}{3}$ of what's left = $\frac{1}{2}$ of the original crumble.



Example:

If you want to multiply mixed numbers, you first need to convert the mixed number into a vulgar fraction. Check to see if your final answer is a vulgar fraction; if it is, you need to convert it into a mixed number:

Calculate $2\frac{3}{5} \times \frac{10}{12}$

First, change the mixed number into a vulgar fraction.

So $2\frac{3}{5} \times \frac{10}{12} = \frac{13}{5} \times \frac{10}{12}$

Then do the multiplication sum: $\frac{13}{5} \times \frac{10}{12} = \frac{130}{60}$

Simplify the answer and since it is a vulgar fraction, convert it into a mixed number: $\frac{13}{6} = 2\frac{1}{6}$

Answer: $\frac{13}{5} \times \frac{10}{12} = 2\frac{1}{6}$

If you are multiplying a fraction or a mixed number by a whole number, you need to turn the whole number into a vulgar fraction.

This is easily done, by **always** sticking the whole number over the number **1**.

So $3 = \frac{3}{1}$ and $14 = \frac{14}{1}$ and $103 = \frac{103}{1}$

First Example

What is $\frac{2}{5}$ multiplied by **4**?

$\frac{2}{5}$ stays the same but $4 = \frac{4}{1}$

Then do the multiplication sum: $\frac{2}{5} \times \frac{4}{1} = \frac{8}{5}$

Since the answer is a vulgar fraction, change it into a mixed number: $\frac{8}{5} = 1\frac{3}{5}$

Answer: $\frac{2}{5} \times 4 = 1\frac{3}{5}$

Second Example

Design students Jane and Karen consume a lot of confectionery. Jane ate 8 pieces of chocolate; each piece of chocolate was $\frac{1}{12}$ of the whole bar.

How much of the chocolate bar did Jane eat?

$$8 \times \frac{1}{12} = \frac{8}{1} \times \frac{1}{12}$$

$$= \frac{8}{12}$$

$$= \frac{2}{3}$$ of the chocolate bar was eaten by Jane.

Third Example

Karen ate $\frac{2}{7}$ of a box of 21 toffees. How many toffees did she eat?

Remember: 'Of' in maths means multiply:

$$= \frac{2}{7} \times 21$$

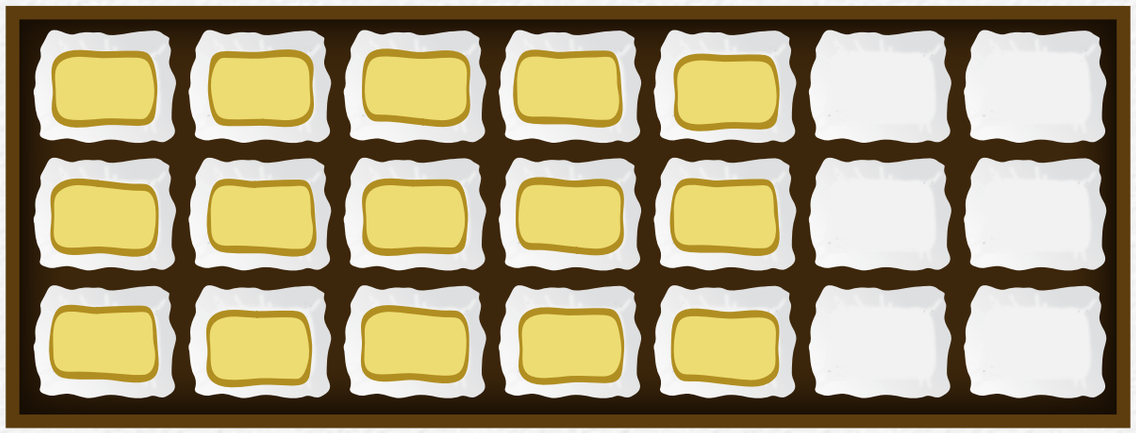
$$= \frac{2}{7} \times \frac{21}{1}$$

Toffees eaten by Karen $= \frac{42}{7} = \frac{6}{1} = 6$

Answer: **Karen ate 6 toffees, out of the box of 21 toffees.**

That leaves 15 toffees uneaten.

What Karen ate is shown as the empty part of the box of toffees in the following picture.



Exercises

Always simplify your answer, where possible.

19) Multiply $\frac{4}{5}$ with $\frac{3}{8}$

20) Tania found she had $\frac{2}{3}$ of a bottle of red wine left from the night before. She and her date, graphic designer Frank, drank $\frac{3}{4}$ of this with their supper. What fraction of the bottle of wine did Tania and Frank drink? (Hint: They drank $\frac{3}{4}$ **of** $\frac{2}{3}$ ... Remember 'of' means multiply)

21) Fausto's retro ice cream van sold $2\frac{1}{2}$ times more ice cream on Bank Holiday Monday than it does on a normal Monday. If the van sells 180 ice creams on a normal Monday, how many ice creams did it sell on the Bank Holiday?

22) Sarah whispers to Jenny that $\frac{3}{4}$ of the men at her housewarming party are single. Jenny knows that $\frac{3}{5}$ of the people at the party are men. What fraction of the people at the party are single men?

23) With her savings plus a bank loan, Ruth buys half of her brother's share in her family's fruit and veg company. Ruth's brother owned $\frac{2}{3}$ of the company. How much of the company has Ruth purchased?

Fausto's ice cream van.

How to Divide Fractions

The rule for **dividing fractions** is: **flip the second fraction**, so the bottom number is on top and the top number is on the bottom. Then multiply.

First Example

Divide $\frac{2}{5}$ by $\frac{1}{4}$ = $\frac{2}{5} \div \frac{1}{4}$, then flip the fraction, so it becomes $\frac{4}{1}$

So the calculation now reads = $\frac{2}{5} \times \frac{4}{1}$ Remember to change the ÷ to x

Now, simply do the multiplication = $\frac{2}{5} \times \frac{4}{1} = \frac{8}{5}$

As it's a vulgar fraction, change to a mixed number = $1\frac{3}{5}$, which is the answer.

Second Example

Divide $3\frac{1}{4}$ by 4 = $3\frac{1}{4} \div 4$

Change both numbers to vulgar fractions = $\frac{13}{4} \div \frac{4}{1}$

Flip the 2nd fraction, and change ÷ to x = $\frac{13}{4} \times \frac{1}{4}$

Do the multiplication, and the answer is = $\frac{13}{16}$

Exercises

Always simplify you answer, where possible.

24) Divide $\frac{2}{3}$ by $\frac{8}{15}$

25) On the morning after her birthday, Tania takes $\frac{2}{3}$ of a Black Forest Gateau to her office and shares it out equally into **4** pieces for her friends. What fraction of the original cake is each of Tania's pieces?

(Hint: $\frac{2}{3}$ of the gateau is divided into **4** pieces.)

One step at a time.

Hiking up Volcano Teide, Canary Islands.

How to Cross Cancel

What is the point of **cross cancelling**? Multiplication and division with fractions can be made **simpler**, if the cancelling is done at the **beginning** of the calculation rather than at the end. This is especially true when big numbers are included in the fractions.

Cross cancelling is done in the same way as simplifying fractions. Do it just before you **multiply** the sum. **Any top number can be simplified with any bottom number**.

Keep your lines straight.

First Example

$$\frac{5}{6} \times \frac{3}{10} = ?$$

Here the **5** on the top line of the first fraction can be cancelled with the **10** on the bottom line of the second fraction, since they can both be divided by **5**:

$$\frac{{}^{1}\cancel{5}}{6} \times \frac{3}{\cancel{10}_{2}}$$

5 ÷ 5 = 1, so 5 is crossed out and replaced with 1.
10 ÷ 5 = 2, so 10 is crossed out and replaced with 2.

Next, the **6** on the bottom of the first fraction can be cross cancelled in the same way with the **3** on the top of the second fraction, since both are divisible by **3**:

$$\frac{{}^{1}\cancel{5}}{{}_{2}\cancel{6}} \times \frac{\cancel{3}^{1}}{\cancel{10}_{2}}$$

Now multiply the remaining numbers.

$$\frac{{}^{1}\cancel{5}}{{}_{2}\cancel{6}} \times \frac{\cancel{3}^{1}}{\cancel{10}_{2}} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Answer: $\frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$

Second Example

$$\frac{25}{28} \times \frac{16}{100} = ?$$

Simplify **25** and **100**, by dividing both by **25**, and simplify **16** and **28**, by dividing both by **4**:

$$\frac{{}^{1}\cancel{25}}{{}_{7}\cancel{28}} \times \frac{\cancel{16}^{4}}{\cancel{100}_{4}} = \frac{1}{7} \times \frac{4}{4}$$

The new **4** on the top line can be simplified with the new **4** on the bottom line:

$$\frac{1}{7} \times \frac{\cancel{4}^{1}}{\cancel{4}_{1}} = \frac{1}{7} \times \frac{1}{1} = \frac{1}{7}$$

If you do all possible cross cancelling at the multiplication stage, then the answer will be in its simplest form and so will not need cancelling.

Answer: $\frac{25}{28} \times \frac{16}{100} = \frac{1}{7}$

When you cancel by dividing by 10, simply cross out zeros. Stick to the rule that every zero crossed out on the top must be matched by crossing a zero off at the bottom.
It doesn't matter which zero on the TOP is matched with which zero on the BOTTOM.

Affordable holiday?
Skiing in Austria.

Third Example

$\frac{70}{100} \div \frac{1}{30} = ?$

You can only cross cancel in multiplication sums.

Before you cross cancel, do the first stage of a division sum, by flipping the second fraction and changing ÷ to **x**.

$\frac{70}{100} \div \frac{1}{30} = \frac{70}{100} \times \frac{30}{1}$

Now you can cross cancel, by knocking off zeros.

Look below. Two zeros are crossed off the top line AND two zeros are crossed off the bottom line to match, (don't worry about which zero from the top matches which one from the bottom).

$\frac{70}{100} \times \frac{30}{1} = \frac{7}{1} \times \frac{3}{1} = \frac{21}{1} = 21$

Answer: $\frac{70}{100} \div \frac{1}{30} = 21$

Exercises

Use cross cancelling to do the following calculations:

26) $\frac{180}{300} \times \frac{35}{90} =$

27) $\frac{8}{15} \div \frac{16}{55} =$

Holiday destination? Old Havana, Cuba.

Fraction Summary

	Rules	Example
Addition	Make denominators the same. Then add the tops only.	$\frac{1}{3} \times \frac{4}{7} = \frac{(1\times7)}{21} + \frac{(3\times4)}{21} = \frac{7}{21} + \frac{12}{21} = \frac{19}{21}$
Subtraction	Make denominators the same. Then subtract the tops only.	$\frac{3}{4} \times \frac{2}{5} = \frac{(3\times5)}{20} - \frac{(4\times2)}{20} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$
Multiplication	Multiply the tops and multiply the bottoms. (Cross cancel if you can.)	$\frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$
Division	Flip the 2nd fraction, and then change ÷ sign to x sign. Then multiply (see above).	$\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}$
For all sums:	change **mixed numbers** to **vulgar fractions** **BEFORE** you do the sum.	
For all answers:	if your answer can be simplified, **SIMPLIFY IT**. if your answer is a **vulgar fraction**, change it to a **MIXED NUMBER**.	

How to Cheat

If you find you have a lot of fraction calculations to do, it may be worth investing in a scientific calculator, because this is the only type of calculator that has a fractions button.

The fractions button is marked as either [A b/c] or [□/□]

Check how your scientific calculator works, by trying out the following examples. Then experiment. If you're still not clear, check the user guide.

To type in a fraction such as $\frac{1}{4}$ you should tap [1] [A b/c] [4].

Some displays will show this clearly as $\frac{1}{4}$

others will read **1 ⌟ 4**

To type in a mixed number, such as $2\frac{3}{4}$

tap in [2] [A b/c] [3] [A b/c] [4]

On some calculators this may be shown as **2 _ 3 ⌟ 4**.

When you have an answer to a calculation that you typed in using the fractions button, you can change it to a decimal by simply pressing the fractions button again, or [S⇔D].

On basic calculators, the only way to do fraction calculations is to convert the fractions to decimals. A fraction can be converted simply to a decimal on any calculator by typing in the fraction with ÷ in place of the dividing line. For example: $\frac{1}{4}$ would be typed in as [1] [÷] [4]. After you press [=] the screen should read **0.25**, the decimal equivalent to $\frac{1}{4}$.

Attention to detail.

Answers to Part 11

Answers to Equivalent Fractions

1) $\frac{1}{4} = \frac{2}{8}$ (You multiply both top and bottom by **2**)

2) $\frac{3}{9} = \frac{1}{3}$ (You divide both top and bottom by **3**)

3) $\frac{12}{16} = \frac{3}{4}$ (You divide both top and bottom by **4**)

Nature's geometric design.

4) Simplify the following fractions to their simplest forms:

a) $\frac{4}{20} = \frac{2}{10} = \frac{1}{5}$ (You divide both top and bottom by **2**, and then by **2** again. Or divide both by **4**)

b) $\frac{7}{28} = \frac{1}{4}$ (You divide both top and bottom by **7**)

c) $\frac{11}{77} = \frac{1}{7}$ (You divide both top and bottom by **11**)

d) $\frac{12}{18} = \frac{6}{9} = \frac{2}{3}$ (You divide both top and bottom by **2**, and then **3**. Or divide both by **6**)

e) $\frac{35}{63} = \frac{5}{9}$ (You divide both top and bottom by **7**)

f) $\frac{48}{88} = \frac{24}{44} = \frac{12}{22} = \frac{6}{11}$

(You divide both top and bottom by **2**, then by **2** again and then by **2** again.
Or divide both by **4**, then by **2**.
Or divide both by **8**)

Answers to Comparing Fractions

5) Which fraction is bigger, $\frac{2}{9}$ or $\frac{2}{10}$?

$$\frac{2}{9} \times \frac{2}{10} = \frac{(2\times10)}{90} \quad \frac{(2\times9)}{90} = \frac{20}{90} \quad \frac{18}{90}$$

Answer: $\frac{20}{90}$ is bigger than $\frac{18}{90}$, so the answer is:

$\frac{2}{9}$ is bigger than $\frac{2}{10}$

6) Which fraction is bigger, $\frac{3}{11}$ or $\frac{1}{4}$?

$$\frac{3}{11} \times \frac{1}{4} = \frac{(3\times4)}{44} \quad \frac{(1\times11)}{44} = \frac{12}{44} \quad \frac{11}{44}$$

Answer: $\frac{12}{44}$ is bigger than $\frac{11}{44}$, so the answer is: **$\frac{3}{11}$ is bigger than $\frac{1}{4}$**

Answers to Adding Fractions

7) Add $\frac{1}{4}$ and $\frac{2}{5}$ together.

$$\frac{1}{4} + \frac{2}{5} = \frac{1}{4} \times \frac{2}{5} = \frac{(1\times5)}{20} + \frac{(4\times2)}{20} = \frac{5}{20} + \frac{8}{20}$$

$$= \frac{13}{20}$$

8) What is $\frac{1}{6} + \frac{3}{7}$?

$$\frac{1}{6} + \frac{3}{7} = \frac{1}{6} \times \frac{3}{7} = \frac{(1\times7)}{42} + \frac{(6\times3)}{42} = \frac{7}{42} + \frac{18}{42}$$

$$= \frac{25}{42}$$

9) Lily shared $\frac{1}{2}$ of her whole packet of wine gums with her friends after school. She then ate $\frac{1}{3}$ of the whole packet of wine gums while waiting for her bus. What fraction of Lily's packet of wine gums had been eaten by the time she got on the bus?

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{(1\times3)}{6} + \frac{(1\times2)}{6} = \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6}$$ **of the wine gums were eaten.**

Answers to Fraction Addition and Subtraction

10) What is $\frac{4}{5} - \frac{1}{2}$?

$$\frac{4}{5} - \frac{1}{2} = \frac{4}{5} \times \frac{1}{2} = \frac{(4 \times 2)}{10} - \frac{(5 \times 1)}{10} = \frac{8}{10} - \frac{5}{10}$$

$$= \frac{3}{10}$$

11) Lily made an enormous strawberry meringue gateau for a family picnic. $\frac{5}{8}$ was eaten at the picnic and $\frac{1}{3}$ of the original gateau was eaten after they returned home.

a) How much of the gateau was eaten altogether?

This is an addition, since the question says "altogether".

$$\frac{5}{8} + \frac{1}{3} = \frac{5}{8} \times \frac{1}{3} = \frac{(5 \times 3)}{24} + \frac{(1 \times 8)}{24} = \frac{15}{24} + \frac{8}{24}$$

$$= \frac{23}{24}$$

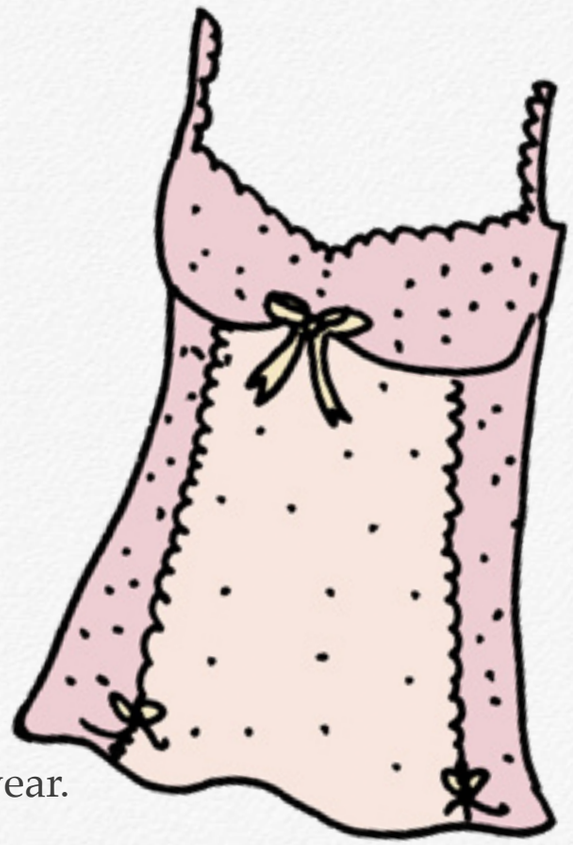
Answer: $\frac{23}{24}$ **of the gateau was eaten**.

b) How much of the gateau was left over? (Hint: look back at the Example of subtracting a fraction from a whole thing.)

To work out how much was left over, subtract $\frac{23}{24}$ from a whole gateau $\frac{24}{24}$

$$\frac{24}{24} - \frac{23}{24} = \frac{1}{24}$$

Answer: $\frac{1}{24}$ **of the gateau was left**.



You need maths to design underwear.

12) Ruth runs a travel agency and only has time to cook at the weekends. One Saturday, Ruth's delicious raspberry tart was shared out between her two children and her mother. Ruth's mother took a quarter of the tart, leaving $\frac{3}{4}$ for Ruth's children. At tea time, Ruth's children ate $\frac{2}{5}$ of the original tart . How much of the tart was left over for Ruth?

$$\frac{3}{4} - \frac{2}{5} = \frac{3}{4} \times \frac{2}{5} = \frac{(3 \times 5)}{20} - \frac{(2 \times 4)}{20} = \frac{15}{20} - \frac{8}{20}$$

$$= \frac{7}{20}$$

Answer: $\frac{7}{20}$ **of the raspberry tart was left for Ruth.**

An alternative way of working out this answer is to add the fractions that Ruth's mother and Ruth's children ate, to see how much was eaten altogether:

$$\frac{1}{4} + \frac{2}{5} = \frac{1}{4} \times \frac{2}{5} = \frac{(1 \times 5)}{20} + \frac{(2 \times 4)}{20} = \frac{5}{20} + \frac{8}{20}$$

$$= \frac{13}{20}$$ was eaten in all.

To work out how much was left over for Ruth, subtract $\frac{13}{20}$ from a whole tart, $\frac{20}{20}$:

$$\frac{20}{20} - \frac{13}{20} = \frac{7}{20}$$

Answer: $\frac{7}{20}$ **of the raspberry tart was left for Ruth.**

13) In the art college canteen, Jane and Karen shared a banoffee pie. Jane ate $\frac{3}{5}$ and Karen ate $\frac{1}{4}$

How much more of the banoffee pie did Jane eat than Karen?

$$\frac{3}{5} - \frac{1}{4} = \frac{3}{5} \times \frac{1}{4} = \frac{(3 \times 4)}{20} - \frac{(1 \times 5)}{20} = \frac{12}{20} - \frac{5}{20}$$

$$= \frac{7}{20}$$

Answer: **Jane ate** $\frac{7}{20}$ **more of the banoffee pie than Karen.**

14) Rachel's dad is a schoolmaster and runs the local scouts group. For a scouts' supper party buffet, Rachel made three big ham and cheese quiches. Afterwards, Rachel collected the leftovers. $\frac{1}{4}$ of one quiche was left and $\frac{2}{5}$ of the other two quiches were left. How much ham and cheese quiche was left over for Rachel's pig?

Add the leftover quiche together:

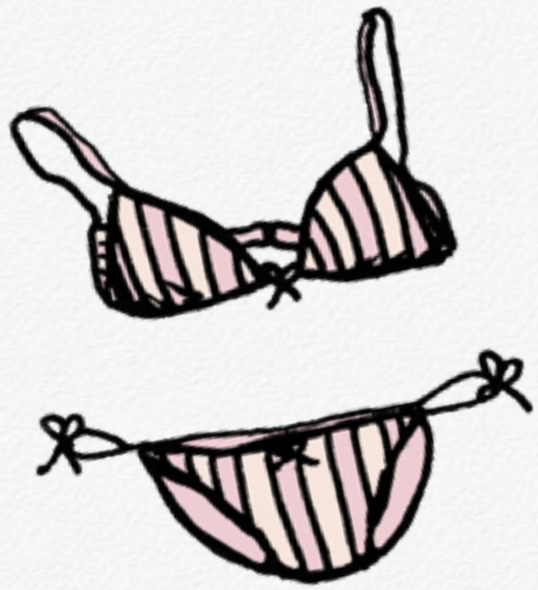
$$\frac{1}{4}+\frac{2}{5}+\frac{2}{5}=\frac{1}{4}+\frac{4}{5}=\frac{1}{4}\times\frac{4}{5}=\frac{(1\times5)}{20}+\frac{(4\times4)}{20}=\frac{5}{20}+\frac{16}{20}$$

$$=\frac{21}{20}$$

$\frac{21}{20}$ = one whole quiche plus one extra twentieth slice

Answer: $\frac{21}{20}$ (or $1\frac{1}{20}$) **of quiche.**

was left for Rachel's lucky pig.



Answers to Changing between Vulgar Fractions and Mixed Numbers

15) What is $\frac{7}{2}$ as a mixed number?

$\frac{7}{2} = 7 \div 2 = 3$ remainder $1 = 3\frac{1}{2}$

16) What is $5\frac{1}{6}$ as a top-heavy fraction?

$5\frac{1}{6} = \frac{31}{6}$

(Numerator calculation is $(6 \times 5) + 1 = 31$)

17) How much wine is left over from a supper party, if one bottle is left $\frac{2}{3}$ full, and another is left $\frac{3}{4}$ full?

$$\frac{2}{3}+\frac{3}{4}=\frac{2}{3}\times\frac{3}{4}=\frac{(2\times4)}{12}+\frac{(3\times3)}{12}=\frac{8}{12}+\frac{9}{12}$$

$= \frac{17}{12}$ bottles of the wine were left.

$\frac{17}{12} = 17 \div 12 = 1$ remainder 5

Answer: $= 1\frac{5}{12}$ **bottles of wine**.

18) Rachel cuts apple pies into quarters, to share out amongst the scout group. She has $3\frac{3}{4}$ pies to cut up.

How many quarters of pie will Rachel have altogether?

$3\frac{3}{4} = \frac{15}{4}$ (Numerator calculation is: **(4 x 3) + 3 = 15**)

Answer: **Rachel will have 15 quarters $\left(\frac{15}{4}\right)$ of apple pie to share out**.

Patterns in nature.

Answers to Multiplying Fractions

19) Multiply $\frac{3}{8}$ with $\frac{4}{5}$

$\frac{4}{5} \times \frac{3}{8} = \frac{12}{40}$

$\frac{12}{40}$ can be simplified: $\frac{12}{40} = \frac{6}{20} = \frac{3}{10}$

Answer: $\frac{3}{10}$

20) Tania found she had $\frac{2}{3}$ of a bottle of red wine left from the night before. She and her date, graphic designer Frank, then drank $\frac{3}{4}$ of this with their supper. What fraction of the bottle of wine did Tania and Frank drink?

(Hint: They drank $\frac{3}{4}$ **of** $\frac{2}{3}$, remember 'of' means multiply)

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

This can be simplified: $\frac{6}{12} = \frac{1}{2}$

Answer: **Tania and Frank drank $\frac{1}{2}$ of the bottle of wine with their supper**.

21) Fausto's ice cream van sold $2\frac{1}{2}$ times more ice cream on Bank Holiday Monday than it does on a normal Monday. If the van sells **180** ice creams on a normal Monday, how many ice creams did it sell on the Bank Holiday?

You need to calculate $2\frac{1}{2}$ of **180** ice creams.

$2\frac{1}{2} = \frac{5}{2}$ and $180 = \frac{180}{1}$ as a fraction. 'Of' = **x**.

So the calculation is $\frac{5}{2} \times \frac{180}{1} = \frac{900}{2}$

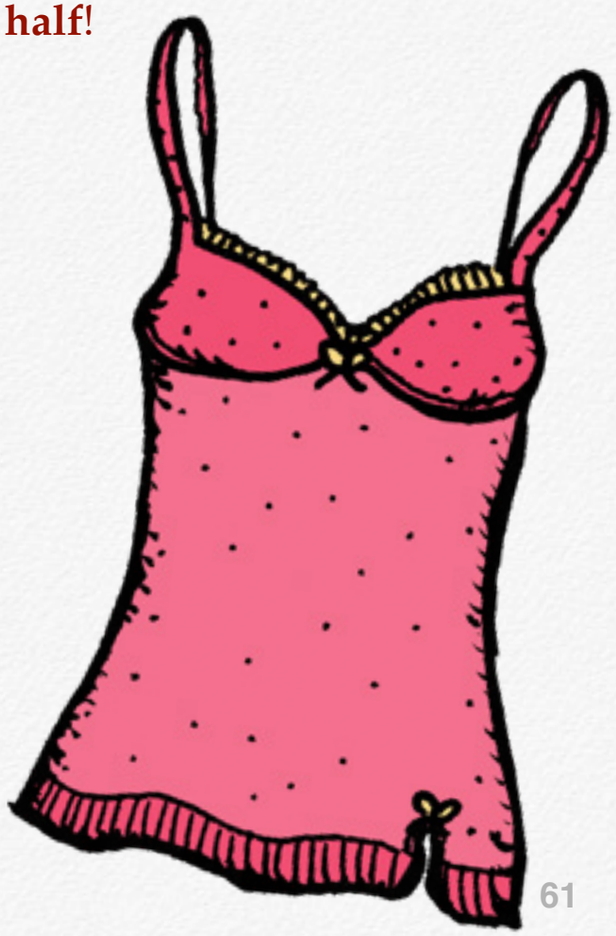
$\frac{900}{2} = \frac{450}{1} = 450$

Answer: **450 ice creams were sold on the Bank Holiday Monday**.

22) Sarah whispers to Jenny that $\frac{3}{4}$ of the men at her housewarming party are single. Jenny knows that $\frac{3}{5}$ of the people at the party are men. What fraction of the people at the party are single men?

$\frac{3}{4}$ of $\frac{3}{5}$ are single men. Multiply the fractions together, because 'of' means multiply: $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

Answer: $\frac{9}{20}$ **of the people at the party are single men, in other words just less than half!**



23) With her savings plus a bank loan, Ruth buys half of her brother's share in her family's fruit and veg company. Ruth's brother owned $\frac{2}{3}$ of the company. How much of the company has Ruth purchased?

Ruth buys $\frac{1}{2}$ of $\frac{2}{3}$ of the company.

Multiply, because 'of' means multiply: $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$

Simplify the answer: $\frac{2}{6} = \frac{1}{3}$

Answer: **Ruth bought $\frac{1}{3}$ of the fruit and veg company.**

Answers to Dividing Fractions

24) Divide $\frac{2}{3}$ by $\frac{8}{15}$

$\frac{2}{3} \div \frac{8}{15} = \frac{2}{3} \times \frac{15}{8} = \frac{30}{24}$

$\frac{30}{24} = \frac{15}{12} = \frac{5}{4}$ when simplified.

Since $\frac{5}{4}$ is a vulgar fraction, change it to a mixed number.

$\frac{5}{4} = 1\frac{1}{4}$

Answer: $1\frac{1}{4}$

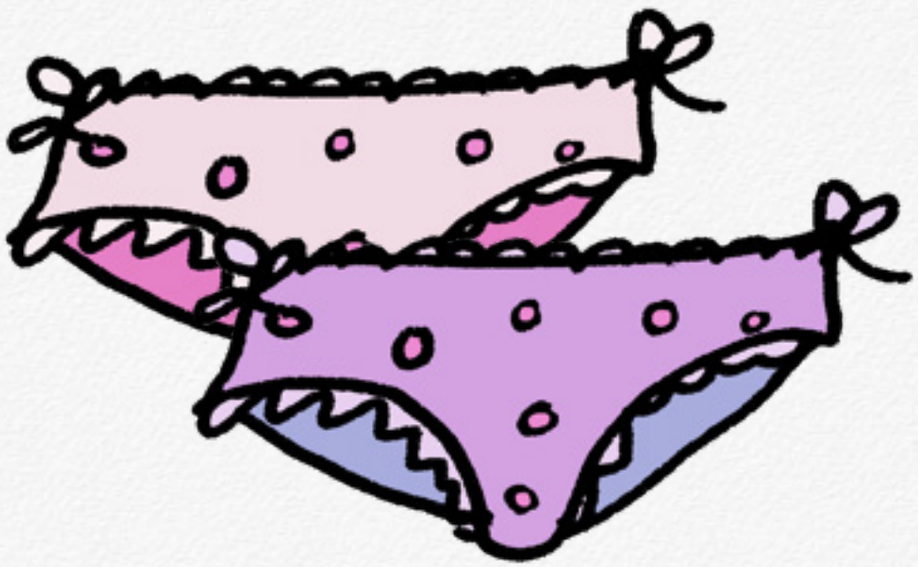
25) On the morning after her birthday, Tania takes $\frac{2}{3}$ of a Black Forest Gateau to her office and shares it out equally into 4 pieces for her friends. What fraction of the original cake is each of Tania's pieces?

(Hint: $\frac{2}{3}$ of the gateau is divided into 4 pieces.)

$\frac{2}{3}$ is divided by **4** $= \frac{2}{3} \div 4 = \frac{2}{3} \div \frac{4}{1}$

$= \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

Answer: **Each of Tania's pieces is $\frac{1}{6}$ of the original gateau.**



Answers to Multiplying and Dividing Fractions with Cross Cancelling

26) $\frac{180}{300} \times \frac{35^{7}}{90_{18}} = \frac{18}{30} \times \frac{7}{18}$

$\frac{{}^{1}18}{30} \times \frac{7}{18_{1}} = \frac{1}{30} \times \frac{7}{1} = \frac{7}{30}$

Answer: $\frac{180}{300} \times \frac{35}{90} = \frac{7}{30}$

27) $\frac{8}{15} \div \frac{16}{55} = \frac{8}{15} \times \frac{55}{16}$

$\frac{{}^{1}8}{15} \times \frac{55}{16_{2}} = \frac{1}{15} \times \frac{55}{2}$

$\frac{1}{{}_{3}15} \times \frac{55^{11}}{2} = \frac{1}{3} \times \frac{11}{2} = \frac{11}{6} = 1\frac{5}{6}$

Answer: $\frac{8}{15} \div \frac{16}{55} = 1\frac{5}{6}$

YOUR BRAIN WORKOUT

Q1.

Choose the fraction which is not equivalent to the other three.

- **A.** $\frac{3}{12}$
- **B.** $\frac{2}{8}$
- **C.** $\frac{5}{20}$
- **D.** $\frac{1}{5}$

YOUR BRAIN WORKOUT

Q2.

Choose the fraction which is not equivalent to the other three.

- **A.** $\frac{15}{20}$
- **B.** $\frac{10}{15}$
- **C.** $\frac{3}{4}$
- **D.** $\frac{6}{8}$

YOUR BRAIN WORKOUT

Q3.

Choose the fraction which is not equivalent to the other three.

- **A.** $\frac{2}{5}$
- **B.** $\frac{6}{15}$
- **C.** $\frac{6}{30}$
- **D.** $\frac{8}{20}$

YOUR BRAIN WORKOUT

Q4.

Choose the fraction which is not equivalent to the other three.

- **A.** $\frac{15}{20}$
- **B.** $\frac{10}{15}$
- **C.** $\frac{3}{4}$
- **D.** $\frac{6}{8}$

YOUR BRAIN WORKOUT

Answers

Q1. $\frac{3}{15}$

Q2. $\frac{1}{5}$

Q3. $\frac{6}{30}$

Q4. $\frac{10}{15}$

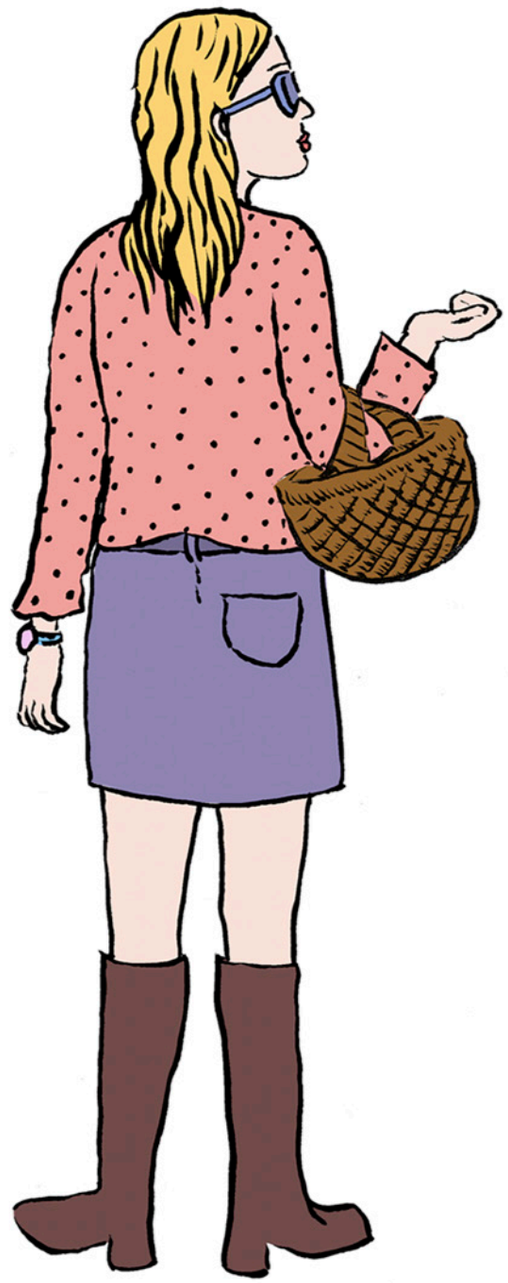
PART 12
DECIMALS

Singer Beyoncé, "We have to reshape our perception... of how we view ourselves."

Feminists 2014

Tamara Mellon, who launched Jimmy Choo shoes, "It's difficult for women – even successful women – to understand and accept their value. But you can't just work hard and wait to be noticed. If you don't like something – you have to speak up for yourself – quickly. 'I'm not happy with this,' is better than eventually expressing pent up rage."

Quick Quiz



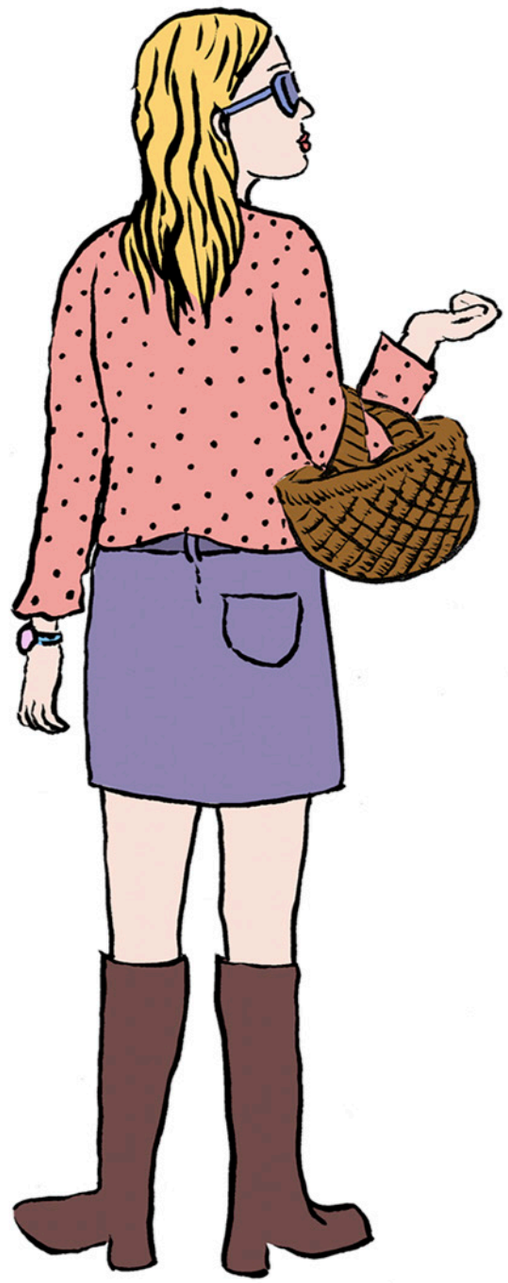
Q1.

What is the answer to?

$-2 - 4 = ?$

- A. 6
- B. -6
- C. -2
- D. 2

Quick Quiz

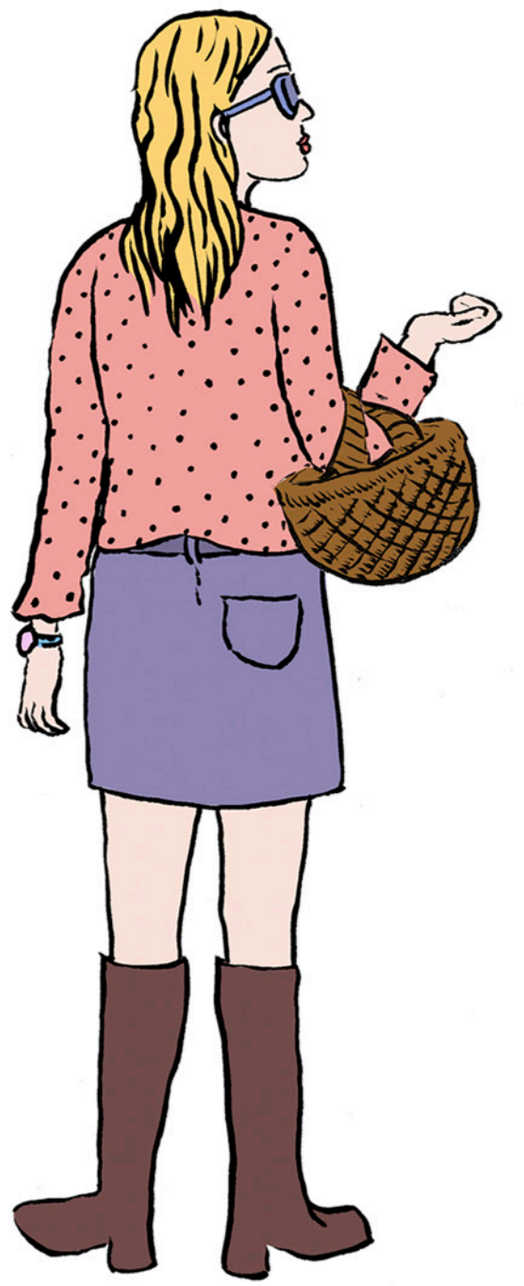


Q2.

Which of the following fractions is bigger than a half?

- **A.** $\frac{3}{12}$
- **B.** $\frac{2}{8}$
- **C.** $\frac{5}{20}$
- **D.** $\frac{3}{5}$

Quick Quiz

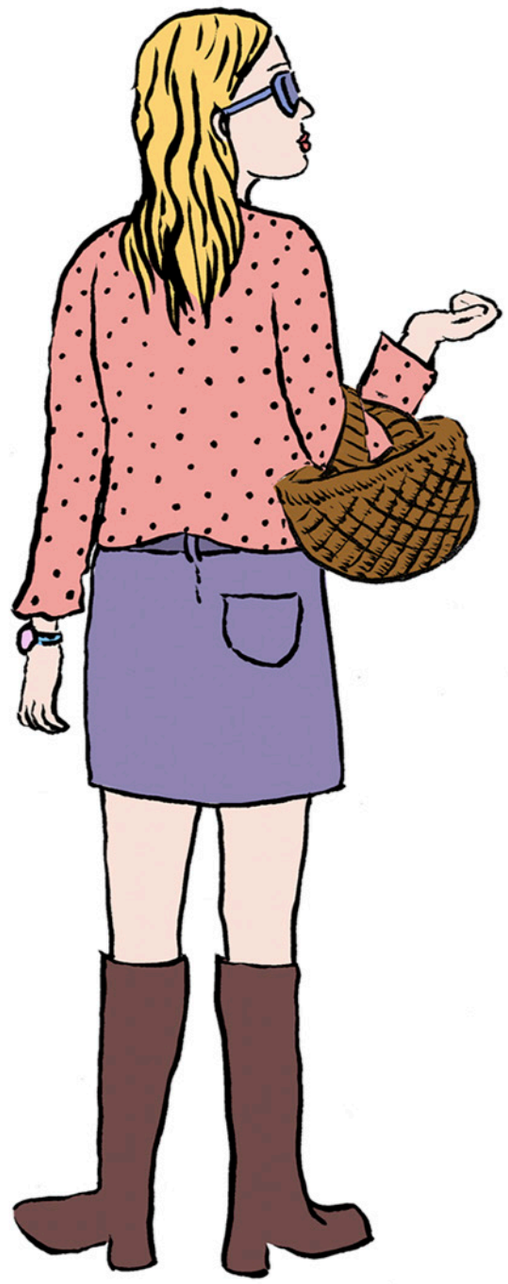


Q3.

If there are **30** guests expected at the tea party, which calculation should be used to work out the number of cupcakes to order, if on average each guest will eat two?

- **A.** 30 x 2 =
- **B.** 30 + 2 =
- **C.** 30 ÷ 2 =
- **D.** 30 – 2 =

Quick Quiz

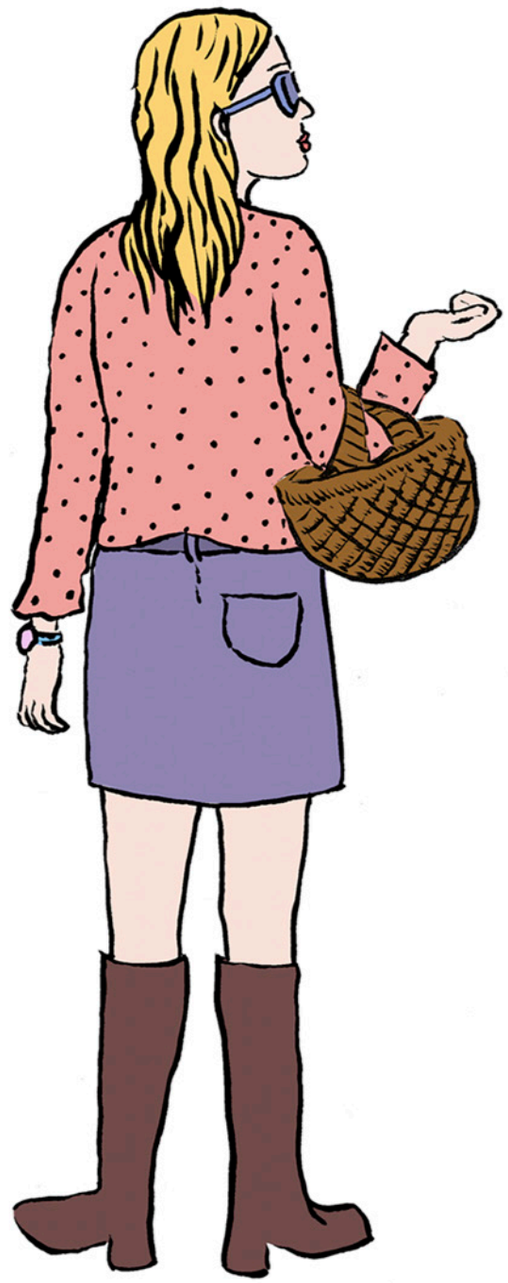


Q4.

24, 28, 32, 36, are from which times table?

- **A.** x12
- **B.** x8
- **C.** x6
- **D.** x4

Quick Quiz



Answers

Q1. -6

Q2. $\frac{3}{5}$

Q3. 30 x 2 =

Q4. X4

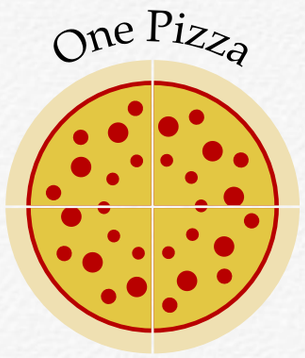
Decimals

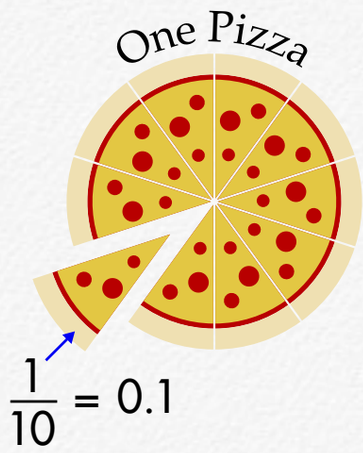
Decimals are an alternative to fractions. Decimal numbers also represent bits of things as well as whole things. In Real Life, a printed sentence about money might use a decimal **and** a fraction in the same sentence.

Decimals are numbers that contain a **decimal point**, such as 3.25 or 0.2. The part of the number which lies to the right of the decimal point is called the **decimal fraction**. The number on the left of the decimal point is a whole number.

The decimal point is the marker that shows where the whole numbers end, and the tenths ($\frac{1}{10}$) begin.

(Incidentally, the word 'decimal' comes from the Latin word **decima**, which means 'one tenth').





The decimal system is based on using 10 as a unit of division for whole numbers. If you divide this pizza into 10 pieces and ten people are given a slice, everyone gets one tenth part of the pizza, which is written as 0.1 of a pizza.

If you eat 1 slice, you have eaten 0.1 of the whole pizza.
If you eat 2 slices, you have eaten 0.2 of the whole pizza.
If you eat 3 slices, you have eaten 0.3 of the whole pizza.
If you eat 9 slices, you have eaten 0.9 of the whole pizza.
If you eat 10 slices, you have gobbled 1.0 pizza, the whole pizza.

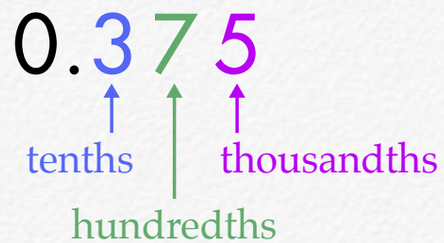
Notice that, after 0.9, the next number is 1.0, one **whole** unit.

Counting up in 1s	Counting up in 0.1s
7	0.7
8	0.8
9	0.9
10	1.0
11	1.1

Start using the next column to the left, after you reach 9.

Start using the next column to the left after you reach 0.9. Here the 'next column' is on the other side of the decimal point.

Some decimals have more than one digit after the decimal point. For example, money usually has two digits after the decimal point, as in **£2.16** or **£3.99**.



If each **0.1** slice of a giant-sized pizza is divided into **10** pieces, each of these tiny slivers is a hundredth (**0.01** or $\frac{1}{100}$), because **100** of these tiny slivers together would make up one whole pizza.

If it were possible, each hundredth piece of pizza might **again** be divided into ten pieces: each of these pieces would be a thousandth (**0.001** or $\frac{1}{1000}$).

And so it could go on..... getting smaller and smaller in what is called **powers of 10**.

The Amazing Powers of Ten

10 to the power of 3 is written in maths shorthand as 10^3 which is 10 x 10 x 10 = 1,000

10^4 is 10 x 10 x 10 x 10 = 10,000

10^6 is 10 x 10 x 10 x 10 x 10 x 10 = 1,000,000

10^3 is 1 plus 3 zeros

10^4 is 1 plus 4 zeros

10^6 is 1 plus 6 zeros

Small decimal quantities are not used to sub-divide pizzas, but they're used in laboratory work, financial institutions, etc.

How to Compare Decimals to Fractions

Half a pizza is equal to 0.5 of a pizza ($\frac{1}{2}$ pizza = 0.5 pizza).

One tenth of a pizza is equal to 0.1 of a pizza ($\frac{1}{10}$ pizza = 0.1 pizza).

Two tenths of a pizza is equal to 0.2 of a pizza ($\frac{2}{10}$ pizza = 0.2 pizza).

Three tenths of a pizza is equal to 0.3 of a pizza ($\frac{3}{10}$ pizza = 0.3 pizza).

Look back at $\frac{2}{10}$ pizza = 0.2 pizza. You know that $\frac{2}{10}$ simplified is $\frac{1}{5}$ so $\frac{1}{5}$ pizza **also** equals 0.2 of a pizza.

Similarly: $\frac{4}{10}$ pizza simplified is $\frac{2}{5}$ pizza = **0.4** pizza.

$\frac{6}{10}$ pizza simplified is $\frac{3}{5}$ pizza = **0.6** pizza.

Use your calculator to change fractions to decimals:

If you put a fraction into your calculator, using the ÷ button in place of the fraction dividing line, you get a decimal answer.

First Example

To input $\frac{3}{8}$ into a calculator, type [3] [÷] [8] [=]

The answer will be given as a decimal, so $\frac{3}{8} = 0.375$

Second Example

$\frac{2}{5} = 2 \div 5 = 0.4$, so $\frac{2}{5} = 0.4$

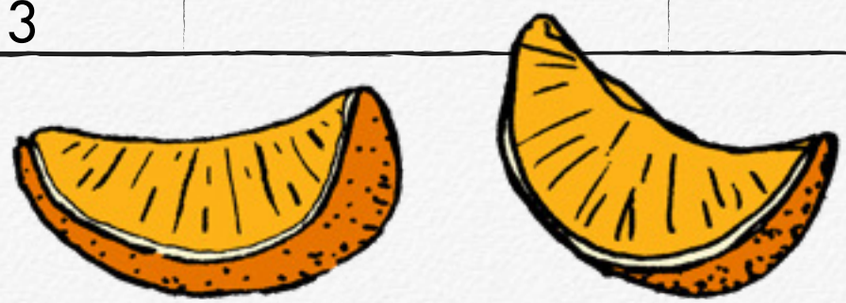
Numbers are needed for jewellery design.
Indonesian bride with ceremonial headdress.

Decimal Equivalents of Fractions

Exercises

Copy this table, then fill it in, **using your calculator** to find out what some of the most common fractions are, as decimals.

Fraction	Calculator Input	Decimal
$\frac{1}{4}$	1 ÷ 4 =	**0.25**
$\frac{1}{2}$		
$\frac{1}{5}$		
$\frac{3}{4}$		
$\frac{1}{3}$		



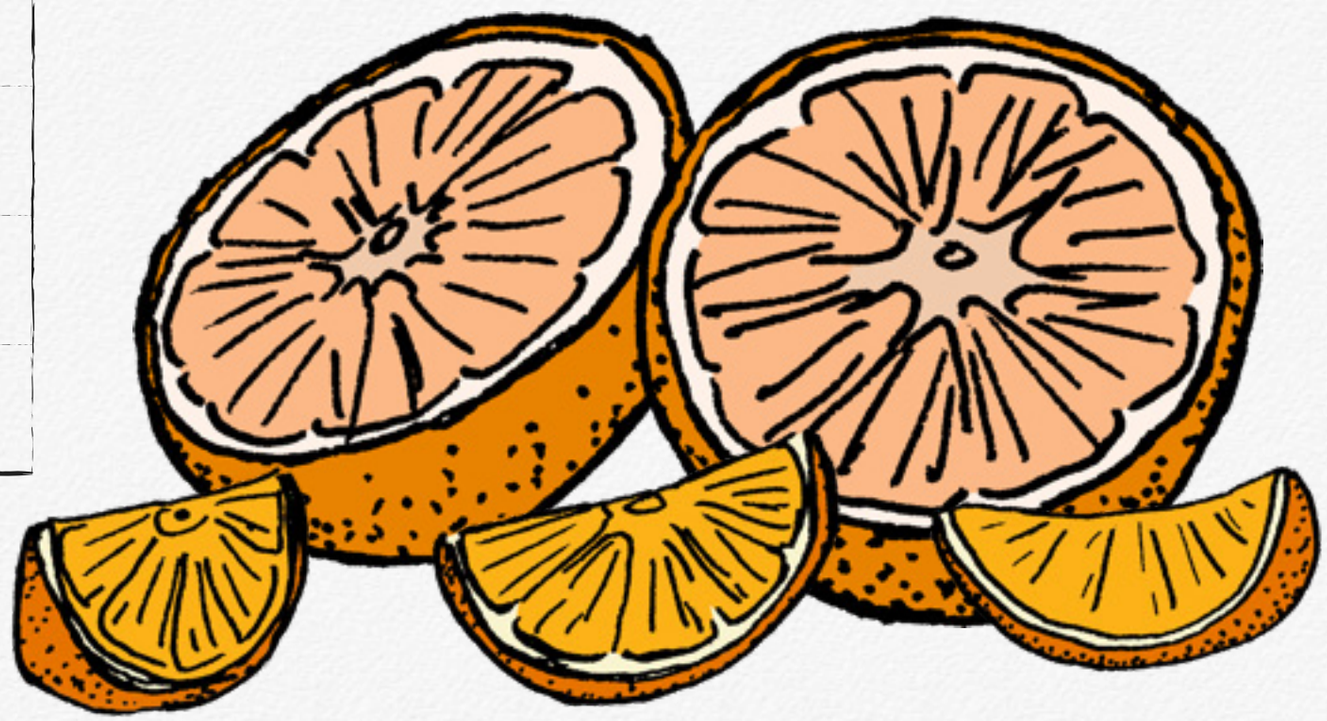
In print, a decimal point is easy to confuse with a full stop – but if you stay alert, you can tell which is which.

The correct place for the decimal point is not at the bottom of a number, but at the middle. **This is possible when you are writing by hand but not in print.**

2·2 ← correct

2.2 ← incorrect

Remember that in print, the decimal point is at the bottom of the text – as written above – but in handwriting you should write it at middle height of the text.



Recurring Decimal Numbers

Theoretically, fractions can be more accurate than decimals. But decimal calculations are easier and more practical for Real Life.

Some numbers are not accurately divisible in the decimal system. Like $\frac{1}{3}$ in the last exercise. The equivalent decimal number is **0.333333**... Here the threes will go on forever – it is a **recurring decimal**.

If you divide one pound - **100** pence - between **3** students, the answer as a decimal is:

$$3\overline{)100} \quad 33.333333333$$

33.333333333 (often written as **33.3** recurring)

In Real Life, each student would get **33p**, leaving one spare penny, which you can't practically divide into **3** equal bits.
Similarly, using fractions, you cannot practically divide **£1** by **3**.

In theory each student gets $33\frac{1}{3}$ pence

but in Real Life there are no $\frac{1}{3}$ pence coins.

Every coin counts.

Be aware when using decimals, some decimal fractions go on to infinity. In Real Life, this won't bother you.

$\frac{2}{3}$ written in decimals is **0.66666666**… to infinity (when the last number repeats like this, it is called 'recurring').

$\frac{1}{9}$ written in decimals is **0.11111111** recurring.

Try doing this on your calculator:

to work out $\frac{2}{3}$ type in [2] [÷] [3] [=]

When such numbers are used, they need to be rounded.

Caucasian ceremonial wedding headdress. Mask optional.

How to Compare Decimals

It is easier to compare decimal numbers than it is to compare fractions. With decimals, all you do is make the length of both numbers the same, which is easily done by adding zeros. Any number of zeros **added to the end** of a decimal number will not change the value of the number.

First Example

Which is the larger number, 0.13 or 0.3?

Add a zero on the end of 0.3, so that both numbers contain 2 digits after the decimal point.

In this example, you only need to add one nought, so 0.3 becomes 0.30.

Now you can compare the decimal numbers directly:
Question: **Which is bigger, 0.13 or 0.30?**

Focus on the digits after the decimal points. Is 13 a bigger number than 30? No, 30 is bigger than 13, so 0.30 is the larger decimal.

Answer: 0.3 is larger than 0.13.

Second Example

Put the following decimal numbers in order, smallest first:

5.3, 5.12, 5.203, 5.02

First, add on the zeros to make all the numbers as long as the longest number (5.203)

5.203 = 5.203 (this is the longest number, so no need to add zeros)

5.3 = 5.300 (with two added zeros)

5.12 = 5.120 (with one added zero)

5.02 = 5.020 (with one added zero)

In this example, the whole number part of each number is 5, so compare only the numbers that follow the decimal point. The correct order from smallest to largest numbers is: 020, 120, 203, 300.

Answer: The decimal numbers in order, with smallest numbers first:

5.02, 5.12, 5.203, 5.3

Remember to check the bill.

Exercises

1) Which is smallest, 0.7 or 0.53?

2) Put the following three numbers in order, smallest to biggest:

0.82 0.2 0.8

3) Put the following numbers in order, smallest to biggest:

2.4 2.042 2.24 4.2

Self-Confidence Tip 7

MORE ON GETTING POSITIVE

From Noah in his ark to Napoleon leading the French army, leaders have always known that the power of positive thought is essential for success: people who switch on their positive energy are more successful, negative energy people are not.

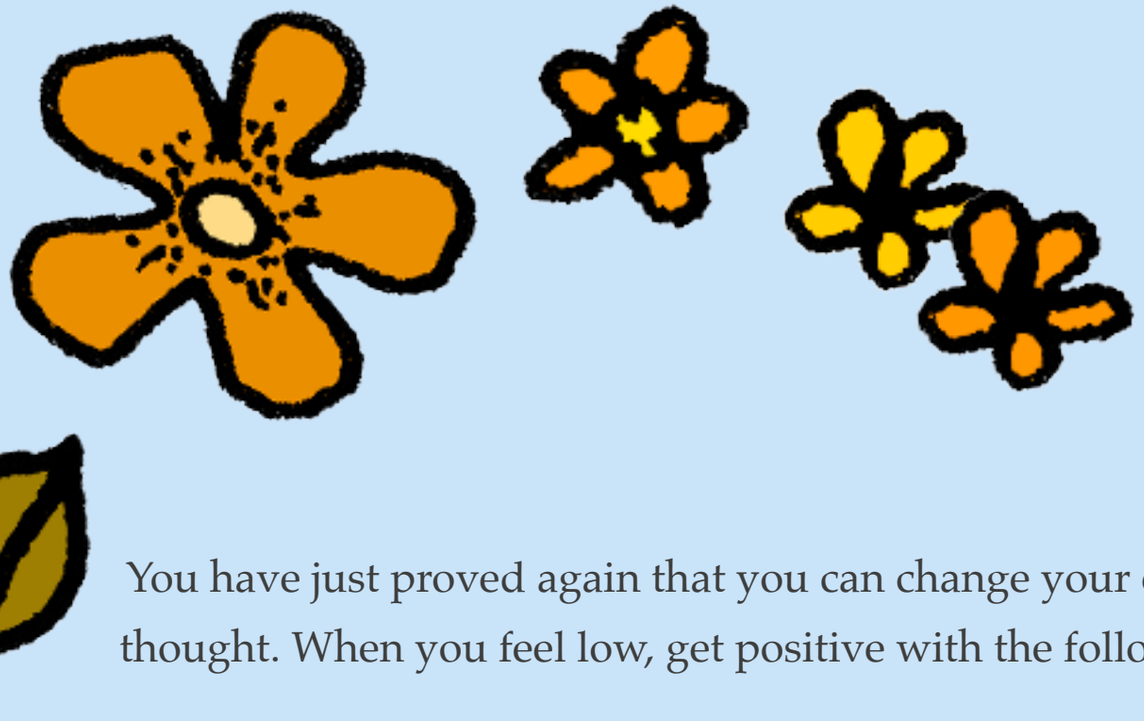
TV talent contestants are positive; they often say, “I really want this…I know I can do it”. And they get on TV, even if they don’t win the contest.

To become a more positive person, spend a minute in front of the mirror. Close your eyes and mentally see yourself as you are when you get out of bed in the morning: you’re still sleepy, you haven’t cleaned your teeth…not much energy…yawn. How do you feel? Low energy?

Next, open your eyes and see yourself as you are at this moment. What’s your energy level? Nothing special?

Now close your eyes again and see yourself at a time when you were looking your best; perhaps you were ready to go to a party, wearing a favourite dress, with hair and face at their best, feeling excited, with plenty of energy.

Now open your eyes again and keep that mood.

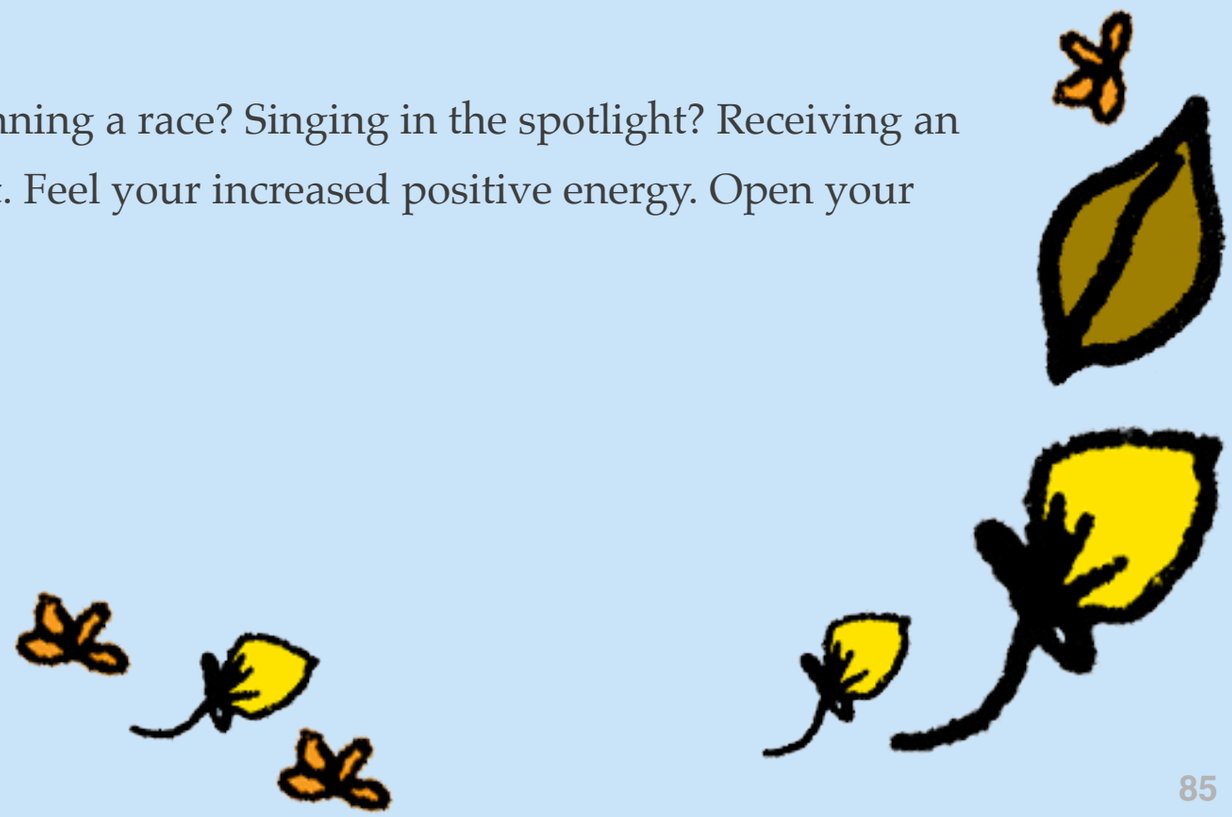


You have just proved again that you can change your energy level…simply by using the power of thought. When you feel low, get positive with the following mirror exercise.

MORE ON HOW TO RAISE YOUR ENERGY LEVEL

Decide on a goal. Say it aloud to yourself in the mirror. Then close your eyes and imagine yourself reaching that goal: see yourself from top to toe.

What are you wearing? What are you doing? Winning a race? Singing in the spotlight? Receiving an award? Stay with this mental picture and enjoy it. Feel your increased positive energy. Open your eyes, and enjoy your higher energy level.



How to Add & Subtract Decimals

The key to adding and subtracting decimals is to line up the numbers correctly with the decimal points above each other. Then you can do the calculation, as usual (look back to Addition and Subtraction of Big Numbers in STEP 1).

First Example

Calculate 13.4 + 72.73

Step 1.

Write the numbers, one on top of the other, with the decimal points in line:

	1	3	·	4	
+	7	2	·	7	3

Step 2.

Fill in any empty spaces in the columns with zeros:

	1	3	·	4	0
+	7	2	·	7	3

Step 3.

Now add each column in turn, starting at the right (ignore the decimal point):

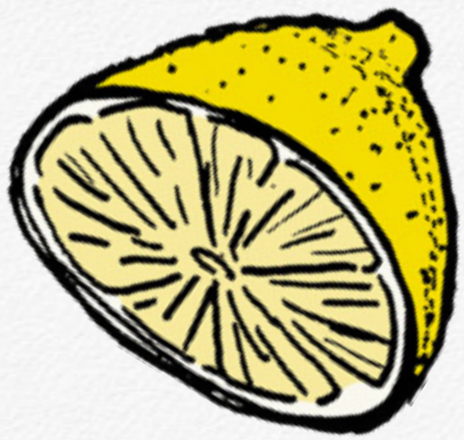
	1	3	·	4	0
+	7	2	·	7	3
	8	6	·	1	3
		1			

Step 4.

Lastly, insert the decimal point in the answer, in line with the decimal points above:

	1	3	·	4	0
+	7	2	·	7	3
	8	6	·	1	3
		1			

Answer: 86.13



Second Example

Calculate 0.07 + 52 + 8.1

If a number has no decimal point shown, then add a decimal point on the right hand side of the number. So 52 = 52. which also equals 52.0. Every whole number in the Universe has an invisible decimal point, followed by zeros.

Step 1.

Write the numbers above each other, with decimal points lined up:

		0	.	0	7
	5	2	.		
		8	.	1	

+

Step 2.

Fill in any empty spaces in the columns with zeros:

		0	.	0	7
	5	2	.	0	0
		8	.	1	0

+

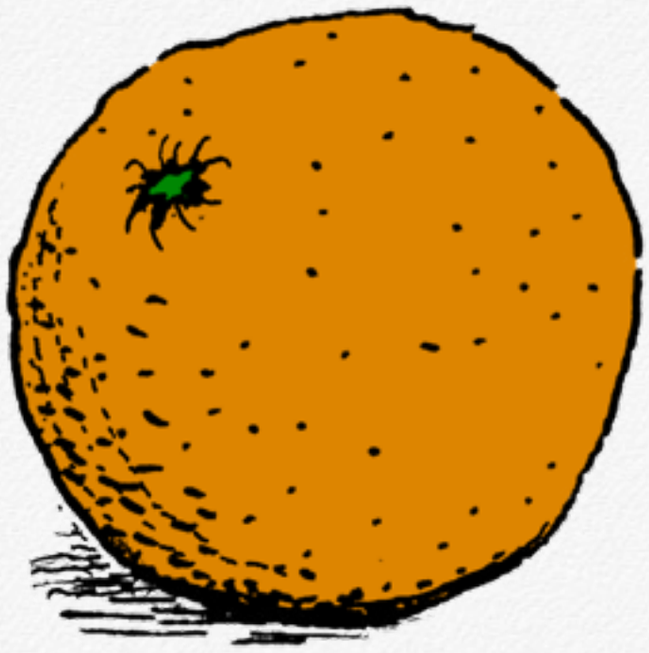
Step 3.

Now do the addition, and insert the decimal point in the answer, in line with the decimal points above:

		0	.	0	7
	5	2	.	0	0
		8	.	1	0
	6	0	.	1	7
		1			

+

Answer: 60.17



Third Example

Calculate 97.6 – 8.52

Step 1.

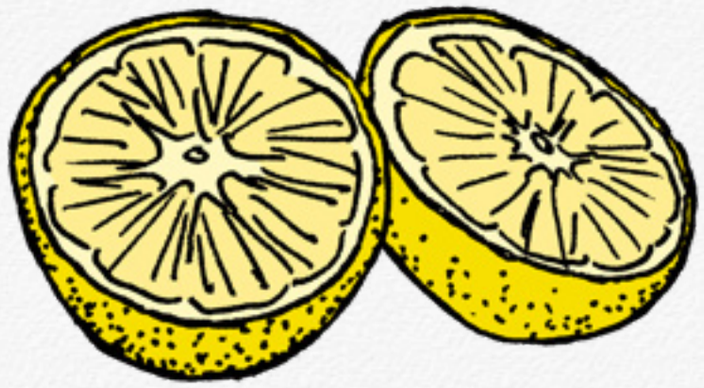
Set out as in addition, with numbers on top of each other, decimal points lined up:

	9	7	· 6		–
		8	· 5	2	

Step 2.

Fill in empty spaces in the columns with zeros:

	9	7	· 6	0	–
		8	· 5	2	



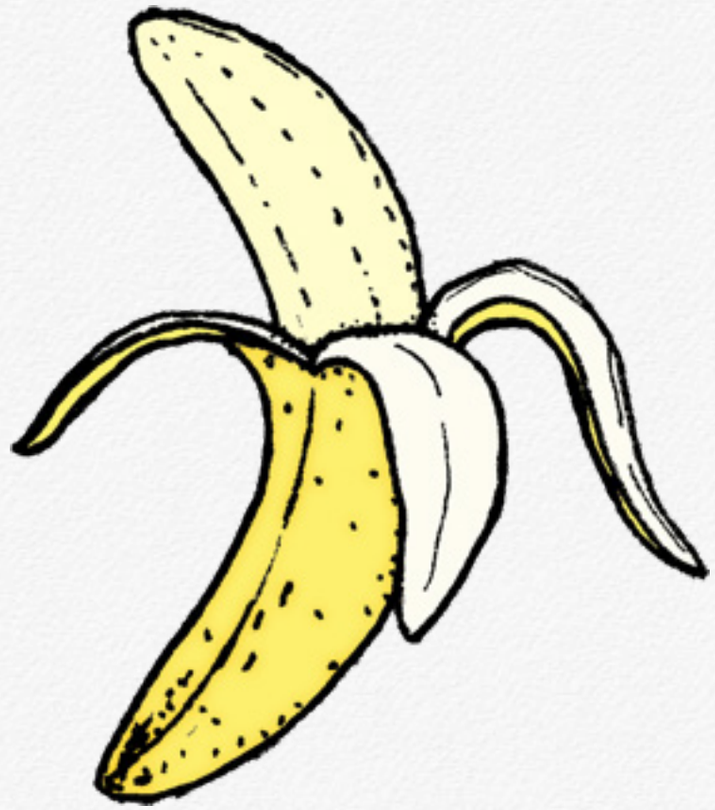
Step 3.

Next, do the subtraction, starting with the column furthest to the right, and borrowing where necessary (see STEP 1 to refresh your memory).

	8	1	5	1	
	~~9~~	7	· ~~6~~	0	–
		8	· 5	2	
	8	9	· 0	8	

Now, insert the decimal point in line with the points in the numbers above.

Answer: 89.08



Exercises

4) Add **34.2** and **4.67** together.

5) Tania's parents have one table **1.5** metres long, another **0.75** metres long and another **2** metres long. If Tania puts the three tables together for a party, how long will they measure?

6) Subtract **5.73** from **17.8**.

7) Tania purchased candles that cost **£12.83** out of the **£100** she had saved for the party. How much did Tania have left? (Hint: For a reminder of how to borrow from a string of zeros, see Third Example of Subtraction in STEP 1.)

How to work out your own style.

Save for a good quality jacket. Buy the rest from good high street shops like Topshop or Zara, plus charity and vintage shops to add originality.

How to Multiply Decimals

If you know how to multiply whole numbers, then you can multiply decimals. The same rules of short and long multiplication apply; the only extra work is deciding where to insert the decimal point in your answer.

To know where the decimal point goes, count how many digits follow **all** the decimal points in the question; the same number of digits must follow the decimal point in the answer.

Note

Whichever method you use for multiplying, **insert the decimal point as shown in step 2 and 3** in the following examples.

She's finished working on her style.
Japanese fashionista makes her statement.

First Example

2.41 x 0.9 = ?

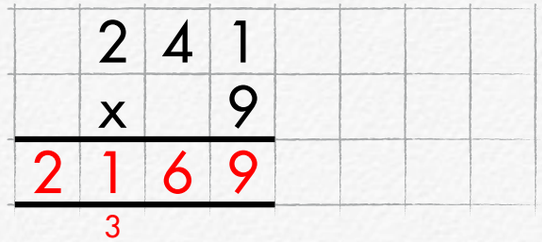
Step 1

Remove the decimal points from the numbers in the sum.

So 2.41 x 0.9 becomes 241 x 09

09 is the same as 9, so the multiplication sum is 241 x 9.

Now, do the multiplication sum (for a reminder of how to do this, see Short Multiplication in Part 3, STEP 1)



So 241 x 09 = 2169.

Step 2

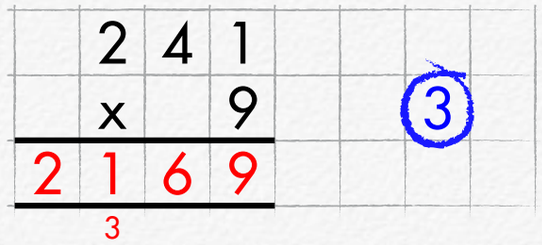
Next, look back at the original question, at the two numbers being multiplied. Count how many digits there are to the right of BOTH decimal points.

2.41 contains two digits to the right of the decimal point.

0.9 contains one digit to the right of the decimal point.

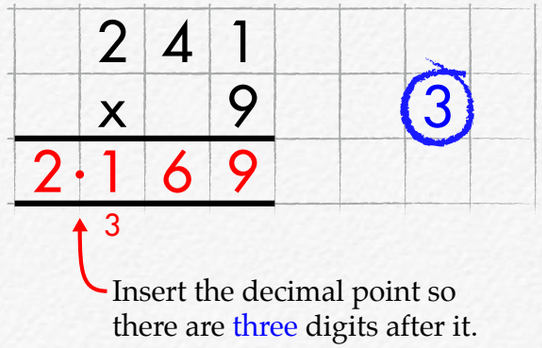
That's a total of **three** digits to the right of the decimal points.

So, jot down 3 in a circle next to the calculation:



Step 3

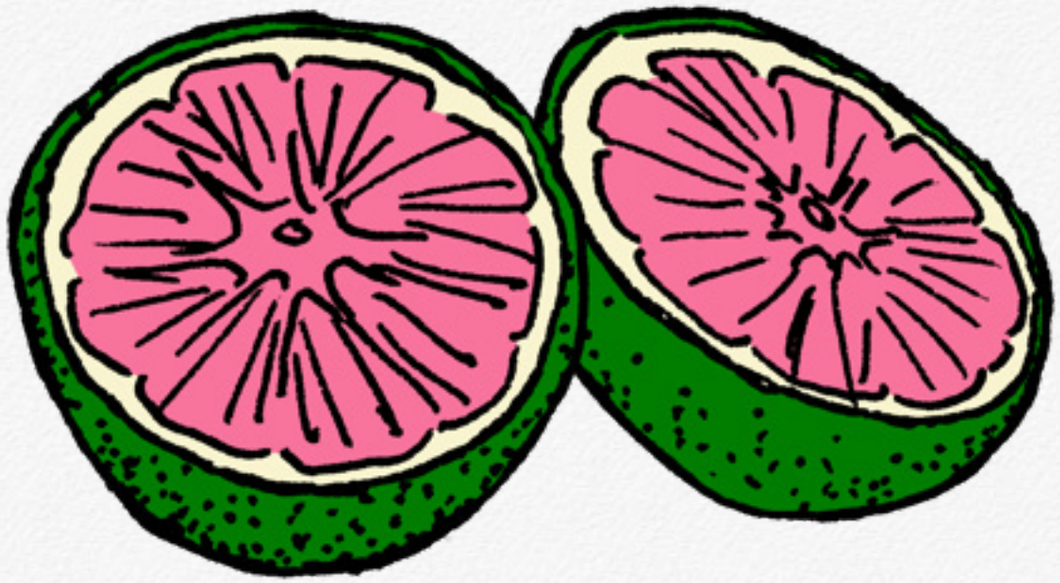
Now use your number in the circle to tell you where to place the decimal point in the answer. Since the number is three, insert the decimal point so there are three digits that follow it.



Answer: 2.41 x 0.9 = 2.169

Quick Tip

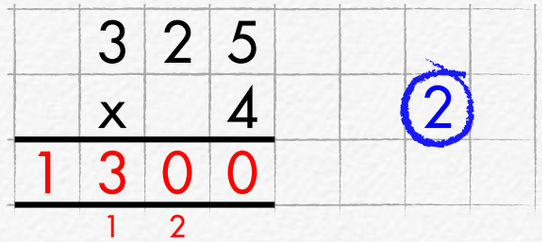
When you reach your final answer, remember: always do a common-sense check. Go back to the question: 'two-and-a-bit multiplied by nearly-one is roughly **2 x 1 = 2**', so the answer should be about two.



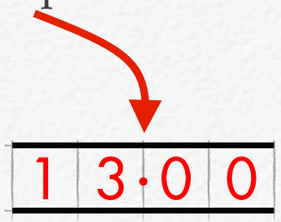
Second Example

Multiply **3.25 x 4**

Set out the short multiplication sum without the decimal points. Do the sum.
In a circle next to it, scribble the total number of digits to the right of both decimal points.



Then, insert the decimal point in the answer so that there are **two** digits after it.



Answer: **3.25 x 4 = 13.00** or **13**

Third Example

42.5 x 0.23 = ?

Write out the calculation, without the decimal points, and jot down in a circle the number of digits that follow the decimal points.

	4	2	5			
	x	2	3			(3)
1	2	7^{1}	5			
8	5^{1}	0	0			
9	7	7	5			

Next, insert the decimal point in the answer, so that there are **three** digits after it:

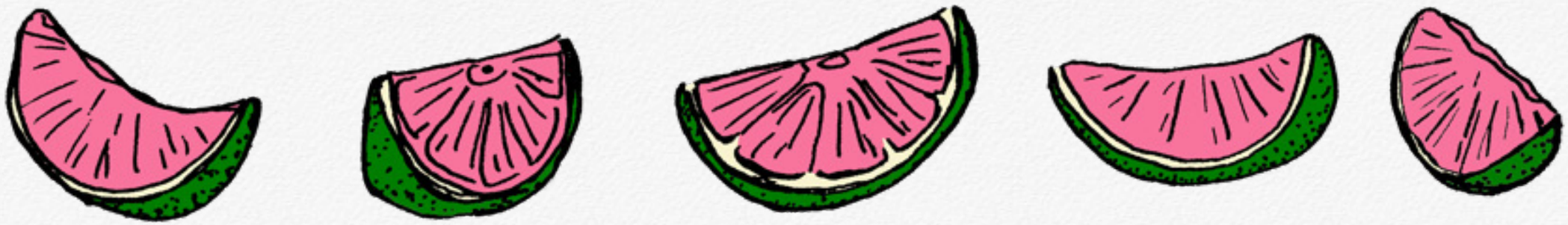
Answer: **42.5 x 0.23 = 9.775**

Exercises

8) Multiply **7.4** by **0.3**

9) In a Hereford school of **560** pupils, **0.8** of the pupils live less than **3** miles from the school. How many pupils live within **3** miles of the school? (Hint: you need to calculate **0.8** of **560** pupils. Remember 'of' in maths means multiply).

10) Lily's father is a restaurant critic for a magazine. To celebrate the end of term, he takes Lily to the new fish brasserie, where lobster costs **£28.75** per kilogram. Lily chooses a lobster that weighs **0.62** kilograms. How much will Lily's lobster cost?



Self-Confidence Tip 8

NICE STUFF

Everyone learns by doing something wrong, until they get it right. Between your first try and your achievement, there can be a lot of NO! NO!... **WHY** can't I get this right? This sort of negativity can easily become a bad habit. If you focus on negative words and criticism… you may create a negative habit that undermines your confidence and your ability to learn new things.

If you worry about what's wrong with yourself, and never think about what's right about yourself, you may spend a lot of time in negativity.

What boosts your confidence? Remembering the nice things people say to you, and about you.

In your maths notebook, scribble **at the back** a list of NICE STUFF that your family, friends, parents, teachers and work mates have said about you. By doing this, you focus on your strengths, and put them in the spotlight. From now on, add good things about yourself to your NICE STUFF list.

Read the list often, to remind yourself of your good points. This reassurance will create a good habit of self-confidence.

How to Multiply Decimals by the Big Noughts

As you know, when you multiply a whole number by **10**, you simply add one nought; when you multiply by **100**, add two noughts and when you multiply by **1,000**, add three noughts. And so on.

To multiply a decimal number by 10, the simplest way is to move the decimal point, one place to the right.

The decimal point is moved according to the number of zeros in the multiplier: one place for **x10**, two places for **x100** and three places for **x1,000**, etc.

First Example

3.45 x 10 = ?

move the decimal point
one place to the right

So the decimal point ends up between the **4** and the **5**.

Answer: 3.45 x 10 = 34.5

Second Example

0.23 x 100 = ?

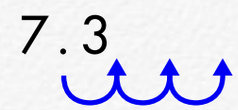
move the decimal point
two places to the right

The decimal point ends up at the end of the number, so there's no need to write it in the answer.

Answer: 0.23 x 100 = 23

Third Example

7.3 x 1,000 = ?



move the decimal point
three places to the right

Insert zeros into any empty troughs formed by the arrows:

Answer: 7.3 x 1,000 = 7,300

BAD ATTITUDE

My brother is:
Ambitious,
Brave,
Determined,
Persistent.

Everyone says
He will go far.

But I
Live in a dream world,
Take unnecessary risks,
Don't know when to stop,
Won't shut up.

Everyone says
I am difficult.

I think
They make it difficult
For a girl to be
A success.

I don't need
Put-me-downs,
I need
ENCOURAGEMENT.

How to Divide Decimals by the Big Noughts

When dividing by 10, 100, 1,000 etc, you move the decimal point in the opposite direction to multiplication, so to the left.

First Example

98.4 ÷ 10 = ?

9 8 . 4 = 9 . 8 4

move the decimal point one place to the left

Answer: 98.4 ÷ 10 = 9.84

Second Example

5.3 ÷ 1,000 = ?

5 . 3

move the decimal point three places to the left

Insert zeros into the empty troughs.

0 0 5 . 3 = . 0 0 5 3

A number should not **begin** with only a decimal point, so add an extra zero before the decimal point.

Answer: 5.3 ÷ 1,000 = 0.0053 (not .0053)

Wrist or neck?

Exercises

By moving the decimal point, calculate the following:

11) 7.64 x 100 =

12) 12.4 ÷ 10 =

13) 0.45 x 1,000 =

14) 7 ÷ 100 =

Quick Tip

If you forget which way to move the decimal point, do a common-sense check.

For **multiplication** by a number bigger than 1, your answer should be **larger** than the original number in the calculation, so the decimal point moves to the **right**.

If you are **dividing** by a number bigger than 1, your answer should be **smaller** than the first number in the calculation, so the decimal point moves to the **left**.

Long neck = beauty in Pradang Tribe, Burma.

How to Divide Decimals

There are two situations to consider when dividing decimals:

A) when the number being divided is a decimal, as in $6\overline{)2.43}$

B) when the number doing the dividing is a decimal, as in $6.24\overline{)35}$

(To refresh yourself on short and long division see STEP 1).

Fencer awaits her turn at tournament, Turin, Italy.

A. How to divide a decimal number by a whole number

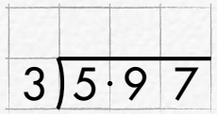
Here, the decimal point is simply inserted into the answer, directly above the position of the decimal point in the calculation:

First Example

5.97 ÷ 3 = ?

Step 1

Write the calculation as a short division sum:

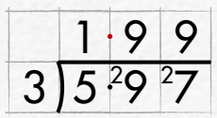


Step 2

Next, do the division, as if the decimal point were not there.

Step 3

Now, insert the decimal point, in line with
the decimal point in the number being divided.



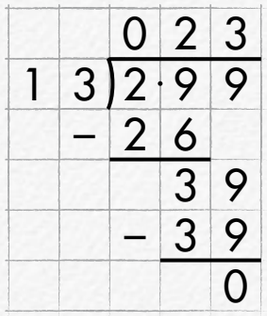
Answer: 1.99

Second Example

Single mum Annabel is decorating a birthday cake for her little daughter, Daisy. A packet of 13 pink sugar mice costs £2.99. How much does each sugar mouse cost?

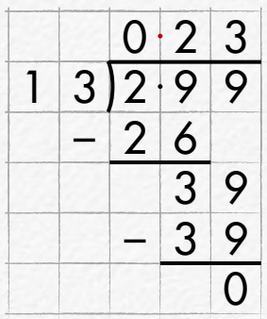
Step 1

Write the calculation as a long division sum and
do the division sum as if the decimal point were not there.



Step 2

Now, insert the decimal point **into the answer**:



Answer: **Each pink sugar mouse costs £0.23 or 23p.**

Third Example

2.5 ÷ 4 = ?

Step 1

Do the division sum, and add the decimal point:

```
  0.6   (plus a remainder of 1)
4)2.5
```

This time there is a remainder. With decimals, it doesn't make sense to talk about a remainder, so NEVER leave the answer with a remainder.

Instead, add four zeros to the end of the decimal number being divided. Adding zeros to the end of the number **after** the decimal point will not change the value of this decimal number: Example: 2.5 = 2.50 = 2.5000 = 2.50000000.

Step 2

Extend the dividing box and add four zeros to the end of the number being divided. If you find that four zeros is not enough for your sum, you can add more later. If four zeros is too many, it doesn't matter.

```
  0.6
4)2.5 0 0 0 0
```

Step 3

Now, write the remainder by the first zero, and divide.
In this case you are dividing 4 into 10 (see below). Continue to carry the remainders until you have no remainders (see below) or until you decide to round the answer
(not necessary in this example – see Fourth Example).

```
  0.6 2 5
4)2.5 1 0 2 0 0 0
```

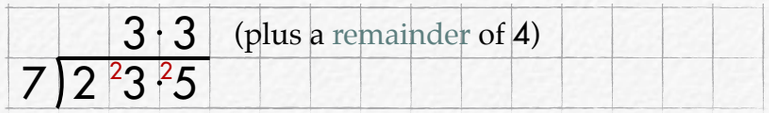
Answer: 0.625

Fourth Example

23.5 ÷ 7 = ?

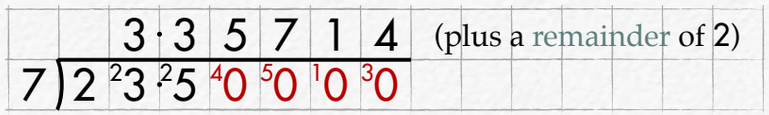
Step 1

Do the division sum, and add the decimal point:



Step 2

Extend the dividing box and add four zeros to the end of the number being divided, and continue to carry the remainders.



Step 3

At this point, you have a remainder of **2**. Each time you have another remainder, add a zero to the number being divided. But with some numbers, this can go on forever. Alternatively, you can **round the number**. In this example, if you round to three decimal places, the answer is **3.357**. If you round to two decimal places, the answer is **3.36**.

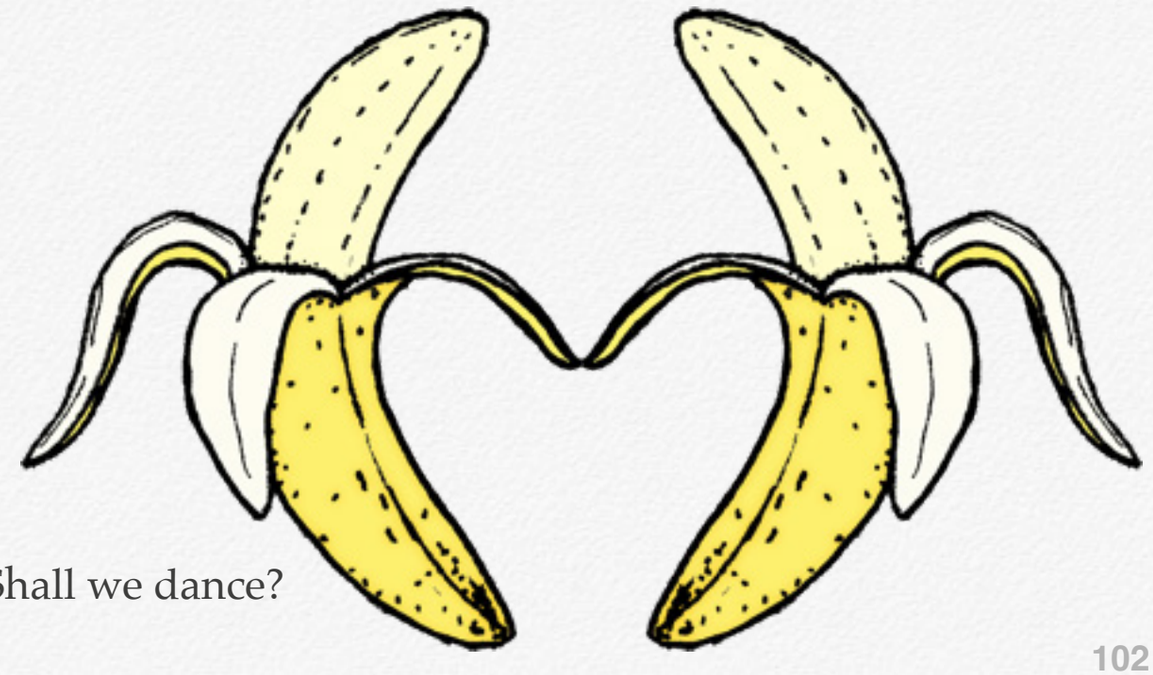
Why **3.36** not **3.35**? Because the following number, **7**, is five-or-more. **(See MONEY STUFF STEP 1, Different Ways to Round Numbers)**

Answer: **3.36** (to **2** d.p.)

Exercises

15) Four friends gather their junk for a car boot sale stall. They make **£75.24** profit. If the profit is shared equally between them, how much does each friend get?

16) Retired actress, Sally has a tomato plant that grew **1.2** metres in a week: she must be feeding it on special Frankenstein food. How much did the tomato plant grow on average, each day that week? Give your answer to two decimal places (**2** d.p.).



Shall we dance?

B. How to divide any number by a decimal number

Don't be tempted to try a shortcut.

When dividing by a decimal, adjust the calculation **before** you start, as in the following examples.

First Example

34 ÷ 0.4 = ?

The dividing number is a decimal number, so first, you multiply it to make it a whole number. Multiply by 10 until you eliminate the decimal point in the dividing number, so making it a whole number: 0.4 x 10 = 4.

34 ÷ 0.4 = ?
×10 ×10
340 ÷ 4 = ?

If you multiply one number in the calculation, you need to multiply the other number in the same way.

Now continue, with a short division sum using **340 ÷ 4**:

```
   0 8 5
 4 3 4 2 0
   3 2
     2 0
     2 0
       0
```

The answer to this division sum is the answer to the original question. There is no need to adjust the answer in any way.

Answer: **34 ÷ 0.4 = 85**.

Second Example

0.098 ÷ 0.28 = ?

Since the dividing number has **2** decimal places, you need to multiply by **10 twice** to get a whole number.

0.098 ÷ 0.28 = ?
×10 ×10
0.98 ÷ 2.8 = ?
×10 ×10
9.8 ÷ 28 = ?

The calculation new reads:

As long as the dividing number is a whole number, you can now do a normal long division sum, or divide by factors since **28** can be split into the factors **4 x 7**, which is much easier. **(See MONEY STUFF STEP 1, Division by Factors)**

Using the factors, first divide **9.8** by the first factor, **4**.

```
     2 . 4  5
 4 | 9 . 8  2  0  0
     8
     1 8
     1 6
       2 0
       2 0
         0
```

Then divide the answer (**2.45**) by the second factor, **7**.

```
     0 . 3  5
 7 | 2 . 4  5
     2 1
       3 5
       3 5
         0
```

```
     0 . 3  5
 7 | 2 . 4  5
 4 | 9 . 8  2  0  0
     8
     1 8
     1 6
       2 0
       2 0
         0
```

Answer: **0.098 ÷ 0.28 = 0.35**.

Exercises

17) Divide **2.52** by **0.3**.

18) Divide **0.45** by **0.16** (to refresh your memory, look back to MONEY STUFF STEP 1, Division by Factors).

Self-Confidence Tip 9

TALK THE TALK

Watch your words. Some people say it's cool to be bad at maths, only nerds are good at maths. Some people say, "I'm hopeless at maths", as if this were something to be proud of.
But it isn't smart to be stupid. Nobody says proudly, "I can't read."

Do you ever put yourself down? Have you ever groaned, "I'm no good at maths…maths is boring", or "I'll never be able to pass this exam", or "I hate maths!". If so, start using new words.
Try, "I wasn't taught maths well, but now I'm getting better"…"I can see my maths is improving"…

Never tell yourself, "I must get this right", or "I ought to pass this exam".

Instead, tell yourself you'll be delighted to get it right, but it's not the end of the world if you don't.

The words you use will make a difference to how you feel. So keep telling yourself "I'm getting better at maths", or "When I finish this exercise, I'll feel a little thrill of achievement". Because you will.

WALK THE TALK

How can you tell a confident person?

What do you notice about a confident person's body language?

Does a confident person:

- sit upright, or slump over her desk or table?
- stand tall, or slouch?
- walk with a firm step, or shuffle, dragging her feet?
- look pleasant, or look empty and exhausted?

DO YOU LIKE BEING WITH CONFIDENT PEOPLE?

Confident people are certainly more popular.

If you want to be confident, start by **acting** like a confident person. Walk with confidence. Stand tall. Hold your head straight. Shoulders back, as if you're trying to touch shoulder blades. Breathe deeper. Now take 3 deep, slow breaths: inhale energy, exhale tension. You will find that just by **acting** confident, you will begin to feel more confident.

Decimal Remainders

When dividing whole numbers, you will often be left with a leftover number… a remainder. This remainder can be presented as a decimal number.

When you end up with a remainder,
calculate the decimal answer as follows.

First Example

33 ÷ 8 = ?

```
  0 4   (with a remainder of 1)
8|3 3
```

The answer is **4** plus a remainder of **1**.

To calculate the answer as a decimal number,
add decimal points on the end of the number inside the division box, and and to the number above the division box.

Add four zeros to the end of the number inside
the division box (**33** is the same as **33.0000**).

```
  0 4 .
8|3 3 . 0 0 0 0
```

Put the remainder beside the first zero.
Continue the division:

```
  0 4 . 1 2 5
8|3 3 . 1 2 4 0 0
```

Answer: **33 ÷ 8 = 4.125**

Second Example

234 ÷ 7 = ?

```
  0 3 3   (with a remainder of 3)
7|2 3 4
```

Add the two decimal points (inside the box and above it).
Add four zeros after the decimal points inside the box.
Now continue the calculation:

```
  0 3 3 . 4 2 8 5
7|2 3 4 . 3 2 6 4 5 0
```

In this example, you can continue the calculation FOREVER, by adding more zeros to the bottom number.
When this situation occurs, simply round the answer to a sensible degree of accuracy. Two decimal places is normally enough.

Answer: **234 ÷ 7 = 33.43** (to **2** d.p.)

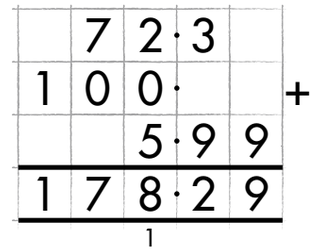
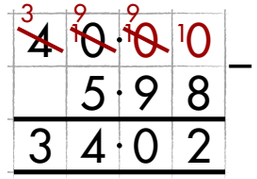
Exercises

19) How many scoops of blueberry ice cream
can each schoolgirl have at a sleepover, if there are **4** girls
and only **7** scoops?

20) **8** friends share the cost of a bottle of champagne
for their friend Lukas, as a **21**st birthday present.
The champagne costs **£30**,
so how much will each of the friends need to pay?

You need maths
in the market.

Decimals Summary

	Rule	Example
Addition	Line up the decimal points, then add the columns. The answer has a decimal point in line with the decimal points above it.	
Subtraction	Line up the decimal points, then subtract the columns, with borrowing if necessary. The answer has a decimal point in line with the decimal points above it.	

Multiplication	First, multiply the numbers while ignoring the decimal points. Next, count how many numbers after the decimal points there were **in total,** in the original question. Then place the decimal point in the answer, so that there is the same number of digits after it as there was in the original question.	Question: 2.5 x 0.47 = ? Use long multiplication to calculate that 25 x 47 = 1175 There are 3 numbers to the right of the decimal points in the question: 2.5 x 0.47 = ? Place the decimal point in the answer, so that there are 3 numbers to the right of it in the answer: 1.175
Division	**A) Dividing a decimal by a whole number:** Divide as if whole numbers; ignore the decimal point. After you complete the division, place the decimal point directly above the decimal point in the box. **B) Dividing by a decimal:** Multiply **both** numbers in the question by 10 until you have eliminated the decimal point from the **dividing number.** Then divide, as in part A.	**Question A:** 75.3 ÷ 4 = ? 18.825 4 | 75.3 10 20 Answer: 75.3 ÷ 4 = 18.825 **Question B:** 45.93 ÷ 0.6 = ? Multiply both numbers by 10 until you have eliminated the decimal point in the dividing number. Question B now reads: 459.3 ÷ 6 = ? 76.55 6 | 459.3 30 Answer: 45.93 ÷ 0.6 = 76.55

Answers to Part 12

Answers to Decimal Equivalents of Fractions

Fraction	Decimal
$\frac{1}{4}$	**0.25**
$\frac{1}{2}$	**0.5**
$\frac{1}{5}$	**0.2**
$\frac{3}{4}$	**0.75**
$\frac{1}{3}$	**0.33333333**

Split-second timing.

Dancer in the National Ballet of China.

Answers to comparing decimals

1) Which is smaller, **0.7** or **0.53**?

0.53 = **0.53** (no need to add zero, because **0.53** is the longer number)

0.7 = **0.70** (with one zero added, as you need **2** digits after the decimal point)

Answer: **0.53 is smallest**.

2) Put the following three numbers in order, smallest to biggest:

0.82 0.2 0.8.

0.82 = 0.82 (no zeros added, because this is the longest number, with 2 digits after the decimal point)

0.2 = 0.20 (with one zero added to make 2 digits after the decimal point)

0.8 = 0.80 (with one zero added to make 2 digits after the decimal point)

Focus on the two-digit numbers after the decimal points: 20, 80 and 82 in order of size.

Answer: 0.2 0.8 0.82

3) Put the following numbers in order, smallest to biggest:

2.4 2.042 2.24 4.2

2.4 = 2.400 (with two zeros added, to make 3 digits after the decimal point)

2.042 = 2.042 (no zeros added because this is the longest number, with 3 digits after the decimal point)

2.24 = 2.240 (with one zero added to make 3 digits after the decimal point)

4.2 = 4.200 (with two zeros added to make 3 digits after the decimal point)

Now you can see that 4.2 is the biggest number, because it starts with the biggest whole number. For the other 3 numbers, focus on the three-digit numbers after the decimal points, since they all begin with the same whole number: 042, 240 and 400, in order of size.

Answer: 2.042 2.24 2.4 4.2

Answers to Addition and Subtraction of Decimals

4) Add **34.2** and **4.67** together.

$$\begin{array}{r} 34.20 \\ +\ \ 4.67 \\ \hline 38.87 \end{array}$$

Answer: **34.2 + 4.67 = 38.87**

5) Tania's parents have one table **1.5** metres long, another **0.75** metres long and another **2** metres long. If Tania puts the three tables together for a party, how long will they measure?

$$\begin{array}{r} 1.50 \\ 0.75 + \\ 2.00 \\ \hline 4.25 \\ {\scriptstyle 1} \end{array}$$

Answer: **4.25** metres

6) Subtract **5.73** from **17.8**.

$$\begin{array}{r} 1\ 7.\overset{7}{\cancel{8}}\ {}^{1}0 \\ -\ \ 5.7\ 3 \\ \hline 1\ 2.0\ 7 \end{array}$$

Answer: **17.8 – 5.73 = 12.07**

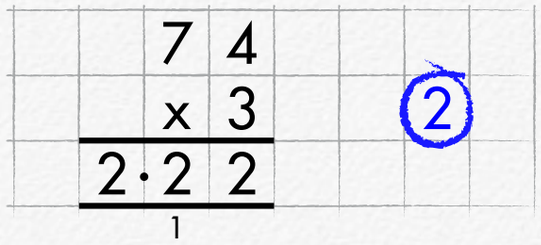
7) Tania purchased candles that cost **£12.83** out of the **£100** she had saved for the party. How much did Tania have left?

$$\begin{array}{r} \cancel{1}\ \overset{9}{\cancel{0}}.\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\ {}^{1}0 \\ -\ \ 1\ 2.8\ 3 \\ \hline 8\ 7.1\ 7 \end{array}$$

Answer: **£100 – £12.83 = £87.17**

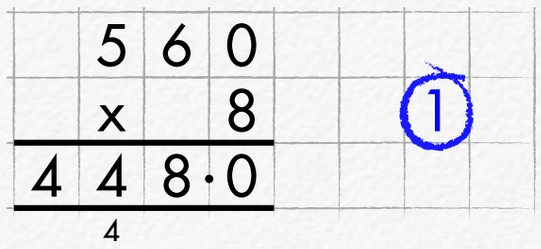
Answers to Multiplication of Decimals

8) Multiply **7.4** by **0.3**



Answer: **7.4 x 0.3 = 2.22**

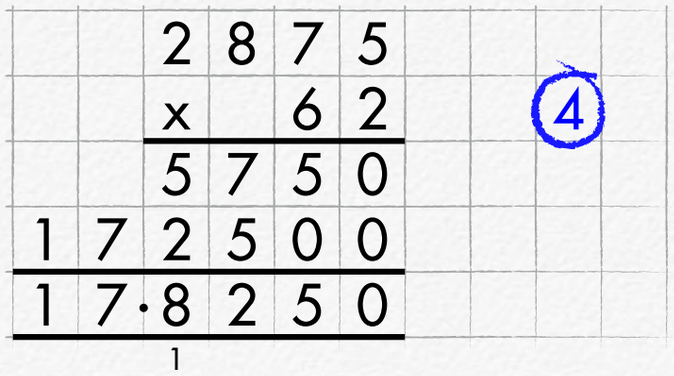
9) In a Hereford school of **560** pupils, **0.8** of the pupils live less than **3** miles from the school. How many pupils live within **3** miles of the school? (Hint: you need to calculate **0.8** of **560** pupils. Remember 'of' in maths means multiply).



Answer: **0.8 x 560 = 448**.

448 pupils live within 3 miles of the school.

10) Lily's father is a restaurant critic for a magazine. To celebrate the end of term, he takes Lily to the new fish brasserie, where lobster costs **£28.75** per kilogram. Lily chooses a lobster that weighs **0.62** kilograms. How much will Lily's lobster cost?



Answer: Lily's lobster will cost her dad **£17.825**, or **£17.83**

Answers to Multiplying and Dividing by the Big Noughts

11) 7.6 4 x 100 = 764.0 or 764

12) 1 2.4 ÷ 10 = 1.24

13) 0.4 5 0 x 1000 = 450 (put zero in the empty trough)

14) 0 7 ÷ 100 = 0.07 (put zero in the empty trough and add an extra zero before the decimal point, as a number should not be written .07)

Sally's tomato plant.

Answer to Dividing Decimals

15) Four friends gather their junk for a car boot sale stall. They make **£75.24** profit. If the profit is shared equally between them, how much does each friend get?

Divide **£75.24** (profit) by **4** (friends).

```
    1 8.8 1
  ┌─────────
4 │7 ³5.³2 4
```

Answer: **The four friends made £18.81 each.**

16) Retired actress Sally has a tomato plant that grew **1.2** metres in a week: she must be feeding it on special Frankenstein food. How much did the tomato plant grow on average, each day that week? Give your answer to two decimal places (**2**d.p.).

Divide **1.2** metres (height) by **7** (number of days in a week).

```
    0.1 7 1 4 ...
  ┌─────────────
7 │1.¹2 ⁵0 ¹0 ³0 ...
```

Answer: **The tomato plant grew 0.17m each day.**

17) Divide **2.52** by **0.3**

First, adjust the calculation by multiplying both numbers by **10**.

The calculation becomes **25.2 ÷ 3**.

$$\begin{array}{r} 0\,8.4 \\ 3\overline{)\,2\,{}^{2}5\,{}^{1}2} \end{array}$$

Answer: **2.52 ÷ 0.3 = 8.4**

18) Divide **0.45** by **0.16**

Adjust the calculation by multiplying both numbers by **100**. (Or multiply by **10** and then by **10** again.)

The calculation becomes **45 ÷ 16**.

Divide by factors of 16, either **4 x 4** or **8 x 2**.

In this example I used **4 x 4**:

$$\begin{array}{r} 0\,2.8\,1\,2\,5 \\ 4\overline{)\,1\,{}^{1}1\,{}^{3}2\,5\,{}^{1}0\,{}^{2}0} \\ 4\overline{)\,4\,5\,{}^{1}0\,{}^{2}0} \end{array}$$

Answer: **0.45 ÷ 0.16 = 2.8125**

Answer to Decimal Remainders

19) How many scoops of blueberry ice cream can each schoolgirl have at a sleepover, if there are **4** girls and only **7** scoops?

$$\begin{array}{r} 1.7\,5\,\,\, \\ 4\overline{)\,7\,{}^{3}0\,{}^{2}0\,0\,0} \end{array}$$

Answer: **Each girl will get 1.75 scoops of blueberry ice cream.**

20) **8** friends share the cost of a bottle of champagne for their friend Lukas, as a 21st birthday present.

The champagne costs **£30**,

so how much will each of the friends need to pay?

$$\begin{array}{r} 3.7\,5\,\,\, \\ 8\overline{)\,3\,{}^{3}0\,{}^{6}0\,{}^{4}0\,0\,0} \end{array}$$

Answer: **Each of the friends will need to pay £3.75 towards the champagne.**

YOUR BRAIN WORKOUT

Q1. What is the answer to?

$$3 + 0.42 = ?$$

YOUR BRAIN WORKOUT

Q2. What is the answer to?

$$5 + 1.7 = ?$$

YOUR BRAIN WORKOUT

Q3. What is the answer to?

$$8.8 - 0.6 = ?$$

YOUR BRAIN WORKOUT

Q4. What is the answer to?

$$0.6 \times 10 = ?$$

YOUR BRAIN WORKOUT

Q5. What is the answer to?

1.7 x 1,000 = ?

YOUR BRAIN WORKOUT

Q6. What is the answer to?

$$430 \div 100 = ?$$

YOUR BRAIN WORKOUT

Q7. What is the answer to?

$$7 \div 10 = ?$$

YOUR BRAIN WORKOUT

Answers

Q1. 3.42

Q2. 6.7

Q3. 8.2

Q4. 6

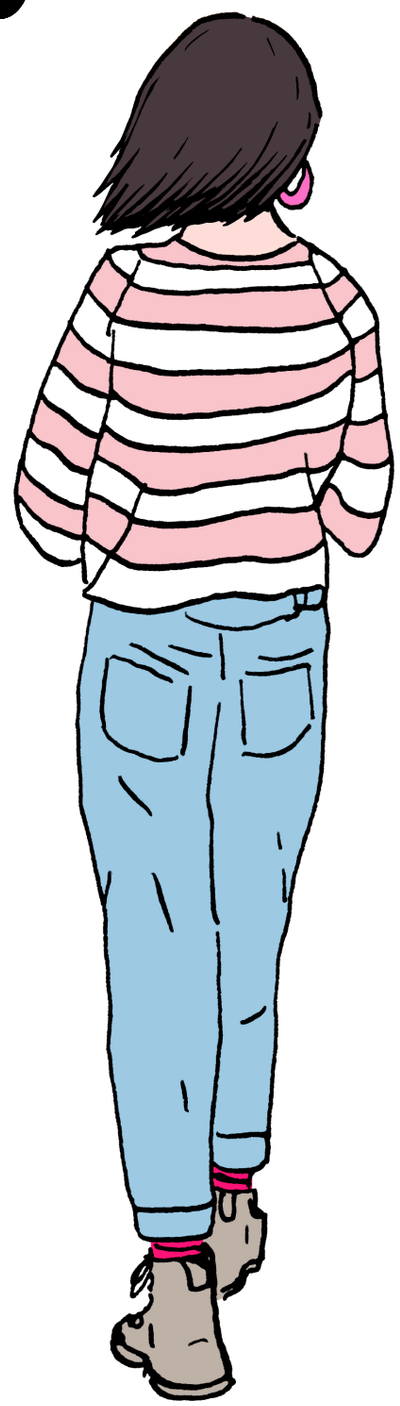
Q5. 1,700

Q6. 4.3

Q7. 0.7

PART 13
PERCENTAGES

Quick Quiz

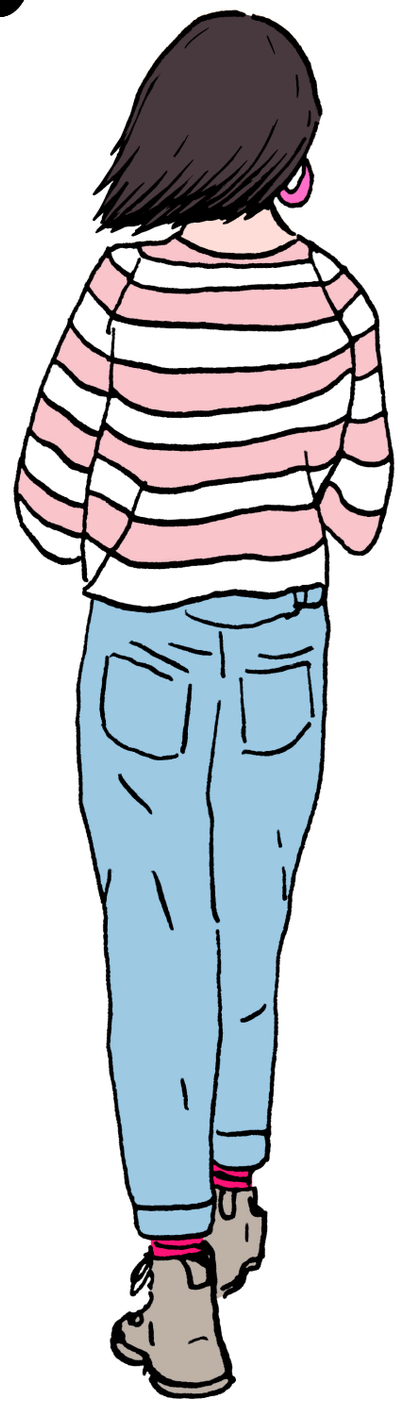


Q1.

What is 9 squared?

- A. 18
- B. 81
- C. 72
- D. 99

Quick Quiz

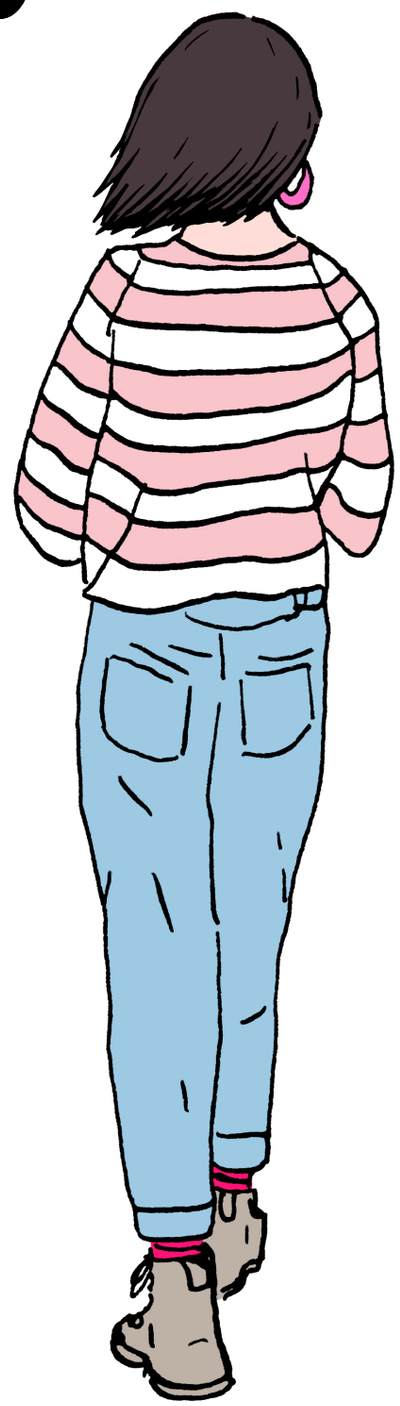


Q2.

Which of the following fractions is not equal to a half?

- A. $\frac{6}{12}$
- B. $\frac{4}{8}$
- C. $\frac{5}{11}$
- D. $\frac{3}{6}$

Quick Quiz



Q3.

Which is the smallest number below?

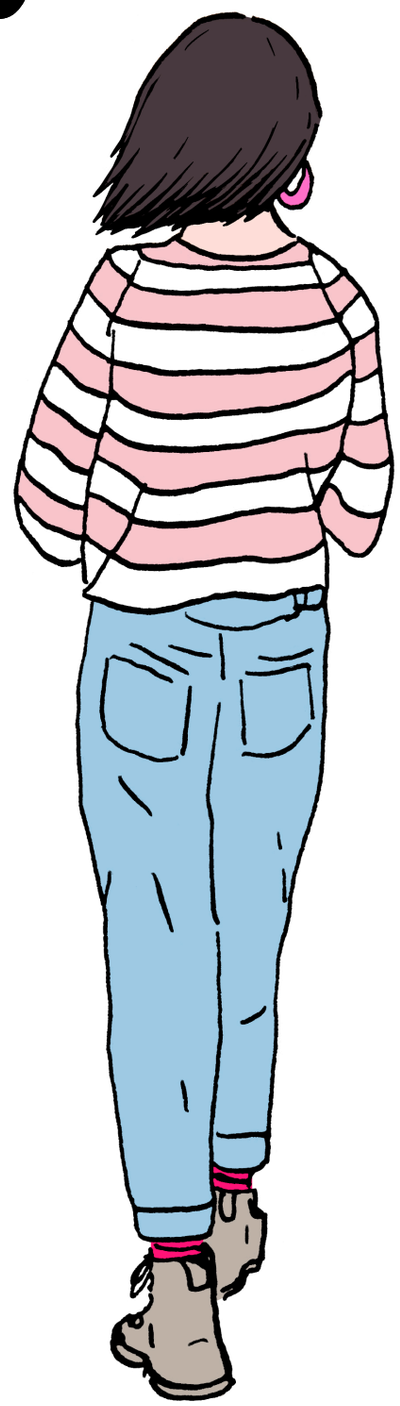
A. 0.03

B. 0.013

C. 0.3

D. 0.1

Quick Quiz

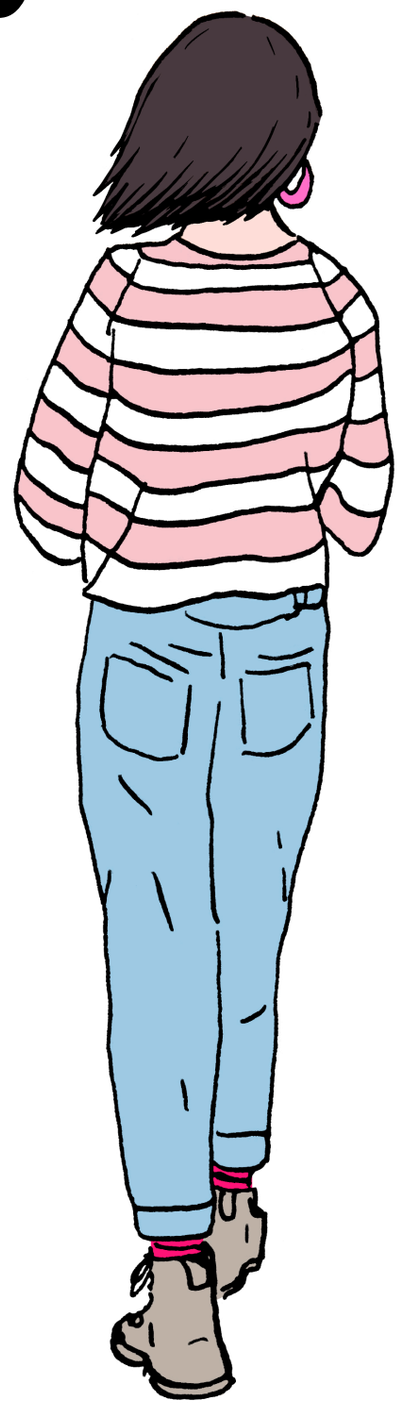


Q4.

What is the value of the **5** in **72,453**?

- **A.** Five
- **B.** Five Hundred
- **C.** Five Thousand
- **D.** Fifty

Quick Quiz



Answers

Q1. 81

Q2. $\frac{5}{11}$

Q3. 0.013

Q4. Fifty

Percentages

In Real Life every business is about money. In business, everyone talks, thinks and breathes percentages, because this is the **easiest** way to compare figures. It's also the easiest way to bamboozle women who don't understand percentages, which is why credit card companies and banks may use it to **their** advantage – but not yours.

'Per cent' comes from the Latin, meaning 'of a hundred'. The shorthand symbol for 'per cent' is %.

So 5% means 5 in every hundred, whether you're talking about 100 meringues, 100 Martinis or 100 helicopters in a fleet.
6% means 6 in every hundred meringues, Martinis or helicopters.
25% means 25 in every hundred meringues, Martinis or helicopters.
99% means 99 in every hundred meringues, Martinis or helicopters.

So percentages represent **a)** part of a thing or **b)** part of a group of things.
A complete whole thing in percentages is represented by 100%, whereas with fractions and decimals a whole thing is represented by 1.

Private helicopter.

1) Juliet ordered 100 meringues for her birthday party. Her naughty brother ate six of them = 6%

25 of the meringues ordered were built into a pile and smothered with chocolate sauce = 25%.

99 of the 100 meringues delivered were perfect, but one had been squashed = 99% perfect meringues.

2) Carla married the President of France. The French Air Force has a fleet of 100 helicopters.

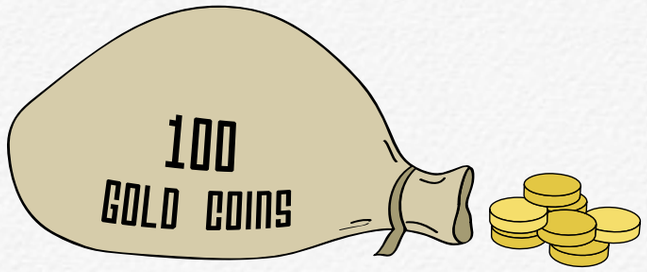
6% of the helicopters were used to fly foreign VIPs from the airport to the President's palace in Paris = 6 helicopters.

25% of the French helicopter fleet was on alert in case there was a terrorist attack at the wedding = 25 helicopters.

99% of the French helicopters flew perfectly = 99 helicopters. But one helicopter had engine trouble, so it was grounded:

1% of the French helicopter fleet could not fly on the day Carla married the French President.

Now, imagine that you have a bag of **100** gold coins.
All the gold coins **= 100%** of the gold coins.



100%

If the gold coins are divided into two piles of fifty coins each, then each pile is **50%** of the gold coins.

50%

50 Gold Coins

50%

50 Gold Coins

Say, **10** gold coins are spent on a burglar-proof safe, then **10%** of the gold coins have been spent to protect the remaining **90** gold coins, which is **90%** of all the gold coins.

There you have the basic concept of percentages, which can refer to anything: a watermelon, a field, a group of dancers or Napoleon's army.

100% is **all** of the gold coins, watermelons, army or whatever.

0% is **none** of it.

50% is **half** of it.

110% is **all** of it, plus a **bit more** (more later).

Using Percentages

A percentage gives a good general idea of the size of something. '99% of the seats on the train were occupied' means it was crammed with people. '2% of the train tickets were sold' means that the train company lost money on that trip.

In Real Life, you calculate accurately by using percentages. You will definitely need percentages to calculate your tax. You may need to show your bank manager (or an investor), the percentage profit that your business made in a year.

To picture any percentage, **visualise in your mind whether it's nearest to 0%, 50% or 100%**. For example, 6% of a class of students is nearest to 0% and means that not many students have turned up for physics. 40% of a bottle of water is nearest to 50%, so 40% is a bit less than half the bottle. 95% of the shoes were sold in the sale is nearest to 100%, which means that nearly all the shoes were sold; so only 5% of the shoes were left in the shop, which is now almost empty.

One simple method can be used for all percentage calculations; the **chart method** is one of the most useful things you'll ever learn, and you'll use it all your life. (You've already come across this in 'How to make a problem much simpler' in STEP 1)

In all percentage calculations, the key fact is knowing what number represents 100%. Remember that 100% is the **whole amount**, whether it is called **the original amount, the starting amount**, or whatever.

All percentage calculations can be separated into 3 different types, which follow.

How to Present your Figures as Percentages

When you know the Real Life figures, but need to show them as percentages.

In a sale, a black leather skirt is reduced from £100 to £70.
By what percentage has the original price been reduced?
The price of the skirt has been reduced by 30%.

More examples on the following pages.

In America, Greek-born writer and political commentator, Arianna Huffington, started her own online newspaper.

Five years later, Arianna reputedly sold her shares in The Huffington Post for half a billion dollars
(Remember, a billion is 9 zeros, so Arianna got 500,000,000 dollars – with 8 zeros).

First Example

In a Salisbury school of **160** pupils, **24** cycle to school. What percentage of pupils cycle to school?

First, decide which number represents **100%**:

Here, the whole school is **100% = 160** pupils.

Next, draw the chart. One column heading will always be **%**, the other column will be the item found in the question: money, sweets, people etc. Here, it's pupils:

Insert the equivalent amounts that are known already: '**160** pupils = **100%**'

Insert the '**24** cycling pupils' in the correct column. The percentage that cycle to school = ?

Pupils	%
160	100
24	?

Use the chart rule: **Multiply the diagonal numbers, then divide by the remaining number**.

Pupils	%
160	100
24	?

Multiply the diagonal numbers

Divide by the remaining number

Applying this rule, do the calculation on your calculator: **100 x 24 ÷ 160 = 15**

Since the answer replaces the ? in the percentage column, the answer is also a percentage.

Answer: **15% of the Salisbury pupils cycle to school**.

Second Example

Frugal Jenny always hunts for a bargain. She bought an emerald suede jacket in a sale for **£60**. The jacket is normally sold for **£80**. What percentage of the original price did Jenny pay for the jacket?

The original amount the emerald suede jacket was sold for was **£80**, so this is your **100%**.

Insert the equivalent amounts that are known already: '**£80 = 100%**'

Insert the **£60** in the correct column.
The percentage of the original price = ?

%	£
100	80
?	60

%	£
100	**80**
?	60

Multiply the diagonal numbers

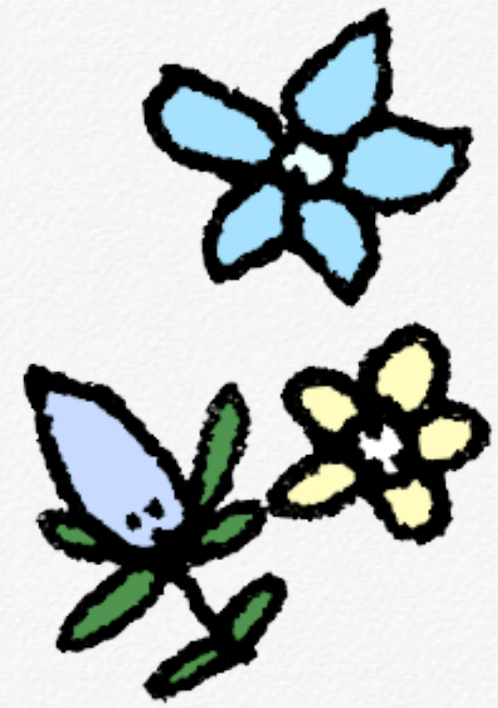
Divide by the remaining number

Say the chart rule aloud: **Multiply the diagonal numbers, then divide by the remaining number.**

Then do the calculation on your calculator: 100 x 60 ÷ 80 = 75

Since the answer replaces the ? which is in the percentage column, the answer is also a percentage.

Answer: **Jenny bought her emerald jacket for 75% of its original price.**



Third Example

Last year there were **40** members in the salsa class, this year there are **56**. By what percentage has the class increased?

100% is the original amount in the class: **40** people.

This question is, "By what percentage has the class increased?". The class has increased by **16** people (from **40** to **56**), what percentage is this?

Insert the equivalent amounts that are known already: '**40** people = **100%**'

Insert the amount by which the class has increased in the correct column. The class increased by **16** people. The percentage increase = ?

People	%
40	100
16	?

People	%
40	100
16	?

Multiply the diagonal numbers

Divide by the remaining number

Say the chart rule aloud. **Multiply the diagonal numbers, then divide by the remaining number**. Then do the calculation on your calculator: 100 x 16 ÷ 40 = 40

Answer: **The salsa class increased by 40%.**

Wish you were there?
French café.

Exercises

Use the chart method and the chart rule, plus a calculator to solve the following problems. Remember to underline and highlight the main facts in each exercise.

1) Gemma scored **57** marks out of a possible **65** in a French exam. Her parents have promised her a weekend in Paris if she gets over **80%**.
Should Gemma pack?

2) Jenny bought a pair of jeans on eBay for **£18** (including postage). This particular pair of jeans would have cost **£90** if she'd bought them new in the high street shop.
What percentage saving has Jenny made on her jeans?

How to Work Out the Quantities Represented by Percentages

The chart method will also help you with these calculations.

Often, you will be told the percentage, but you need to find out what that means in Real Life quantities.

Examples

In a shoe sale, how much cheaper will the boots you want be, when reduced by **35%**?

If the **£120** red jacket is reduced by **20%**, how much will it cost you?

Tania bought her silver sequin dress in the sale (see exercise 3).

First Example

15% of the meals on a Boeing 747 flight to New York will be vegetarian. A total of 540 meals are needed, so how many will be vegetarian meals?

100% is the total amount of meals needed: 540 meals.

Insert the equivalent amounts that are known already: '540 meals = 100%'

Insert the percentage that are vegetarian, in the correct column. '15% = ?'

Meals	%
540	100
?	15

Meals	%
540	100
?	15

Multiply the diagonal numbers

Divide by the remaining number

Say the chart rule aloud: **Multiply the diagonal numbers, then divide by the remaining number.**

Then do the calculation on your calculator: 15 x 540 ÷ 100 = 81

Answer: **81 vegetarian meals are needed on the flight.**

Second Example

Advertising account manager Tania needs to pay **22%** income tax.

How much of the **£660** she earned this week will be paid to the tax man?

Tania's whole salary, £660 = 100%

22% = ?

£	%
660	**100**
?	22

Multiply the diagonal numbers

Divide by the remaining number

The calculation is: 22 x 660 ÷ 100 = 145.20

Answer: **Tania will pay £145.20 to the tax man this week.**

It's always worth quickly checking the tax deducted from your pay, in case too much has been deducted.

Third Example

Jenny sees a pair of Armani jeans on sale, '30% off!' The jeans normally cost £79.50. Jenny decides she'll buy those jeans if they cost less than £50. Will Jenny buy the jeans?

First, calculate the value of the jeans in percentage terms:

The Armani jeans are reduced by 30% from 100%, **so they cost 70% of their normal value**. 100 – 30 = 70

Use 70% in your chart because you want to find the cost of the jeans (they cost 70% of their normal value).

Do not use 30% in your chart, because 30% is the reduction in price.

The original price, £79.50 = 100%

70% = ?

£	%
79.50	100
?	70

Multiply the diagonal numbers

Divide by the remaining number

The calculation is: 70 x 79.50 ÷ 100 = 55.65

Answer: **The Armani jeans cost £55.65 in the sale**. Therefore, Jenny shouldn't buy the jeans, since they cost more than **£50**. But she does anyway, since she is going to stay with newly-wed Sarah for the weekend.

Fourth Example

Melanie wants to buy a starter box of wine for her bring-a-bottle-party. The prices marked on the items at the cash-and-carry don't include VAT (Value Added Tax). Melanie takes a box of **6** bottles of wine, labelled **£34.80**, to the checkout. If VAT is **20%**, how much will Melanie's total bill be for the box of wine?

First, calculate the value of the box of wine in percentage terms:

The wine will cost **100%** of the price listed **plus** the VAT of **20%**, so the wine will cost **120%** total, in percentage terms.
So use 120% in the chart.

The original price, £34.80 = 100%

120% = ?
Since this is an **increase** in value, add the percentage increace to 100%

£	%
34.80	**100**
?	120

Multiply the diagonal numbers

Divide by the remaining number

So the calculation is: 120 x 34.80 ÷ 100 = 41.76

Answer: **Melanie's total bill for the box of wine will be £41.76 when VAT is added on**. That'll get the party going.

Fifth Example

Sally's second-hand, scarlet convertible has depreciated in price by **40%** since she bought it, three years ago.

Sally bought it for **£9,000**. What is the value of her scarlet convertible today?

First, calculate the value of the car in percentage terms. Depreciation is a **reduction** in price, so the value has fallen by **40%** from **100%**. **40%** reduction: **100 – 40 = 60%**.

The original price, £9,000 = 100%

The value of Sally's car is 100 – 40 = 60%

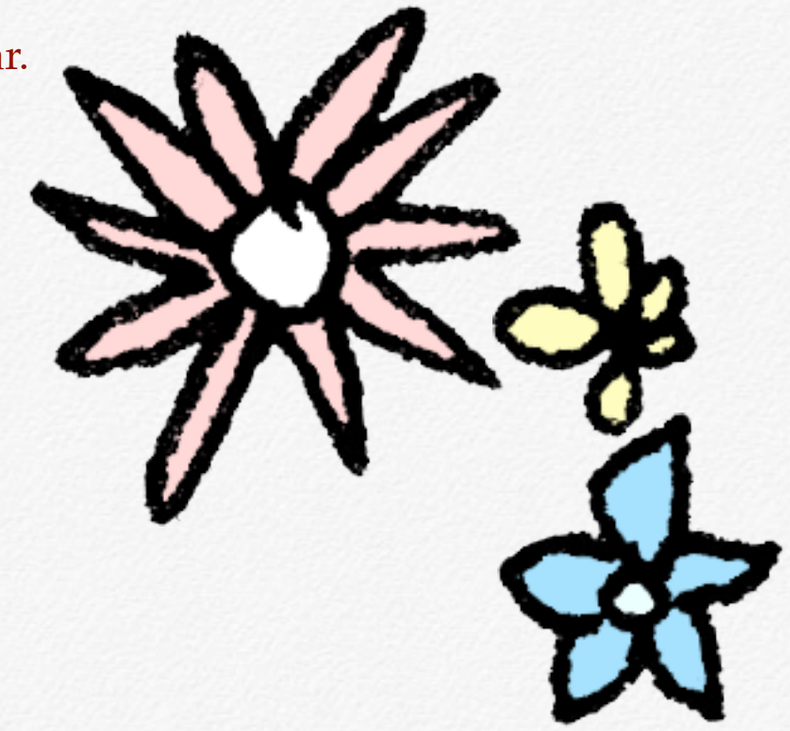
£	%
9,000	**100**
?	60

Multiply the diagonal numbers

Divide by the remaining number

So the calculation is: 60 x 9,000 ÷ 100 = 5,400

Answer: **Sally's scarlet convertible is now worth £5,400**, but it's still a great car.



Exercises

3) For weeks, Tania has longed for a silver sequin dress that she simply can't afford; it is perfect for clubbing in the West End. The price of the dress is **£325**, but Tania only has **£190** saved. However, today the silver dress is in the shop window, marked 'on sale, **35%** off'. Can Tania afford the sale price? Remember, the price of the dress has fallen by **35%** from **100%**.

4) Gemma hears her father say that the value of their flat in Chelsea has increased by **17%** in one year. The value of the flat one year ago was **£450,000**. What is the value of the flat now? Remember to calculate the value of the flat in percentage terms first. (As in the fourth example about Melanie's wine.)

FACT!

In the workplace, it is essential to be able to calculate percentages, bosses say. You need to be able **to understand what a percentage means**, and **to calculate a percentage**.

Example: Head office wants to see **20%** more perfume sales next November. Any sales girl who reaches this target will receive a Christmas bonus in her next month's salary.

Hannah wants to know how many bottles of scent she will need to sell in November to get her Christmas bonus. If she sold **80** bottles of scent in October, she will need to sell
80 + (**20%** of **80** = **16**) bottles = **80** + **16** = **96**.
96 bottles will get Hannah her Xmas bonus.

*Ref: In this Part of the Course, references are from: Confederation of British Industry (CBI) Report 'Working on the Three R's: Employers' Priorities for Financial Skills in Maths and English.' Published 2006

“85% of 121 women said they would buy this cream again.”

– TV advertisement for a face cream.

But 85% of 121 women is 102.85 women.

Have you ever spotted 0.85% of a woman?

How dumb
does this beauty firm
think women are?

Dumb enough to buy their face cream?

What the Original Amount was before the Percentage Change

Here, the **100% is the unknown amount**.

First Example

Sally longs for a fake leopard skin coat that now costs **£84** in a '**30%** off' sale. Sally wants to know how much she will be saving if she buys the fake leopard skin coat; so she needs to know how much the coat cost before the sale.

The original cost of the coat **= 100%**, which is what Sally wants to know.

First, calculate the current value of the coat in percentage terms. The coat has been reduced by **30%** from the original **100%**. **100 – 30 = 70**. The coat is now **70%** of its original value. Since we know it costs **£84**, **£84 = 70%**. Use this in the chart:

? = 100%

£84 = 70%

£	%
?	100
84	**70**

Multiply the diagonal numbers

Divide by the remaining number

Say the rule aloud - do you know it now? Then grab your calculator: **100 x 84 ÷ 70 = 120**

Answer: **The fake leopard skin coat originally cost £120**. Seems like a good deal.

Second Example

The value of Tania's one bedroom flat in Shepherds Bush has increased by **12%** since she purchased it last year.

It is now valued at **£190,000**. What was the price of Tania's flat when she purchased it?

First, calculate the current value of the flat in percentage terms. The flat has increased by **12%** from its original value of **100%**.

100 + 12 = 112. The flat is now **112%** of its original value.

The original price, £9,000 = 100%

The value of Sally's car is 100 – 40 = 60%

£	%
9,000	**100**
?	60

Multiply the diagonal numbers

Divide by the remaining number

The calculation to do is: 100 x 190,000 ÷ 112 = 169,642.86

Answer: **Tania's flat was worth approximately £170,000 a year ago**, so it has increased in value by **£20,000** – a good investment.

Third Example

In a survey, **1,080** of the people questioned were smokers who said they would like to give up smoking. The survey reported that **72%** of smokers would like to give up smoking. How many smokers were interviewed in this survey?

1,080 people = the **72%** of smokers who would like to give up smoking.

? = 100%

1,080 people = 72%

People	%
?	100
1,080	**72**

Multiply the diagonal numbers

Divide by the remaining number

The calculation is: 100 x 1,080 ÷ 72 = 1,500

Answer: **A total of 1,500 smokers were interviewed in this survey.**

Exercises

5) Shaheena's savings account received **£135** interest last year. The annual interest rate on her account is **4.5%**. How much money was in Shaheena's bank account before the latest interest payment?

6) Attendance at the Annual Thailand vs England polo match is down by **30%** since last year. If **400** spectators attended this year, how many spectators were there last year?

But were her bargains **really** bargains?

Exercises for all types of percentage problems

7) Betsy runs 'Dream Partners', an exclusive and expensive dating agency in London, which has a great track record. Last year, **570** people signed up with Dream Partners, of which **390** were men under thirty.

a) What percentage of Betsy's clients are men under thirty?

b) Dream Partners advertises that **80%** of its clients find someone special within **6** months of registering. Assuming that is true (because you can't check), how many of last year's members found a partner within **6** months?

c) Betsy also claims that Dream Partners has been responsible for introducing **78** couples in the last three years, who eventually married. This accounts for **2%** of the introductions made by Dream Partners in those **3** years. How many introductions did Dream Partners make in the last three years?

8) This year, in Britain around 25% of 15–year–old girls are smokers.

a) This year there are about 300,000 girls aged 15 in the UK. How many of these girls are smokers?

b) If one girl smoker smokes 2 packets of 20 cigarettes per week for 20 years, and her brand costs her an average of £4.80 a pack during that time, how much will she pay for cigarettes over a 20-year period?

c) If the girl smoker stops smoking after 20 years, name three interesting purchases that the money saved by smoking might be spent on.

d) Approximately 50% of all regular cigarette smokers will eventually be killed by their addiction. If all of the 15–year–old girl smokers in part (a) continue to be regular smokers, how many will eventually die from a smoking-related disease?

Sally longed for a fake leopard-skin coat, like this one worn by one of the Olsen twins, fashion designers of The Row (see first example).

Earning: Gross is not a Dress Size and Net is not for Fishing

The first (small) bit of your yearly income is tax free and is called your **tax allowance**. Income tax is calculated as a percentage of what remains of your earnings.

Your wage before tax is deducted is your **gross** earnings. From your gross earnings, your employer will deduct your tax and also your National Insurance contributions (which pay for the NHS and your small state pension). What's left is yours, and it is called your **net** earnings.

So **gross** is any complete amount of money, and **net** is what's left for you, after any deductions from the gross amount. **What you earn is not what you get!**

Caroline Link of the Thai Polo Team (see exercise 6).

First Example

Cleo earns **£30** per hour as a piano teacher. **24%** of Cleo's earnings will be paid in tax and National Insurance contributions, so how much of the **£30** Cleo earns for a one-hour lesson will be left for her?

24% is paid in tax and NI, so **100 – 24 = 76%** (Cleo's net earnings.)

£30 = Cleo's gross earnings for a one-hour lesson = **100%**.

£30 (gross income) = 100%

? = 76%

£	%
30	100
?	76

Multiply the diagonal numbers

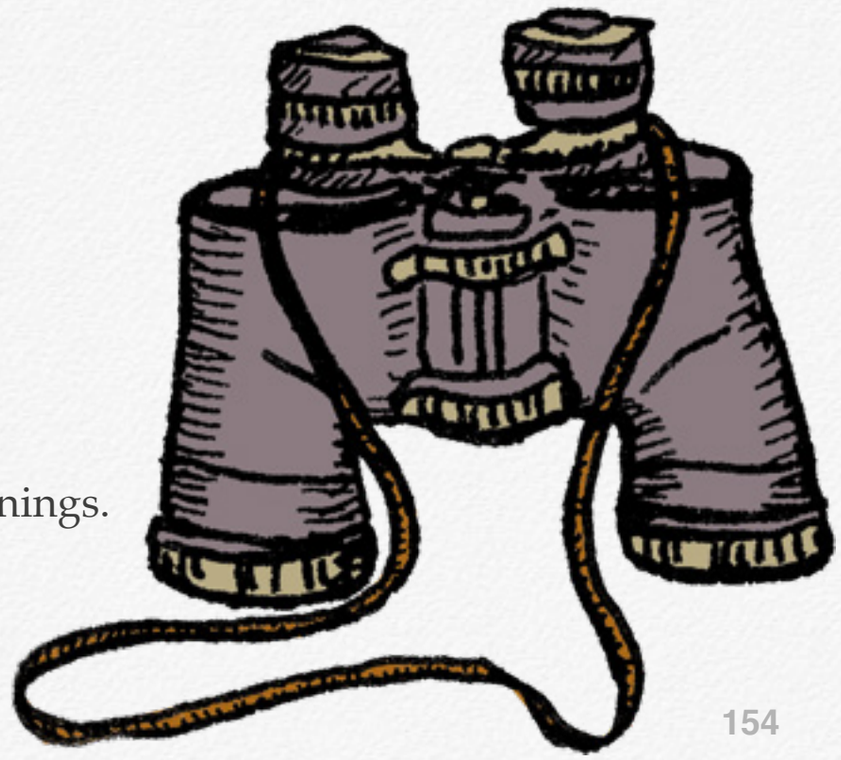
Divide by the remaining number

The calculation is: 30 x 76 ÷ 100 = 22.8

Answer: **For every £30 Cleo earns gross, she earns £22.80 net, which she can call her own.**

£7.20 is paid in tax.

Keep a close eye on your earnings.



Second Example

Cleo, who shares a sea-view flat with three friends in Brighton, calculates that all her expenses for next year will be **£10,000**. This year tax has increased, and now Cleo's tax and National Insurance contributions add up to **27%** of her wages for one year.

Question: What **gross** amount of money does Cleo need to earn next year in order to have an absolute minimum of **£10,000** **net** income?

Here's how to work it out: Cleo's **net** earnings will be **100 – 27** (Cleo's tax and National Insurance) **= 73%** of her income.

If **73% = £10,000**, then what is **100%**?

Gross income (?) = 100%

10,000 = 73%

£	%
?	100
10,000	**73**

Multiply the diagonal numbers

Divide by the remaining number

The calculation is: 100 x 10,000 ÷ 73 = 13,698.63

Round this up to the nearest **£100**.

Answer: **In order to earn £10,000 net, Cleo needs to earn £13,700 gross next year**.

But after the following year, Cleo didn't need to worry whether she would have enough pupils, because Cleo won the lottery. Her first purchase was a larger flat; her three friends then became her tenants and after that, Cleo only played the piano for pleasure.

Career choice?
Three fashion editors wait for the show to start.

Business Expenses

When starting in business, many make the mistake of thinking that the entire cost of business expenses will be deducted from their income tax bill. Sadly, it isn't.
For example, if Rock Star Tamara Pink pays **£2,000** for a stage performance costume, this is a business expense, so **£2,000** is tax deductible.

Tax deductible doesn't mean that the Inland Revenue will deduct that **£2,000** from Tamara Pink's tax bill. What **tax deductible** means is that Tamara Pink will not need to pay tax on this **£2,000** of her gross income that she spent on that stage costume; Tamara Pink's other clothes are **NOT** tax-deductable.

Warning!

What YOU consider a business expense may not be considered a **tax-deductable** business expense by the taxman. No use trying to argue, you must accept his decision. For example, if you are employed, the cost of travelling from home to your place of work and back again is NOT a business expense to the taxman – although it is to you.

First Example

In Manchester, Imogen is now a self-employed part-time PA, aged **22**. Imogen's **gross** earnings were **£10,000** last year and her **tax deductible** business expenses were **£2,000**. Remember, nobody needs pay tax on the first bit of their income, which is called a **tax allowance**. This year, Imogen's tax allowance is **£3,000**. What will Imogen's **net** income be? (Hint: remember you are not taxed on business expenses).

First, calculate how much Imogen's taxable income is, as follows, by subtracting Imogen's business expenses and her non-taxable income (tax allowance):

Gross income less tax deductible expenses: **£10,000 – £2,000 = £8,000**.

Then, deduct **£3,000** tax allowance: **£8,000 – £3,000 = £5,000**, this is Imogen's taxable income.

Next, calculate the tax payable on the taxable income (Imogen's tax level is **20%**):

Taxable income £5,000 = 100%

Tax payable (?) = 20%

£	%
5,000	**100**
?	20

Multiply the diagonal numbers

Divide by the remaining number

The calculation is: 5,000 x 20 ÷ 100 = 1,000

On **£5,000** taxable income, Imogen pays **£1,000** income tax.

Example continues on the next page.

Now that you know how much tax Imogen must pay, calculate her net income as follows:

Gross amount Imogen earned = £10,000
Minus tax deductible expenses = – £2,000
Minus tax allowance = – £3,000

This leaves the taxable income = £5,000
Tax to be paid on this taxable income = £1,000

Imogen's net income is the money she has left to spend:
Gross amount Imogen earned = £10,000
Minus tax deductible expenses = – £2,000
Minus tax to be paid on taxable part of income = – £1,000
This leaves £7,000

Answer: **Imogen has £7,000 net income this year**.
Not enough for a mouse to live on, Imogen thinks gloomily.

Who landed the top job in fashion?

Anna Wintour, editor-in-chief of Vogue USA.

Second Example

Tamara Pink has just reached Number 1 with her latest song. As a reward, Tamara's manager says that she can spend **£15,000** on a new wardrobe for offstage street wear.

Question: How much money does Tamara Pink need to earn **gross** in order to have **£15,000 net** to spend on the designer dresses of her dreams?

Tamara knows that **10%** of her **gross** earnings pay her clever agent, and **15%** of her **gross** earnings go to her vulpine manager. What's left is Tamara's taxable income, **40%** of which goes to the tax man.

For this calculation you need to work backwards. There will be two small sums.

Small sum A

First, calculate how much Tamara's taxable income needs to be for Tamara to have **£15,000** spending money:

As **40%** of Tamara's taxable income pays her income tax, Tamara's **net** earnings (**£15,000**) will be **100 – 40 = 60%** of her taxable income.

60% = £15,000 clothes money.

	£	%
£15,000 = 60%	15,000	60
Taxable income (?) = 100%	?	100

The calculation is: **15,000 x 100 ÷ 60 = 25,000**

Example continues on the next page.

So the answer to small sum A: Tamara's taxable income will need to be **£25,000**.

Small sum B

Next, calculate how much Tamara's **gross** income is, when it includes both the agent and the manager's fees.

We know that the agent's and manager's fees are 10 + 15 = 25% in total.

As 25% of Tamara's **gross** income pay her agent and manager,
Tamara's taxable income (£25,000) is 100 – 25 = 75% of her gross income.

£25,000 = 75% of Tamara's **gross** income.

	£	%
£25,000 = 75%	25,000	75
Gross income (?) = 100%	?	100

The calculation is: 25,000 x 100 ÷ 75 = 33,333.33

Answer: **In order to earn £15,000 spending money, Tamara needs to earn £33,300 gross.**

Over twice as much!

And by the way, 'vulpine' means fox-like, clever and crafty.

Singer AmyJay performs to a crowd in the park.

Exercises

9) After training as a beautician, Chris earns **£15** per hour **gross** at her new job. Chris now thinks nothing of spending **£15** on a bottle of good wine or other little luxuries: after all, it's equivalent to just one hour of her work. Remember, Chris is spending from her **net** earnings, after deduction of her income tax which is **22%**, and her National Insurance which is **5%**. So how much money does Chris need to earn **gross** in order to earn **£15** net?

10) Chris earned **£120** extra this week by staying on late and working a few extra hours for her boss. She feels she can now reward herself by buying the coral suede handbag she has had her eye on; it costs **£97**. Will the **£120** extra she earned cover the cost of the coral handbag?
Chris' income tax and National Insurance is a total of **27%**.

Self-Confidence Tip 10

YOUR CONFIDENCE SNAPSHOT

Think of a time when you felt confident; it can be anywhere, anytime.
This feeling of confidence is in your memory bank forever.

Remember as many details as possible of that snapshot in time when you were confident. WHERE were you? WHAT were you doing? WHAT were you wearing? Were you smiling? How did you feel?

Make sure that it's a coloured snapshot, and give it a title, "Me, singing….at ….", or "Me, at the gym display", or "Me, at my……birthday party", or "Me, after my first dive".

Any time you want to feel more confident, quickly shut your eyes, recall your confidence snapshot, and re-live that feeling of happy confidence.

Before you start a maths session, shut your eyes for a moment and mentally look at your confidence snapshot. Remember the happy feeling, remember your energy at that time. Make an effort to feel that energy NOW.

Percentages in your Head

The 10% method

10% is easy to calculate. Divide your original amount by 10 and this gives you 10% of the original amount. Use this fact as your building block for all quick mental calculations.

Dividing by 10 is easily done if you move the decimal point one place to the left (see Dividing Decimals by the Big Noughts).

No career choices, uphill work.

Examples of dividing by 10

10% of a £40.00 restaurant bill, for the tip = £ 4 0 . 0 0 = £4.00

10% of 27 kg of sausages for the school barbeque = 2 7 . = 2.7 kg

10% of 33 litres of petrol = 3 3 . = 3.3 litres

Since 10% of £40 (the restaurant bill) is £4, then:

20% will be double £4: 2 x £4 = £8
30% will be 3 times £4: 3 x £4 = £12
40% will be 4 times £4: 4 x £4 = £16
50% will be 5 times £4: 5 x £4 = £20
60% will be 6 times £4: 6 x £4 = £24
70% will be 7 times £4: 7 x £4 = £28
80% will be 8 times £4: 8 x £4 = £32
90% will be 9 times £4: 9 x £4 = £36

To work out 5%, simply halve the 10% value.

10% of £40 = £4, so 5% will be half of £4 = £2.

To work out 15%, you add the value for 10% and 5%, so £4 plus £2 in this example = £6.
To work out 25%, you add the value for 20% and 5%, so £8 plus £2 in this example = £10.
To work out 35%, you add the value for 30% and 5%, so £12 plus £2 in this example = £14.
To work out 75%, you add the value for 70% and 5%, so £28 plus £2 in this example = £30.

First Example of using 10% to calculate other percentages

How much cheaper will a **£90** pink tote bag be when reduced by **20%**?

Try it in your head. **10%** of **£90** is **£9**. Double **£9** to get **20%**: **2 x £9 = £18**.

You can write this as:

100% = £90

(÷10) 10% = £9

(x2) 20% = £18

Answer: **20% of £90 is £18, therefore the pink tote bag will be £18 cheaper**.

Second Example

Lily asks her dad to show her how to tip. He tells her to tip nothing for bad service, **10%** for reasonable service, **15%** for excellent service. Then he takes Lily to a small Chinese restaurant, for practice. Lily wants to leave a **15%** tip for the waiter after a delicious meal, well served. The bill comes to **£28**. How much will a **15%** tip be?

Head calculation: **10%** of **£28** is **£2.80**. Half that is **£1.40**. **£2.80 + £1.40 = £4.20**.

Written:

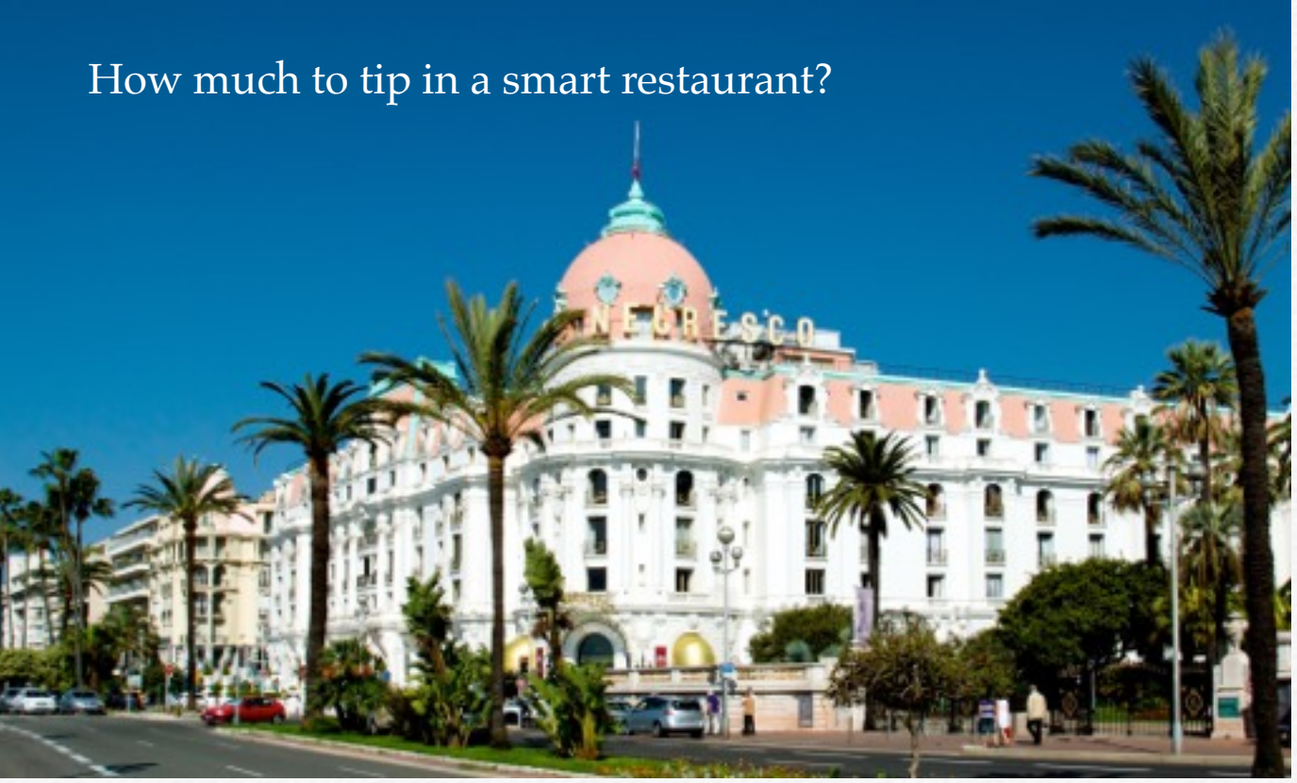
100% = £28

(÷10) 10% = £2.80

(÷2) 5% = £1.40

(10% + 5%) 15% = £4.20

Answer: **15% of £28 is £4.20, so Lily should leave about £4 as a tip**. If you are rounding a tip, don't round it down or you'll look mean. Round it up a bit. Maybe Lily should leave **£4.50**.



Exercises

Use the **10%** method to **mentally** calculate the following. You can highlight and underline the important bits as normal.

11) Gemma is going to a charity ball in aid of a Romanian orphanage. The organisers advertise that **80%** of the cost of each ticket will go to the orphanage. Each ticket costs **£90**. How much of the money Gemma's parents paid for her ticket will go to the orphanage?

Another way to find **25%** of a number is by **halving it twice**.

12) Rachel gets a new Saturday job that pays **65%** commission on all the paintballing tickets she sells. In one Saturday afternoon in the shopping mall in a nearby town, Rachel sold **£300** worth of tickets. How much commission did Rachel make in that one afternoon?

13) Melanie's local off-licence offers **20%** off any three bottles of wine. For a supper party, Melanie wants to buy three bottles of Australian Merlot which would normally cost **£8** each. How much will the wine cost when reduced by the **20%** offer?

14) Penny-pincher Jenny boasts that she made **45%** profit on a pair of turquoise wedge sandals which she purchased at a charity shop for **£11**, then sold on eBay. What price did Jenny's sandals sell for on eBay? (See drawing a bit further on.)

15) The latest successful boy band, SWIPES, ate a delicious Thai meal in Las Vegas. They offered the waitress a free ticket to their next gig, or a **40%** tip. The bill came to **$1,800**. How much would the waitress lose out on as a cash tip if she chooses the concert ticket?

How to Calculate a Tip

Many restaurants automatically add a tip of 12.5% service charge to the end of your bill. If so – you don't need to tip. If not, a little over 10% is enough in Europe. **Here's how to calculate a tip.**

Say your bill is for **£48.99,** take the first figure from the left ONLY, then add 1. Here, the first figure is **4**.

4 + 1 = £5, which is what you tip.

Two other examples

Your bill is **£26.74.** **2 + 1 = 3**, which is what you tip, **£3**.

Your bill is **£37.79.** **3 + 1 = 4,** which is what you tip, **£4.**

For a sum over **£99**, take the first **two** numbers from the left, and add 1.

Two examples

Your bill is **£137.43.**

13 + 1 = 14, so that's what you tip, **£14.**

Your bill is **£346.43.**

34 + 1 = 35, so that's what you tip, **£35.**

Can a Percentage be More than 100%?

Yes, when something **increases**, such as a price.

First Example

When Rachel was **12** years old, she received **£5** pocket money a week if she kept her bedroom tidy. After her fourteenth birthday, Rachel argues for an increase in pocket money to cover her extra, more grown-up expenses, which she had sensibly listed. Her mother eventually agrees to give Rachel **£12** a week. That is an increase of **£7** on what Rachel had originally. What is the percentage increase of Rachel's allowance?

£5 = 100%

£7 increase = ? %

£	%
5	100
7	?

Multiply the diagonal numbers

Divide by the remaining number

Using the chart rule, the calculation is: 7 x 100 ÷ 5 = 140

The answer is a percentage.

Answer: **Rachel received a 140% increase in her pocket money**.

Well done, Rachel. She prepared her arguments well.

Second Example

Aisha purchased a dusty vase in a second-hand shop and sold it to an antique dealer for a **500%** profit.

She bought the vase for **£3**. How much did Aisha sell the vase for?

£3 = 100%

500% profit = ?

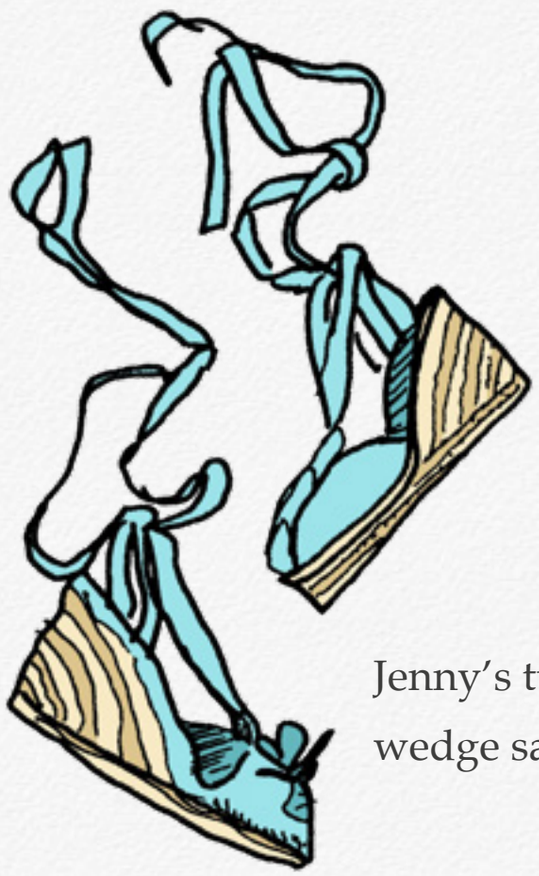
£	%
3	**100**
?	500

Multiply the diagonal numbers

Divide by the remaining number

Using the chart rule, the calculation is: 3 x 500 ÷ 100 = 15 (which is in **£**). So Aisha made a **£15** profit.

Answer: **Aisha sold the vase for £18** (**£15** more than she bought it for). Aisha clearly has a trader's sharp eye for a bargain.



Jenny's turquoise wedge sandals.

Retail Percentage

Mark-ups and Mark-downs

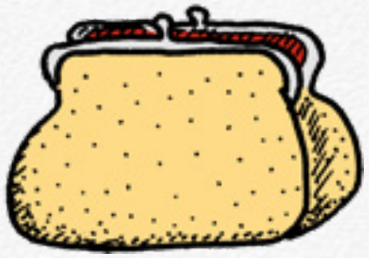
If you want to be a fashion buyer or shop manager… or even if you just want to calculate what a bargain really costs in a sale… you need to understand a few terms.

The **wholesale cost** is the price the manufacturer at the factory charges for a dress.

The expense of running the store plus the cost of making a profit is called the **overheads** of the store.

On top of the wholesale cost of an item, the retailer has to make enough money to cover the **overheads** of running the shop (expense of running the shop and salaries) as well as the **profit.**

The amount the retailer adds on to the **wholesale price** is the **mark-up**.



Add the **wholesale cost** and the **mark-up** to reach the **retail price**, which is what the store charges the customer.

Summary:

Wholesale cost + mark-up (overheads + profit) = retail price

At **sale time**, if any dress remains unsold, the manager needs to calculate by what percentage she can reduce the price of the dress, without making a loss on it.
The percentage reduction is called the **mark-down**.

The **mark-up** percentage of any item is always higher than the **mark-down** percentage needed to get back to its original price.

Examples of mark-up

A blue and green-striped dress that costs £100 wholesale was marked up by three different shops by 20%, 50% and 100%:

Betty's Bargains marked the dress up by 20%.

20% of £100 is £20, so the price of the dress in Betty's Bargains was £120.

Clarice's Classics marked the dress up by 50%.

50% of £100 is £50, so the price of the dress in Clarice's Classics was £150.

Denise's Designer Dresses marked the dress up by 100%.

100% of £100 is £100, so the price of the dress in Denise's Designer Dresses was £200.

Examples

The blue and green-striped dress bought wholesale by three shops is obviously a design failure and remains unsold. So each shop needs to calculate the maximum percentage mark-down for which the dresses can go on sale without their being sold for less than the original **£100**, which would mean a loss for the shop.

At **Betty's Bargains**: The retail price of the dress was **£120**. It must not be reduced by more than **£20**.

Normal price, £120 = 100%

Max. reduction of £20 = ? %

£	%
120	100
20	?

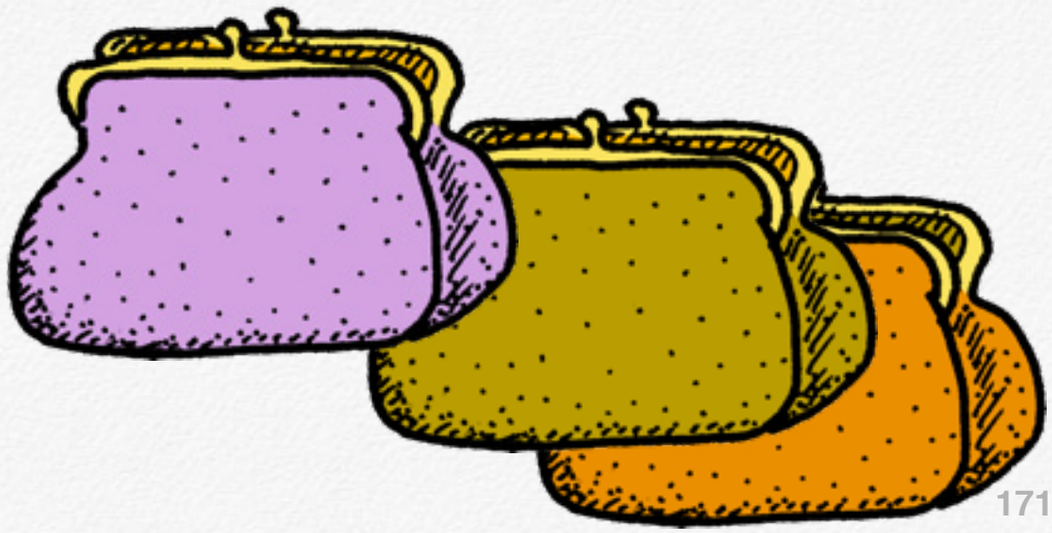
Multiply the diagonal numbers

Divide by the remaining number

Using the chart rule, the calculation is: 20 x 100 ÷ 120 = 16.67%

Round **16.67%** down to **16%**, because if you round up to **17%**, you will reduce the dress by more than **£20**. So the maximum mark-down Betty's Bargains can offer a customer is **16%**. To summarise, if the mark-up to retail price is **20%**, the mark-down to wholesale price is **16%**.

Example continues on the next page.



At **Clarice's Classics**: The retail price of the dress was **£150**. It must not be reduced by more than **£50**.

Normal price, £150 = 100%

Max. reduction of £50 = ? %

£	%
150	100
50	?

Multiply the diagonal numbers

Divide by the remaining number

Using the chart rule, the calculation is: 50 x 100 ÷ 150 = 33.33%

So, with the mark-up of **50%**, the maximum mark-down Clarice's Classics can offer is **33%**.

At **Denise's Designer Dresses**: The retail price of the dress was **£200**. It must not be reduced by more than **£100**.

Normal price, £200 = 100%

Max. reduction of £100 = ? %

£	%
200	100
100	?

Multiply the diagonal numbers

Divide by the remaining number

Using the chart rule, the calculation is: 100 x 100 ÷ 200 = 50%

So, with the mark-up of **100%**, the maximum mark-down Denise's Designer Dresses can offer a customer is **50%**.

Exercises

16) At Edna's Emporium, a fabulous, red sequin top hat was marked up by **200%**. The hat cost **£50** wholesale.

a) What is the price of the red sequin top hat at Edna's Emporium?

b) Unfortunately, the hat did not sell. What is the maximum percentage mark-down at which Edna's Emporium can offer the hat, without making a loss?

17) At Felicity's Fascinations, a peacock feather hair band costs **£70** wholesale. The hair band is marked up by **400%**.

a) What is the price of the peacock feather hair band at Felicity's Fascinations?

b) At sale time, what is the maximum percentage mark-down at which Felicity's Fascinations can offer the hair band without making a loss?

WARNING!

Many a start-up boutique has made a loss because the first-time owner mistakenly thought that the mark-down percentage is the same amount as the mark-up percentage. **It isn't.**

Example: The wholesale price of a yellow dress is **£60**
The markup is **50%**
50% of **£60** = **£30**
So the retail price of the yellow dress is **£60** + **£30** = **£90**

At sale time, the owner of the boutique thinks that a mark-down price of **50%** (the same percentage as the mark-up price) will mean that she doesn't make a loss on the yellow dress.

But **50%** of **£90** is **£45**
£90 – **£45** = **£45**
So the owner sells the yellow dress for **£45** – a loss of **£15** on the wholesale price of **£60**

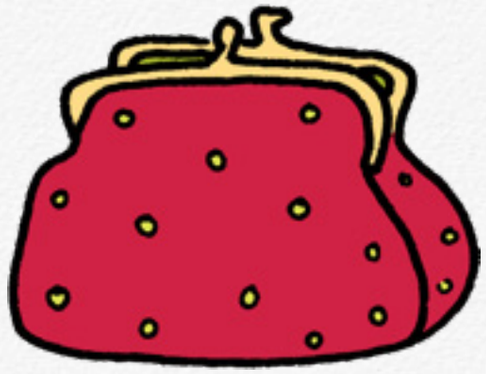
Because the owner made the same mistake with all the clothes in her sale, the owner made a big loss – and so she had to close down her boutique forever.

A crib list is useful to anyone in retail, or anyone purchasing an item in a sale and anyone bargaining, whether it's to buy a blouse or a studio flat.

This **Crib List** shows some mark-up percentages with their maximum mark-down percentage, to avoid making a loss. These figures apply no matter what the original wholesale price of the item was. If it is marked-up by **50%**, to calculate the original cost, you deduct **33%**... which is a third of the marked retail price, incidentally.

Mark-up percentage	Mark-down percentage	
10%	9%	
20%	16%	← see example of Betty's Bargains
25%	20%	
50%	33%	← see example Clarice's Classics
75%	40%	
100%	50%	← see example Denise's Designer Dresses
150%	60%	
200%	66%	
300%	75%	

So if you see a shop advertising reductions of **75%**, you know that probably it originally marked up the wholesale prices by more than **300%**. Incidentally, watch out if a shop advertises **up to 75%** reductions, because there may be only one item reduced by this large amount.



Maths in Real Life: SHOPPING TIPS

18-Year-old Karen: "In theory I can control my money and I always have in my head a rough idea of what I can spend. But when I go shopping, I forget my plans. What can I do?"

Shoe patterns.

Some ideas that Karen liked

1. Before you go shopping, decide how much you want to spend. Don't take more cash than that amount, and leave your credit cards and debit cards at home.

2. Check online to compare prices before you go shopping.

3. Write a list of what you want to buy and how much you think it will cost.

4. Stick your shopping list in a pocket that's easy to reach (**not** in the depths of your bag).

5. **At the shops**. Try to look at everything on your list, before you buy ONE thing. Remember the prices.

6. Have a break. Sit down for an ice cream or coffee – and look at your shopping list. Did you underestimate the prices? Of course you did.

7. Work out (A) what you need most, (B) what you want most, (C) where the best bargains are. Make your choices. Have another look at them.

8. Go back and get **ONLY** what is on your (A) list.

Never buy anything that isn't on your shopping list… unless it's absolutely irresistible, in which case you may have to cross something else off your shopping list.

Percentages Written as Fractions

Because ‘per cent’ means ‘per hundred’ or ‘out of 100’, a percentage is also a fraction. The fraction’s **denominator** (the number written under the line) is always 100, but it is rarely written down. If you decide to write your percentage as a fraction, it can then be cancelled down to the simplest possible fraction. Here’s how:

percentage	**written as a fraction**	
100%	$= \frac{100}{100} = 1$	= one whole thing, say a pizza
50%	$= \frac{50}{100} = \frac{1}{2}$	= half a pizza
75%	$= \frac{75}{100} = \frac{3}{4}$	= three quarters of a pizza
25%	$= \frac{25}{100} = \frac{1}{4}$	= quarter of a pizza
10%	$= \frac{10}{100} = \frac{1}{10}$	= one tenth of a pizza

← Previous page, Spanish Flamenco dance shoes.
This page, Spanish Flamenco dance dress. →

Answers to Part 13

Answers to: A. How to present your Figures as Percentages

1) Gemma scored **57** marks out of a possible **65** in a French exam. Her parents have promised her a weekend in Paris if she gets over **80%**. Should Gemma pack?

%	marks
100	65
?	57

100 x 57 ÷ 65 = 87.69

Answer: **Gemma achieved 87.7% in her French**. Bon voyage!

2) Jenny bought a pair of jeans on eBay for **£18** (including postage). This particular pair of jeans would have cost **£90**, had she bought them new in the high street shop. What percentage saving has Jenny made on her jeans?

Jenny saved **£72** by buying the jeans on eBay. You need to find this saving as a percentage.

%	£
100	90
?	72

100 x 72 ÷ 90 = 80

Answer: **Jenny made a saving of 80% on her eBay jeans purchase**.

Answers to: B.

How to work out the quantities represented by Percentages

3) For weeks, Tania has longed for a silver sequin dress that she simply can't afford; although it is perfect for clubbing downtown. The price of the dress is **£325**, but Tania only has **£190** saved. However, today the silver dress is in the shop window, marked "on sale, **35%** off". Can Tania afford the sale price? Remember, the price of the dress has fallen by 35% from 100%.

First, find the value of the dress in percentage terms.
35% reduction: 100 – 35 = 65%

£	%
325	100
?	65

65 x 325 ÷ 100 = 211.25

Answer: **The sale price of the dress is £211.25**. This is a bit more than **£20** over her budget, but she purchased the dress anyway and economised by walking to work for a week.

4) Gemma hears her father say that the value of their flat in Chelsea has increased by **17%** in one year. The value of the flat one year ago was **£450,000**. What is the value of the flat now?

Remember to calculate the value of the flat in percentage terms first. (Remember the Fourth Example, about Melanie's wine).

First, find the value of the flat in percentage terms.
17% increase: 100 + 17 = 117%

%	£
100	450,000
117	?

117 x 450,000 ÷ 100 = 526,500

Answer: **Gemma's family's Chelsea flat is now worth £526,500**.

In a fashion sale, 10% off means you pay 90% of the original retail price.

Answers to: C. To Calculate a Percentage Change

5) Shaheena's savings account received **£135** interest last year. The annual interest rate on her account is **4.5%**. How much money was in Shaheena's savings account before the latest interest payment?

£135 is the interest, which is **4.5%** of the total amount.

%	£
100	?
4.5	135

100 x 135 ÷ 4.5 = 3,000

Answer: **Shaheena has £3,000 invested in her account, excluding the interest just paid.**

6) Attendance at the Annual Thailand vs England polo match is down by **30%** since last year. If **400** spectators attended this year, how many spectators were there last year?

The number of spectators decreased by **30%**, from **100%** to **70%**.

%	Spectators
100	?
70	400

100 x 400 ÷ 70 = 571.4

Answer: **Last year, there were 571 spectators at the polo match.**

Check that your bargains really are bargains.

Answers to all types of percentage problems.

7) Betsy runs 'Dream Partners', an exclusive and expensive dating agency in London, which has a great track record. Last year, **570** people signed up with Dream Partners, of which **390** were men under thirty.

a. What percentage of Betsy's clients are men under thirty?

Clients	%	
570	100	(Total no of clients)
390	?	(men under 30)

100 x 390 ÷ 570 = 68.42

Answer: **68% of Dream Partners' clients are men under 30**.

b. Dream Partners advertises that **80%** of its clients find someone special within **6** months of registering. Assuming that is true (because you can't check), how many of last year's members found a partner within **6** months?

Clients	%
570	100
?	80

80 x 570 ÷ 100 = 456

Answer: **456 of last year's Dream Partners' clients found someone special within 6 months**.

c. Betsy also claims that Dream Partners has been responsible for introducing **78** couples in the last three years, who eventually married. This accounts for **2%** of the introductions made by Dream Partners in those **3** years. How many introductions did Dream Partners make in the last three years?

Introductions	%
?	100
78	2

100 x 78 ÷ 2 = 3,900

Answer: **Dream Partners made 3,900 introductions in the last 3 years**.

8) This year, in the UK around **25%** of 15-year-old girls are smokers.

a. This year there are about **300,000** girls aged 15 in the UK. How many of these girls are smokers?

15 year old girls	%
300,000	100
?	25

300,000 x 25 ÷ 100 = 75,000

Answer: **Currently in the UK, there are around 75,000 15–year old girls who smoke**.

b. If one girl smoker smokes **2** packets of **20** cigarettes per week for **20** years, and her brand costs her an average of **£4.80** a pack during that time, how much will she pay for cigarettes over a **20**–year period?

Cost per year:

(cost per week:
2 x £4.80 = £9.60)

Weeks	£
1	9.60
52	?

9.60 x 52 ÷ 1 = 499.20

For 20 years:

Years	£
1	499.20
20	?

499.20 x 20 ÷ 1 = 9984

Answer: **20 years of smoking 2 packets a week will cost her approximately £10,000**.

c. If the girl smoker stops smoking after **20** years, name three interesting purchases that the money saved by not smoking might be spent on.

Answer: **The saved money could be spent on travelling, clothes, music or whatever else she chooses**.

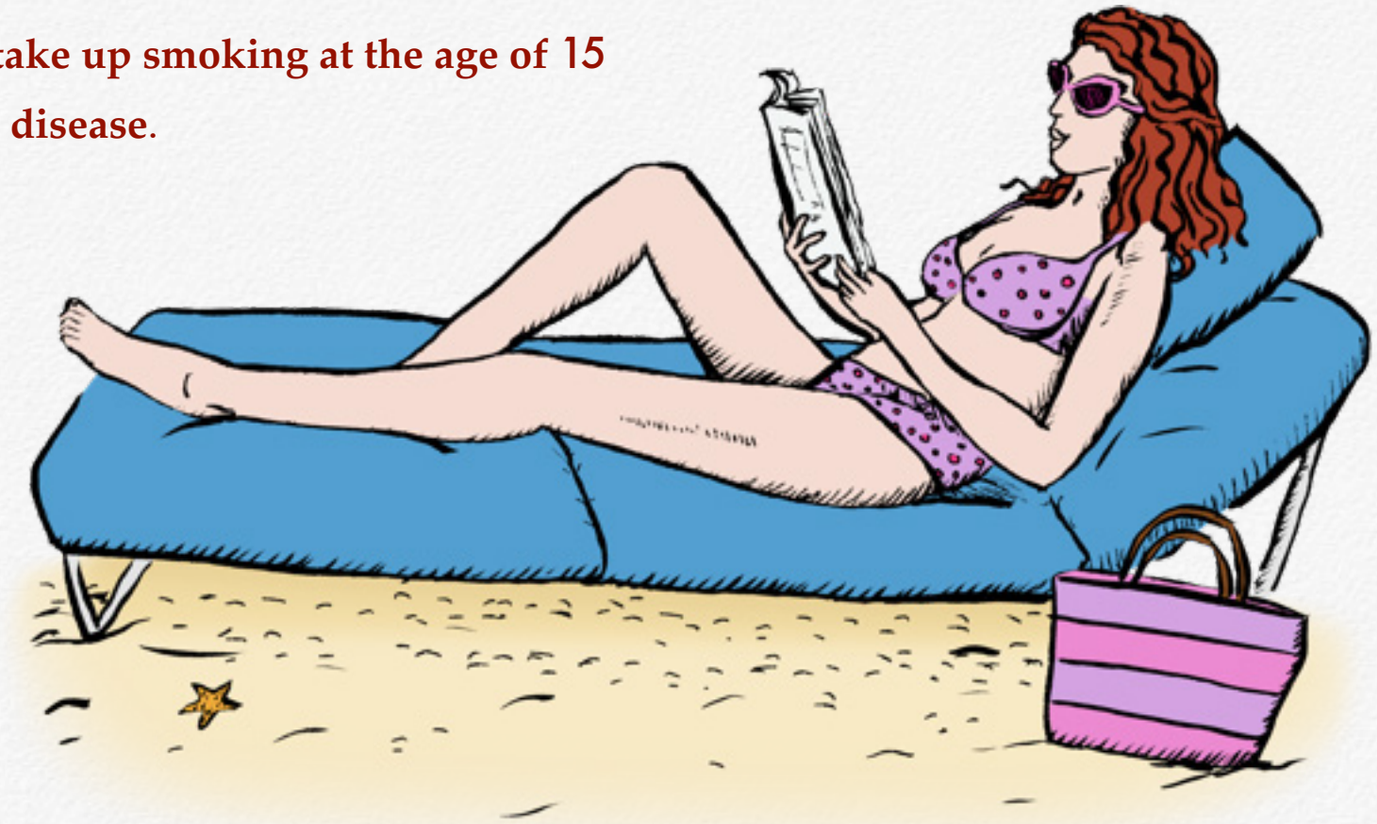
d. Approximately **50%** of all regular cigarette smokers will eventually be killed by their addiction. If all of the **15**–year–old girl smokers in part (a) continue to be regular smokers, how many will eventually die from a smoking-related disease?

15 year old girl smokers	%
75,000	100
?	50

50 x 75,000 ÷ 100 = 37,500

Answer: **Approximately 37,500 girls who take up smoking at the age of 15 will eventually die from a smoking related disease**.

How many holidays can you buy with **£10,000**?



Answers to Earnings: Gross is not a Dress Size and Net is not for Fish

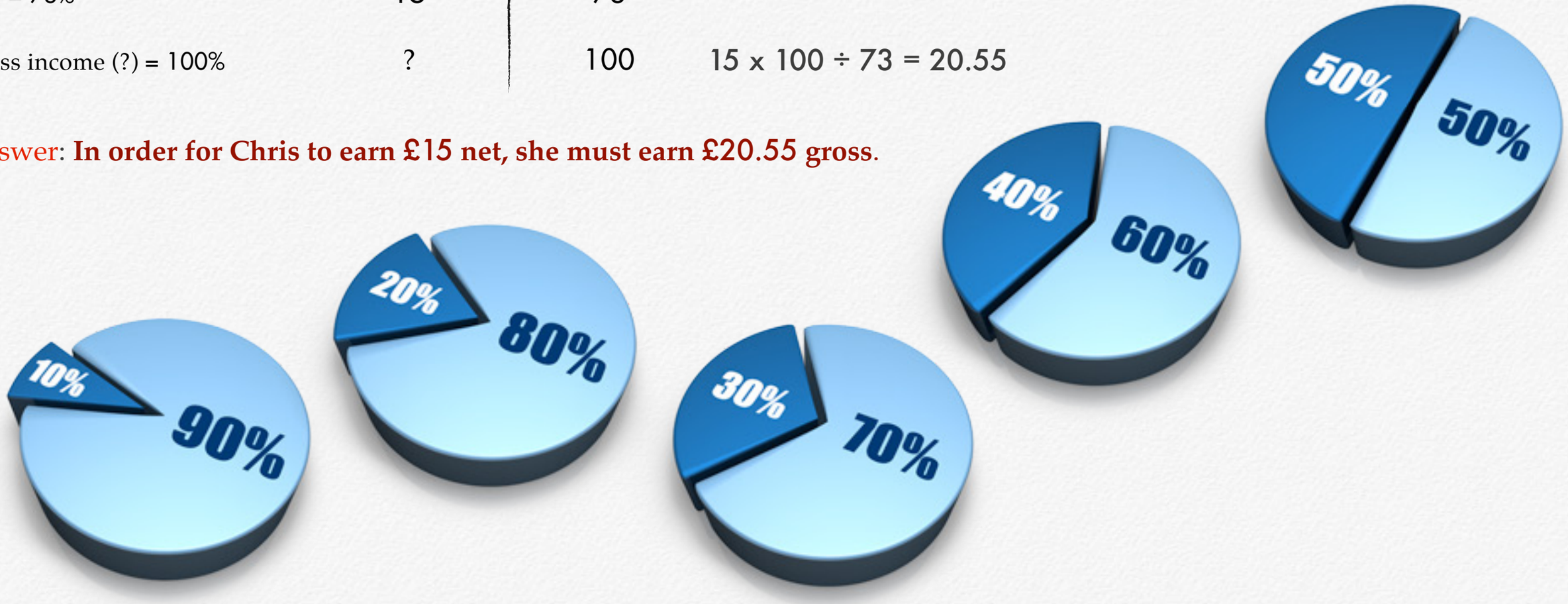
9) After training as a beautician, Chris earns **£15** per hour at her new job. Chris now thinks nothing of spending **£15** on a bottle of good wine or other little luxuries: after all, it's equivalent to just one hour of her work. Remember, Chris is spending from her net earnings, after deduction of her income tax, which is **22%** and her National Insurance which is **5%**.
So how much money does Chris need to earn gross in order to earn **£15** net?

22 + 5 = 27% of Chris's gross income goes to the government, so Chris receives **73% (100 – 27)** of her gross income.

£15 Chris spends on her wine **= 73%** of her gross earnings

	£	%	
£15 = 73%	15	73	
Gross income (?) = 100%	?	100	15 x 100 ÷ 73 = 20.55

Answer: **In order for Chris to earn £15 net, she must earn £20.55 gross**.



10) Chris earned **£120** extra this week by staying on late and working a few extra hours for her boss. She feels she can now reward herself by buying the handbag she has had her eye on, which costs **£97**. Will the **£120** extra she earned cover the cost of the handbag? Chris's income tax and National Insurance is **27%**.

Chris receives **73% (100 – 27)** of her gross income.

Chris's gross income, here, is **£120 = 100%**

	£	%	
£120 = 100%	120	100	
Spending money (net income) (?) = 73%	?	73	120 x 73 ÷ 100 = 87.60

Answer: **The net income Chris earned from the £120 will be £87.60; this does not completely cover the cost of the handbag.**

Answers to Percentages in your Head

11) Gemma is going to a charity ball in aid of a Romanian orphanage. The organisers advertise that 80% of the cost of each ticket will go to the orphanage. Each ticket costs £90. How much of the money Gemma's parents paid for her ticket will go to the orphanage?

100% = £90
(÷10) 10% = £9
(x8) 80% = £72

Answer: **£72 from Gemma's ticket will go to the orphanage**.

12) Having saved enough tigers, Rachel gets a new Saturday job that pays **65%** commission on all the paintballing tickets she sells. In one Saturday afternoon in the shopping mall in a nearby town, Rachel sold **£300** worth of tickets. How much commission did Rachel make in that one afternoon?

100% = £300
(÷10) 10% = £30
(x6) 60% = £180
(10% ÷ 2) 5% = £15
(60% + 5%) 65% = £180 + £15 = £195

Answer: **Rachel made £195 commission in one afternoon in the shopping mall**. Nice work if you can get it!

13) Melanie's local off-licence offers **20%** off any three bottles of wine. For a supper party, Melanie wants to buy three bottles of Australian Merlot which would normally cost **£8** each. How much will the wine cost when reduced by the **20%** offer?

The wine will normally cost **3 x £8 = £24**

100% = £24
(÷10) 10% = £2.40
(x2) 20% = £4.80

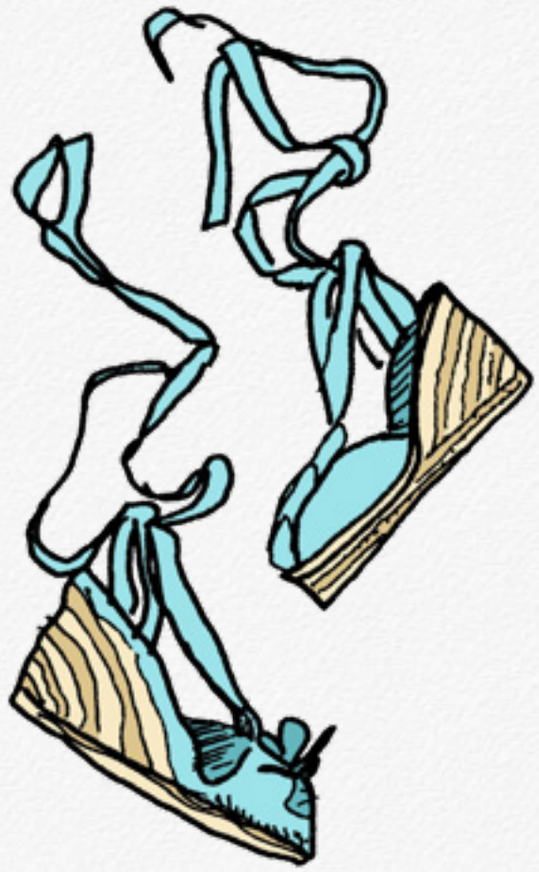
The wine is reduced by **£4.80**,
so it will cost **(£24 – £4.80) = £19.20**

Answer: **The wine will cost £19.20.**

14) Penny-pincher Jenny boasts that she made a **45%** profit on a pair of turquoise wedge sandals which she purchased at a charity shop for **£11**, then sold on eBay.
What price did Jenny's sandals sell for on eBay?

100% = £11
(÷10) 10% = £1.10
(x4) 40% = £4.40
(10% ÷ 2) 5% = £0.55
(40% + 5%) 45% = £4.40 + £0.55 = £4.95 profit.
(+ £11) = £15.95

Answer: **Jenny's turquoise wedge sandals sold for £15.95 on eBay.**



15) The latest successful boy band, SWIPES, ate a delicious Thai meal in Las Vegas. They offered the waitress a free ticket to their next gig or a **40%** tip. The bill came to **$1,800**. How much would the waitress lose out on a cash tip if she chooses the concert ticket?

100% = $1,800
(÷10) 10% = $180
(x4) 40% = $720

Answer: **The waitress would forego a $720 tip if she chooses the ticket**.
Which would you choose?

I choose the tip
to pay for this trip.

16) At Edna's Emporium, a fabulous, red sequin top hat was marked up by **200%**. The hat cost **£50** wholesale.

a) What is the price of the red sequin top hat at Edna's Emporium?

The wholesale (original) price was **£50**. The mark up is **200%**.

	%	£
Wholesale price is 100% = £50	100	50
Mark up of 200% = £?	200	?

Using the chart rule, the calculation is: **200 x 50 ÷ 100 = 100**

The price of the hat **= £100** mark-up **+ £50** wholesale price **= £150** retail.

Answer: **The price of the red sequin top hat at Edna's Emporium is £150**.

b) Unfortunately, the hat did not sell. What is the maximum percentage mark-down at which Edna's Emporium can offer the hat, without making a loss?

The hat's price was **£150**. The price must not decrease by more than **£100**.

	£	%
Normal price, £150 = 100%	150	100
Max reduction of £100 = ? %	100	?

Using the chart rule, the calculation is: **100 x 100 ÷ 150 = 66.6** recurring

66.6 recurring is normally rounded to **67%**, but this is rounding-up which would make the reduction in price slightly more than **£100**. Because the reduction must not be more than **£100**, round the percentage reduction down to **66%**.

Answer: **The maximum percentage mark-down that Edna's Emporium can knock off is 66%**.

17) At Felicity's Fascinations, a peacock feather hairband costs **£70** wholesale. The hair band is marked up by **400%**.

a) What is the price of the peacock feather hairband at Felicity's Fascinations?

The wholesale (original) price was **£70**. The mark up is **400%**.

	%	£
Wholesale price is 100% = £70	100	70
Mark up of 400% = £ ?	400	?

Using the chart rule, the calculation is: **400 x 70 ÷ 100 = 280**

The retail price of the hairband is:
= £280 mark-up **+ £70** wholesale price **= £350** retail.

Answer: **The price of the peacock feather hairband is £350 at Felicity's Fascinations**.

b) At sale time, what is the maximum percentage mark-down at which Felicity's Fascinations can offer the hair band without making a loss?

The hair band's price was **£350**. The price must not decrease by more than **£280**.

	£	%
Normal price, £350 = 100%	350	100
Max reduction of £280 = ? %	280	?

Using the chart rule, the calculation is: **280 x 100 ÷ 350 = 80**

Answer: **The maximum percentage mark-down for which Felicity's Fascinations can offer the hair band is 80%.**

Self-Confidence Tip 11

VISUALISE A WINNER!

Sports coaches use visualisation to train winners. Before an athlete attempts anything, she is asked to close her eyes and imagine that she has already succeeded. Try it yourself: close your eyes, see yourself preparing for this event…performing…and then succeeding! Everyone is applauding. You smile… with delight. You are hot, sweaty and happy.

This sports training exercise succeeds because your powerful mind is able to work 24 hours a day to achieve the goals you set for yourself. Your mind can do this only if you tell it exactly what you want.

Close your eyes while you state your goal aloud.

Write your own goal on a postcard, and read it often.



This method works well, not only for sport, but for anything you want to do in life. State your goal aloud, write it down, then write the steps necessary to achieve it. Next, close your eyes and imagine yourself achieving your goal. That's the first step.

Next step.

See yourself calmly approaching maths with an interested, open mind. See yourself gradually understanding each maths method, then moving on to the next step. Feel your satisfaction every time you turn a page of MONEY STUFF. Feel your exultation every time you 'get it'.

Next, imagine finishing MONEY STUFF. You feel proud of your persistence. You enjoy your success. Everyone tells you they're proud of you. And so they should be.


Doesn't it feel great when you've done something well?

Yelena Isinbayeva jumps for joy.
IAAF World Athletics Finals, Greece.

YOUR BRAIN WORKOUT

Q1.

100% represents?

- **A.** All of something
- **B.** Half of something
- **C.** None of something
- **D.** Double something

YOUR BRAIN WORKOUT

Q2.

50% of 40 children is?

- **A.** 40 children
- **B.** 30 children
- **C.** 20 children
- **D.** 10 children

YOUR BRAIN WORKOUT

Q3.

The amount of money you earn and get to keep after you have paid your income tax and your national insurance contribution is called?

- **A.** Gross earnings
- **B.** Net earnings
- **C.** Business expense
- **D.** Tax allowance

YOUR BRAIN WORKOUT

Answers

Q1. All of something

Q2. 20 Children

Q3. Net earnings

YOUR BRAIN WORKOUT

Q1. What is the answer to?

What is 10% of £80 ?

YOUR BRAIN WORKOUT

Q2. What is the answer to?

What will a £80 dress cost with 10% off ?

YOUR BRAIN WORKOUT

Q3. What is the answer to?

What is 10% of £220 ?

YOUR BRAIN WORKOUT

Q4. What is the answer to?

What is 20% of £220 ?

YOUR BRAIN WORKOUT

Q5. What is the answer to?

What is 10% of
700 people ?

YOUR BRAIN WORKOUT

Q6. What is the answer to?

What is 5% of
700 people ?

YOUR BRAIN WORKOUT

Q7. What is the answer to?

What is 20% of £40 ?

YOUR BRAIN WORKOUT

Answers

Q1. £8

Q2. £72

Q3. £22

Q4. £44

Q5. 70 people

Q6. 35 people

Q7. £8

PART 14
QUICK SWITCH

Quick switch of costume in a Real Life, messy dressing room.

Quick Quiz

Q1.

What is the remainder when 18 is divided by 7?

- A. 2
- B. 3
- C. 4
- D. 5

Quick Quiz

Q2.

What is $-3 + -5$?

A. -2

B. $+2$

C. -8

D. $+8$

Quick Quiz

Q3.

What is 700 x 3,000?

- A. 21,000,000
- B. 2,100,000
- C. 210,000
- D. 21,000

Quick Quiz

Q4.

Which of the following fractions is less than half?

- A. $\frac{9}{20}$
- B. $\frac{5}{8}$
- C. $\frac{5}{10}$
- D. $\frac{3}{5}$

Quick Quiz

Answers

Q1. 4

Q2. -8

Q3. 2,100,000

Q4. $\frac{9}{20}$

How to Switch Quickly between Percentages, Fractions, Decimals

It's time to tell you more about relationships.
Percentages, fractions and decimals are three systems that can all represent bits of things; you can swap one system for another of the three and sometimes this is unavoidable.
For example, when you are buying something, you may be given several bits of information, some in fractions, some in percentages and some in decimals. To clarify your situation, you should translate all that information into one system, such as percentages.

Here's an example: In a passenger survey of train services, 60% of train passengers prefer window seats, $\frac{1}{4}$ prefer aisle seats and 0.15 don't mind where they sit.

If you translate that into one system – I've chosen percentages – that information reads as follows: approximately 60% of train passengers prefer window seats, 25% prefer aisle seats and 15% don't mind where they sit.

You can see that in order to compare the values, you need to know how to convert:

To Percentages:

A. Decimals to Percentages

B. Fractions to Percentages

From Percentages:

C. Percentages to Decimals

D. Percentages to Fractions

From Fractions to Decimals and back again:

E. Fractions to Decimals

F. Decimals to Fractions

Most people find it easiest to deal with percentages, so lets start with them.

SWITCH →

This cake can be equally divided into 10 slices.

One cut slice = $\frac{1}{10}$ of this cake.

One cut slice = 10% of this cake.

The cake diagram shows that $\frac{1}{10}$ = 10%

To Switch Decimals to Percentages

Simply **multiply by 100**. Remember that multiplying by 100 is easy, because you just move the decimal point along 2 places to the right (look back to 'Multiplying Decimals by the Big Noughts', Part 12).

Examples

0.75 multiplied by 100 is 0.7 5 = **75%**

0.5 multiplied by 100 is 0.5 = **50%**

0.043 multiplied by 100 is 0.0 4 3 = **4.3%**

1.24 multiplied by 100 is 1.2 4 = **124%**

Exercises

Convert the following decimals to percentages.

1) 0.2

2) 0.64

This is Lucy Daniels. Lucy's school exam results were so bad – particularly in maths – that when she left school, Lucy could only find work as a cleaner.

When scrubbing floors at an employment agency, Lucy temporarily stood in for someone who was ill.
She proved to be good at business. **When Lucy was 22, she bought her own house.** Later, Lucy started the charity, Working Parents. **Exams aren't always proof of ability.**

Lucy, aged 22, talking to Japanese businessmen.

To Switch Fractions to Percentages

Again, simply **multiply by 100**.

First Example

What is $\frac{2}{5}$ expressed as a percentage?

$\frac{2}{5}$ becomes $\frac{2}{5} \times 100$

Change the whole number – 100 – into a fraction by putting the whole number ON TOP of 1, then do the multiplication. (To remind yourself of how to multiply fractions by whole numbers, see Part 11: How to Multiply Fractions.)

$$\frac{2}{5} \times \frac{100}{1} = \frac{200}{5}$$

$\frac{200}{5}$ is top heavy, so divide the top by the bottom:

$200 \div 5 = 40$

Answer: $\frac{2}{5} = 40\%$

Second Example

What is $\frac{1}{4}$ expressed as a percentage?

$$\frac{1}{4} \text{ becomes } \frac{1}{4} \times 100 = \frac{1}{4} \times \frac{100}{1}$$
$$= \frac{100}{4}$$
$$= 100 \div 4 = 25\%$$

Answer: $\frac{1}{4} = 25\%$

Third Example

What is $\frac{27}{40}$ expressed as a percentage?

$$\frac{27}{40} \text{ becomes } \frac{27}{40} \times 100 = \frac{27}{40} \times \frac{100}{1}$$
$$= \frac{2700}{40}$$
$$= 270 \div 4 = 67.5\%$$

Answer: $\frac{27}{40} = 67.5\%$

Fourth Example

What is $\frac{2}{3}$ expressed as a percentage?

$$\frac{2}{3} \text{ becomes } \frac{2}{3} \times 100 = \frac{2}{3} \times \frac{100}{1}$$

$$= \frac{200}{3}$$

$$= 200 \div 3 = 66.66666\%$$

When you have a recurring decimal number like this, round it.

$= 66.66666\% = 66.7\%$

Answer: $\frac{2}{3} = 66.7\%$

Exercises

Convert the following fractions to percentages.

3) $\frac{7}{10}$

4) $\frac{1}{8}$

To change **from a percentage**, whether to a **decimal** or to a **fraction**, you **divide the percentage by 100**, as follows.

Supermodel
Rosie Huntington-Whiteley
knows her numbers.

To Switch Percentages to Decimals

Here, to **divide by 100**, you move the decimal point two places to the left.

Examples

23.7% divided by 100 is 23.7 % = **0.237**

49% divided by 100 is 49. % = **0.49**

7% divided by 100 is 7. % = **0.07**

350% divided by 100 is 350. = **3.50**

Exercises

Convert the following percentages to decimals.

5) **48%**

6) **17.5%**

Switching jobs.
British women at war in 1941.

To Switch Percentages to Fractions

Again, you **divide by 100**, but this time you put your percentage OVER 100, to create a fraction.

Then you simplify the fraction.

Examples

25% becomes $\frac{25}{100} = \frac{5}{20} = \frac{1}{4}$ Answer: 25% = $\frac{1}{4}$

40% becomes $\frac{40}{100} = \frac{4}{10} = \frac{2}{5}$ Answer: 40% = $\frac{2}{5}$

9% becomes $\frac{9}{100}$ This can't be simplified. Answer: 9% = $\frac{9}{100}$

210% becomes $\frac{210}{100} = \frac{21}{10}$

Since this is top heavy, change it to a mixed number,

$= 2\frac{1}{10}$ Answer: 210% = $2\frac{1}{10}$

If the percentage has a **decimal point** in it, 'multiply up' the fraction before cancelling it down:

12.5% becomes $\frac{12.5}{100} = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$ Answer: 12.5% = $\frac{1}{8}$

The decimal point was eliminated from the fraction, when $\frac{12.5}{100}$ was changed to $\frac{125}{1000}$ by multiplying both top and bottom figures by 10.

Exercises

Convert the following percentages to fractions.

7) **75%**

8) **8%**

Two school teachers train for space in zero gravity.

To Switch Fractions to Decimals

This is easily done with a calculator. Remember that the middle line in a fraction means **divide**. Put the fraction into your calculator, starting with the top number, then divide by the bottom number. When you press [=], you will be given the equivalent decimal to your fraction.

Examples

$\frac{3}{4} = 3 \div 4 = 0.75$

$\frac{5}{16} = 5 \div 16 = 0.3125$

When you need to convert a mixed number that includes a fraction, the first step is to do the fraction part in the calculator.

Example

Express $2\frac{3}{5}$ as a decimal.

$2\frac{3}{5} = 2 + \frac{3}{5}$

First step, $\frac{3}{5} = 3 \div 5 = 0.6$

Now, add the whole number to the decimal number:

$0.6 + 2 = 2.6$

Answer: $2\frac{3}{5} = 2.6$

Exercises

Convert the following fractions to decimals.

9) $\frac{3}{20}$

10) $3\frac{5}{8}$

To Switch Decimals to Fractions

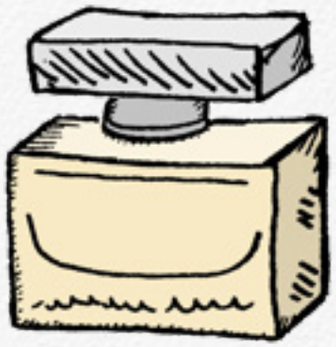
On top of the fraction line, put the number to the right of the decimal point.

First Example

$0.7 = \frac{7}{?}$

For a figure with one number after the decimal point, put 10 as the bottom number. So here, you're working with $\frac{7}{10}$

$0.7 = \frac{7}{10}$ ← the number after the decimal point.
← 10 is the bottom number because there is **one** number after the decimal point in the original decimal number.



For a figure with **two** numbers after the decimal point, you put **two** noughts after the 1 on the bottom.

Second Example

$0.12 = \frac{12}{100}$ ← the number after the decimal point.
← 100 is the bottom number (**two** noughts)

Then you simplify, so $\frac{12}{100} = \frac{6}{50} = \frac{3}{25}$

Third Example

$0.055 = \frac{55}{1000}$ ← 55 is the top number as it equals 055, the number after the decimal point
← 1000 is the bottom number (**three** noughts)

Then simplify: $\frac{55}{1000} = \frac{11}{200}$

Rule: For every digit after the decimal point, you add a nought after the 1 in the bottom line of the fraction.

So $0.12345 = \frac{12345}{100000}$ ← five digits after the decimal point
← five noughts

Note: To put the explanation next to the numbers, sometimes I've reduced the text size.

Another way to do it

Some people find it easier to change the decimal to a percentage, and then change the percentage to a fraction.

Example: 0.35 becomes 0.35 x 100 = 35%

Answer: 35% becomes $\frac{35}{100} = \frac{7}{20}$

Exercises

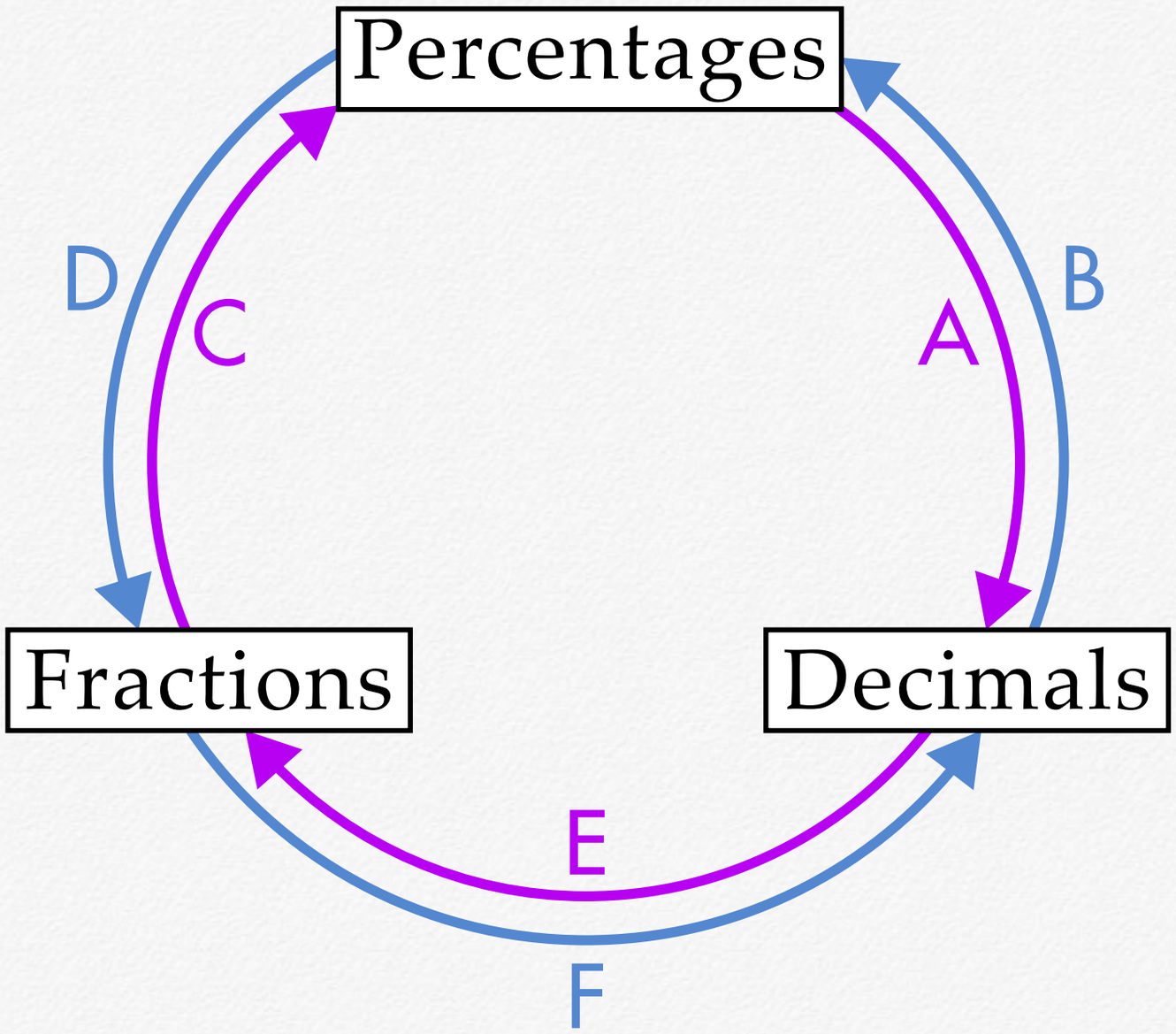
Convert the following decimals to fractions.

11) 0.45

12) 0.375

Japanese astronaut and aerospace engineer can't afford to get her numbers wrong.

The Six-Switch Diagram



Look carefully at these circles.
The purple arrows show three ways of converting (for instance, arrow A converts percentages to decimals).

The blue arrows show the opposite ways (for instance, arrow B converts decimals to percentages).

In the next table, you can see:
the method for A
the method for B
the methods for C, D, E and F.

The Six-Switch Table

A	**Percentage to Decimal**	÷ 100 by moving the decimal point two places to the left.	28% = 2 8 . % = 0.28
B	**Decimal to Percentage**	x 100 by moving the decimal point two places to the right.	0.375 = 0 . 3 7 5 = 37.5%

C	**Fraction to Percentage**	Multiply the fraction by 100	$\frac{4}{5}$ becomes $\frac{4}{5} \times \frac{100}{1} = \frac{400}{5} = 80\%$
D	**Percentage to Fraction**	÷ 100 by putting 100 underneath the percentage. Then cancel down.	$55\% = \frac{55}{100} = \frac{11}{20}$

E	**Decimal to Fraction**	Above the line, put the digits that follow the decimal point. Remember that below the line, after the digit 1, the **number of noughts** must equal the number of **digits above the line** i.e. 10, 100, 1000... Then cancel down.	$0.065 = \frac{065}{1000} = \frac{13}{200}$
F	**Fraction to Decimal**	Divide the number above the line by the bottom number.	$\frac{4}{5} = 4 \div 5 = 0.8$

Exercise

13) When you feel comfortable with the Six-Switch table, try this exercise. On paper, copy the Six-Switch Table, but without the calculations, so in line A you stop after **28% =**

Now fill in the calculations yourself. Do this slowly, one at a time. If you get stuck, check back to the table in the book.

14) Change the following to percentages:

a) $\frac{2}{5}$ b) 0.8 c) $\frac{7}{20}$ d) 0.065

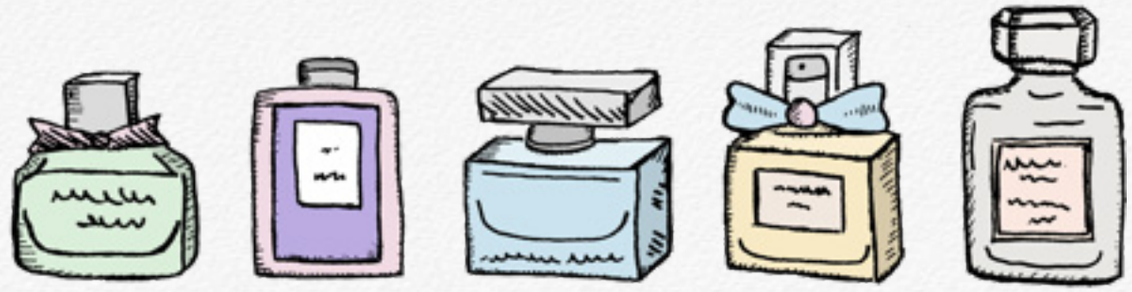
15) Change the following to decimals:

a) $\frac{3}{8}$ b) 23% c) $\frac{17}{40}$ d) 7%

16) Change the following into fractions:

a) 52% b) 0.35 c) 87.5% d) 0.05

Hint: Look back to the examples with a decimal point in the percentage.



17) Two department stores are offering deals on the new designer perfume. Debenhams offers $\frac{1}{3}$ off and Selfridges offers **30%** off. Work out which is the better deal by comparing both the offers as percentages.

18) Chris and Imogen are boasting to each other, on the telephone, about their new TV screens. Chris says her TV screen is **0.8** metres wide, and Imogen says hers is $\frac{3}{4}$ of a metre. Chris is a little confused, but she thinks her TV screen is the wider one. Is Chris correct?

19) Put the following numbers in order, starting with the smallest:

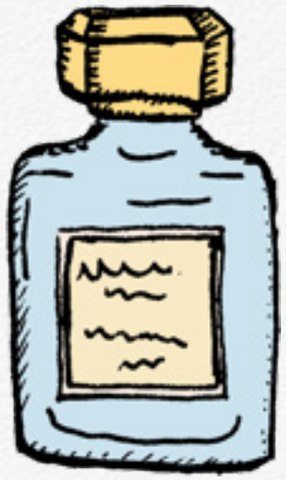
$\frac{3}{4}$ 0.7 $\frac{36}{50}$ 80% 0.08

A few Useful Fraction, Percentage and Decimal Equivalents

Look carefully at these. In time, you will find – to your surprise – that you know the equivalents by heart…, because you will have used them so often in Real Life.

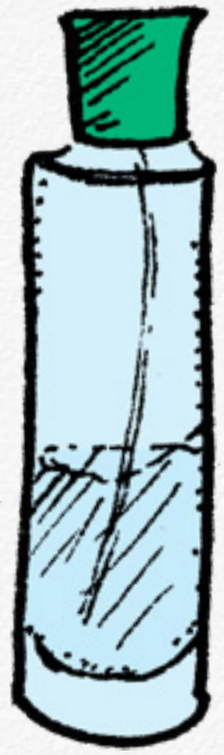
Half and Whole		
$\frac{1}{2}$	50%	0.5
$\frac{1}{1}$	100%	1.0

Quarters		
$\frac{1}{4}$	25%	0.25
$\frac{3}{4}$	75%	0.75



Thirds		
$\frac{1}{3}$	33.3%	0.333
$\frac{2}{3}$	66.7%	0.667

Tenths and Fifths		
$\frac{1}{10}$	10%	0.1
$\frac{2}{10} = \frac{1}{5}$	20%	0.2
$\frac{3}{10}$	30%	0.3
$\frac{4}{10} = \frac{2}{5}$	40%	0.4
$\frac{5}{10} = \frac{1}{2}$	50%	0.5
$\frac{6}{10} = \frac{3}{5}$	60%	0.6
$\frac{7}{10}$	70%	0.7
$\frac{8}{10} = \frac{4}{5}$	80%	0.8
$\frac{9}{10}$	90%	0.9



Answers to Part 14

Answers to A. Changing Decimals to Percentages

1) 0.2 multiplied by 100 = 0.2

Answer: 20%

2) 0.64 multiplied by 100 = 0.64

Answer: 64%

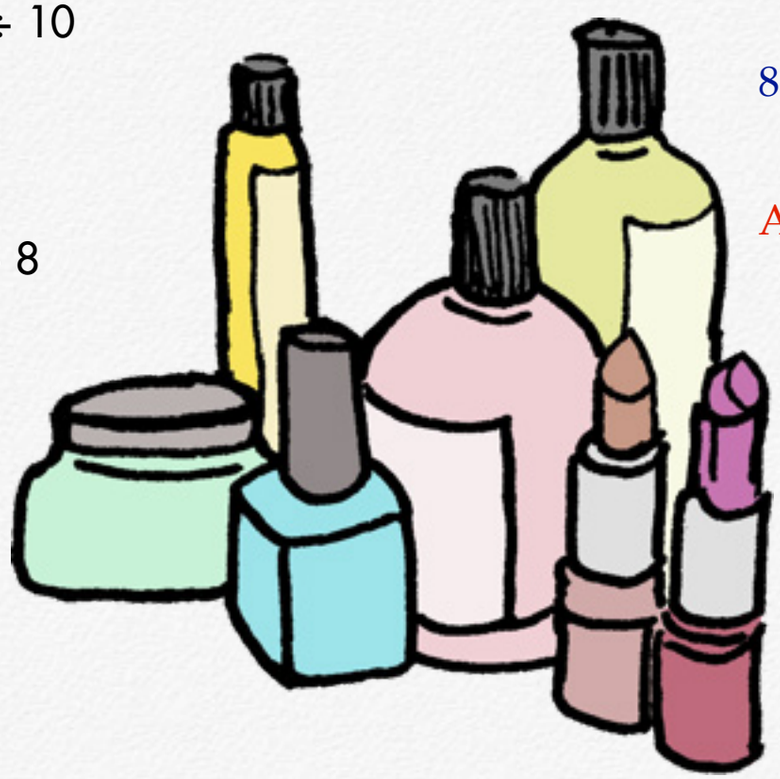
Answers to B. Changing Fractions to Percentages

3) $\frac{7}{10} \times \frac{100}{1} = \frac{700}{10} = 700 \div 10$

Answer: 70%

4) $\frac{1}{8} \times \frac{100}{1} = \frac{100}{8} = 100 \div 8$

Answer: 12.5%



Answers to C. Changing Percentages to Decimals

5) 48% divided by 100 = 0.48

Answer: 0.48

6) 17.5% divided by 100 = 0.175

Answer: 0.175

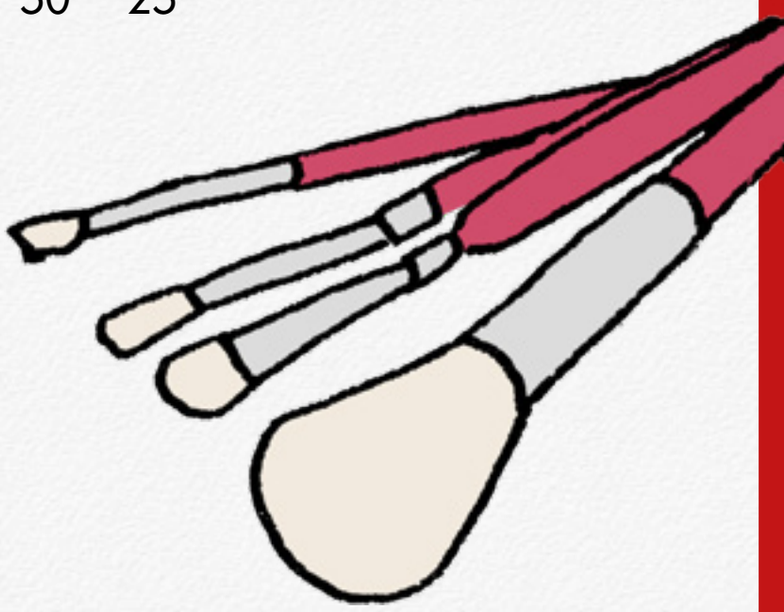
Answers to D. Changing Percentages to Fractions

7) 75% becomes $\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$

Answer: $\frac{3}{4}$

8) 8% becomes $\frac{8}{100} = \frac{4}{50} = \frac{2}{25}$

Answer: $\frac{2}{25}$



Answers to E. Changing Fractions to Decimals

9) $\frac{3}{20} = 3 \div 20$ (Use a calculator)

Answer: 0.15

10) $3\frac{5}{8} = 3 + \frac{5}{8}$

$\frac{5}{8} = 5 \div 8$ (Use a calculator) = 0.625

3 + 0.625

Answer: 3.625

Answers to F. Changing Decimals to Fractions

11) $0.45 = \frac{45}{100} = \frac{9}{20}$

Answer: $\frac{9}{20}$

12) $0.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$

Answer: $\frac{3}{8}$

Answers to Changing Between Percentages, Decimals and Fractions

13) Compare your answers with the six-switch table page.

14) Change the following to percentages:

a) $\frac{2}{5}$ becomes $\frac{2}{5} \times \frac{100}{1} = \frac{200}{5} = 200 \div 5 =$ Answer: 40%

b) 0.8 becomes 0.8 x 100 = Answer: 80%

c) $\frac{7}{20}$ becomes $\frac{7}{20} \times \frac{100}{1} = \frac{700}{20} = 700 \div 20 =$ Answer: 35%

d) 0.065 becomes 0.065 x 100 = Answer: 6.5%

15) Change the following to decimals:

a) $\frac{3}{8} = 3 \div 8 =$ Answer: 0.375

b) 23% = 23 ÷ 100 = Answer: 0.23

c) $\frac{17}{40} = 17 \div 40 =$ Answer: 0.425

d) 7% = 7 ÷ 100 = Answer: 0.07

16) Change the following into fractions:

a) $52\% = \frac{52}{100} = \frac{26}{50} = \frac{13}{25}$ Answer: $\frac{13}{25}$

b) $0.35 = \frac{35}{100} = \frac{7}{20}$ Answer: $\frac{7}{20}$

c) $87.5\% = \frac{87.5}{100} = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$ Answer: $\frac{7}{8}$

d) $0.05 = \frac{5}{100} = \frac{1}{20}$ Answer: $\frac{1}{20}$

17) Two department stores are offering deals on the new designer perfume. Debenhams offers '$\frac{1}{3}$ off' and Selfridges offers '**30%** off'. Work out which is the better deal, by comparing both the offers as percentages.

Change to a percentage by multiplying by **100**:

$\frac{1}{3} \times \frac{100}{1} = \frac{100}{3}$

$\frac{100}{3} = 100 \div 3 = 33.3333333\%$ (or 33.3%)

Answer: **Debenhams has the better offer, as $\frac{1}{3}$ off is more than 30% off.**

18) Chris and Imogen are boasting to each other, on the telephone, about their new TV screens. Chris says her TV screen is **0.8** metres wide, and Imogen says hers is $\frac{3}{4}$ of a metre. Chris is a little confused, but she thinks her TV screen is the widest. Is Chris correct?

$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 300 \div 4 = 75\%$

So, Imogen's screen is **75%** of a metre wide.

0.8 x 100 = 80% So, Chris' is **80%** of a metre wide.

Answer: **Chris' TV screen is the wider one**.

19) Put the following numbers in order, starting with the smallest:

$\frac{3}{4}$ 0.7 $\frac{36}{50}$ 80% 0.08

Change all the numbers to percentages:

$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 300 \div 4 = 75\%$

0.7 becomes **0.7 x 100 = 70%**

$\frac{36}{50} \times \frac{100}{1} = \frac{3600}{50} = 3600 \div 50 = 72\%$

0.08 becomes **0.08 x 100 = 8%**

Answer: 0.08 0.7 $\frac{36}{50}$ $\frac{3}{4}$ and 80%

YOUR BRAIN WORKOUT

Q1. What is the answer to?

What is 10% as a fraction ?

YOUR BRAIN WORKOUT

Q2. What is the answer to?

What is $\frac{3}{4}$ as a decimal ?

YOUR BRAIN WORKOUT

Q3. What is the answer to?

What is 0.43 as a percentage ?

YOUR BRAIN WORKOUT

Q4. What is the answer to?

What is 0.4 as a fraction ?

YOUR BRAIN WORKOUT

Q5. What is the answer to?

What is 19% as a decimal ?

YOUR BRAIN WORKOUT

Q6. What is the answer to?

What is $\frac{1}{100}$ as a percentage ?

YOUR BRAIN WORKOUT

Answers

Q1. $\frac{1}{10}$

Q2. 0.75

Q3. 43%

Q4. $\frac{4}{10}$ or $\frac{2}{5}$

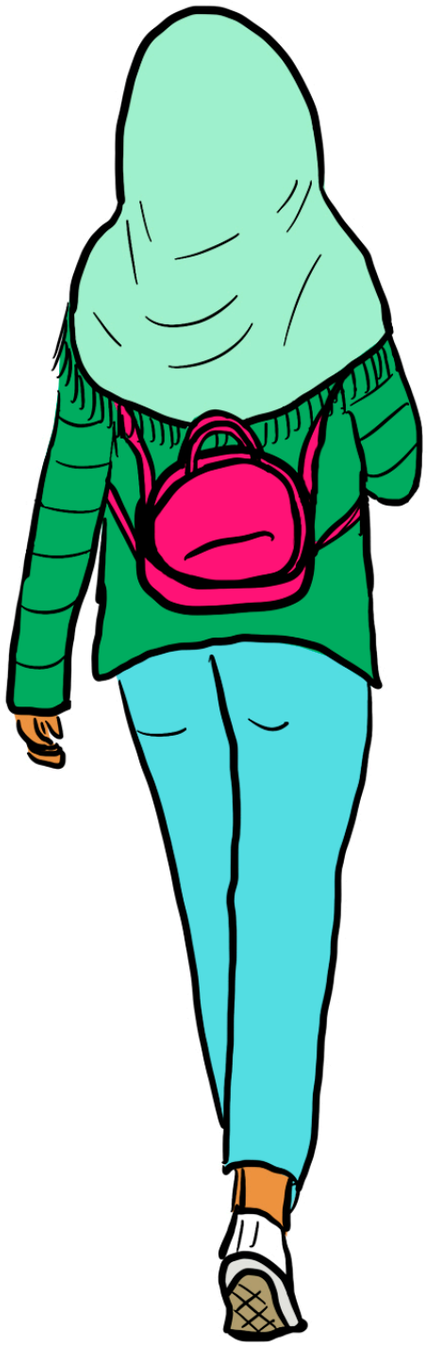
Q5. 0.19

Q6. 1%

PART 15
RATIOS

Careful calculations.

Quick Quiz

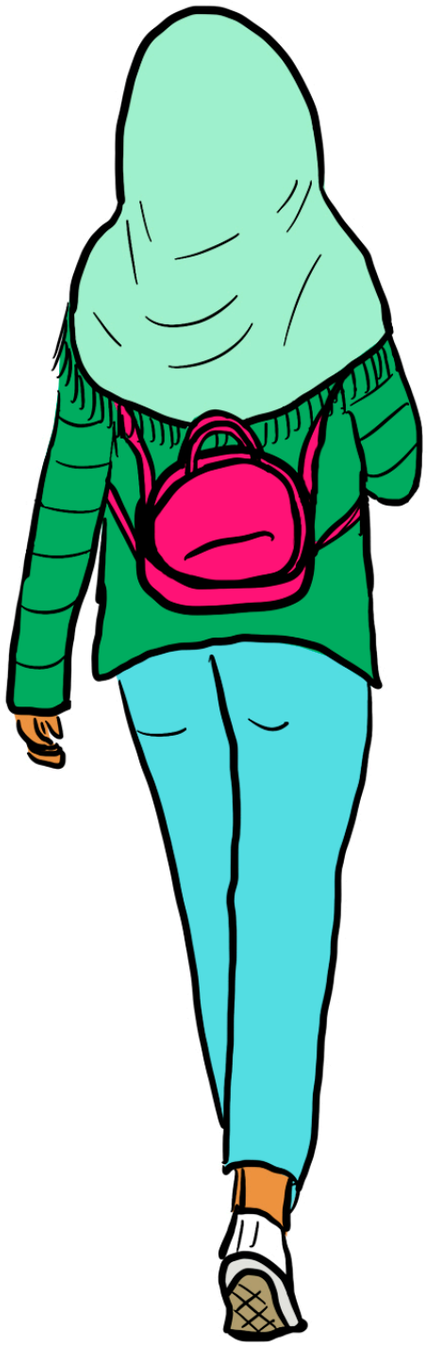


Q1.

15, 18, 21, 24, 27 are from which times table?

- A. x2
- B. x6
- C. x9
- D. x3

Quick Quiz

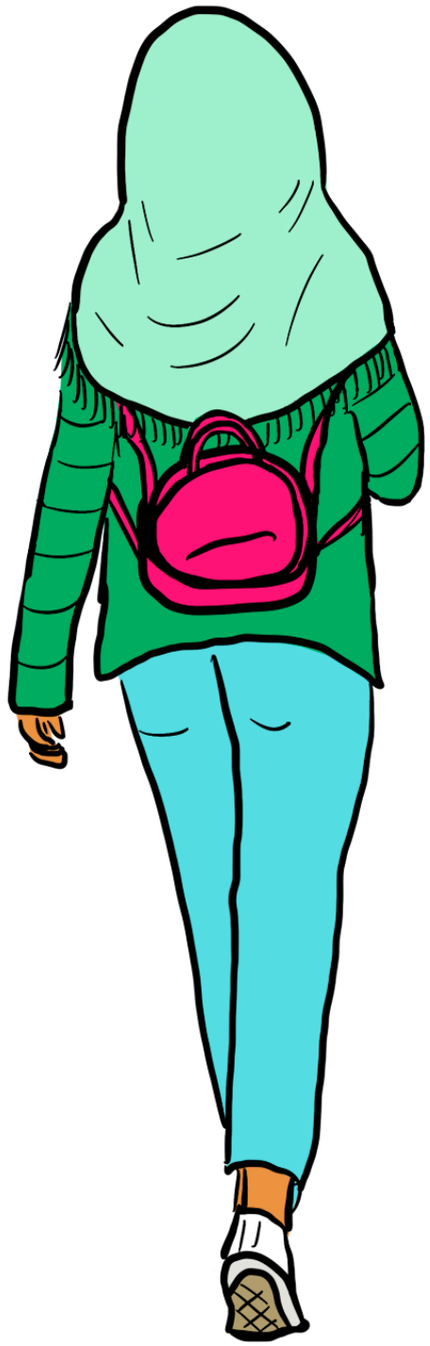


Q2.

If you buy a sandwich for **£3.95**, a coffee for **£2.30** and a muffin for **£2.80** approximately how much will you pay?

- A. £7
- B. £8
- C. £9
- D. £10

Quick Quiz

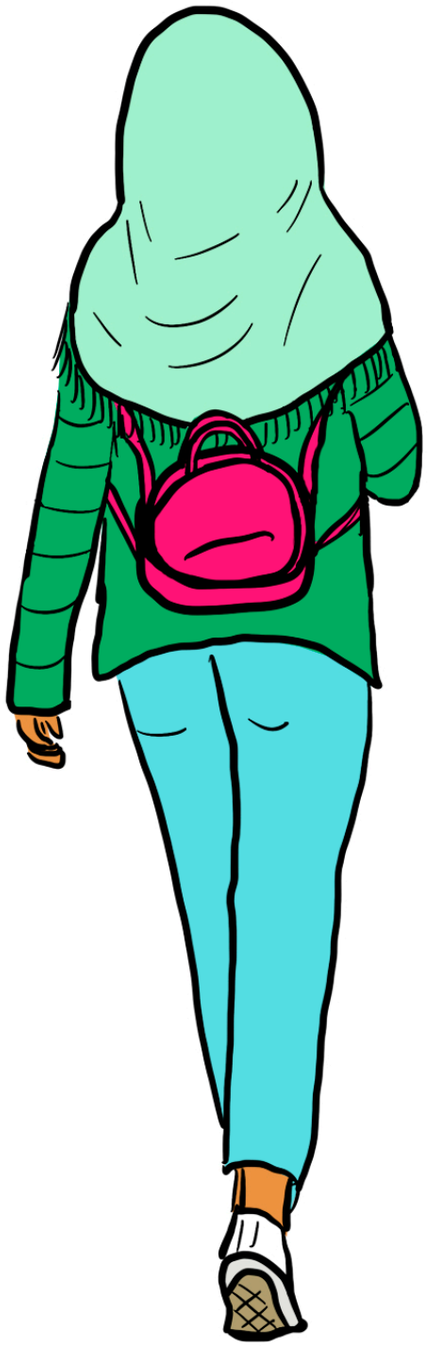


Q3.

What is **20%** of **£250**?

- A. £25
- B. £50
- C. £12.50
- D. £75

Quick Quiz

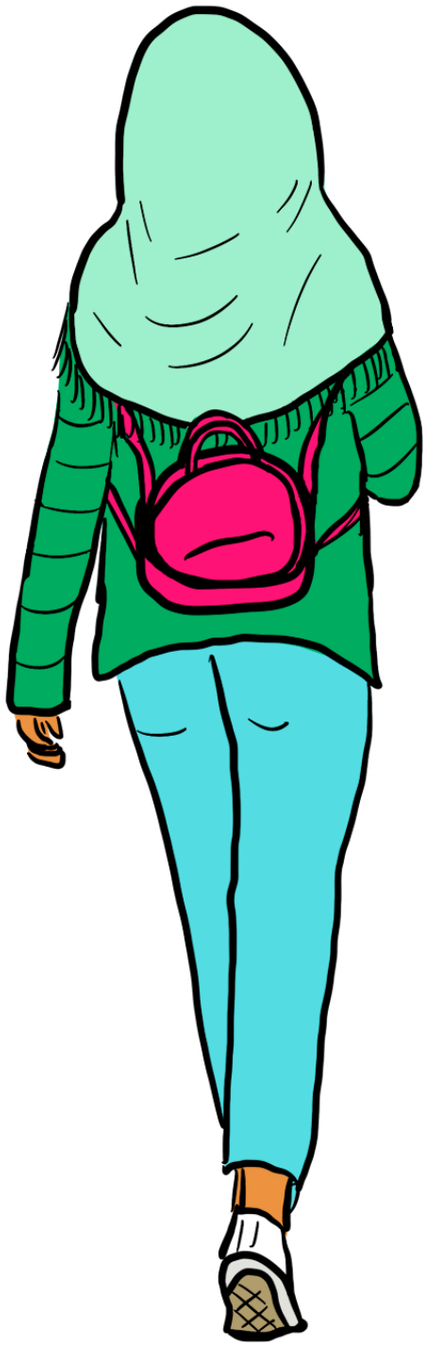


Q4.

Your supermarket bill comes to **£73.60**, but you have a voucher worth **£8**. Which calculation should be used to correctly work out the amount you will pay?

- A. £73.60 - £8
- B. £8 - £73.60
- C. £73.60 ÷ £8
- D. £8 + £73.60

Quick Quiz



Answers

Q1. x3

Q2. £9

Q3. £50

Q4. £73.60 – £8

Ratios

The youngest member of every family knows that life isn't fair. Ratios are useful to calculate the amount of unfairness and to divide a group of things **unequally**. This isn't as mean as it sounds; for example, if you invest twice as much money as Serena in a business, then you expect twice as much profit… which is fair, although it's an **unequal** division.

A **simple ratio** is a comparison of two quantities. For example, if there are **24** students and **3** teachers on a safari trip to Kenya, the ratio of students to teacher is: **24** students to **3** teachers, which is written **24:3**.

This ratio of **24:3** can be simplified. To simplify a **ratio**, **divide** both numbers by the same common factor.

3 is the common factor of **24** and **3**, so **24:3** when simplified becomes **(24÷3):(3÷3) = 8:1**; this means that on the safari trip there are eight students to every one teacher, which sounds a reasonable responsibility for each teacher.

So, the ratio of students to teachers is 8:1.

First Example

At the under-fives play area in the local park, Ruth counts **18** adults and **27** children. What is the ratio of adults to children?

First, write the ratio in words, with the quantities directly underneath. Separate the numbers by a colon:

Adults	:	Children
18	:	27

Simplify the ratio by dividing both numbers by the same common factor. Keep reducing the ratio until you cannot find a common factor, one number that divides both numbers. At this point, you know your ratio is in its simplest form.

	Adults	:	Children
	18	:	27
(÷3)	6	:	9
(÷3)	2	:	3

Answer: **The ratio of adults to children in the play area is 2:3.**

When reading or writing ratios, the numbers are **always** written in the same order as the groups in the text. In the previous example, the ratio of adults to children is 2:3; the first number, 2, refers to the adults, because they are mentioned first; the 3 refers to the children, because they are mentioned second.

It's easy to do a common-sense check. **Question**: ask yourself, which is the bigger number in the written sentence, 18 adults or 27 children?

Answer: 27 children

So in the ratio 2:3, the bigger number should also be of children: 2 adults : 3 children

Second Example

In a school with 210 students and 3 teachers, what is the ratio of students to teachers?

	Students	:	Teachers
	210	:	3
(÷3)	70	:	1

Answer: **The ratio of students to teachers is 70:1**. A class of 70 students might be too many for one teacher to teach everyone something useful.

Third Example

Natalia's special raspberry-peach smoothie recipe is made from 200ml of raspberries, 100ml of orange juice and 150ml of peach yogurt. How should Natalia write the recipe as a ratio of raspberries to orange juice to peach yogurt?

Here, you need 3 columns. First, write the ratio in words, with the quantities directly underneath:

	Raspberries	:	Orange Juice	:	Peach Yogurt
	200	:	100	:	150
(÷10)	20	:	10	:	15
(÷5)	4	:	2	:	3

Answer: **The ratio of ingredients for Natalia's raspberry-peach smoothie is 4:2:3 for raspberries, orange juice and peach yogurt**.

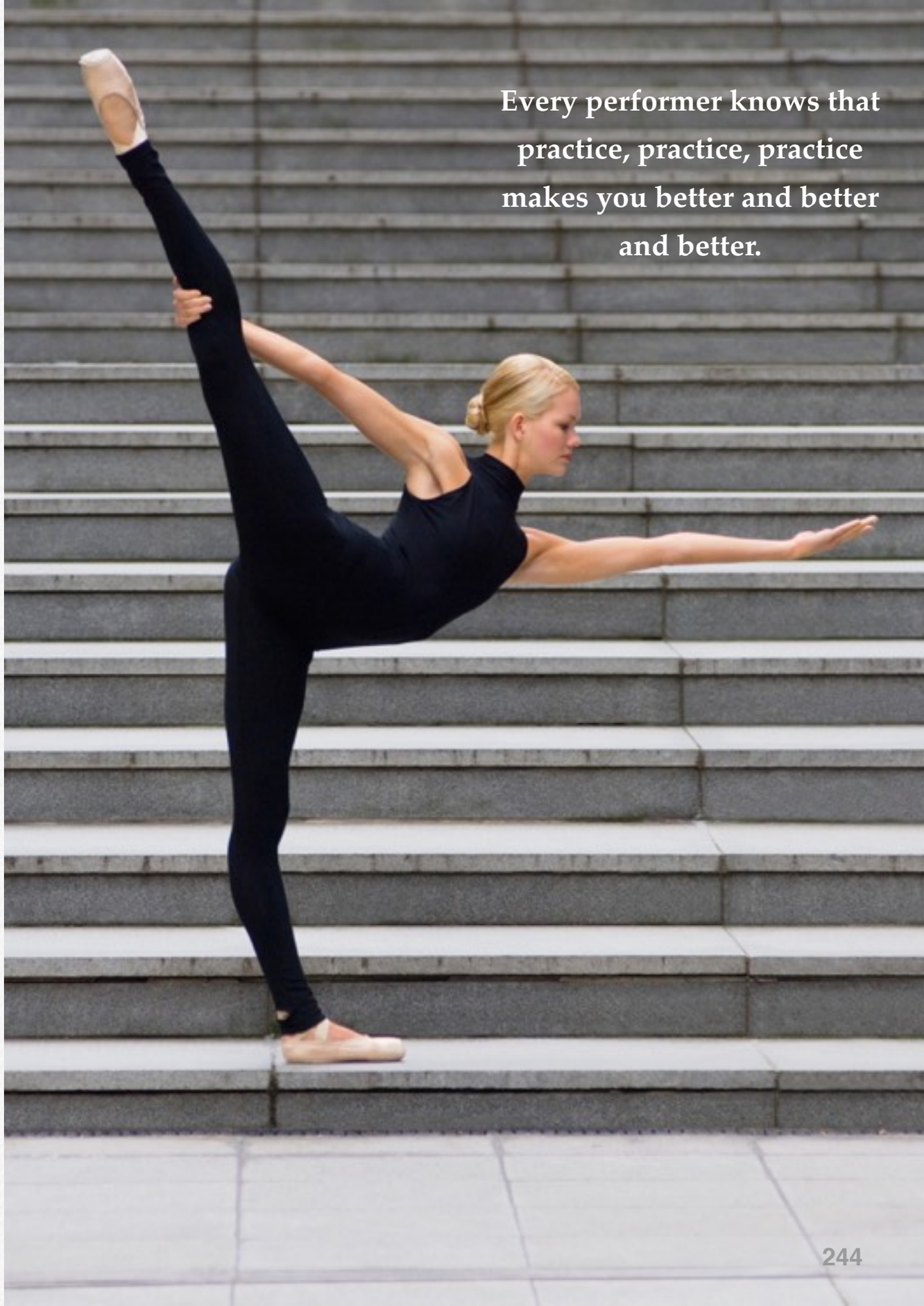
Exercises

1) The breakdown of expenses for the Nurses charity ball for **250** people includes **£500** for hire of Claridges ballroom, **£17,500** for the catering (including wine) and **£2,500** for the band.
Show these expenses as ratios.

2) When making her special salad dressing, Carla puts two tablespoons of olive oil to one tablespoon of vinegar, adds a quarter of a tablespoon of sugar and a quarter of a tablespoon of mustard powder. Put this recipe in ratio form, using **1** for the smallest quantity.

Hint: The smallest quantity here is less than **1** (a quarter). To change this to **1**, you will need to multiply rather than divide.

3) Betsy can't decide what to wear for a first date with one of her clients, a surgeon called Simon who says he has no time to meet anyone. When Betsy looks through her wardrobe, she finds she has more red dresses than any other colour: she has **4** red dresses and **6** in other colours. What is the ratio of red to other-coloured dresses in Betsy's wardrobe?



Sometimes, ratios don't cancel down neatly.
If this is the case, use decimals.

Example

Travel agent Ruth is asked by a fussy customer to compare the annual rainfall of Transylvania and Albania. Ruth finds out that the average annual rainfall in Transylvania is 1702mm and in Albania is 1485mm.
Ruth tells her customer that the ratio of rainfall in Albania to Transylvania is 1:1.15. The customer then knows that the rainfall is similar in both places, but slightly higher in Transylvania.

Use the famous **basic chart method** in your calculations for these ratios. Remember, you have already used the basic chart method to solve problems.
(See MONEY STUFF, STEP 1, Part 9: How to Make a Problem Much Simpler. For calculating percentages, turn to Part 13).
Now you use **the basic chart method** to calculate ratios.

First Example

For a geography project, Rachel is comparing Jamaica and Haiti. The population of Jamaica is approximately 2,780,000 and the population of Haiti is estimated at 8,706,000. How should Rachel present the ratio of the populations of Jamaica and Haiti?

First, draw the chart and fill in the numbers that you know:

	Jamaica	Haiti
Population	2,780,000	8,706,000
Ratio numbers.	1	?

Put 1 under the smaller number.

Next, use the basic chart rule to work out the ratio number for Haiti: **Multiply the diagonals and divide by the remaining number**:

1 x 8,706,000 ÷ 2,780,000 = 3.13

As you cannot have 0.13 of a live person, round the number to the nearest whole number = 3 when rounded.

Answer: **The ratio of the population of Jamaica to the population of Haiti is roughly 1:3**.
(That's one Jamaican for every three Haitians.)

Second Example

Rachel has been asked to make a 3D model – a diorama – of the solar system. One of the calculations she needs is to compare the diameter of the Earth to that of Jupiter.
The diameter (width) of the Earth is **7,926** miles, which is considerably smaller than the diameter of the biggest planet, Jupiter, which is **88,846** miles.
Show how much bigger Jupiter is than the planet Earth, by giving their diameters in the ratio Earth to Jupiter.

First, draw the chart and fill in the numbers you know, including a **1** for Earth's ratio:

	Earth	Jupiter
Diameter (miles)	**7,926**	88,846
Ratio numbers	1	?

Next, use the basic chart rule to work out the ratio number for Jupiter: **Multiply the diagonals and divide by the remaining number**:

1 x 88,846 ÷ 7,926 = 11.21

Answer: **The ratio of the diameters of Earth to Jupiter is 1:11.2**, so Jupiter has a diameter approximately **11** times bigger than the Earth.

Exercises

4) Cleo's cream Mini is chugging up the hill at **30** miles per hour, when Betsy in her navy blue Ferrari overtakes Cleo at **75** miles per hour. Show the ratio of the speed, of both cars, in the form **1**: ….

5) Carla is moving from her **32** square metre, studio flat into a new apartment – nearer to Carla's Sandwich Bar – which measures **100** square metres. Give the areas of the two flats in a ratio, in the form **1**: …. Give your answer to **1**d.p.

6) Walking in Hyde Park, Carla buys a delicious toffee ice cream. She is told by the good-looking ice cream vendor, Fausto, that he owns the retro ice cream van and makes his own ices. Carla asks him to supply her sandwich bar.
In the park on the following Saturday, Fausto sold **23** chocolate ice creams and **276** toffee ice creams. Show Fausto's chocolate-to-toffee ice cream sales in a ratio, in the form **1**:

Self-Confidence Tip 12

YOUR BENEFITS

When you are confident in maths, that will have a positive effect on many other day-to-day areas of your life. I don't know you, but I do know that it will bring life-changing benefits to you, and you will be able to see these benefits very quickly.

Some money-saving areas where you will be able to use your new knowledge and maths expertise are:

- To compare prices, especially reductions in sales.
- To find the best mobile phone deal.
- To understand the different interest rates of credit cards, and choose the best.
- To check your pay.

You will not only save money: you will feel in control.

How to Use Ratios to Solve a Problem

This is the basic chart method plus a column for the totals, (see below).

First Example

At a singles party there is a ratio of **2:1**, men to women. There are **42** people at the party.

Question: How many men and how many women are there at the party?

First, draw up the chart with the headings according to the information: 'men', 'women', plus an extra column 'Total'.

Men	Women	Total

Put the ratio numbers on the first line of each column. The first number in the ratio goes with the first group, so in a ratio of **2:1**, men to women, **2** refers to the number of men, and **1** refers to the number of women.

Then add the two numbers in the ratio and write it in the 'Total' column.

	Men	Women	Total
Ratio numbers	2	1	3

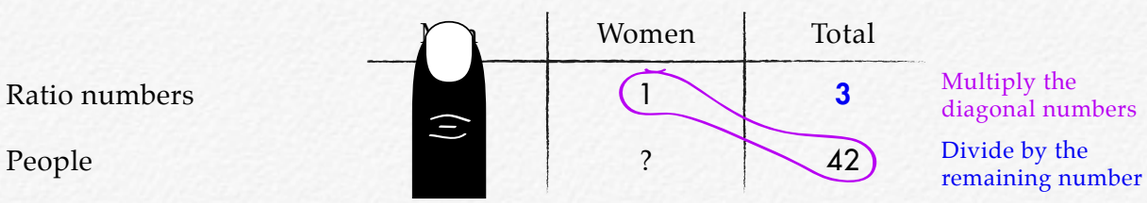
Next, fill in the other piece of information given: 'There are **42** people at the party.' You need to decide which column the **42** belongs in. Does it refer to the number of men, women or to the total number of people present?
It refers to the total number of people present, so put the **42** in the 'Total' column.

	Men	Women	Total
Ratio numbers	2	1	3
People			42

Now use the basic chart rule:

Multiply the numbers that are diagonal to each other and divide by the other number.

You can only use the basic chart rule with two columns, so ignore the third column.



The calculation is: 1 x 42 ÷ 3 = 14

That tells you there are 14 women at the party.

Insert this into the chart.

	Men	Women	Total
Ratio numbers	2	1	3
People		14	42

There are two alternative ways to fill in the last space on the chart:

a) You could subtract the number of women from the total number of people at the party, in order to find out how many men are present:

42 - 14 = 28

b) You could again use the basic chart method in the 'men' and 'women' columns:

2 x 14 ÷ 1 = 28

Answer: **There are 28 men and 14 women at the party**.

Lucky ladies.

Second Example

A small, specialist jewellery business shares its profits between the two owners, Mallory and Antonia, according to the amount of money they each invested. Mallory invested **£40,000** and Antonia invested **£30,000**, so the profit share ratio is **4:3**.

Question: The profit for April is **£3,500**.
How much of this money should each partner receive?

First, draw the chart, filling in as much information as possible:

	Mallory	Antonia	Total
Ratio numbers	4	3	7
Profit (£)			3,500

Use the basic chart rule on the last two columns:
3 x 3,500 ÷ 7 = 1,500

Fill in the **£1,500** in the correct column (Antonia), and then see what's left for Mallory:

	Mallory	Antonia	Total
Ratio numbers	4	3	7
Profit (£)	2,000	1,500	3,500

3,500 – 1,500 = 2,000 left for Mallory.

Answer: **Mallory receives £2,000 of the profit,**
Antonia receives £1,500 of the April profit of £3,500.

Architect Heidi's model chalet for her granddaughter, Daisy.

Third Example

Architect Heidi has made a dolls' house - which is a scale model of her own Swiss chalet in Wengen - as a Christmas gift for her granddaughter, Daisy. The ratio of the size of the model and the size of the house is **1:10**.

Question: If the model is **0.8**m high, how tall is the real chalet?

Draw the chart and fill in the numbers you have:

	Model	House
Ratio numbers	1	10
Height (m)	0.8	

10 x 0.8 ÷ 1 = 8

Answer: **The real chalet is 8 metres high**. Daisy thought it was her best present ever, until Annabel – who had sold several snowy paintings as Christmas gifts – produced the carefully wrapped but unmistakeable outline of Daisy's first bicycle.

Fourth Example

In Manchester, Chris and Imogen hold a joint Christmas party. They share the cost of the party in the ratio **3:5** because Imogen invited more guests. The total cost of the party was **£384**, which Imogen put on her credit card.

Question: How much does Chris owe Imogen for the party?

The chart will be as follows:

	Chris	Imogen	Total
Ratio numbers	3	5	8
Cost (£)			384

Since you only need to calculate what Chris owes, cover up the 'Imogen' column and use the basic chart rule on the 'Chris' and 'Total' columns:

	Chris	Imogen	Total
Ratio numbers	3		8
Cost (£)			384

So the calculation is **3 x 384 ÷ 8 = 144**

Answer: **Chris owes Imogen £144 for her share of the Christmas party**.

Exercises

7) The ratio of butter and flour in Natalia's shortcrust pastry is **4:9**. Natalia has **125**g of butter.
So how much flour will Natalia need if she is to use all the butter to make pastry?

8) Family tease, Aunt Anastasia, gives Christmas money to Natalia, her younger brother Conrad and their little sister Maria, in a ratio of **7:4:2** according to their ages. If Natalia received **£21**, how much do Conrad and Maria receive?

9) Rachel is going on a school geography field trip to the Lake District; the student–to–teacher ratio for a field trip is **8:1**. There are **64** students.

a) How many teachers will be needed to accompany the students?

b) How many people from the school, in total, will be travelling to the Lake District?

10) Natalia works out that the ratio of her age to her mother's age is currently **1:3**. Natalia also says that her age and her mother's age, added together, total **68** years.
How old is Natalia and how old is her mother?

Chris and Imogen give a party (see fourth example).

Self-Confidence Tip 13

YOUR NEW LIFE SKILLS

In life, nothing is isolated; every new experience connects to your other experiences in the past and in the future.

For instance in a dance, each new step that you learn builds on the steps you have previously learned…perhaps your footwork, your timing or your rhythm. Your future dance steps will build on what you are learning now.

You will find that you can use your new maths skills to learn anything new, throughout your life, not only maths.

Every time you practice these methods, they will become easier, a more natural part of you.

To test this, think of one non-maths area in which you'd like to feel more confident; this might be:

a) I'd like to be able to talk to someone new more easily.

b) I'd like to say 'no' politely, when I feel I'm being pushed into something.

c) I'd like to be able to stand up for myself in an unfair situation.

If you want to be all-round successful, always **appear** calm and confident (not overconfident and cocky). **ACT IT**.

If you don't feel confident in some new area, re-read and practice the self-confidence tips in this Course.

Practice will make you a confident person who doesn't fall into **SCARED MODE** when anything new comes her way.

Pick two other areas for further practice – one at a time.

You need to do this exercise in three different areas – including maths – to get into the habit of being successful. This will reinforce your maths confidence...

...and anything else you choose to do.

What every performer knows: practice improves performance.

Country ‘n’ Western singer Emmylou Harris in 1970.

Answers to Part 15

1) The breakdown of expenses for the Nurses charity ball for **250** people includes **£500** for hire of Claridges ballroom, **£17,500** for the catering (including wine) and **£2,500** for the band. Show these expenses as a ratio.

	Venue	:	Catering	:	Music
	500	:	17,500	:	2,500
(÷100)	5	:	175	:	25
(÷5)	1	:	35	:	5

Answer: **The ratio for expenses for the venue, catering and music for the charity ball was 1:35:5.**

2) When making her special salad dressing, Carla puts two tablespoons of olive oil to one tablespoon of vinegar, adds a quarter of a tablespoon of sugar and a quarter of a tablespoon of mustard powder. Put this recipe in ratio form, using 1 for the smallest quantity.

Hint: The smallest quantity here is less than **1** (a quarter). To change this to **1**, you will need to multiply rather than divide.

	Olive Oil	:	Vinegar	:	Sugar	:	Mustard Powder
	2	:	1	:	0.25	:	0.25
(x4)	8	:	4	:	1	:	1

Answer: **The ratio of ingredients for Carla's salad dressing is 8:4:1:1 of olive oil, vinegar, sugar and mustard powder.**

Hint: mix in the olive oil as the last ingredient.

3) Betsy can't decide what to wear for a first date with one of her clients, a surgeon called Simon who says he has no time to meet anyone. When Betsy looks through her wardrobe, she finds she has many more red dresses than any other colour: she has **4** red dresses and **6** in other colours.

What is the ratio of red to other-coloured dresses in Betsy's wardrobe?

	Red Dresses	:	Other Dresses
	4	:	6
(÷2)	2	:	3

Answer: **Betsy owns 2:3 red dresses to other-coloured dresses**.

She decided to wear a black slinky dress – not that Simon noticed what she was wearing: he fell for her immediately.

4) Cleo's cream Mini is chugging up the hill at **30** miles per hour, when Betsy in her navy blue Ferrari overtakes Cleo at **75** miles per hour. Show the ratio of the speed, of both cars, in the form **1:** ….?

	Mini	Ferrari
Speeds (miles per hour)	30	75
Ratio numbers	1	?

1 x 75 ÷ 30 = 2.5

Answer: **The ratio of speeds of Cleo's Mini to Betsy's Ferrari was 1 : 2.5**.

5) Carla is moving from her **32** square metre, studio flat into a new apartment – nearer to Carla's Sandwich Bar – which measures **100** square metres. Give the areas of the two flats in a ratio, in the form **1:** ….?

Give your answer to 1d.p.

	Studio Flat	New Apartment
Area (square meters)	32	100
Ratio numbers	1	?

1 x 100 ÷ 32 = 3.125

Answer: **The ratio of the areas of Carla's old and new flats is 1 : 3.1.**

6) Walking in Hyde Park, Carla buys a delicious toffee ice cream. She is told by the good-looking ice cream vendor, Fausto, that he owns the retro ice cream van and makes his own ices. Carla asks him to supply her sandwich bar.
On the following Saturday, Fausto sold **23** chocolate ice creams and **276** toffee ice creams. Show Fausto's chocolate to toffee ice cream sales in a ratio, in the form **1:** ….?

	Chocolate Ice Creams	Toffee Ice Creams
Ice creams sold	23	276
Ratio numbers	1	?

1 x 276 ÷ 23 = 12

Answer: **The ratio of Fausto's chocolate to toffee ice cream sales is 1:12.**

7) The ratio of butter and flour in Natalia's shortcrust pastry is **4:9**. Natalia has **125**g of butter. So how much flour will Natalia need if she is to use all the butter to make pastry?

Draw the chart; fill in the ratios and the amount of butter.

	Butter	Flour	Total
Ratio numbers	4	9	13
Ingredient (g)	125		

Now use the butter and flour columns to calculate the amount of flour needed:

125 x 9 ÷ 4 = 281.25

Answer: **Natalia will need just over 281g of flour.**

The 'Total' column was not really necessary for this calculation, but it's a good working habit always to include it.

8) Family tease, Aunt Anastasia, gives Christmas money to Natalia, her younger brother Conrad and their little sister Maria, in a ratio of **7:4:2** according to their ages. If Natalia received **£21**, how much do Conrad and Maria receive?

Draw and fill in the chart. **£21** is what Natalia was given, so put it in Natalia's column. A 'Total' column is not necessary here, but it doesn't matter if it is drawn in.

	Natalia	Conrad	Maria
Ratio numbers	7	4	2
Christmas money (£)	21		

Use the 'Natalia' and 'Conrad' columns to calculate Conrad's money:

21 x 4 ÷ 7 = 12

Cover up the Conrad column and use the Natalia and Maria columns to calculate Maria's money:

21 x 2 ÷ 7 = 6

	Natalia	Conrad	Maria
Ratio numbers	7	4	2
Christmas money (£)	21	12	6

Answer: **Conrad received £12 and Maria received £6 from Aunt Anastasia.**

9) The student-to-teacher ratio for a field trip to the Lake District is **8:1**. There are **64** students.

a) How many teachers will be needed to accompany the students?

b) How many people from the school, in total, will be travelling to the Lake District?

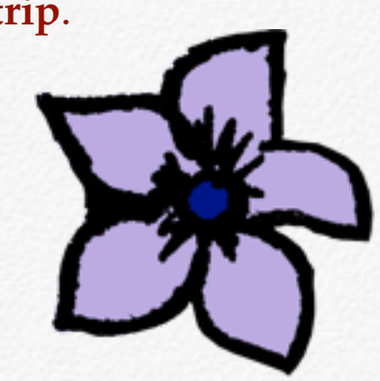
Draw the chart; fill in the ratios and the number of students.

	Students	Teachers	Total
Ratio numbers	8	1	9
People	64		

To work out the number of teachers needed:
64 x 1 ÷ 8 = 8 teachers

To work out the total number of people on the trip, add the students and teachers: **64 + 8 = 72**

Answer: a) **8 teachers will be needed for the trip**.
b) **A total of 72 people will be on the field trip**.



10) Natalia works out that the ratio of her age to her mother's age is currently **1:3**. She says that her age and her mother's age, added together, total **68** years. How old is Natalia and how old is her mother?

Draw the chart; fill in the ratios and the total of Natalia and her mother's ages.

	Natalia	Natalia's mum	Total
Ratio numbers	1	3	4
Age (years)			68

First, calculate Natalia's mum's age, using that column and the Total column:

3 x 68 ÷ 4 = 51 years

Now you can calculate Natalia's age by taking away her mum's age from the total:

68 - 51 = 17 years

Answer: **Natalia is currently 17 years old and her mother is 51**.

YOUR BRAIN WORKOUT

Q1.

If there are 20 men and 10 women at a singles party, what is the ratio of men to women?

YOUR BRAIN WORKOUT

Q2.

If there are 6 teachers and 30 children on a field trip, what is the ratio of children to teachers?

YOUR BRAIN WORKOUT

Q3.

If there are 3 sheep dogs and 150 sheep, what is the ratio of dogs to sheep?

YOUR BRAIN WORKOUT

Q4.

If there are 4 broken toys for every 100 produced, what is the ratio of broken toys to toys produced?

YOUR BRAIN WORKOUT

Q5.

If there are 37 boys and 37 girls, what is the ratio of boys to girls?

YOUR BRAIN WORKOUT

Q6.

If there are 5 cancellations for every 300 passengers, what is the ratio of cancellations to passengers?

YOUR BRAIN WORKOUT

Q7.

If the ratio of flour to sugar is 2:1
how much sugar do I need
for 100g of flour?

YOUR BRAIN WORKOUT

Q8.

If the ratio of porridge oats to water is 1:2.5, how many cups of water do I need for 4 cups of oats?

YOUR BRAIN WORKOUT

Q9.

If the ratio of gin to tonic is 1:4, how much tonic do I need for a 500ml bottle of gin?

YOUR BRAIN WORKOUT

Q10.

If the ratio of sand to cement is 3:1, how much sand do I need for a 30kg sack of cement?

YOUR BRAIN WORKOUT

Answers

Q1. 2:1

Q2. 5:1

Q3. 1:50

Q4. 1:25

Q5. 1:1

Q6. 1:60

Q7. 50g

Q8. 10g

Q9. 2000ml or 2 litres

Q10. 90kg

Self-Confidence Tip 14

FAME & AMBITION

Many young people want to be rich and famous.
They are optimistic. They think, YES, it **might** happen to me!
On TV you see some ordinary person become famous just for being themselves.
They appear on a reality show, get a sponsorship deal, become a celebrity. You think, Why not **me**?

- **Could that happen to you? What are your chances?**

When American opinion polls asked a lot of young people for their two most important life goals, 81% said 'to get rich', 51% said 'to get famous'. As there are over 42 million Americans aged 16-25, the chance that an ordinary American girl will get famous in that way are about **one in ten million**. Girls from other countries are unlikely to have a higher chance.

What else did they want?
30% said their biggest concern in the Real World is **how to pay their bills**.
18% said getting a **good education** is their biggest concern.
16% said **careers and jobs** is their biggest area of concern.

- So, can you do anything to help yourself get rich and famous? **!!! YES !!!**

YOUR PERSONALITY QUIZ

In the past, your sex, your colour and your class dictated your identity.
Today, you can carve out your own identity.
In this quiz, YOU work out who YOU are.

When you write your answers, just write what comes into your head and don't let anyone see it. This is private.
Remember, best not to give as an answer, "because it's boring." **Only boring people are bored**. Smart people are never bored, they can always find something to interest themselves.

LONG TERM

- What is your most important life goal?
- Why?
- What sort of person are you? (quiet/popular/noisy/shy/sporty/whatever)
- What sort of work do you do? (school work/dentist's receptionist/shop assistant)
- What sort of work would you like to do? (Be a pop singer/nurse/work for a fashion magazine)
- Why?

FOR NOW

- What do you like? (happy people, sunshine, music festivals, shopping)
- Why?
- What do you like doing? (dancing, video games, swimming)
- Why do you like these things?
- What do you not like? (tidying my room/flat, being pressured to go to an event, exams)
- Why?

SHOPPING

- What branded stuff do you buy? (Nike trainers, Gucci tote bag, Adidas backpack)
- Why?
- What stuff do you buy that isn't branded?
- Why?

YOUR KIND OF PEOPLE

- Who are your friends?
- Why? (Kelly, because she's a generous, kind person and understands when I'm feeling down.)
- What people do you not like? Why? (Rude people/my sister, because she badmouths me.)
- Do you have any strong opinions? (Save the Whale, Climate change, Oxfam).
- Do you do anything to help these causes? (Buy the T-shirt, run for charity, give my time?)
- Who do you admire? (This can be someone you know, someone in your family or some famous achiever that you've never met.)
- Why?
- Now read what you've written. You have clarified who you are and what you want.
- Now... TELL THE WORLD

BE A CELEB IN YOUR OWN WORLD

Today, a girl can be a celebrity in her own world. You can create your own online reality show. You can post videos, photographs, drawings, poems or similar things on a social network.

When you put your creativity, your time, your energy and your focus into presenting yourself on the Web in this way, you're saying, "This is who I am. THIS IS ME".

- You're creating your own, online self-portrait.
- You are establishing your own personality.
- You're making your mark on the world.

This helps to increase your self-worth – and so it should – although you don't get the intimacy that happens with face-to-face communication, and the interaction that you get in a traditional friendship.

RESPECT

There's another sort of fame: Bullies and gangs get what they call 'respect' – but it isn't respect, it's fear.

Their motto is: "If I make you look smaller, then you make me look bigger". This makes bullies feel less fearful, but a quick fix doesn't bring real respect to a bully. If you respect someone, you're not afraid of him or her.

To get RESPECT from those around you – at home and at work – is fairly easy to achieve - if you want to do that - because you know what they would like you to do:

At home: Help with the house-work.
Keep the noise down.

At work: Be interested in your job.
Be reliable.
Be punctual.
That's all.

These things are always noticed by employers and everyone else at work.

HOW TO BE AN ACHIEVER

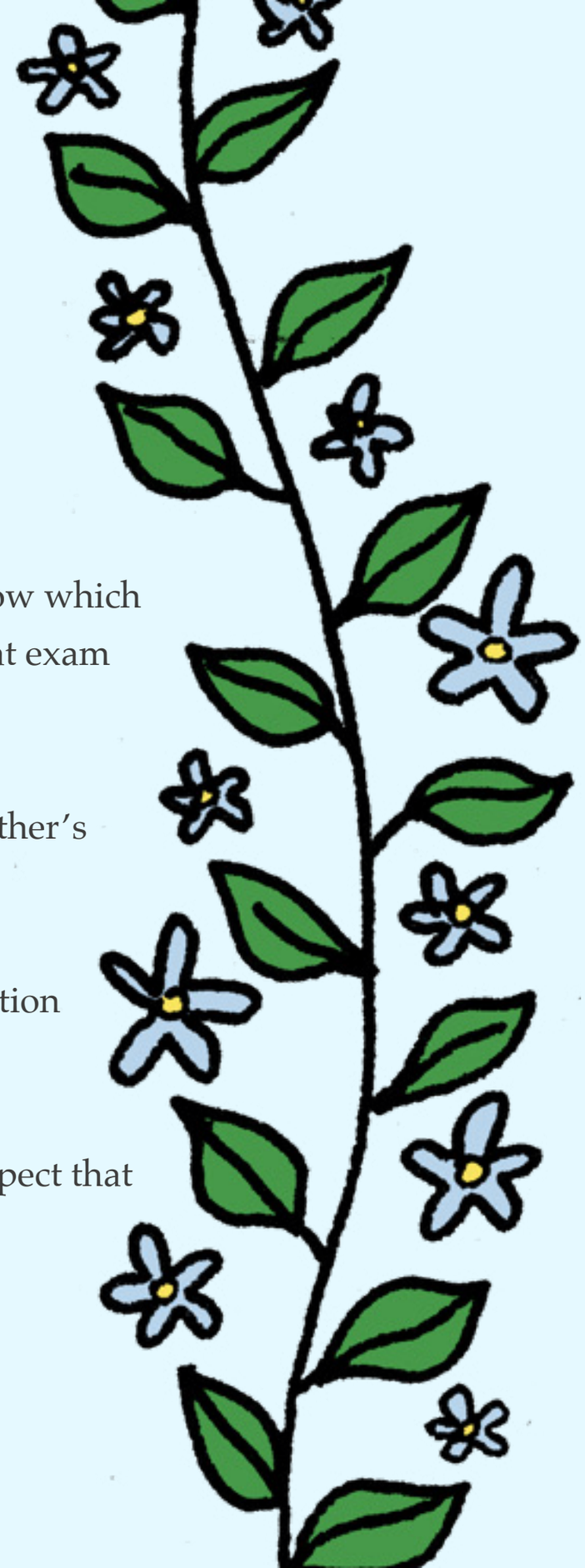
Achievers get things done, whether it's passing an exam or making a great cake.

When you get yourself to do something – you get that good feeling, that inner glow which comes when you know that you've done something well, whether it's passing that exam or making that cake.

I shall never forget landing my first job. I still remember the taste of my grandmother's chocolate cake.

That is true achievement, and it's a much safer investment of your time and ambition than just… wishing… to be famous.

You might – or you might not – become famous. As an achiever, you will gain respect that will last as long as you do.



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Money Stuff

4 STEP MATHS PLAN

I judge myself competent in the following:

FRACTIONS AND DECIMALS
Addition, subtraction, multiplication, division, simplifying & comparing.

PERCENTAGES
All percentages calculations, including basic mental calculations.
Converting numbers between fractions, decimals and percentages.

RATIOS
Calculating ratios and using ratios.

Signed .....

Date .....

Money Stuff

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